Semihard processes and BFKL resummation

Alessandro Papa

Università della Calabria & INFN - Cosenza

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Outline

1. Introductory remarks
   - Gluon Reggeization in perturbative QCD
   - BFKL in the leading order
   - BFKL in the next-to-leading order
   - Other BFKL topics
2. Phenomenology
   - General scheme
   - Processes at e^+e^-, $e\gamma$ colliders
   - Processes at hadron colliders
   - Processes at ep colliders
3. Conclusions

Semihard collision processes

Collision processes with the following scale hierarchy:

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD}$$

Q **is the hard scale of the process (*e.g.* photon virtuality, heavy quark mass, jet/hadron transverse momentum,** t**, *etc.*)**

- **large** $Q \implies \alpha_s(Q) \ll 1 \implies$ **perturbative QCD**
- **large** $s \implies$ **large energy logs** $\implies \alpha_s(Q) \log s \sim 1 \implies$ **resummation**

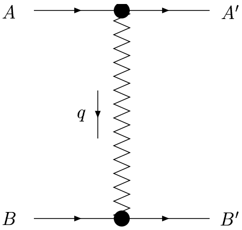
The Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach provides with the general framework for this resummation.

Outline

Gluon Reggeization in perturbative QCD

Elastic scattering process $A + B \longrightarrow A' + B'$

- **gluon quantum numbers in the t-channel: octet color representation, negative signature**
- **Regge limit: $s \simeq -u \to \infty$, t fixed (i.e. not growing with s)**
- **all-order resummation:**
  leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$



$$\left(\mathcal{A}_8^-\right)_{AB}^{A'B'} = \Gamma^c_{A'A} \left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)} \right] \Gamma^c_{B'B}$$

$$j(t) = 1 + \omega(t) \,, \qquad j(0) = 1$$

$\omega(t)$ **– Reggeized gluon trajectory**

$$\Gamma^c_{A'A} = g \langle A' | T^c | A \rangle \Gamma_{A'A}$$

T^c **fundamental (quarks) or adjoint (gluons)**

Gluon Reggeization in perturbative QCD

Interlude: Sudakov decomposition

$$p = \beta p_1 + \alpha p_2 + p_\perp \; , \qquad p_\perp^2 = -\vec{p}^{\,2}$$

(p_1, p_2) **light-cone basis of the initial particle momenta plane**

$$p_A = p_1 + \frac{m_A^2}{s} p_2 \; , \qquad p_B = p_2 + \frac{m_B^2}{s} p_1 \; , \qquad 2\, p_1 \cdot p_2 = s \qquad \blacksquare$$

The gluon Reggeization has been first verified in fixed order calculations, then rigorously proved

- **in the LLA** [Ya.Ya. Balitsky, V.S. Fadin, L.N. Lipatov (1979)]

$$\Gamma^{(0)}_{A'A} = \delta_{\lambda_{A'}\lambda_A} \; , \qquad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{D-1}} \frac{N}{2} \int \frac{d^{D-2}k_\perp}{k_\perp^2 (q-k)_\perp^2} = -g^2 \frac{N\Gamma(1-\epsilon)}{(4\pi)^{D/2}} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}^{\,2})^\epsilon$$

$$D = 4 + 2\epsilon \; , \qquad t = q^2 \simeq q_\perp^2$$

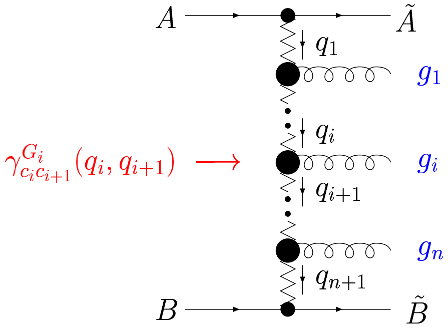
- **in the NLA** [V.S. Fadin, R. Fiore, M.G. Kozlov, A.V. Reznichenko (2006)]

$$\Gamma^{(1)}_{A'A} = \delta_{\lambda_{A'}\lambda_A} \Gamma^{(+)}_{AA} + \delta_{\lambda_{A'},-\lambda_A} \Gamma^{(-)}_{AA} \; , \qquad \omega^{(2)}(t)$$

Outline

BFKL in the LLA

Inelastic scattering process $A+B \;\; \rightarrow \;\; \tilde{A}+\tilde{B}+n$ **in the LLA**



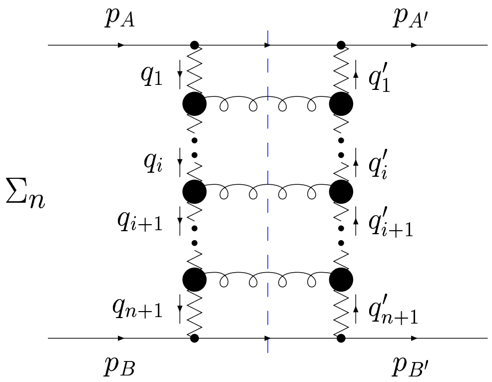
multi-Regge kinematics

$$\mathbf{Re}\mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\,\Gamma^{c_1}_{\tilde{A}A}\left(\prod_{i=1}^{n}\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma^{c_{n+1}}_{\tilde{B}B}$$

s_R **energy scale, irrelevant in the LLA**

BFKL in the LLA

Elastic amplitude $A + B \longrightarrow A' + B'$ in the LLA via s-channel unitarity

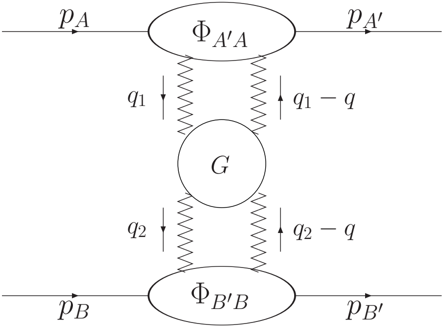


$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} \,, \qquad \mathcal{R} = \mathbf{1}\ \textbf{(singlet)},\ \mathbf{8}^-\ \textbf{(octet)}, \ldots$$

