Effective Transverse Radius of Nucleon in High-Energy Elastic Diffractive Scattering

Anton Godizov

Institute for High Energy Physics, Protvino, Russia

WE-Heraeus Physics School
on Diffractive and Electromagnetic Processes at High Energies
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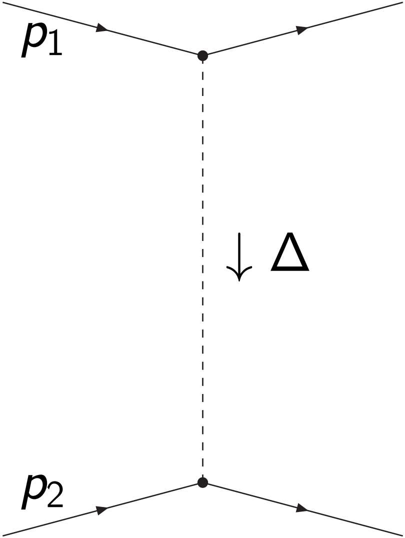
Precursors

S. Kato, *Strong-Interaction Form Factor of Proton in High-Energy Proton-Proton Elastic Scattering*, Nuovo Cim. A **10** (1972) 48

H.I. Miettinen and G.H. Thomas, *Evidence That Hadronic Interiors Have a Denser Matter Than Charge Distribution*, Nucl. Phys. B **166** (1980) 365

Single-meson-exchange contribution



$$\sim \frac{(p_1 p_2)^j}{m_j^2 - \Delta^2}$$

The Born amplitude for HEEDS

L. Van Hove, Phys. Lett. B **24** (1967) 183:

$$\delta^{\pm}(s,t) = \sum_{m_j}\sum_{j}(1 \pm e^{-i\pi j})\times$$

$$\times J^{(1)}_{\alpha_1\ldots\alpha_j}(p_1,\Delta)\frac{D^{\alpha_1\ldots\alpha_j\,,\,\beta_1\ldots\beta_j}(\Delta)}{m_j^2-\Delta^2}J^{(2)}_{\beta_1\ldots\beta_j}(p_2,\Delta)\,,$$

$$s \equiv (p_1+p_2)^2\,,\qquad t \equiv \Delta^2\,,\qquad m_j^2 \equiv M_j^2 - iM_j\Gamma_j$$

Reggeization (the Sommerfeld-Watson transform)

$$\sum_j \to \oint dj + O(s^{-1/2}) \to j = \alpha_{\rm R}(t):$$

$$J^{(1)}_{\alpha_1\dots\alpha_j}(p_1,\Delta)D^{\alpha_1\dots\alpha_j\,,\,\beta_1\dots\beta_j}(\Delta)J^{(2)}_{\beta_1\dots\beta_j}(p_2,\Delta)\to$$

$$\to g^{(1)}_{\rm R}(t)\,g^{(2)}_{\rm R}(t)\left(\frac{s}{2}\right)^{\alpha_{\rm R}(t)}\left[1+O\left(\frac{1}{s}\right)\right]\quad,$$

$$\frac{1}{m_j^2-t}\to\frac{d\alpha_{\rm R}(t)}{dt}\quad,\qquad (1\pm e^{-i\pi j})\to\pi\frac{\mp 1-e^{-i\pi\alpha_{\rm R}(t)}}{\sin(\pi\alpha_{\rm R}(t))}\quad,$$

$$\sum_{m_j}\to\sum_{\rm R}$$

Reggeon-exchange contributions to the Born amplitude

$$\delta^+(s,t) = \sum_{\mathrm{R}_+}\left(i + \tan\frac{\pi(\alpha_{\mathrm{R}_+}(t)-1)}{2}\right)\pi\alpha'_{\mathrm{R}_+}(t)\times$$

$$\times\, g^{(1)}_{\mathrm{R}_+}(t) g^{(2)}_{\mathrm{R}_+}(t)\left(\frac{s}{2}\right)^{\alpha_{\mathrm{R}_+}(t)}\left[1 + O\left(\frac{1}{s}\right)\right],$$

$$\delta^-(s,t) = \sum_{\mathrm{R}_-}\left(i - \cot\frac{\pi(\alpha_{\mathrm{R}_-}(t)-1)}{2}\right)\pi\alpha'_{\mathrm{R}_-}(t)\times$$

$$\times\, g^{(1)}_{\mathrm{R}_-}(t) g^{(2)}_{\mathrm{R}_-}(t)\left(\frac{s}{2}\right)^{\alpha_{\mathrm{R}_-}(t)}\left[1 + O\left(\frac{1}{s}\right)\right]$$

