

Empirical Studies on the Ratio of the Elastic to the Total Hadronic Cross Sections

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Outline

- Introduction
- Motivation
- Asymptotic Scenarios
- Parametrization and Fit Procedures
- Fit Results
- Summary and Conclusions

Motivation

Why study the ratio $X \equiv \frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} ?$

- **Central opacity** in Hadron-Hadron Collisions

Elastic scattering amplitude (Impact Parameter representation)

$$F(s, q) = i \int_0^\infty b db J_0(qb) \Gamma(s, b) \quad (\text{azimuthal symmetry})$$

- $\Gamma(s, b)$: Profile function
- $\Gamma(s, b = 0) \equiv \Gamma_0(s) \longrightarrow$ Central opacity
- Naive geometrical models

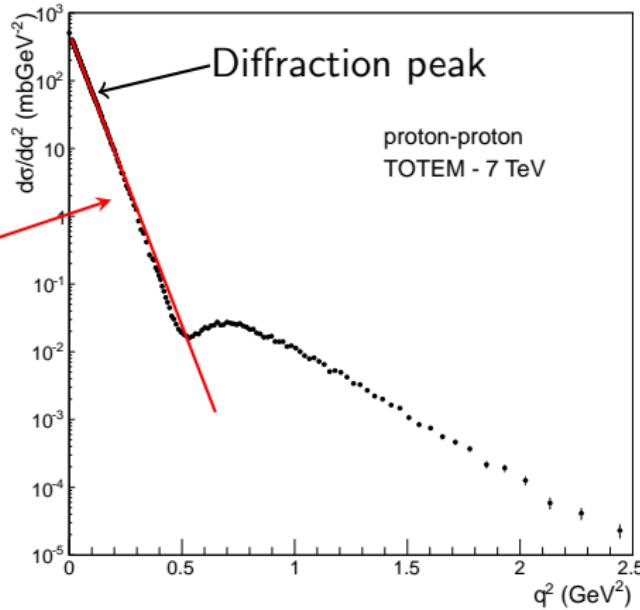
$$X(s) = \begin{cases} \Gamma_0(s)/2, & \text{Gray disk} \\ \Gamma_0(s)/4, & \text{Gaussian} \end{cases}$$

- Black disk: $\Gamma_0 \rightarrow 1 \Rightarrow X \rightarrow 1/2$

Motivation

- **Unitarity**
- σ_{tot} and σ_{el} : from Elastic Scattering

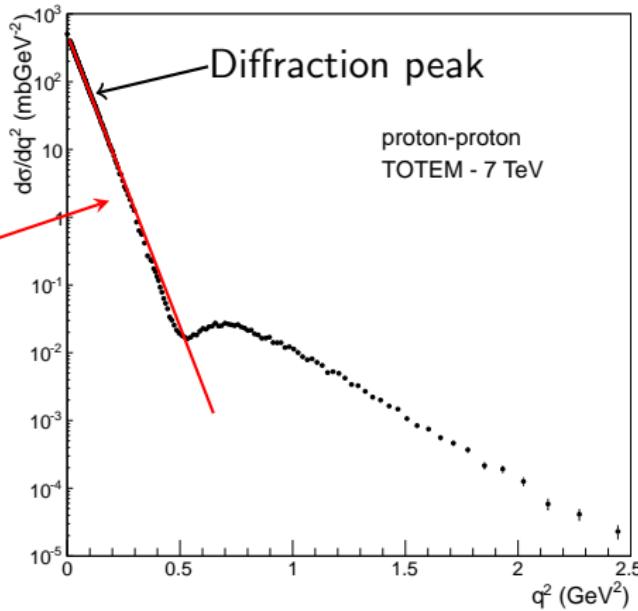
$$\frac{d\sigma}{dq^2} = \left[\frac{(1 + \rho^2)}{16\pi} \sigma_{\text{tot}}^2 \right] e^{-Bq^2}$$
$$\sigma_{\text{el}} = \int_0^{q_0^2} \frac{d\sigma}{dq^2} dq^2$$



Motivation

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$$\sigma_{\text{el}} = \int_0^{q_0^2} \frac{d\sigma}{dq^2} dq^2$$



- Unitarity: $\sigma_{\text{inel}} = \sigma_{\text{tot}} - \sigma_{\text{el}}$
(Free from models assumptions)

- It gives the fraction of inelastic scattering:

$$\frac{\sigma_{\text{inel}}}{\sigma_{\text{tot}}}(s) = 1 - X(s)$$

Motivation

- Connection with ratio $\frac{\sigma_{\text{tot}}}{B}$
 - Important for cosmic rays experiments (EAS studies)
 - Needs extrapolation from accelerator experiments (but it has large uncertainties).

$$X \simeq \frac{1}{16\pi} \frac{\sigma_{\text{tot}}}{B}$$

However...

Motivation

Elastic Scattering

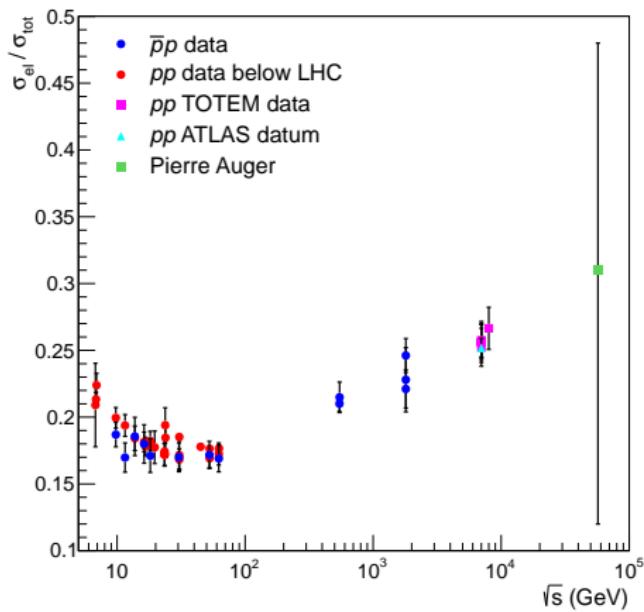
- Soft strong interaction → non-perturbative QCD
- Presently → lack global/complete description from first principles
- One way to look for phenomenological insights/inputs:

Empirical Approach

- Empirical parametrization for $X(s)$?
- Let us look to data...

Motivation

Experimental data (pp , $\bar{p}p$) $\rightarrow X$ data **rise** at $\sqrt{s} \gtrsim 100$ GeV



Motivation

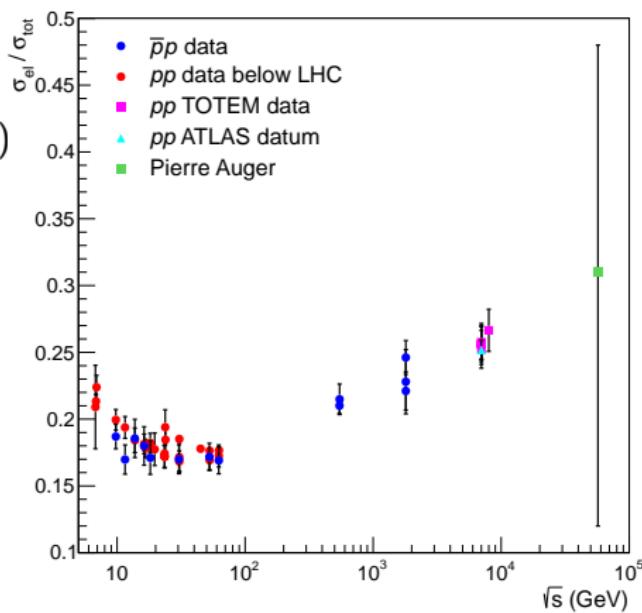
Experimental data $(pp, \bar{p}p) \rightarrow X$ data **rise** at $\sqrt{s} \gtrsim 100$ GeV

- Positive curvature
- Expected Asymptotic limit (all contexts)

↓
a constant

$$\boxed{\lim_{s \rightarrow \infty} X(s) = A}$$

$$A = \begin{cases} 1 & \text{(maximum unitarity)} \\ 1/2 & \text{(black disk)} \end{cases}$$



Motivation

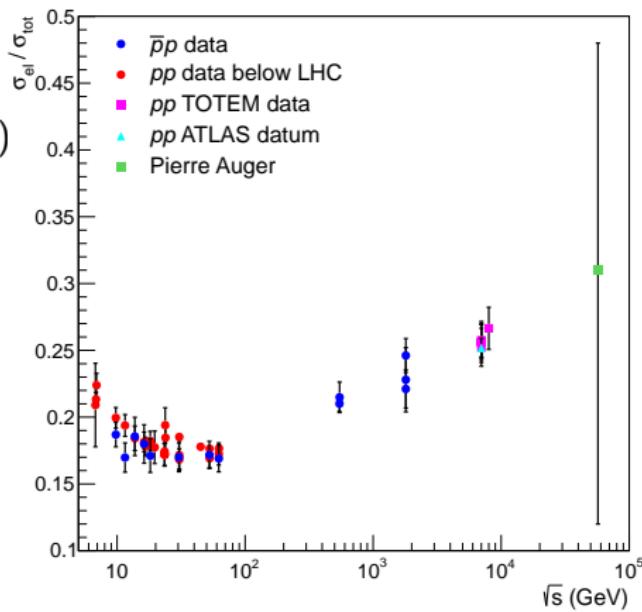
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Rise and saturation (constant value) $\xrightarrow{\text{demand}}$ **change of curvature**