The 8^- color representation is important for the bootstrap, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange

BFKL in the LLA

Structure of the amplitude:



$$\mathbf{Im}_s(A_{\mathcal{R}})_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q})^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q})^2} \sum_{\nu} \Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1;\vec{q})$$

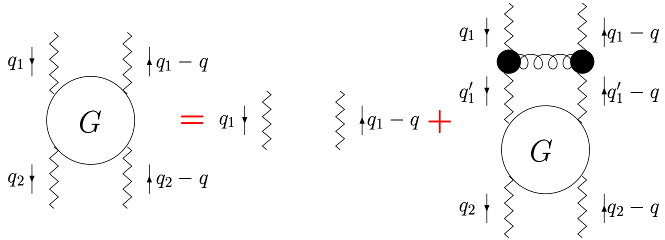
$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2, \vec{q}) \right] \Phi_{B'B}^{(\mathcal{R},\nu)}(-\vec{q}_2; -\vec{q})$$

BFKL in the LLA

- $G_\omega^{(\mathcal{R})}$ – **Mellin transform of the Green's functions for Reggeon-Reggeon scattering**

$$\omega G_\omega^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2, \vec{q}) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{q})^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int \frac{d^{D-2}q_r}{\vec{q}_r^{\,2}(\vec{q}_r - \vec{q})^2} \mathcal{K}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_r; \vec{q})\, G_\omega^{(\mathcal{R})}(\vec{q}_r, \vec{q}_2; \vec{q})$$

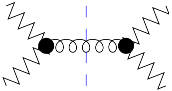
BFKL equation: $t = 0$ **and singlet color representation**
[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]



BFKL in the LLA

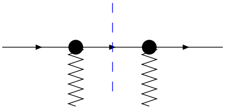
$$\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1, \vec{q}_2; \vec{q}\right) = \left[\omega\left(-\vec{q}_1^{\,2}\right) + \omega\left(-(\vec{q}_1 - \vec{q})^{\,2}\right)\right]\delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \mathcal{K}_r^{(\mathcal{R})}\left(\vec{q}_1, \vec{q}_2; \vec{q}\right)$$

In the LLA: $\omega(t) = \omega^{(1)}(t)$, $\mathcal{K}_r = \mathcal{K}_{RRG}^{(B)}$



- $\Phi_{A'A}^{(\mathcal{R},\nu)}$ **– impact factors in the** t**-channel color state** $(\mathcal{R},\nu)$

$$\Phi_{A'A} = \sum_{\{f\}} \int \frac{d\kappa}{2\pi} d\rho_f \Gamma^{c}_{\{f\}A} (\Gamma^{c'}_{\{f\}A'})^*$$



constant in the LLA

BFKL Pomeron in the LLA

Pomeron channel: $t = 0$ **and singlet color representation in the** t**-channel**

Redefinition: $G_\omega(\vec{q}_1, \vec{q}_2) \equiv \frac{G_\omega^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}$, $\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}$

$$\omega G_\omega\left(\vec{q}_1, \vec{q}_2\right) = \delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \int d^{D-2}q_r \mathcal{K}\left(\vec{q}_1, \vec{q}_r\right) G_\omega\left(\vec{q}_r, \vec{q}_2\right)$$

$$\mathcal{K}\left(\vec{q}_1, \vec{q}_2\right) = 2\omega(-\vec{q}_1^{\,2})\delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \mathcal{K}_r\left(\vec{q}_1, \vec{q}_2\right)$$

Infrared divergences cancel in the singlet kernel

$\mathcal{K}\left(\vec{q}_1, \vec{q}_2\right)$ **is scale-invariant** $\longrightarrow$ **its eigenfunctions are powers of** $\vec{q}_2^{\,2}$**:**

$$\int d^{D-2}q_2 \mathcal{K}\left(\vec{q}_1, \vec{q}_2\right)\left(\vec{q}_2^{\,2}\right)^{\gamma-1} = \frac{N\alpha_s}{\pi}\chi(\gamma)(\vec{q}_1^{\,2})^{\gamma-1}$$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\,, \quad \psi(\gamma) = \frac{\Gamma'(\gamma)}{\Gamma(\gamma)}$$

The set of functions $(\vec{q}_2^{\,2})^{\gamma-1}$**, with** $\gamma = 1/2 + i\nu$**,** $\nu \in (-\infty, +\infty)$ **is complete.**

BFKL Pomeron in the LLA

$$\sigma_{AB}(s) = \int_{-\infty}^{+\infty} d\nu \left[\int \frac{d^2\vec{q}_A}{2\pi}\Phi_A(\vec{q}_A)\frac{(\vec{q}_A^{\,2})^{-i\nu-3/2}}{\pi\sqrt{2}}\right]\left(\frac{s}{s_0}\right)^{\bar{\alpha}_s\chi(\nu)}\left[\int \frac{d^2\vec{q}_B}{2\pi}\Phi_B(-\vec{q}_B)\frac{(\vec{q}_B^{\,2})^{i\nu-3/2}}{\pi\sqrt{2}}\right]$$

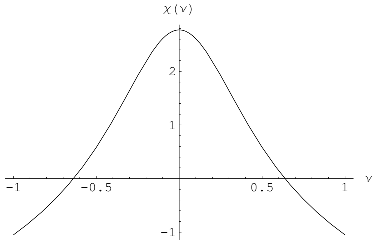
$$\bar{\alpha}_s \equiv \frac{N\alpha_s}{\pi}\,, \qquad \chi(\nu) \equiv \chi(1/2 + i\nu)$$

Saddle point approximation:

$\chi(\nu) = 4\ln 2 + \psi''\left(\frac{1}{2}\right)\nu^2 + O(\nu^4)$

$$\sigma_{AB}(s) \sim \frac{s^{4\bar{\alpha}_s \ln 2}}{\sqrt{\ln s}}$$

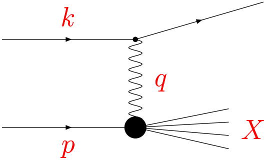
$\omega_P = 4\bar{\alpha}_s \ln 2 \simeq 0.40$ **for** $\alpha_s = 0.15$



- **unitarity is violated; BFKL cannot be applied at asymptotically high energies**
- **the scale of s and the argument of the running coupling constant are not fixed in the LLA $\longrightarrow$ NLA**

BFKL and DIS

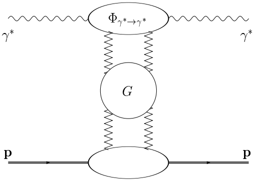
Deep inelastic electron-proton scattering: $e + p \to e + X$



$$Q^2 = -q^2 > 0$$
$$s = (p+k)^2$$
$$W^2 = (p+q)^2$$

$$x = \frac{Q^2}{2p \cdot q} \simeq \frac{Q^2}{Q^2 + W^2}\,, \qquad W^2 \simeq \frac{Q^2(1-x)}{x}$$

$$W^2 \gg Q^2 \gg M_P^2 \quad \longrightarrow \quad x \ll 1$$



$$F_2(x, Q^2) \sim \sigma_{\textbf{tot}}(\gamma^* p) = \frac{\mathcal{I}m\mathcal{A}(\gamma^* p \to \gamma^* p)|_{t=0}}{W^2}$$
$$\sim x^{-4\bar{\alpha}_s \ln 2}$$

Outline

BFKL in the NLA

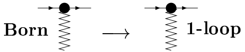
Production amplitudes keep the simple factorized form

$$\mathrm{Re}\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{P_i}(q_i,q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

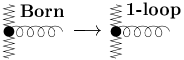
but, with respect to the LLA case, one replacement is allowed among the following:

multi-Regge kinematics (one α_s more)

- $\omega^{(1)} \longrightarrow \omega^{(2)}$
- $\Gamma_{P'P}^{c\,\textbf{(Born)}} \longrightarrow \Gamma_{P'P}^{c\,\textbf{(1-loop)}}$



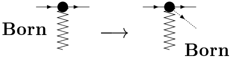
- $\gamma_{c_i c_{i+1}}^{G_i\textbf{(Born)}} \longrightarrow \gamma_{c_i c_{i+1}}^{G_i\textbf{(1-loop)}}$



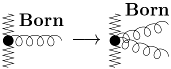
BFKL in the NLA

quasi-multi-Regge kinematics **(one** log s **less)**

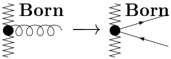
- $\Gamma^{c\,\textbf{(Born)}}_{P'P} \longrightarrow \Gamma^{c\,\textbf{(Born)}}_{\{f\}P}$



- $\gamma^{G_i\textbf{(Born)}}_{c_i c_{i+1}} \longrightarrow \gamma^{Q\overline{Q}\textbf{(Born)}}_{c_i c_{i+1}}$



- $\gamma^{G_i\textbf{(Born)}}_{c_i c_{i+1}} \longrightarrow \gamma^{GG\textbf{(Born)}}_{c_i c_{i+1}}$



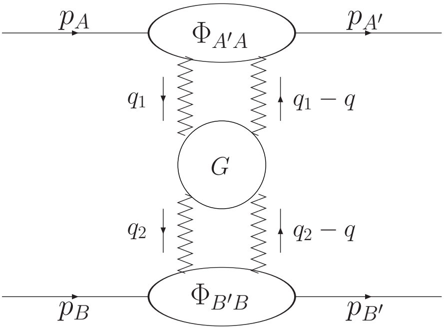
This is the program of calculation of radiative corrections to the LLA BFKL

[V.S. Fadin, L.N. Lipatov (1989)]

BFKL in the NLA

- $\omega^{(2)}(t)$ [V.S. Fadin (1995)] [V.S. Fadin, R. Fiore, M.I. Kotsky (1995)]
  [V.S. Fadin, R. Fiore, A. Quartarolo (1996)]
  [V.S. Fadin, R. Fiore, M.I. Kotsky (1996)]
  [V.S. Fadin, M.I. Kotsky (1996)]
- $\gamma^{G_i}_{c_i c_{i+1}}$ **(1-loop)** [V.S. Fadin, L.N. Lipatov (1993)]
  [V.S. Fadin, R. Fiore, A. Quartarolo (1994)]
  [V.S. Fadin, R. Fiore, M.I. Kotsky (1996)]
  [V.S. Fadin, R. Fiore, A. P. (2001)]
- $\Gamma^{c}_{P'P}$ **(1-loop)** [V.S. Fadin, R. Fiore (1992)] [V.S. Fadin, L.N. Lipatov (1993)]
  [V.S. Fadin, R. Fiore, A. Quartarolo (1994)]
  [V.S. Fadin, R. Fiore, M.I. Kotsky (1995)]
- $\gamma^{Q\overline{Q}}_{c_i c_{i+1}}$ **(Born)** [V.S. Fadin, R. Fiore, A. Flachi, M.I. Kotsky (1997)]
  [S. Catani, M. Ciafaloni, F. Hautmann (1990)]
  [G. Camici, M. Ciafaloni (1996)]
- $\gamma^{GG}_{c_i c_{i+1}}$**(Born)** [V.S. Fadin, L.N. Lipatov (1996)]
  [V.S. Fadin, M.I. Kotsky, L.N. Lipatov (1997)]

Structure of the amplitude:



$$\mathrm{Im}_s (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2} q_1}{\vec{q}_1^{\,2} (\vec{q}_1 - \vec{q})^2} \int \frac{d^{D-2} q_2}{\vec{q}_2^{\,2} (\vec{q}_2 - \vec{q})^2} \sum_{\nu} \Phi_{A'A}^{(\mathcal{R},\nu)} (\vec{q}_1; \vec{q}; s_0)$$