High-energy scattering of nucleons

The eikonal representation for the elastic scattering amplitude:

$$T(s,t) = 4\pi s \int_0^\infty db^2 J_0(b\sqrt{-t}) \frac{e^{2i\delta(s,b)} - 1}{2i} \quad \left(\frac{d\sigma}{dt} = \frac{|T(s,t)|^2}{16\pi s^2} \right)$$

$$\delta(s,b) = \frac{1}{16\pi s} \int_0^\infty d(-t) J_0(b\sqrt{-t}) \delta(s,t)$$

The Pomeron-exchange approximation for the Born amplitude:

$$\delta_{\rm P}(s,t) = \left(i + \tan\frac{\pi(\alpha_{\rm P}(t) - 1)}{2} \right) \pi\alpha'_{\rm P}(t)\, g^2_{\rm P}(s_0,t) \left(\frac{s}{2s_0} \right)^{\alpha_{\rm P}(t)}$$

$$s_0 = 1 \text{ GeV}^2$$

A QCD-inspired restriction on $\alpha_{\rm P}(t)$

1. If Pomeron does not contain valence quarks, then, in the limit of high momentum transfers, the exchange by Pomeron turns into the exchange by 2 gluons, due to asymptotic freedom.

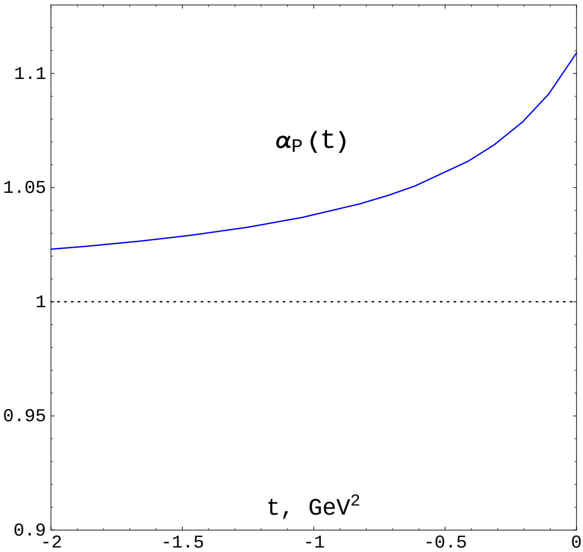
2. H. Cheng and T.T. Wu, Phys. Rev. Lett. **22** (1969) 666:

   2-photon (2-gluon) exchange Born amplitudes behave as $T^{\rm Born}(s,t)\sim s\,. \quad \Rightarrow \quad \lim_{t\to-\infty}\alpha_{\rm P}(t)=1\,.$

3. Hence, an extra assumption $\{\alpha'_{\rm P}(t)>0$ at $t<0\}$ leads to

$$\alpha_{\rm P}(t)>1 \;\; {\rm at} \;\; t<0\,.$$

Expected behavior of the Pomeron Regge trajectory



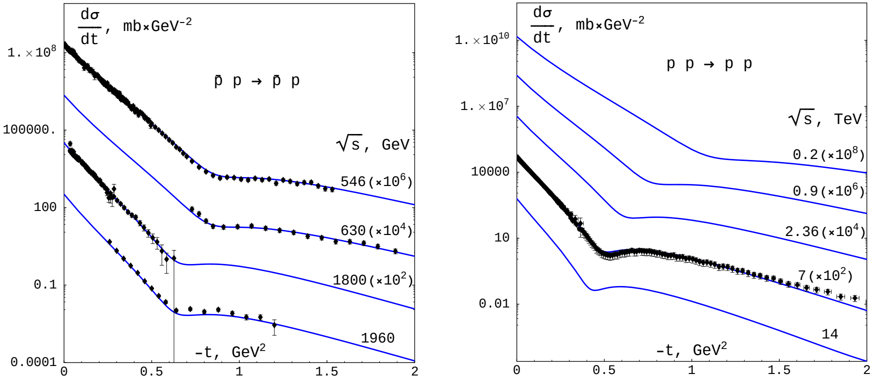
Test parametrizations for $\alpha_{\mathrm{P}}(t)$ and $g_{\mathrm{P}}(t)$