Motivation

Basic suitable assumption

$$X(s) = \textcolor{red}{A} \textcolor{blue}{f}(s) \quad \text{with} \quad \lim_{s \rightarrow \infty} f(s) = 1$$

↓ ↗ change of curvature

Asymptotic ratio

Motivation

Basic suitable assumption

$$X(s) = \underset{\downarrow}{A} f(s) \quad \text{with} \quad \lim_{s \rightarrow \infty} f(s) = 1$$

 change of curvature

Asymptotic ratio

- $f(s)$ as a composite function:

$$f(s) = S(g(s))$$

Motivation

Basic suitable assumption

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Asymptotic ratio

- $f(s)$ as a composite function:

$$f(s) = S(g(s))$$

$S = S(g)$ → sigmoid (“S-shaped”) function → global behaviour

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Asymptotic ratio

- $f(s)$ as a composite function:

$$f(s) = S(g(s))$$

$S = S(g)$ → sigmoid ("S-shaped") function → global behaviour

- $g(s)$ {
- account for deviations from pure sigmoid shape
 - economic number of parameters
 - function of the soft variable $\ln s$

Introduction

In this presentation

- we study the ratio $\sigma_{\text{el}}/\sigma_{\text{tot}}$ by means of an empirical analysis
- with the results, we discuss the consequences to some of the other ratios involving σ_{diff} , σ_{inel} and σ_{tot}

Asymptotic Scenarios

$$A = \lim_{s \rightarrow \infty} X(s)$$

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Black-disk: $A = 0.5$ (Eikonalized models)
(BD)

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Above BD:

$$\begin{cases} A = 1.0 & (\text{Maximum Unitarity}) \\ A = 0.75 & \begin{array}{l} \text{Saturation of} \\ (s \rightarrow \infty) \end{array} \quad \begin{array}{l} \sigma_{\text{tot}}(s) \leq \frac{\pi}{m_\pi^2} \ln^2 s \\ \sigma_{\text{inel}}(s) \leq \frac{\pi}{4m_\pi^2} \ln^2 s \end{array} \\ A > 1/2 & (U\text{-Matrix}) \text{ [Troshin, Tyurin]} \end{cases}$$

Black-disk: $A = 0.5$ (Eikonalized models)
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Asymptotic Scenarios: $A = \lim_{s \rightarrow \infty} X(s)$

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Black-disk: $A = 0.5$ (Eikonalized models)
(BD)

Below BD:
$$\begin{cases} A = 0.436 & \text{TOTEM/COMPETE } (\sigma_{\text{el}}/\sigma_{\text{tot}}) \\ A = 0.3 & \begin{array}{l} \text{Fagundes-Menon-Silva lowest value} \\ (\sigma_{\text{tot}}, \rho, \sigma_{\text{el}}) [\text{JPG,IJMPA 2013}] \end{array} \end{cases}$$

Sigmoid function $S(g)$

Choices for Sigmoid Functions

- Sigmoid Function, $S(g)$:
 - “S-shaped” functions
 - Finite limits for $x \rightarrow \pm\infty$, with $\lim_{x \rightarrow \infty} S(x) = 1$

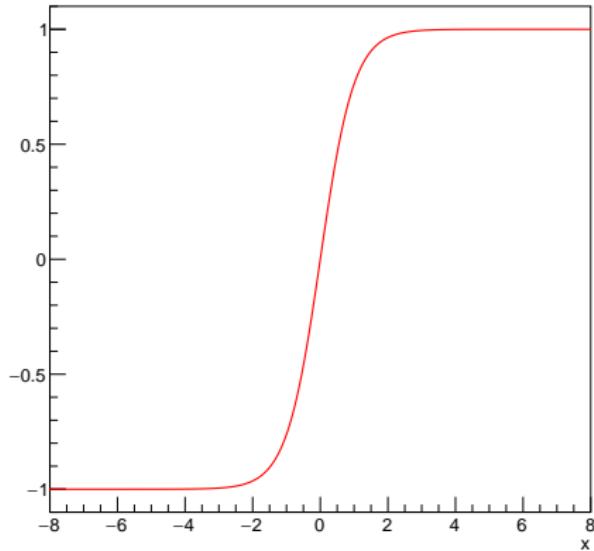
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- Sigmoid Function, $S(g)$:
 - “S-shaped” functions
 - Finite limits for $x \rightarrow \pm\infty$, with $\lim_{x \rightarrow \infty} S(x) = 1$

- Here we consider one choice:

- Hiperbolic Tangent:

$$S_{HT}(x) = \tanh(x) = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$



Function $g(s)$

Choices for $g(s)$

- Function of $\ln(s/s_0)$, where s_0 is a fixed energy scale

$$g(s) = \alpha + \beta \ln(s/s_0) + \gamma h_i(s)$$

α, β, γ are real free fit parameter

Choices for $g(s)$

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- Two choices for $h_i(s)$

Power-Law

$$h_p(s) = \ln^\delta(s/s_0)$$

Log-Law

$$h_l(s) = \ln \ln(s/s_0)$$

- Energy scale $s_0 = 4m_p^2 = 3.521 \text{ GeV}^2$

Analysis with $s_0 = 4m_p^2$

A collection of variants...

- Fits with **Hiperbolic Tangent**
- **Power-Law** and **Log-Law** with A fixed and A free
- For A fixed, all scenarios previously discussed:
0.3, **0.436**, **0.5**, **0.75** and 1.0
- For A free: fits for all cases above

Analysis with $s_0 = 4m_p^2$ - Dataset and Fit Procedures

• Dataset

- Only accelerator data on pp and $\bar{p}p$, $\sqrt{s} \geq 5$ GeV (PDG)
- TOTEM data: 4 points at 7 TeV, 1 point at 8 TeV
- ATLAS datum: 1 point at 7 TeV
- Total number points: 42 (pp : 29, $\bar{p}p$: 13)

• Fit Procedures

- Data reduction: TMinuit of ROOT Framework ($CL \approx 68\%$)
- Goodness of fit: χ^2/ν and $P(\chi^2, \nu)$
- Nonlinearity demands initial values for free parameters

Results

- A fixed \rightarrow impose asymptotic scenario
 - Fit TanH: Power-Law
 - Fit TanH: Log-Law
- A free \rightarrow select an asymptotic scenario

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A fixed - General Aspects

- All cases with $\chi^2/\nu < 1$, where ν = degrees of freedom
- χ^2/ν increase with A
- $P(\chi^2, \nu)$ decrease with A

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- Power-Law and Log-Law → same description of data

A fixed - General Aspects

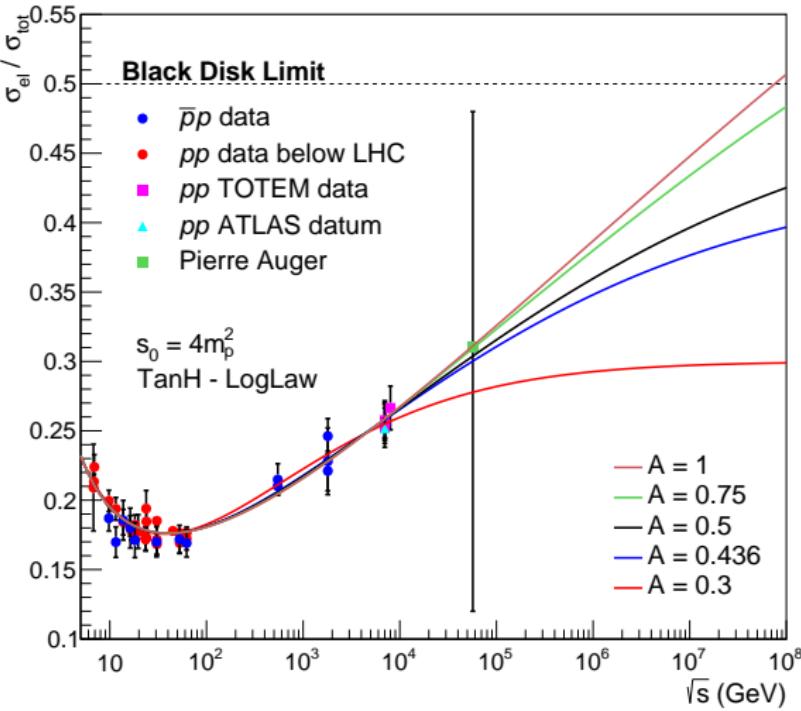
- All cases with $\chi^2/\nu < 1$, where ν = degrees of freedom
- χ^2/ν increase with A
- $P(\chi^2, \nu)$ decrease with A
- Power-Law and Log-Law → same description of data
- Log-Law → less parameters → better error estimation (in general)
Log-Law → preferable parametrization
- Example: TanH with Log-Law

