Hadron structure-Highlights from lattice QCD

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DESY

WE - Heraeus Physics School

Diffractive and electromagnetic processes at high energies

Bad Honnef, August 17 - 21, 2015

Outline

1. **Introduction**
   - Computer and algorithmic developments
   - Hadron spectrum
   - Hyperons and Charmed baryons
2. **Excited states, resonances**
   - ρ-meson width
   - Scalar mesons
3. **Nucleon structure**
   - Scalar, axial and tensor charges $\rightarrow$ axial charges of hyperons
   - Electromagnetic form factors
   - Generalized Parton Distributions
   - ...
4. **Conclusions**

Introduction

Quantum ChromoDynamics (QCD)

QCD-Gauge theory of the strong interaction
Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f\right) \psi_f$$

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2} A^a_\mu$$


Harald Fritzsch


Murray Gell-Mann


Heinrich Leutwyler

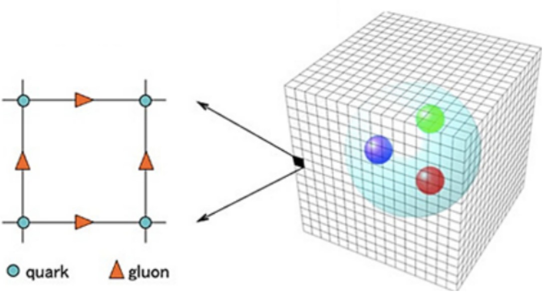
This “simple” Lagrangian produces the amazingly rich structure of strongly interacting matter in the universe.

Numerical simulation of QCD provides essential input for a wide class of complex strong interaction phenomena
→ In this talk: Highlights in hadron structure related to topics discussed in this meeting

Introduction of QCD on the lattice

QCD Lagrangian: formulated in terms of quarks and gluons

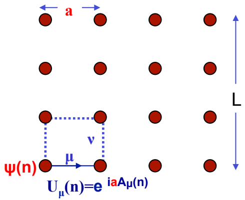
$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left[i\gamma^\mu \left(\partial_\mu - igA_\mu \right) - m_f \right] \psi_f$$



- Gauge invariant discretization of QCD on a space-time lattice
- Finite lattice spacing a provides an ultraviolet cutoff at π/a
- Lattice provides a non-perturbative regularization
  $\rightarrow$ "lattice regularization" well suited for an asymptotically free theory like QCD
- Theory described by a discrete action S: $S = S_G + S_F$ where $S_F = \sum_x \bar{\psi}(x) D \psi(x)$
  $\longrightarrow$ fermions can be integrated out of the path integral to yield $\det(D[U])$

Introduction of QCD on the lattice

Lattice QCD Lagrangian: formulated in terms of quarks and gluons



- For numerical evaluation:
  - Finite box $L^3 \times T$
  - Fermion degrees of freedom integrated out
  - Rotate into imaginary time - most drastic operation
- Path integral over gauge fields:

  Partition function: $Z = \int \mathcal{D}U_\mu(x) \prod_f det(D_f[U])\, e^{-S_G[U]}$
  - Monte Carlo simulation to produce a representative ensemble of $\{U_\mu(x)\}$ using the largest supercomputers
  - Computation of observables: $\langle \mathcal{O} \rangle = \sum_{\{U_\mu\}} O(D_f^{-1}, U_\mu)$ need inverse of Dirac matrix, typically of $10^8 \times 10^8$ dimensions

COURTESY: FORSCHUNGSZENTRUM JÜLICH

5.0 Pflop/s (10^{15} flop/s), second biggest in Europe

Systematic uncertainties

- Finite lattice spacing a - take the continuum limit $a \to 0$
- Finite volume L - take infinite volume limit $L \to \infty$
- Identification of hadron state of interest - depends on observable
- Simulation at physical quark masses - now feasible
- Inclusion of quark loop contributions - now feasible

Fermion actions

Observables: $\langle \mathcal{O} \rangle = \sum_{\{U_\mu\}} O(D_f^{-1}, U_\mu)$

Several $\mathcal{O}(a)$-improved fermion actions, K. Jansen, Lattice 2008
$\langle O \rangle_{\rm cont} = \langle O \rangle_{\rm latt} + \mathcal{O}(a^2)$

Action	Advantages	Disadvantages
Clover improved Wilson	computationally fast	breaks chiral symmetry needs operator improvement
Twisted mass (TM)	computationally fast automatic improvement	breaks chiral symmetry violation of isospin
Staggered	computational fast	four doublers (fourth root issue) complicated contractions
Domain wall (DW)	improved chiral symmetry	computationally demanding needs tuning
Overlap	exact chiral symmetry	computationally expensive