$$\alpha_{\mathrm{P}}(t) = 1 + \frac{\alpha_{\mathrm{P}}(0) - 1}{1 - \frac{t}{\tau_a}} \qquad\qquad g_{\mathrm{P}}(t) = \frac{g_{\mathrm{P}}(0)}{(1 - a_g t)^2}$$

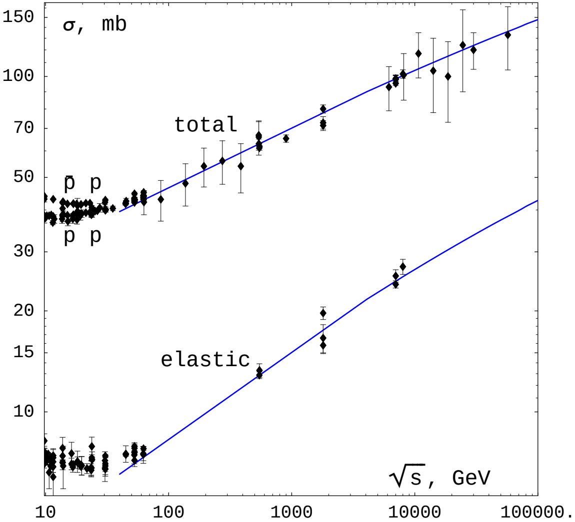
The parameter values obtained via fitting to the high-energy differential cross-section data:

Parameter	Value
$\alpha_{\mathrm{P}}(0) - 1$	0.109 ± 0.017
τ_a	(0.535 ± 0.057) GeV^2
$g_{\mathrm{P}}(0)$	(13.8 ± 2.3) GeV
a_g	(0.23 ± 0.07) GeV^{-2}

Differential cross-sections



Total and elastic cross-sections



Comparison of the characteristic scales

The effective transverse (pomeronic) radius of nucleon:

$$\sqrt{8a_g} \sim 0.2 \div 0.3 \text{ fm} \ll \sqrt{2B} \approx 1.3 \text{ fm (at the LHC)}\,.$$

The transverse size of the diffractive interaction region:

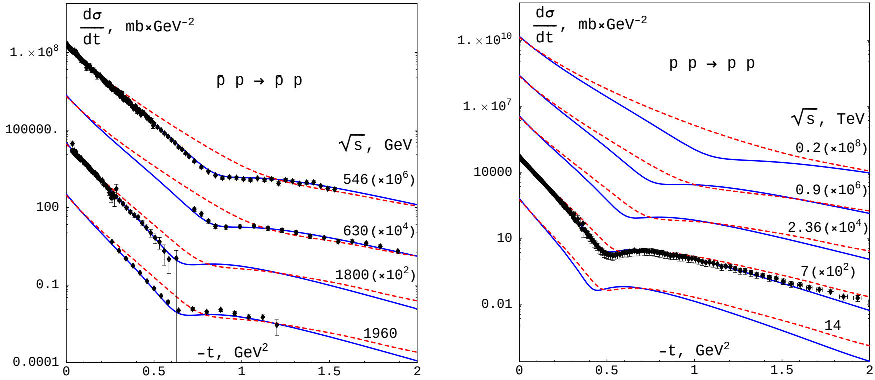
$$2B \approx \frac{\int db^2\, b^2\, \mathrm{Im}\, T(s,b)}{\int db^2\, \mathrm{Im}\, T(s,b)}\,.$$

The radius of Pomeron (P.D.B. Collins (1977)):

$$\langle R_{\mathrm{P}}^2 \rangle \;\sim\; \left\langle \frac{1}{R_{\mathrm{P}}^2} \right\rangle^{-2} = \alpha'_{\mathrm{P}}(t)(2\alpha_{\mathrm{P}}(t)+1) \sim [0.1 \div 0.2 \text{ fm}]^2\,.$$

The transverse charge radius of proton: $\sqrt{2/3}R_E^{(p)} \approx 0.7$ fm .