$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(\mathcal{R})} (\vec{q}_1, \vec{q}_2, \vec{q}) \right] \Phi_{B'B}^{(\mathcal{R},\nu)} (-\vec{q}_2; -\vec{q}; s_0)$$

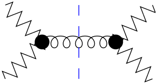
BFKL in the NLA

- $G_\omega^{(\mathcal{R})}$ – **Mellin transform of the Green's functions for Reggeon-Reggeon scattering**

$$\omega G_\omega^{(\mathcal{R})}(\vec q_1, \vec q_2, \vec q) = \vec q_1^{\,2}(\vec q_1 - \vec q)^{\,2}\delta^{(D-2)}(\vec q_1 - \vec q_2) + \int \frac{d^{D-2}q_r}{\vec q_r^{\,2}(\vec q_r - \vec q)^{\,2}} \mathcal{K}^{(\mathcal{R})}(\vec q_1, \vec q_r; \vec q)\, G_\omega^{(\mathcal{R})}(\vec q_r, \vec q_2; \vec q)$$

$$\mathcal{K}^{(\mathcal{R})}(\vec q_1, \vec q_2; \vec q) = \left[\omega\left(-\vec q_1^{\,2}\right) + \omega\left(-(\vec q_1 - \vec q)^{\,2}\right)\right]\delta^{(D-2)}(\vec q_1 - \vec q_2) + \mathcal{K}_r^{(\mathcal{R})}(\vec q_1, \vec q_2; \vec q)$$

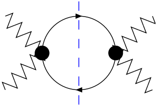
In the NLA: $\omega(t) = \omega^{(1)}(t) + \omega^{(2)}(t)$, $\quad \mathcal{K}_r = \mathcal{K}_{RRG}^{(B)} + \mathcal{K}_{RRG}^{(1)} + \mathcal{K}_{RRQ\overline{Q}}^{(B)} + \mathcal{K}_{RRGG}^{(B)}$



$\mathcal{K}_{RRG}^{(1)}$

$t = 0$: **[V.S. Fadin, L.N. Lipatov (1993)]**
[V.S. Fadin, R. Fiore, A. Quartarolo (1994)]
[V.S. Fadin, R. Fiore, M.I. Kotsky (1996)]

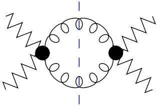
$t \neq 0$: **[V.S. Fadin, R. Fiore, A. P. (2001)]**



$\mathcal{K}_{RRQ\overline{Q}}^{(B)}$

$t = 0$: **[V.S. Fadin, R. Fiore, A. Flachi, M.I. Kotsky (1997)]**

$t \neq 0$: **[V.S. Fadin, R. Fiore, A. P. (1999)]**



$\mathcal{K}_{RRGG}^{(B)}$

$t = 0$: **[V.S. Fadin, L.N. Lipatov, M.I. Kotsky (1997)]**

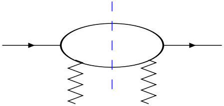
$t \neq 0$: **[V.S. Fadin, D.A. Gorbachev (2000)]**
[V.S. Fadin, R. Fiore (2005)]

BFKL in the NLA

- $\Phi_{A'A}^{(\mathcal{R},\nu)}$ **– impact factors in the t-channel color state $(\mathcal{R},\nu)$**

$$\Phi_{A'A} = \sum_{\{f\}} \int \frac{d\kappa}{2\pi} d\rho_f \Gamma^c_{\{f\}A} (\Gamma^{c'}_{\{f\}A'})^*$$

$$\times \left(\frac{s_0}{\vec{q}_1^{\,2}}\right)^{\frac{\omega(-\vec{q}_1^{\,2})}{2}} \left(\frac{s_0}{(\vec{q}_1 - \vec{q})^2}\right)^{\frac{\omega(-(\vec{q}_1-\vec{q})^2)}{2}}$$



$-$ counterterm

non-trivial momentum and scale-dependence

BFKL Pomeron in the NLA

Pomeron channel: $t = 0$ **and singlet color representation in the** t**-channel**

$$\left(\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}} \;,\; \gamma = \frac{1}{2} + i\nu \right)$$

$$\int d^{D-2} q_2 \mathcal{K}\left(\vec{q}_1, \vec{q}_2\right) \left(\vec{q}_2^{\,2}\right)^{\gamma-1} = \frac{N\alpha_s(\vec{q}_1^{\,2})}{\pi} \left(\chi(\gamma) + \frac{N\alpha_s(\vec{q}_1^{\,2})}{\pi} \chi^{(1)}(\gamma) \right) \left(\vec{q}_1^{\,2}\right)^{\gamma-1}$$

- **broken scale invariance**
- **large corrections:** $\quad -\left.\frac{\chi^{(1)}(\gamma)}{\chi(\gamma)}\right|_{\gamma=1/2} \simeq 6.46 + 0.05 \frac{n_f}{N} + 0.96 \frac{n_f}{N^3}$

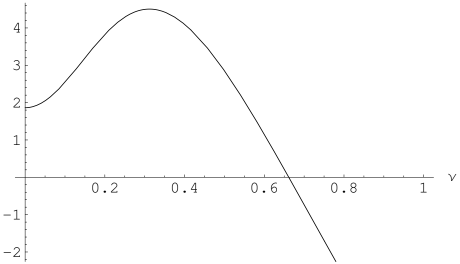
  [V.S. Fadin, L.N. Lipatov (1998)] [G. Camici, M. Ciafaloni (1998)]

BFKL Pomeron in the NLA

$\chi(\nu) + \bar{\alpha}_s(\vec{q}_1^{\,2})\chi^{(1)}(\nu)$ *vs* ν

$\bar{\alpha}_s(\vec{q}_1^{\,2}) \equiv \frac{\alpha_s(\vec{q}_1^{\,2})N}{\pi} = 0.15$