Results: Fit TanH with Log-Law

$$X(s) = A \operatorname{tanh}[g(s)]; \quad g(s) = \alpha + \beta \ln(s/s_0) + \gamma \ln \ln(s/s_0)$$

A (fixed)	χ^2/ν	$P(\chi^2, \nu)$
0.3	0.796	0.813
0.436	0.822	0.777
0.5	0.831	0.763
0.75	0.847	0.737
1.0	0.853	0.729

$(\nu = 39)$



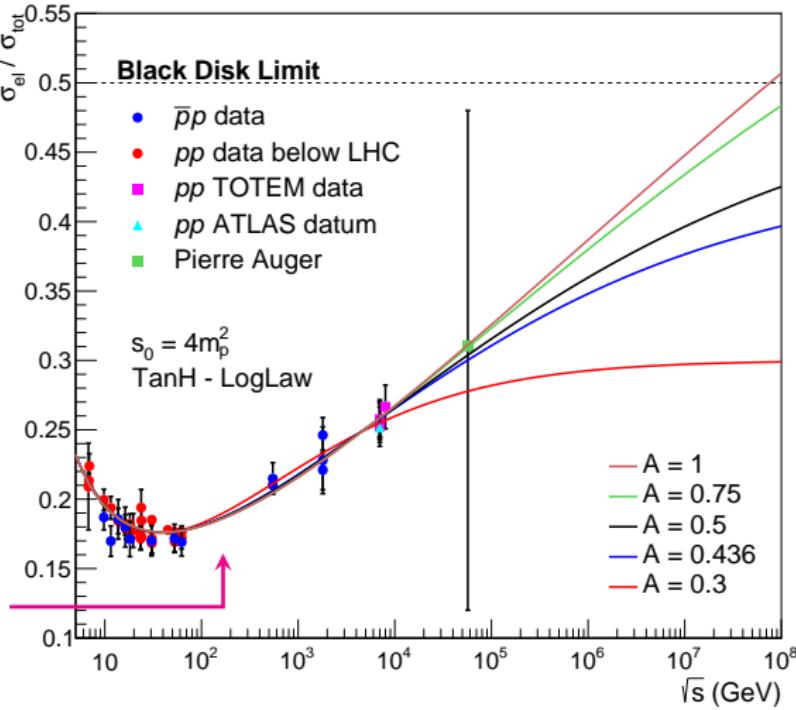
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Consistent description of data
for all cases considered
(do not select a scenario)



Results

- A fixed \rightarrow impose asymptotic scenario
 - Fit TanH: Power-Law
 - Fit TanH: Log-Law
- A free \rightarrow select an asymptotic scenario

Results: Fits with A free

With A free \rightarrow select the scenario that better describe the data

- For each $g(s) \rightarrow$ 5 different Initial Values
- All cases converged to results indicating **the same asymptotic scenario!**

Table: Values obtained for parameter A

$g(s) \setminus S(g)$	A	χ^2/ν
Power-Law	0.31(10)	0.838
Log-Law	0.312(35)	0.815

Results: Fits with A free

With A free \rightarrow select the scenario that better describe the data

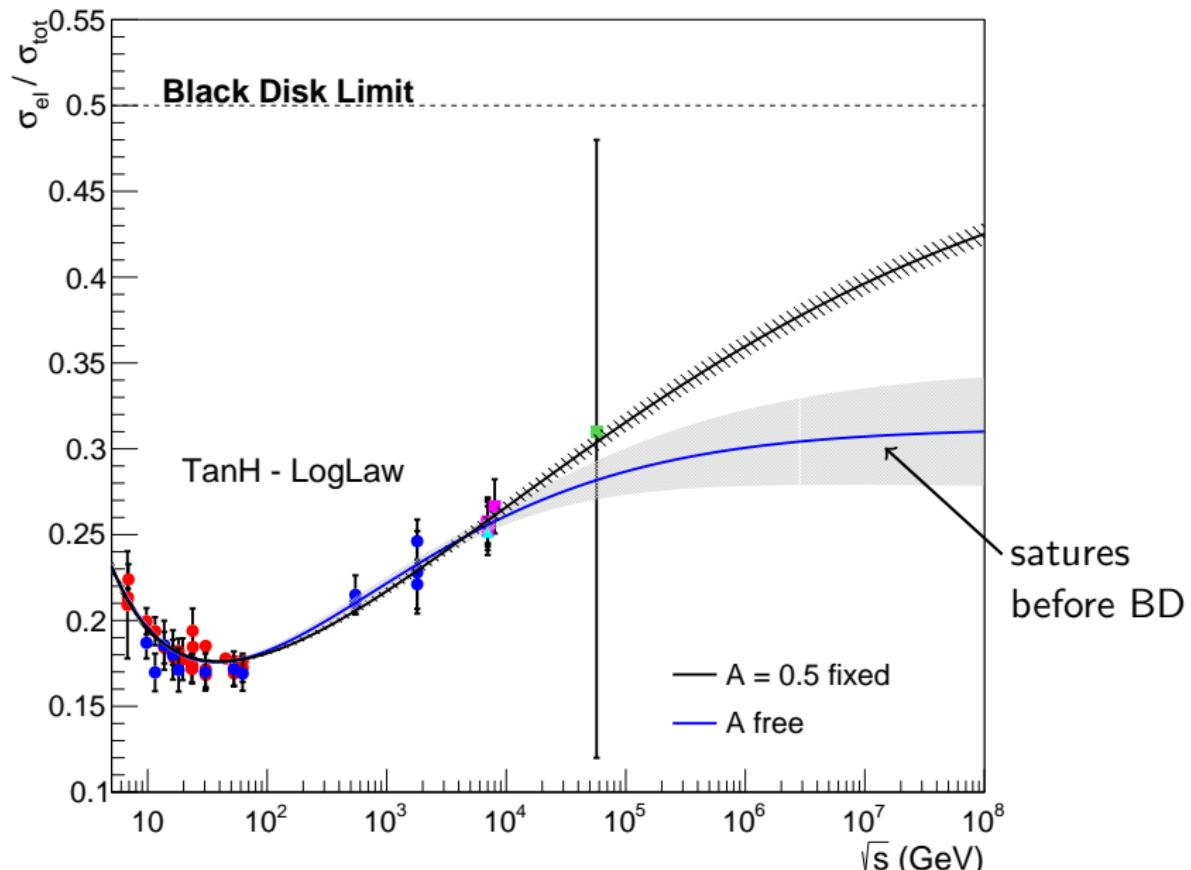
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- All **below** the black disk limit: $A \sim 0.3$

Results: Fits with A free



Predictions and Possible Implications

- Results are not in disagreement with **Pumplin bound** [PRD (1973)]:

$$\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} + \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \leq \frac{1}{2}$$

where $\sigma_{\text{diff}} = \sigma_{\text{SD}} + \sigma_{\text{DD}}$

SD: Single Diffractive
DD: Double Diffractive

- In fact, if we suppose that Pumplin is **saturated**:

Table: Asymptotic values for $\sigma_{\text{diff}}/\sigma_{\text{tot}}$

$g(s) \setminus S(g)$	TanH
Power-Law	0.19(10)
Log-Law	0.188(35)

- We can also predict $\sigma_{\text{el}}/\sigma_{\text{tot}}$, $\sigma_{\text{inel}}/\sigma_{\text{tot}}$ and $\sigma_{\text{diff}}/\sigma_{\text{tot}}$ (Pumplin bound saturated)

Predictions and Possible Implications

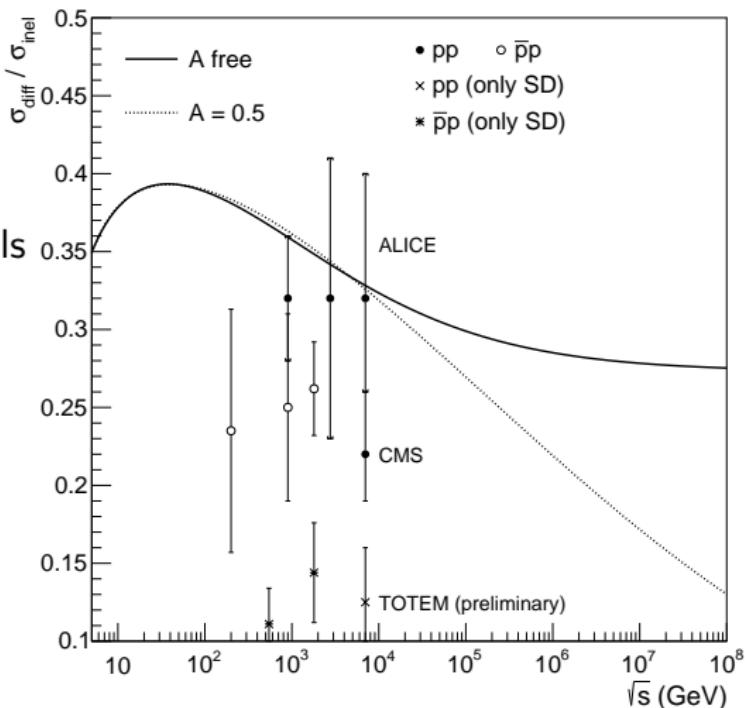
From Pumplin Bound and Unitarity

$$R(s) \equiv \frac{\sigma_{\text{diff}}}{\sigma_{\text{inel}}} \leq \frac{1/2 - X(s)}{1 - X(s)}$$

For all fits: PL and LL are equals

Curves represent upper bound

Trend of data (?)