The nucleon size influence on the diffractive pattern



The dashed lines correspond to the value $a_g = 0$ instead of $a_g = 0.23\ \text{GeV}^{-2}$.

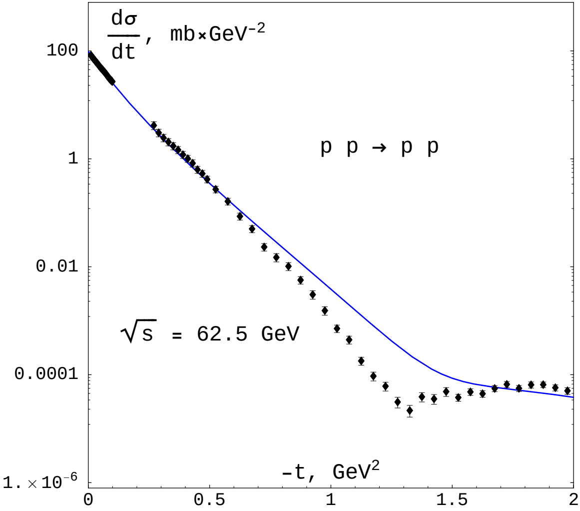
Conclusion

- It seems that Pomeron interacts with a very small region inside proton, comparable with the size of Pomeron itself.
  As a consequence, at ultrahigh energies the diffractive pattern of the nucleon-nucleon elastic scattering rather weakly depends on the nucleon shape.

Thank you for attention!

Backup slides

The influence of secondaries



The quality of description of the angular distribution data

$\sqrt{s}$, GeV	**Number of points**	χ^2
546 ($\bar{p}\,p$; UA1, UA4, CDF)	231	253
630 ($\bar{p}\,p$; UA4)	17	11
1800 ($\bar{p}\,p$; E710)	51	16
7000 ($p\,p$; TOTEM, ATLAS)	201	188
Total	500	468

Predictions for the pp observables

$\sqrt{s}$, GeV	σ_{tot}, mb	σ_{el}, mb	B, GeV^{-2}
62.5	43.0 ± 4.4	7.4 ± 1.2	14.5 ± 0.8
200	53.3 ± 3.8	10.2 ± 1.2	16.0 ± 0.8
546	63.8 ± 3.3	13.2 ± 1.0	17.3 ± 0.9
1800	78.5 ± 3.4	17.8 ± 1.0	19.1 ± 1.0
7000	98.4 ± 5.4	24.5 ± 1.8	21.4 ± 1.1
8000	100.5 ± 5.7	25.3 ± 1.9	21.6 ± 1.1
13000	108.6 ± 6.9	28.2 ± 2.5	22.5 ± 1.2
14000	109.9 ± 7.1	28.6 ± 2.5	22.6 ± 1.2
32000	124.9 ± 9.7	34.1 ± 3.7	24.2 ± 1.4
100000	148.0 ± 14.1	42.8 ± 5.6	26.6 ± 1.7

The quality of description of the data not included into the fitting procedure

$\sqrt{s}$, GeV	**Number of points**	χ^2
1960 ($\bar{p}\,p$; D0)	17	55
1960 ($\bar{p}\,p$; D0, multiplied by 0.92)	17	29
1800 ($\bar{p}\,p$; CDF)	26	178
1800 ($\bar{p}\,p$; CDF, multiplied by 0.88)	26	45
8000 ($p\,p$; TOTEM)	30	175
8000 ($p\,p$; TOTEM, multiplied by 0.958)	30	73
8000 ($p\,p$; TOTEM, multiplied by 0.9)	30	18

References

A.A. Godizov, Eur. Phys. J. C **75** (2015) 224

A.A. Godizov, Phys. Lett. B **735** (2014) 57