(omitted terms with the first derivative)



Double maxima $\longrightarrow$ oscillations in momentum space after ν-integration

Ways out:

- **rapidity veto** **[C.R. Schmidt (1999)]** **[J.R. Forshaw, D.A. Ross, A. Sabio Vera (1999)]**
- **collinear improvement** **[G. Salam (1998)] [M. Ciafaloni, D. Colferai (1999)]**
- **renormalization with a physical scheme** **[S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov, G.B. Pivovarov (1999)]**
- **...**

Outline

Other BFKL topics

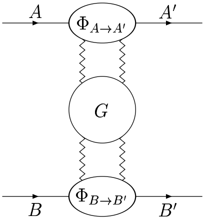
- Reggeization in other color channels [I.P. Ivanov (2005)]
- "diffusion" in the infrared
- unitarization problem
  - non-linear generalization of the BFKL equation, based on the idea of saturation of parton densities
    [L.V. Gribov, E.M. Levin, M.G. Ryskin (1983)]
    [I. Balitsky (1996)] [Yu. Kovchegov (1999)]
  - gauge-invariant effective theory for Reggeized gluon interactions
    [L.N. Lipatov (1995)]
- BFKL in coordinate representation and conformal invariance
  [V.S. Fadin, R. Fiore, A.P. (2007,2012)]
  [V.S. Fadin, R. Fiore, A.V. Grabovsky, A.P. (2007)]
  [V.S. Fadin, R. Fiore, A.V. Grabovsky, A.P. (2011)]
- BFKL in $N = 4$ SUSY [A.V. Kotikov, L.N. Lipatov (2000) and many others]

Outline

General scheme

Scattering $A + B \longrightarrow A' + B'$ in the Regge kinematical region $s \to \infty$, t fixed

BFKL approach: convolution of the Green's function of two interacting Reggeized gluons and of the impact factors of the colliding particles, valid both in the LLA and in the NLA.

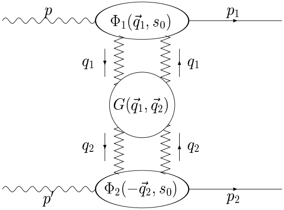


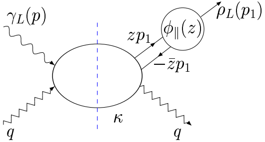
- **The BFKL Green's function is universal and takes care of the energy dependence**
- **Impact factors are process-dependent and depend on the hard scale, but not on the energy**

The list of processes which can be studied within NLA BFKL depends on the list of available NLO impact factors calculated so far.

Outline

$\gamma^*\gamma^* \to VV$, $V = \rho, \omega, \phi$





[D.Yu. Ivanov, M.I. Kotsky, A.P. (2004)]

Hard scales: photon virtualities, $Q_{1,2}$

- **Longitudinally polarized vector mesons are produced by longitudinally polarized photons; other helicity amplitudes power suppressed by $\sim m_\rho/Q_{1,2}$;**
- **forward scattering, i.e. zero transverse momenta of the produced mesons**

$$\frac{Q_1 Q_2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s} = \frac{1}{(2\pi)^2}\alpha_s(\mu_R)^2\left[b_0+\sum_{n=1}^{\infty}\bar{\alpha}_s(\mu_R)^n\, b_n\left(\ln\left(\frac{s}{s_0}\right)^n + d_n(s_0,\mu_R)\ln\left(\frac{s}{s_0}\right)^{n-1}\right)\right]$$

$$D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}}\sqrt{N_c^2-1}\,, \qquad e_q \longrightarrow \frac{e}{\sqrt{2}}\,,\quad \frac{e}{3\sqrt{2}}\,,\quad -\frac{e}{3}$$

$$\rho^0\,,\qquad \omega\,,\qquad \phi$$

$Q_1 = Q_2 \equiv Q$ **"pure" BFKL regime** [D.Yu. Ivanov, A.P. (2006)]

LLA: b_n coefficients (Q-independent)

$$b_0 = 17.0664 \quad b_1 = 34.5920 \quad b_2 = 40.7609 \quad b_3 = 33.0618 \quad b_4 = 20.7467$$
$$b_5 = 10.5698 \quad b_6 = 4.54792 \quad b_7 = 1.69128 \quad b_8 = 0.554475$$

NLA: $d_n(s_0, \mu_R)$ coefficients ($s_0 = Q^2 = \mu_R^2$, $n_f = 5$)

$$d_1 = -3.71087 \quad d_2 = -11.3057 \quad d_3 = -23.3879 \quad d_4 = -39.1123$$
$$d_5 = -59.207 \quad d_6 = -83.0365 \quad d_7 = -111.151 \quad d_8 = -143.06$$

NLA: $d_n^{\rm imp}(s_0, \mu_R)$ coefficients ($s_0 = Q^2 = \mu_R^2$, impact factor contribution)

$$d_1^{\rm imp} = -3.71087 \quad d_2^{\rm imp} = -8.4361 \quad d_3^{\rm imp} = -13.1984 \quad d_4^{\rm imp} = -18.0971$$
$$d_5^{\rm imp} = -23.0235 \quad d_6^{\rm imp} = -27.9877 \quad d_7^{\rm imp} = -32.9676 \quad d_8^{\rm imp} = -37.9618$$

Large NLA corrections!

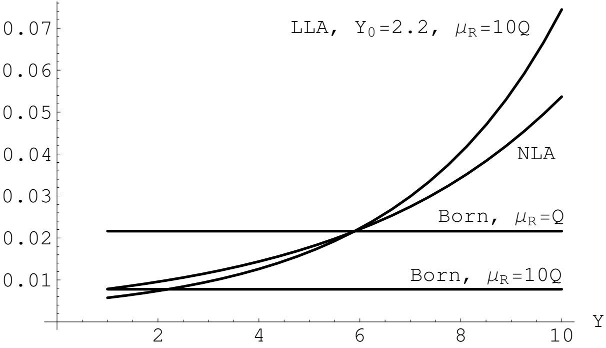
d_n coefficients negative and increasingly large in absolute value.
The contribution from the kernel dominates only for $n \geq 4$.