Summary and Conclusions

- Empirical Parametrization

$$X(s) = A f(s) \quad \text{with} \quad f(s) = S(g(s))$$

A → Asymptotic value

$S(g)$ → Sigmoid Function: Hiperbolic Tangent

$g(s)$ → 2 variants: Power-Law [$\ln^\delta(s/s_0)$] and Log-Law [$\ln \ln(s/s_0)$]

Energy scale: $s_0 = 4m_p^2 = 3.521 \text{ GeV}^2$

- Only 5 (PL) / 4 (LL) free parameters

Summary and Conclusions

- Fit results to pp and $\bar{p}p$ data, $\sqrt{s} : 5 \text{ GeV} - 8 \text{ TeV}$

- A fixed

0.3, 0.436, 0.5, 0.75, 1.0 → all results consistent with exp. data

Cannot discriminate asymptotic scenarios → still open problem

Black-disk limit → does not represent an unique solution

Summary and Conclusions

- Fit results to pp and $\bar{p}p$ data, $\sqrt{s} : 5 \text{ GeV} - 8 \text{ TeV}$

- A free

In each variant: unique solution \longrightarrow below the black-disk

$$X(s) \rightarrow 0.312 \pm 0.035, \quad (\text{TanH}) \quad (s \rightarrow \infty)$$

- Within uncertainties \longrightarrow agreement with:
 - FMS amplitude analyses (σ_{tot} , ρ , σ_{el})
 - Recent phenomenological analysis by Kohara-Ferreira-Kodama:
[EJPC; JPG (2014)]

$$X \rightarrow \frac{1}{3} \quad (s \rightarrow \infty)$$

Summary and Conclusions

- Recalling that $X(s) \leftrightarrow$ Central opacity
- For $A < 1/2 \rightarrow$ proton behaves asymptotically as a gray disk at $b = 0$
- Based in #free parameters, Log-Law is the best choice for $g(s)$
- Future: comparison with new data at 13 TeV



Special thanks to



THANK YOU!!

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Backup Slides

Introduction

In a scattering process, the total cross section can be written

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}}$$

For pp and $\bar{p}p$ scattering, the inelastic cross section can be divided:

$$\sigma_{\text{inel}} = (\sigma_{\text{SD}} + \sigma_{\text{DD}}) + \sigma_{\text{CD}} + \sigma_{\text{nD}}$$

$\sigma_{\text{diff(soft)}}$

el = elastic

SD = single diffractive

DD = double diffractive

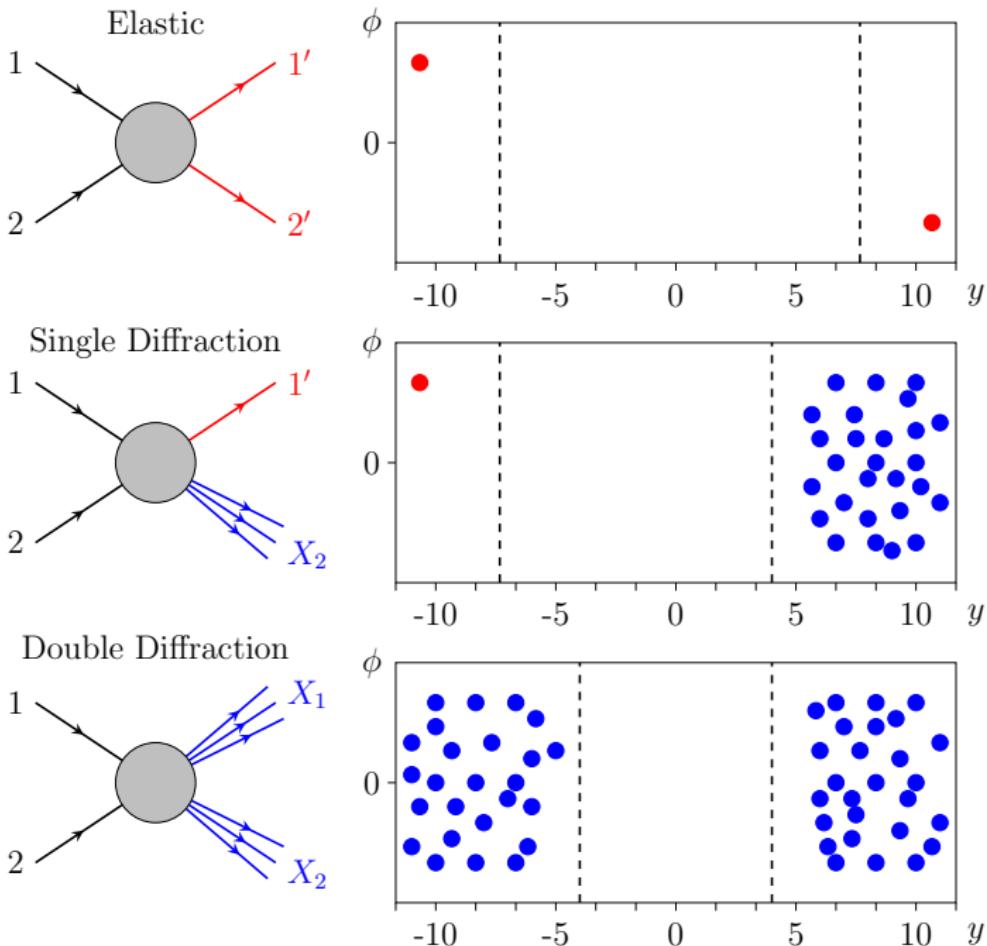
CD = central diffractive

nD = non diffractive \longrightarrow

}

- exchange of color singlets
- shows *rapidity gap*

- exchange of others quantum numbers



Central Opacity

- Profile function: $\Gamma(s, b) = 1 - e^{i\chi(s, b)}$
- Eikonal: $\chi(s, b)$
- Opacity: $\Omega(s, b) = \text{Im } \chi(s, b)$
- Considering only the imaginary part of the eikonal:

$$e^{-\Omega(s, b)} = 1 - \Gamma(s, b)$$

- Central collisions: $e^{-\Omega(s, b=0)} = 1 - \Gamma(s, b=0)$

Relation between σ_{tot}/B and $\sigma_{\text{el}}/\sigma_{\text{tot}}$

- Differential cross section (forward peak): $\frac{d\sigma}{dq^2} = \left. \frac{d\sigma}{dq^2} \right|_{q^2=0} e^{-Bq^2}$
- Optical point: $\left. \frac{d\sigma}{dq^2} \right|_{q^2=0} = \frac{(1 + \rho^2)}{16\pi} \sigma_{\text{tot}}^2$
- Integrated elastic cross section: $\sigma_{\text{el}} = \int_0^{q_0^2} \frac{d\sigma}{dq^2} dq^2$
- With assumption $1 + \rho^2 \approx 1$, taking limit $q_0^2 \rightarrow \infty$ and using the optical point:

$$\sigma_{\text{el}}(s) = \frac{1}{B(s)} \frac{\sigma_{\text{tot}}^2(s)}{16\pi} \Rightarrow \boxed{\frac{\sigma_{\text{tot}}(s)}{B(s)} = 16\pi \frac{\sigma_{\text{el}}(s)}{\sigma_{\text{tot}}(s)}}$$

σ_{tot}/B in Extensive Air Showers (EAS)

- Connection between hadron-hadron and hadron-nucleus scattering
- **Glauber formalism:** elastic + quasi-elastic cross section for hadron-nucleus (hA) scattering

$$\sigma_{\text{el}}^{hA}(s) + \sigma_{\text{qel}}^{hA}(s) = \int d^2b \left| 1 - \prod_{j=1}^A [1 - a_j(s, \mathbf{b} - \mathbf{b}_j)] \right|^2 \left[\prod_{j=1}^A \tau(\mathbf{r}_j) d^3r_j \right]$$

$$a_j(s, \mathbf{b}_j) = \frac{[1 + \rho(s)]}{4\pi} \frac{\sigma_{\text{tot}}(s)}{B(s)} e^{-\mathbf{b}_j^2/2B(s)} \rightarrow \begin{array}{l} \text{nucleon-nucleon impact} \\ \text{parameter amplitude (profile function)} \end{array}$$

- **Main ingredients:**
 - possible configurations for the nucleus
 - $a_j(s, \mathbf{b}_j)$ → it is model and extrapolation dependent
- \mathbf{r}_j and \mathbf{b}_j are the coordinate and impact parameter of the individual nucleons
- $\tau(\mathbf{r}_j)$ is the single nucleon density
- \mathbf{b} is the impact parameter of the cosmic-ray hadron

Black-disk model

- Impact parameter formalism (azimuthal symmetry)

$$F(s, q) = i \int_0^\infty b db J_0(qb) \Gamma(s, b)$$

Gray-disk (Profile function):

$$\Gamma(s, b) = \begin{cases} \Gamma_0(s), & b \leq R(s) \\ 0, & b > R(s) \end{cases}$$

$$\sigma_{\text{tot}} = 2\pi R^2 |\Gamma_0|^2$$

$$\sigma_{\text{el}} = \pi R^2 \operatorname{Re} \Gamma_0$$

$$\frac{d\sigma}{dq^2} = \frac{|\Gamma_0|^2 R^4}{16\pi s^2} \left| \frac{J_1(qR)}{qR} \right|^2$$

- Black-disk: $\Gamma_0 \rightarrow 1$

$$\boxed{\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} = \frac{1}{2}}$$

(black-disk limit)

Black-disk (BD): below, at and beyond

Analogy to optics

In the **b** plane

- **Gray-disk:**

$$\Gamma(s, b) = \Gamma_0(s) \rightarrow \text{constant in } b \text{ for } b < R(s)$$

- We have elastic scattering and absorption



- **Black-disk:**

$$\text{Limit } \Gamma_0(s) \rightarrow 1 \Leftrightarrow \Omega_0(s) \rightarrow \infty \rightarrow \text{maximum opacity}$$

- We have elastic scattering and maximum absorption



- **Beyond Black-disk:**

anti-shadowing \rightarrow Reflective scattering for some
 $b < R_0(s)$

- We now have elastic scattering, absorption and reflective scattering [Troshin-Tyurin]



Pumplin Bound in a beyond black-disk scenario

- Pumplin bound (for $X \leq 1/2$):

$$\sigma_{\text{diff}}(s, b) \leq \frac{1}{2} \sigma_{\text{tot}}(s, b) - \sigma_{\text{el}}(s, b) \xrightarrow{\int b db} \sigma_{\text{diff}}(s) \leq \frac{1}{2} \sigma_{\text{tot}}(s) - \sigma_{\text{el}}(s)$$

where $\sigma_i(s, b) = \frac{1}{4\pi} \frac{d\sigma_i}{db^2}$ and $i = \text{diff, el, tot}$.

- For $X > 1/2$, the Pumplin bound for integrated cross-sections is *only valid* for reduced cross sections¹:

$$\bar{\sigma}_{\text{diff}}(s, b) \leq \frac{1}{2} \bar{\sigma}_{\text{tot}}(s, b) - \bar{\sigma}_{\text{el}}(s, b)$$

where $\bar{\sigma}_i(s) = \sigma_i(s) - 8\pi \int_0^{R_0(s)} b db \sigma_i(s, b)$.

- The range $0 < b < R_0(s)$ is where the reflective scattering occurs

¹S.M. Troshin, N.E. Tyurin, arXiv:1408.6946 [hep-ph]

Asymptotic Scenarios I: The Black-Disk Limit

- **Black-Disk**

$$\lim_{s \rightarrow \infty} X(s) = \frac{1}{2}$$

- Standard phenomenological expectation
- Typical of eikonal models (unitarized by construction): Chou-Yang, Bourrely-Soffer-Wu, Block-Halzen, etc

Asymptotic Scenarios II: Below the Black-Disk

(1) Amplitude Analyses: Fagundes, Menon, Silva [JPG, IJMPA, JPG (2013)]

$$\sigma_{\text{tot}}(s) = \text{Regge terms} + \alpha + \beta \ln^{\gamma}(s/s_h)$$

- Fits σ_{tot} and ρ data (pp , $\bar{p}p$) using DDR, $\sqrt{s} \geq 5$ GeV
 - $\gamma = 2$ fixed and γ as a free fit parameter ($\gamma > 2$)
- Extension to σ_{el} data ($\gamma = 2$ and $\gamma > 2$)
- Several distinct fit variants
- All cases: $\lim_{s \rightarrow \infty} X(s) < 1/2$ (within uncertainties)

Lowest central value: $X \rightarrow 0.3$

Asymptotic Scenarios II: Below the Black-Disk

(2) COMPETE and TOTEM parametrizations

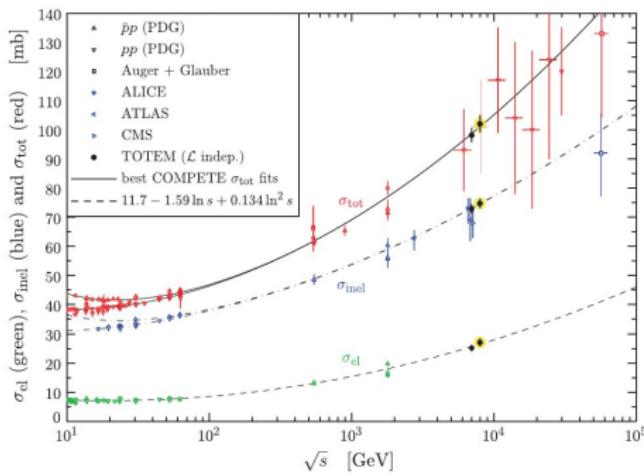
COMPETE highest-rank result for $\sigma_{\text{tot}}(s)$ [PRL (2002)]:

$$\sigma_{\text{tot}}(s) = \text{Regge} + 35.5 + 0.307 \ln^2(s/29.1 \text{ GeV}^2)$$

TOTEM empirical fit to σ_{el} data:
[PRL (2013)]

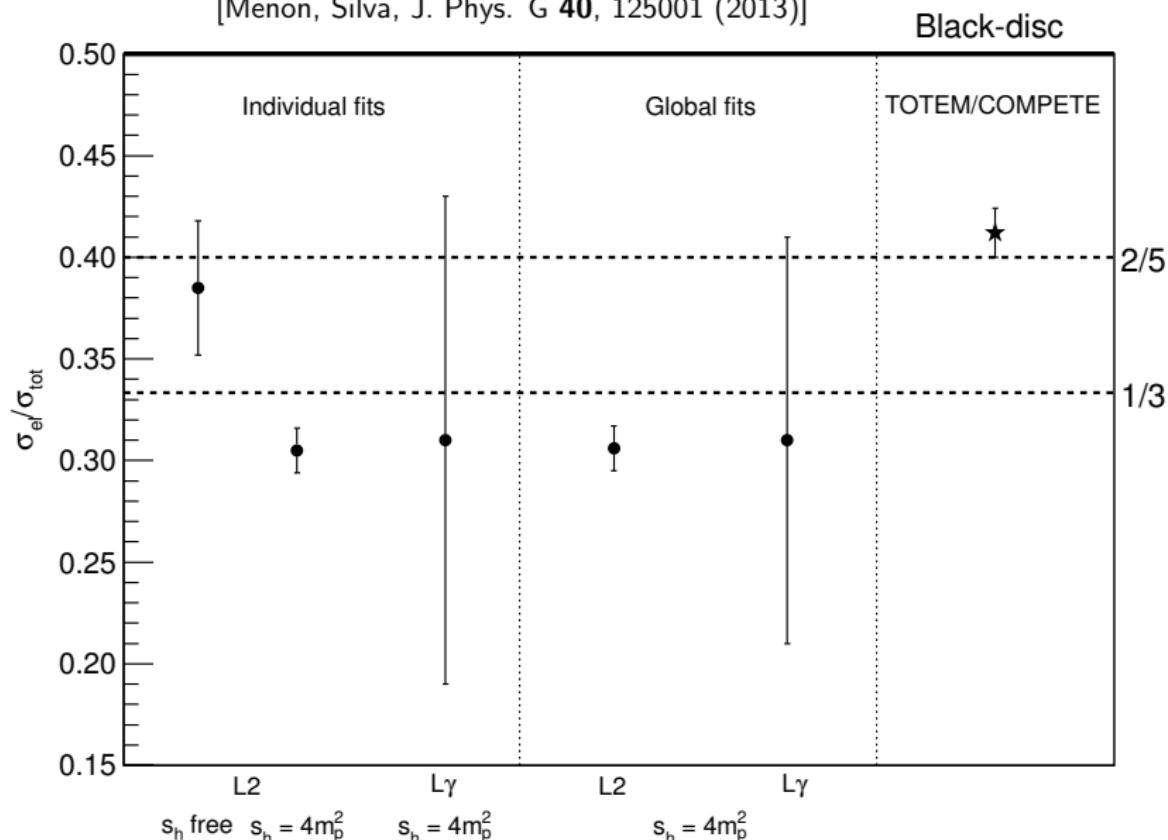
$$\sigma_{\text{el}}(s) = 11.7 - 1.59 \ln s + 0.134 \ln^2 s$$

$$\lim_{s \rightarrow \infty} X(s) = 0.436$$



Asymptotic Scenarios II: Below the Black-Disk

[Menon, Silva, J. Phys. G **40**, 125001 (2013)]



Asymptotic Scenarios III: Above the Black-Disk

(1) Maximum Unitarity bound:

$$\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} + \frac{\sigma_{\text{inel}}}{\sigma_{\text{tot}}} = 1 \quad \Rightarrow \quad \boxed{X \rightarrow 1} \quad (s \rightarrow \infty)$$

(2) Phenomenological approach

Troshin, Tyurin [PLB (1993), IJMPA (2007)]

U -matrix unitarization

$$\boxed{X > \frac{1}{2}}$$

Asymptotic Scenarios III: Above the Black-Disk

(3) Possible Formal Limit

Formal results ($s \rightarrow \infty$)