Optimization of the perturbative expansion needed!
Principle of minimal sensitivity (PMS) [P.M. Stevenson (1981)]: require the minimal sensitivity to the change of both s_0 and μ_R.

$\frac{Q^2}{D_1 D_2} \frac{\mathcal{I}m_s \mathcal{A}}{s}$ vs Y

$Y \equiv \log s/Q$
$Y_0 \equiv \log s_0/Q$

Q^2=24 GeV^2, $n_f = 5$



Lessons

- **The Born approximation does not give necessarily the estimate from below.**
- **The optimal values for μ_R are “unnaturally” larger than Q (new scale or nature of the BFKL series?).**

$\longrightarrow$ Since NLA corrections are large and since the exact amplitude should be renorm- and energy scale invariant, the NNLA terms should be large and of the opposite sign with respect to the NLA.
If the NNLA corrections were known and we would apply PMS to the NNLA amplitude, we would obtain more natural values of μ_R.

$\frac{Q^2}{D_1 D_2} \frac{\mathcal{I}m_s\mathcal{A}}{s}$ vs Y

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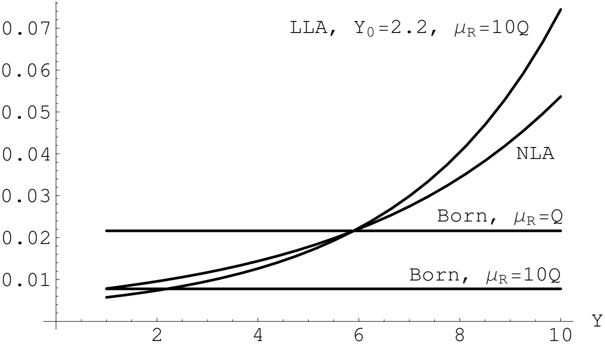

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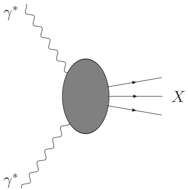
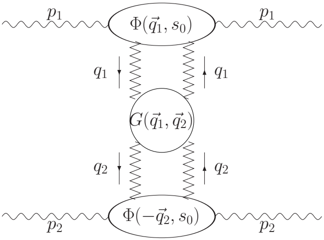
$\gamma^*\gamma^* \to$ hadrons





Hard scales: photon virtualities, $Q_{1,2}$

The NLO impact factor, a long story ... **[J. Bartels, S. Gieseke, C.F. Qiao (2001)]**
[J. Bartels, S. Gieseke, A. Kyrieleis (2002)]
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[J. Bartels, A. Kyrieleis (2004)]

... till the recent breakthrough **[I. Balitsky, G.A. Chirilli (2013)]**
[G.A. Chirilli, Yu.V. Kovchegov (2014)]

Compatibility between the different approaches not yet established.

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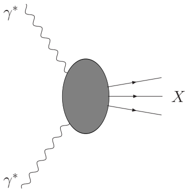

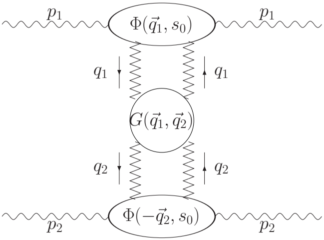

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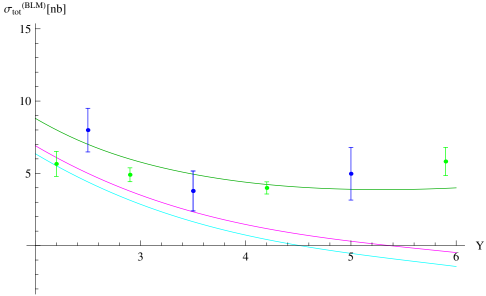
$Q_1^2 = Q_2^2 \equiv Q^2$=17 GeV^2

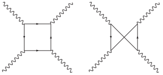
$Y \equiv \log s/Q$

- **OPAL** (Q^2=18 GeV^2)
- **L3** (Q^2=16 GeV^2)



Green: NLA Green's function + LO impact factors (PMS scale setting)
[F. Caporale, D.Yu. Ivanov, A.P. (2008)]

Cyan/Magenta: full NLA (BLM scale) **[D.Yu. Ivanov, B. Murdaca, A.P. (2014)]**

- **quark box included** 
- **[S.J. Brodsky, G.P. LePage, P.B. MacKenzie (1983)] (BLM) scale setting: μ_R set at the value that makes the β_0-dependent terms in the amplitude vanish.**

$\longrightarrow$ **High-energy linear colliders needed!**

$Q_1^2 = Q_2^2 \equiv Q^2$**=17 GeV^2**

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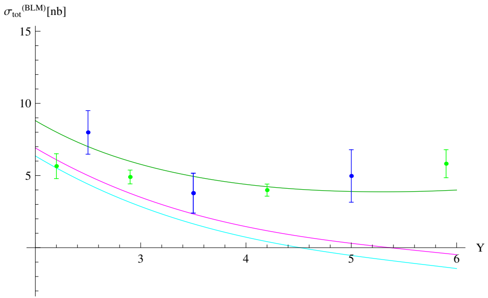

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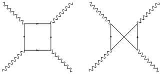
- **quark box included** 
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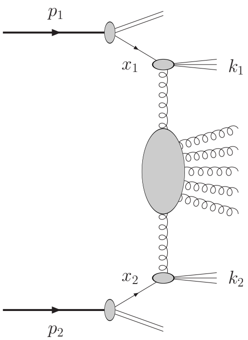
$\longrightarrow$ High-energy linear colliders needed!