[Froissart, PR (1961); Martin, Lukaszuk, NC (1966); Martin, PRD (2009)]

$$\sigma_{\text{tot}}(s) \leq \frac{\pi}{m_\pi^2} \ln^2 s$$

and

$$\sigma_{\text{inel}}(s) \leq \frac{\pi}{4m_\pi^2} \ln^2 s$$

If both limits saturate as $s \rightarrow \infty$:

$$\frac{\sigma_{\text{inel}}}{\sigma_{\text{tot}}} \rightarrow \frac{1}{4} \xrightarrow{\text{Unitarity}} X \rightarrow \frac{3}{4} = 0.75$$

Estimation² of $\sigma_{\text{diff}}/\sigma_{\text{tot}}$ at 7 TeV

- TOTEM (indep. lum.):

$$\sigma_{\text{tot}} = 98.0 \pm 2.5 \text{ mb}, \sigma_{\text{el}} = 25.1 \pm 1.1 \text{ mb}, \sigma_{\text{inel}} = 72.9 \pm 1.5 \text{ mb},$$
$$\sigma_{\text{el}}/\sigma_{\text{tot}} = 0.256 \pm 0.013$$

- ALICE: Fraction of single (SD) and double (DD) diffraction in inelastic collisions:

$$\frac{\sigma_{\text{SD}}}{\sigma_{\text{inel}}} = 0.20^{+0.04}_{-0.07} \quad \text{and} \quad \frac{\sigma_{\text{DD}}}{\sigma_{\text{inel}}} = 0.12^{+0.05}_{-0.04}$$

With $\sigma_{\text{diff}} = \sigma_{\text{SD}} + \sigma_{\text{DD}}$:

$$\frac{\sigma_{\text{diff}}}{\sigma_{\text{inel}}} = 0.32^{+0.06}_{-0.08}$$

²P. Lipari, M. Lusignoli, Eur. Phys. J. C **73**, 2630 (2013)

Estimation³ of $\sigma_{\text{diff}}/\sigma_{\text{tot}}$ at 7 TeV

- Combining TOTEM and ALICE results:

$$\sigma_{\text{diff}} = 23.3^{+4.4}_{-5.9} \text{ mb} \quad \text{and} \quad \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \simeq 0.24^{+0.05}_{-0.06}$$

$$\boxed{\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} + \frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} = 0.496^{+0.05}_{-0.06}}$$

- Indicates saturation of Pumplin bound at LHC energy

³P. Lipari, M. Lusignoli, Eur. Phys. J. C **73**, 2630 (2013)

Minijets

- Jets → particles produced in a region of space (usually a cone) from the hadronization of a parton in hard collisions.
- Experimental measurements → depends on the algorithm used and the p_T limit that the experiment is able to detect
- Minijets → jets measured with low p_T limit (or E_T , transverse energy)
- Example: UA1 Collab → minijets with $E_T > 5$ GeV

Minijets

- Cross section (pQCD) [p_{Tmin} is defined as the region of validity of pQCD]⁴:

$$\sigma_{\text{jet}}^{AB}(s, p_{Tmin}) = \int_{p_{Tmin}}^{\sqrt{s}/2} dp_T \int_{4p_T^2/s}^1 dx_1 \int_{4p_T^2/(x_1 s)}^1 dx_2 \sum_{i,j,k,l} f_{i|A}(x_1, p_T^2) f_{j|B}(x_2, p_T^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_T}$$

$f_{i|A}(x_1, p_T^2)$: PDFs where i, j, k, l denote the partons

x_i : fraction of the parent momentum carried by the parton

$\sqrt{\hat{s}} = \sqrt{x_1 x_2 s}$: CM energy of two parton system

$\hat{\sigma}$: hard parton scattering cross section

⁴D.A. Fagundes *et al*, Phys. Rev. D **91**, 114011 (2015).

Motivation

Basic suitable assumption

$$X(s) = A f(s) \quad \text{with} \quad \lim_{s \rightarrow \infty} f(s) = 1$$

↓ ↗ change of curvature

Asymptotic ratio

- Previous Empirical Ansatz: Fagundes and Menon [NPA (2012)]

$f(s)$ in terms of $\ln s$ (standard soft variable)

↳ Trial and error:
$$f_{\text{FM}}(s) = \tanh[a + b \ln(s/s_0) + c \ln^2(s/s_0)]$$

- Fits $X(s)$ data only 3 free parameters: a, b, c ($s_0 = 1 \text{ GeV}^2$ fixed)
 - $A = 1/2$ and $A = 1$ (fixed parameters)
 - only pp data: $\sqrt{s}_{\min} = 10 \text{ GeV}$, $\sqrt{s}_{\max} = 7 \text{ TeV}$ (1st TOTEM datum)
 - Prediction to σ_{tot}/B (uncertainties for EAS studies)

Ref.: A. Martin, Phys. Rev. D **80**, 065013 (2009)

After obtaining the limit $\sigma_{\text{inel}} < \frac{\pi}{4m_\pi^2} \ln^2 s$ (pag. 3):

“This ends the rigorous part of this paper. Now comes the fact that most theoreticians believe that the worse that can happen at high energies is that the elastic cross section reaches half of the total cross section, which corresponds to an expanding black disk.”

End of the paper (pag. 3), concerning the limit $\sigma_{\text{el}}/\sigma_{\text{tot}} > 1/2$, $s \rightarrow \infty$:

"To say the least, this seems to me extremely unlikely and, therefore, I tend to believe that we have"

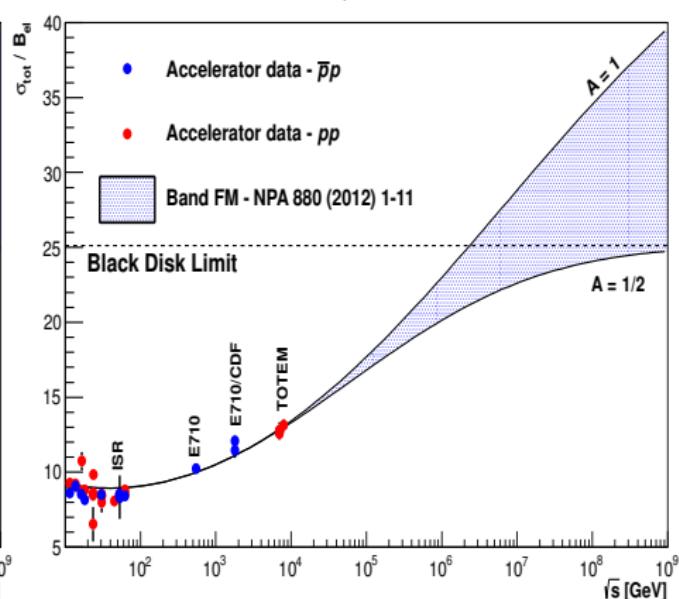
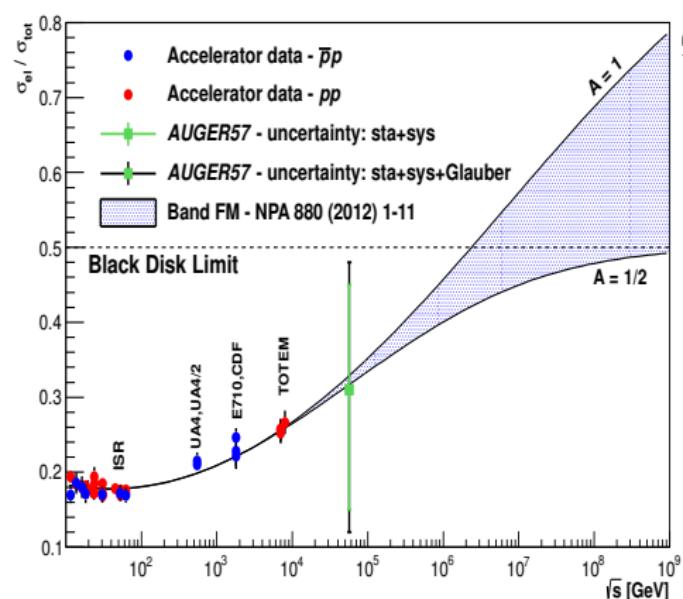
$$\sigma_{\text{tot}} < \frac{\pi}{2m_\pi^2} \ln^2 s.$$

Some authors assume this result:

- M.M. Block, F. Halzen, Phys. Rev. Lett. **107**, 212002 (2011)
- N. Cartiglia, *Measurement of the proton-proton total, elastic, inelastic and diffractive cross sections at 2, 7, 8 and 57 TeV*, arXiv:1305.6131v3 [hep-ex]
- I.M. Dremin, *Hadron structure and elastic scattering*, arXiv:1311.4159 [hep-ph]

Previous Results

- First work: Fagundes and Menon (2011)
 - $\delta = 2$ fixed with $s_0 = 1 \text{ GeV}^2$. Only pp data. One point at 7 TeV
 - only two scenarios considered: $A = 0.5$ and $A = 1.0$
 - only Hyperbolic Tangent
 - extrapolation to higher energies and results for σ_{tot}/B