Outline

Mueller-Navelet jets

$$\mathrm{proton}(p_1) + \mathrm{proton}(p_2) \to \mathrm{jet}_1(k_1) + \mathrm{jet}_2(k_2) + X$$

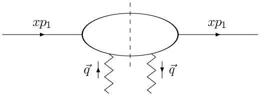


- **large jet transverse momenta (hard scales):** $\vec{k}_1^2 \sim \vec{k}_2^2 \gg \Lambda^2_{\mathrm{QCD}}$
- **large rapidity gap between jets,** $\Delta y \equiv Y = y_{J_1} - y_{J_2}$**,**
  which requires large c.m. energy of the proton collisions, $s = 2p_1 \cdot p_2 \gg \vec{k}_{1,2}^2$

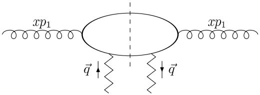
[A.H. Mueller, H. Navelet (1987)]

- **Step 0**: take the impact factors for colliding partons

[V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)]
[M. Ciafaloni and G. Rodrigo (2000)]

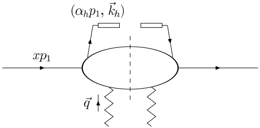


quark impact factor

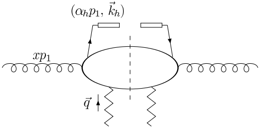


gluon impact factor

- **Step 1**: "open" one of the integrations over the phase space of the intermediate state to "allow" one parton to generate the jet



quark jet vertex

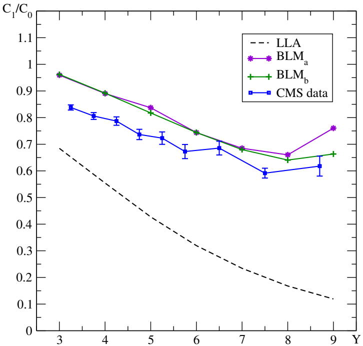


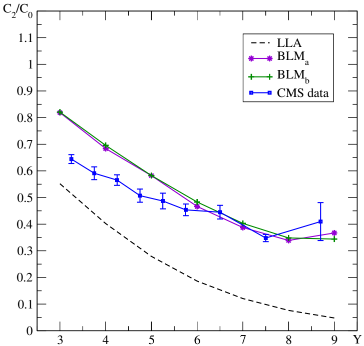
gluon jet vertex

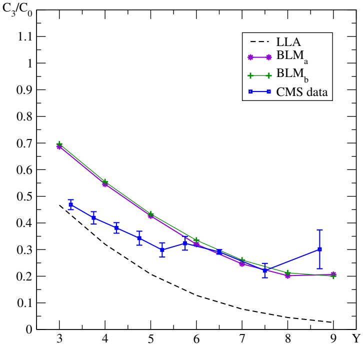
- **Step 2**: take the convolution with leading-twist PDFs

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark jet vertex}) \qquad + \qquad f_g \otimes (\text{gluon jet vertex})$$

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P., A. Perri (2011)]
[D.Yu. Ivanov, A.P. (2012)] (small-cone approximation)





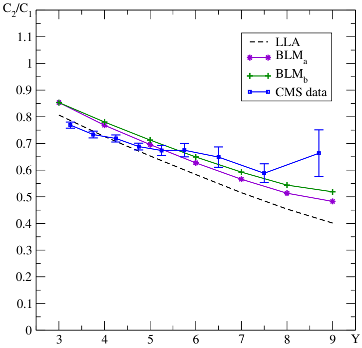


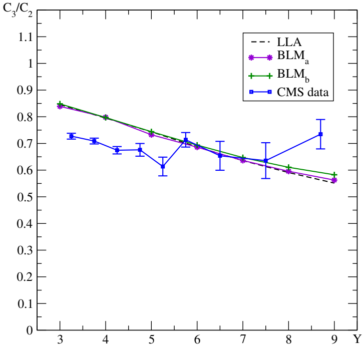
$C_n/C_0 = \langle \cos[n(\phi_{J_1} - \phi_{J_2} - \pi)]\rangle$
vs $Y = y_{J_1} - y_{J_2}$

small-cone approximation
BLM scale setting

□ **CMS (7 TeV; $|\vec{k}_1|, |\vec{k}_2| \geq 35$ GeV)**

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P. (2014)]





$C_n/C_m = \frac{\langle\cos(n\phi)\rangle}{\langle\cos(m\phi)\rangle}$ *vs* $Y = y_{J_1} - y_{J_2}$ **[A. Sabio Vera (2006)]**

[A. Sabio Vera, F. Schwennsen (2007)]

small-cone approximation; BLM scale setting

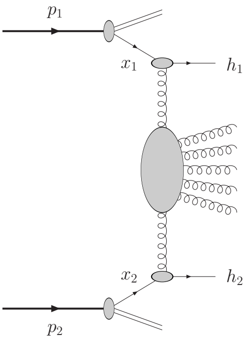
□ **CMS (7 TeV;** $|\vec{k}_1|, |\vec{k}_2| \geq 35$ **GeV)**

Similar results obtained with the exact jet vertices
[B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]

Discrimination between BFKL and fixed-order (DGLAP) by asymmetric cuts in jet transverse momenta **[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (2015)]**

Di-hadrons with "Mueller-Navelet" kinematics

$$\mathrm{proton}(p_1) + \mathrm{proton}(p_2) \;\to\; \mathrm{hadron}_1(k_1) + \mathrm{hadron}_2(k_2) + X$$



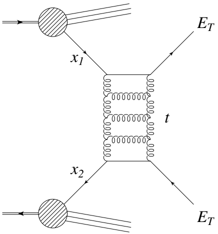
- **large hadron transverse momenta (hard scales):** $\vec{k}_1^{\,2} \sim \vec{k}_2^{\,2} \gg \Lambda^2_{\rm QCD}$
- **large rapidity gap between the identified hadrons,** $\Delta y \equiv Y = y_{h_1} - y_{h_2}$
- **The relevant NLO impact factor is known** **[D.Yu. Ivanov, A.P. (2012)]**
- **Same observables as for Mueller-Navelet jets**
  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (in progress)]

Other processes

- **Mueller-Tang jets**

$$\text{proton} + \text{proton} \to \text{jet}_1 + \text{jet}_2 + \text{gap}$$

  - **Test of BFKL at $t \neq 0$**
  - **The relevant NLO impact factor is known [M. Hentschinski, J.D. Madrigal Martinez, B. Murdaca, A. Sabio Vera (2014)]**
  - **Non-perturbative gap survival probability to be accounted for**



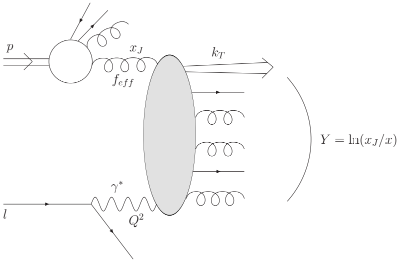
[from S. Wallon, arXiv:0710.0833]

- **Inclusive central + forward jet production, Mueller-Navelet + central jet production, *etc.***

Outline

Forward jets in DIS

$$\gamma^* + \text{proton} \to \text{jet} + \text{X}$$



[from O. Kepka *et al.*, EPJ C55 (2008) 259]

- **large photon virtuality and jet transverse momentum (hard scales)**
- **large** $Y \equiv \log\left(\frac{x_J}{x_B}\right)$
- **Past analyses based on LO forward jet vertex and LO photon impact factor agree with HERA data**
  [O. Kepka, C. Royon, C. Marquet, R.B. Peschanski (2008)]
- **Inclusion of NLO corrections in the forward jet impact factor in progress**
  [F. Caporale, F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (in progress)]
- **Similar process: identified hadron instead of the jet**

Conclusions

- The BFKL approach gives a common basis for the description of semihard processes; it is based on a remarkable property of perturbative QCD, the gluon Reggeization.
- Physical amplitudes in NLA are written in terms of a universal Green's function and of process-dependent impact factors of the colliding particles.
- Several impact factors have been calculated with NLO accuracy, thus allowing for tests of the NLA BFKL approach in several processes.
- The advent of LHC has allowed for some successful tests of NLA BFKL in the jet sector; previously, also HERA had shown compatibility with partially-NLA BFKL approaches. High-energy linear colliders could consolidate these results and open the way to further tests.

My collaborators

Dmitry Yu. Ivanov

Francesco Caporale **Francesco G. Celiberto** **Beatrice Murdaca**

BACKUP

NLO BFKL kernel "eigenvalues"

Pomeron channel: $t = 0$ and singlet color representation in the t-channel

$$\left(\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}} \; , \; \gamma = \frac{1}{2} + i\nu\right)$$

$$\int d^{D-2} q_2 \mathcal{K}\left(\vec{q}_1, \vec{q}_2\right) \left(\vec{q}_2^{\,2}\right)^{\gamma - 1} = \frac{N\alpha_s(\vec{q}_1^2)}{\pi} \left(\chi(\gamma) + \frac{N\alpha_s(\vec{q}_1^2)}{\pi} \chi^{(1)}(\gamma)\right) \left(\vec{q}_1^{\,2}\right)^{\gamma - 1}$$

$$\chi^{(1)}(\nu) = -\frac{\beta_0}{8\, N_c} \left(\chi^2(\nu) - \frac{10}{3}\chi(\nu) - i\chi'(\nu)\right) + \bar{\chi}(\nu) \; , \qquad \beta_0 = \frac{11\, N_c}{3} - \frac{2 n_f}{3}$$

$$\begin{aligned} \bar{\chi}(\nu) \quad &= \quad -\frac{1}{4}\left[\frac{\pi^2 - 4}{3}\chi(\nu) - 6\zeta(3) - \chi''(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} \right. \\ &+ \quad \left. \frac{\pi^2 \sinh(\pi\nu)}{2\nu \cosh^2(\pi\nu)} \left(3 + \left(1 + \frac{n_f}{N_c^3}\right) \frac{11 + 12\nu^2}{16(1 + \nu^2)}\right) + 4\phi(\nu)\right] \end{aligned}$$

$$\phi(\nu) \; = \; 2 \int_0^1 dx \, \frac{\cos(\nu \ln(x))}{(1 + x)\sqrt{x}} \left[\frac{\pi^2}{6} - \mathrm{Li}_2(x)\right]$$

BLM method

Strategy [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]:

- **finite renormalization to the MOM-scheme (ξ=0)**

$$\alpha_S^{\overline{\rm MS}} \to \alpha_S^{\rm MOM}\left[1+T_{MOM}(\xi=0)\frac{\alpha_S^{\rm MOM}}{\pi}\right], \qquad T_{MOM}(\xi=0)=T_{MOM}^{conf}+T_{MOM}^{\beta}$$

$$T_{MOM}^{conf}=\frac{N_C}{8}\frac{17}{2}I\,, \quad T_{MOM}^{\beta}=-\frac{\beta_0}{2}\left[1+\frac{2}{3}I\right], \quad I=-2\int_0^1 dx\,\frac{\ln(x)}{x^2-x+1}\simeq 2.3439$$

- **μ_R chosen to make the β_0 terms in the amplitude vanish**

In semihard processes, the BLM scale depends on the energy and is given by the implicit solution of an equation stating that a certain integral over ν must vanish.

Approximate applications:
(a) **- scale fixed from the correction to the impact factor**
(b) **- scale fixed from the correction to the kernel**
(c) **- vanishing of the integrand of the "exact" integral equation**

[F. Caporale, B. Murdaca, D.Yu. Ivanov, A.P. (2015)]