Previous Results

- Fagundes, Menon and Silva (2014, 2015)
 - New pp data at 7 and 8 TeV (TOTEM and ATLAS)
 - Tests (only pp) with δ free, A fixed and $s_0 = 25 \text{ GeV}^2$ fixed $\rightarrow \delta \sim 0.5$
 - Inclusion of $\bar{p}p$ data.
 - Fits with A free and $\delta = 1/2$ fixed (pp and $\bar{p}p$ data) $\longrightarrow A < 1/2$
- So far, we have tested two values for s_0
 - 1 GeV^2
 - 25 GeV^2 \longrightarrow cutoff energy
- Now, we try a more physical value \rightarrow Threshold energy for elastic scattering

$$s_0 = 4m_p^2 = 3.521 \text{ GeV}^2$$

Analysis with $s_0 = 4m_p^2$ - Initial Values

TanH



IV_0 : result with A fixed, $\delta = 0.5$ fixed and $s_0 = 25 \text{ GeV}^2$

Discussion: Change of Curvature

- **Change of curvature**

- We looked for a function that shows a change of curvature
- What is the energy that this change occurs?
- We must have (at least)

$$\frac{d^2 X(s)}{ds^2} = 0$$

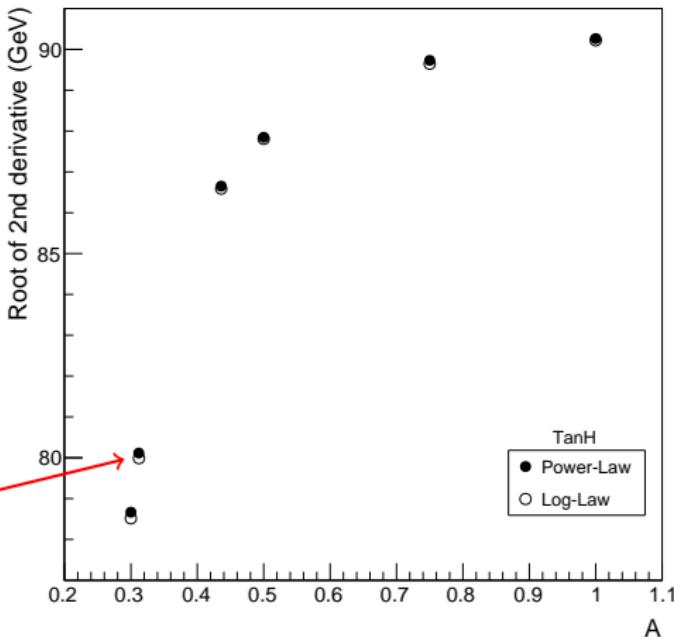
- The root (or roots, if more than one is possible) of this equation tells us when the change of curvature occurs.
- The solution is numerical → not so easy to estimate errors

Discussion: Change of Curvature

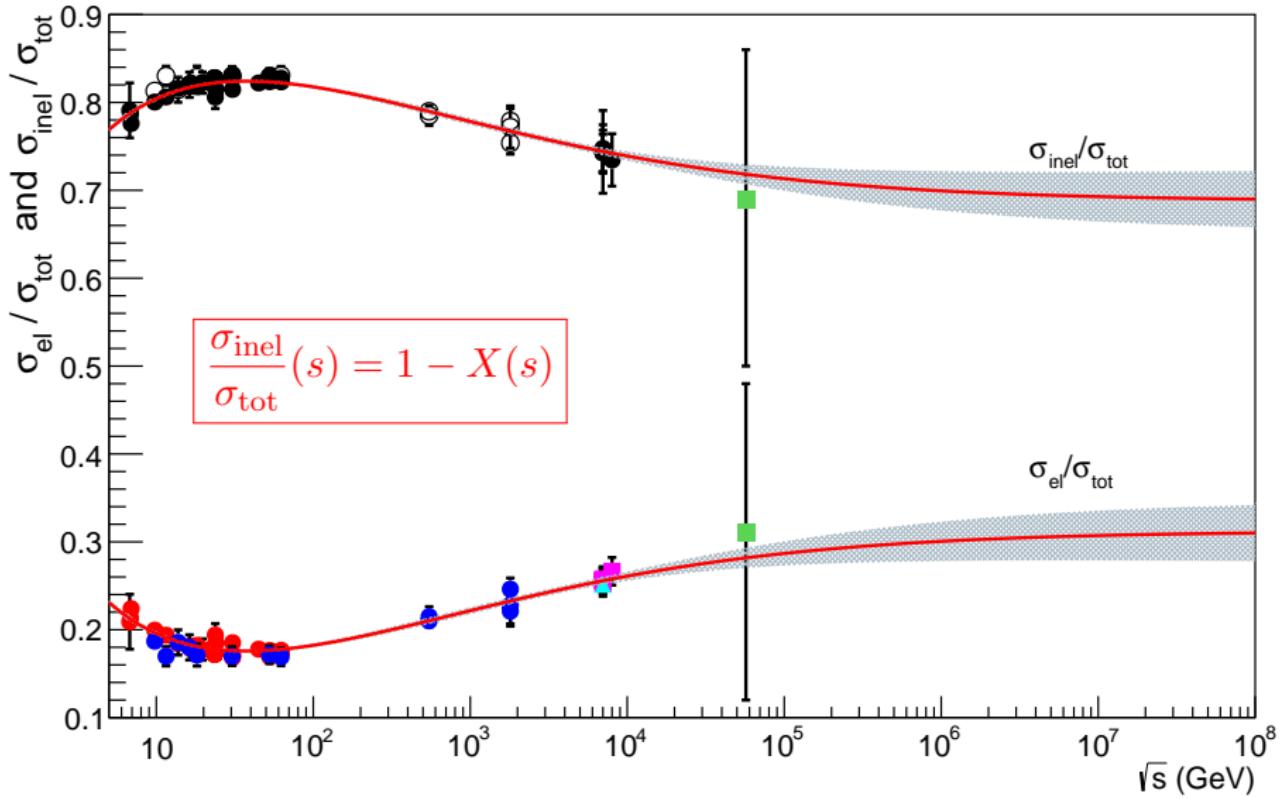
- In the physical domain of interest here, we have **only one root**
- It corresponds to a *non-stationary point of inflection* ($dX(s)/ds \neq 0$)
- The result clearly depends on the value of A

Table: Inflection points for TanH in GeV

A	Power-Law	Log-Law
0.3	78.7	78.5
0.436	86.7	86.6
0.5	87.8	87.8
0.75	89.7	89.6
1.0	90.3	90.2
free	80.1	80.0



Predictions and Possible Implications



Results

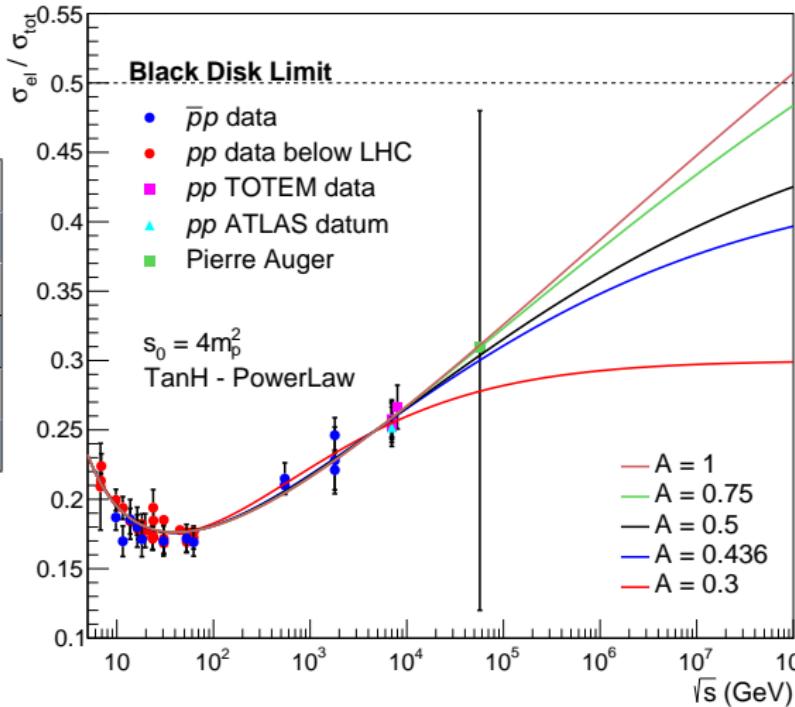
- A fixed and A free
 - Fit TanH: Power-Law
 - Fit TanH: Log-Law

Fit Results: Fit TanH with Power-Law

$$X(s) = A \tanh[g(s)]; \quad g(s) = \alpha + \beta \ln(s/s_0) + \gamma \ln^\delta(s/s_0)$$

A (fixed)	χ^2/ν	$P(\chi^2, \nu)$
0.3	0.818	0.780
0.436	0.844	0.740
0.5	0.853	0.724
0.75	0.870	0.697
1.0	0.875	0.687

$(\nu = 38)$



Fit Results: Fit TanH with Power-Law - A fixed

$$X(s) = A \tanh[g(s)]; \quad g(s) = \alpha + \beta \ln(s/s_0) + \gamma \ln^\delta(s/s_0)$$

Table: Fit with TanH and $h_{PL}(x)$: δ free and A fixed ($\nu = 38$), $s_0 = 4m_p^2$.

A fixed	α	β	γ	δ	χ^2/ν	P_{χ^2}
0.3	125.23(24)	0.13243(93)	-123.95(24)	$6.31(48) \times 10^{-3}$	0.818	0.780
0.436	169.21(13)	0.0588(35)	-168.50(13)	$2.23(15) \times 10^{-3}$	0.844	0.740
0.5	211.44(10)	0.0477(28)	-210.83(10)	$1.380(96) \times 10^{-3}$	0.853	0.724
0.75	68.06(14)	0.0283(16)	-67.67(14)	$2.56(18) \times 10^{-3}$	0.870	0.697
1.0	83.50(10)	0.0204(16)	-83.22(10)	$1.51(10) \times 10^{-3}$	0.875	0.687

Fit Results: Fit TanH with Power-Law - A free

$$X(s) = A \tanh[g(s)]; \quad g(s) = \alpha + \beta \ln(s/s_0) + \gamma \ln^\delta(s/s_0)$$

Table: Fit with TanH and $h_{PL}(x)$: δ and A free ($\nu = 37$), $s_0 = 4m_p^2$.

A initial	A free	α	β	γ	δ	χ^2/ν	P_{χ^2}
0.3	0.31(10)	125.18(43)	0.12(11)	-123.99(43)	$5.7(4.9) \times 10^{-3}$	0.838	0.746
0.436	0.31(10)	169.45(43)	0.12(11)	-168.25(42)	$4.2(3.8) \times 10^{-3}$	0.838	0.746
0.5	0.312(49)	211.73(18)	0.118(52)	-210.54(17)	$3.3(1.4) \times 10^{-3}$	0.837	0.746
0.75	0.31(10)	68.42(48)	0.12(11)	-67.23(48)	$1.04(0.93) \times 10^{-2}$	0.838	0.745
1.0	0.31(10)	83.92(46)	0.12(11)	-82.73(45)	$8.5(7.4) \times 10^{-3}$	0.838	0.746

Results

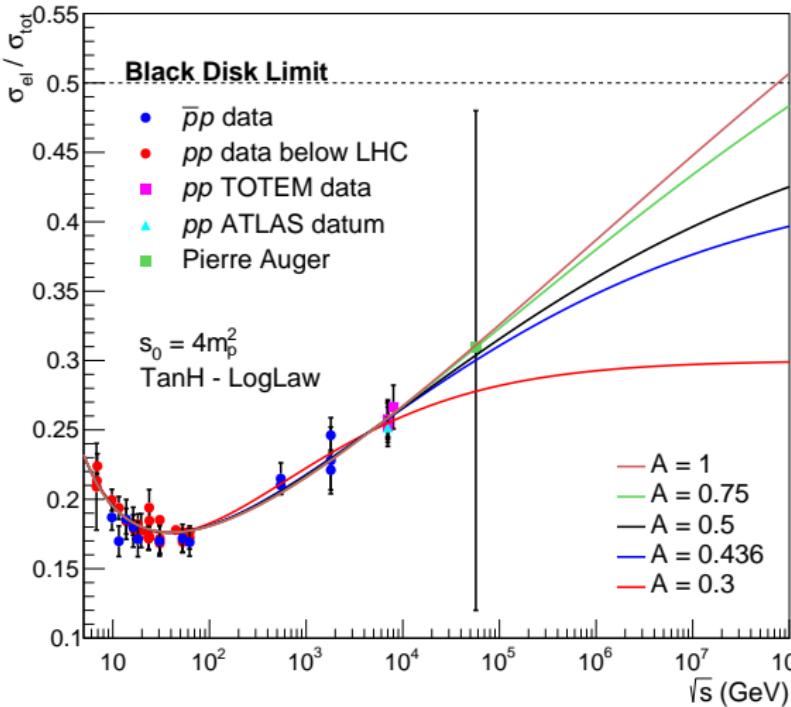
- A fixed and A free
 - Fit TanH: Power-Law
 - Fit **TanH**: Log-Law

Results: Fit TanH with Log-Law

$$X(s) = A \operatorname{tanh}[g(s)]; \quad g(s) = \alpha + \beta \ln(s/s_0) + \gamma \ln \ln(s/s_0)$$

A (fixed)	χ^2/ν	$P(\chi^2, \nu)$
0.3	0.796	0.813
0.436	0.822	0.777
0.5	0.831	0.763
0.75	0.847	0.737
1.0	0.853	0.729

$(\nu = 39)$



Results: Fit TanH with Log-Law - A fixed

$$X(s) = A \text{tanh}[g(s)]; \quad g(s) = \alpha + \beta \ln(s/s_0) + \gamma \ln \ln(s/s_0)$$

Table: Fit with TanH and $h_{LL}(x)$: A fixed ($\nu = 39$), $s_0 = 4m_p^2$.

A fixed	α	β	γ	χ^2/ν	P_{χ^2}
0.3	1.289(54)	0.1317(90)	-0.786(58)	0.796	0.813
0.436	0.718(25)	0.0587(35)	-0.358(25)	0.822	0.777
0.5	0.605(20)	0.0476(28)	-0.291(20)	0.831	0.763
0.75	0.381(12)	0.0283(16)	-0.174(12)	0.847	0.737
1.0	0.2811(89)	0.0204(11)	-0.1259(87)	0.853	0.729

Results: Fit TanH with Log-Law - A free

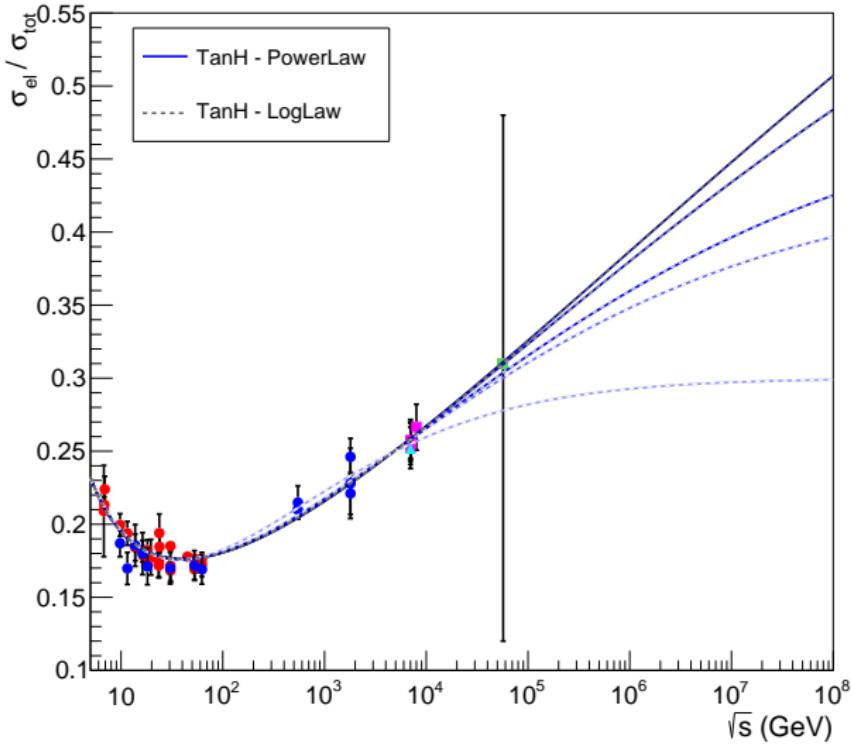
$$X(s) = A \text{tanh}[g(s)]; \quad g(s) = \alpha + \beta \ln(s/s_0) + \gamma \ln \ln(s/s_0)$$

Table: Fit with TanH and $h_{LL}(x)$: A free ($\nu = 38$), $s_0 = 4m_p^2$.

A initial	A free	α	β	γ	χ^2/ν	P_{χ^2}
0.3	0.312(35)	1.19(25)	0.117(37)	-0.70(21)	0.815	0.783
0.436	0.312(48)	1.19(35)	0.117(51)	-0.70(29)	0.815	0.783
0.5	0.312(35)	1.19(25)	0.117(36)	-0.70(21)	0.815	0.783
0.75	0.312(35)	1.19(25)	0.117(37)	-0.70(21)	0.815	0.783
1.0	0.312(35)	1.19(25)	0.117(37)	-0.70(21)	0.815	0.783

Results with A fixed

All cases → consistent description of experimental data (analyzed)
(do not select a scenario)



Roots of 1st and 2nd derivatives of $X(s)$ - TanH

Table: Roots of 1st and 2nd derivative of X (in GeV) for Power-Law and Log-Law with $s_0 = 4m_p^2$ and TanH. Root of 2nd derivative corresponds to inflection point.

	Power-Law		Log-Law	
	1st derivative	2nd derivative	1st derivative	2nd derivative
$A = 0.3$	37.1	78.7	37.0	78.5
$A = 0.436$	39.7	86.7	39.7	86.6
$A = 0.5$	40.1	87.8	40.1	87.8
$A = 0.75$	40.7	89.7	40.6	89.6
$A = 1.0$	40.8	90.3	40.8	90.2
A free	37.6	80.1	37.5	80.0