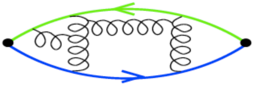
Several collaborations:

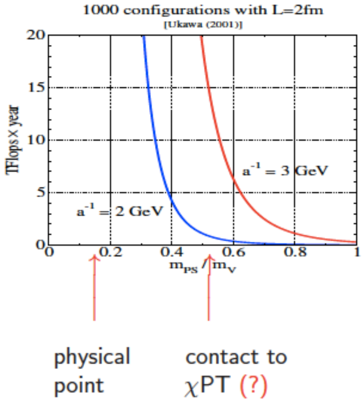
Clover	QCDSF, BMW, ALPHA, CLS, PACS-CS, NPQCD
Twisted mass	ETMC
Staggered	MILC
Domain wall	RBC-UKQCD, JLQCD
Overlap	JLQCD

Cost of simulations

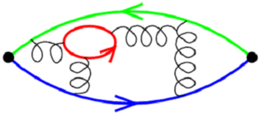
Main cost due to fermions

Cost of dynamical quark simulations in 2001





Before the 21st century: neglect pair creation (quenched QCD)



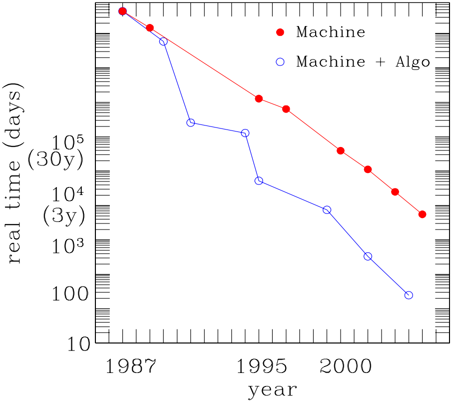
21st century: Dynamical quark simulations

Simulation cost: $C_{\rm sim} = C \left(\frac{300\text{MeV}}{m_\pi}\right)^{c_m} \left(\frac{L}{3\text{fm}}\right)^{c_L} \left(\frac{0.1\text{fm}}{a}\right)^{c_a}$

For $N_f = 2$ Wilson fermion simulations in 2001: Number of inversions, step size in molecular dynamics and autocorrelations $\sim \frac{1}{m_q}\frac{1}{m_q}\frac{1}{m_q} \rightarrow (m_\rho/m_\pi)^6$ scaling.

Computer and algorithmic development

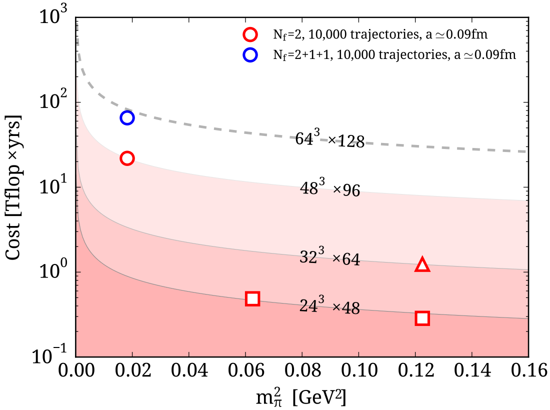
Algorithm development has been decisive



Simulation on a $32^3 \times 64$ lattice, 5000 configurations

ETMC simulations with physical quark masses

European Twisted Mass Collaboration (ETMC): $N_f = 2$ and $N_f = 2+1+1$ twisted mass Wilson fermions



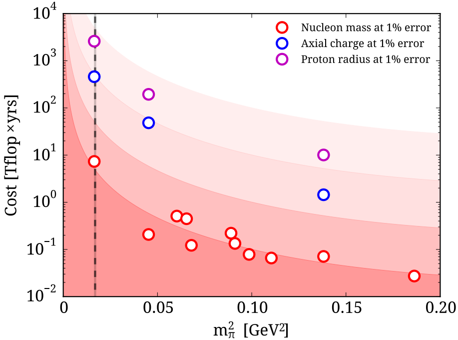
Simulation cost: $C_{\rm sim} \propto \left(\frac{300\rm MeV}{m_\pi}\right)^{c_m} \left(\frac{L}{3\rm fm}\right)^{c_L} \left(\frac{0.1\rm fm}{a}\right)^{c_a}$

We find $c_L \sim 4.5$ and $c_m \sim 2$ for a fixed lattice spacing.

A. Abdel-Rehim *et al.* (ETMC), arXiv:1507.05068

Observables at physical quark mass

European Twisted Mass Collaboration (ETMC): $N_f = 2$ and $N_f = 2+1+1$ twisted mass Wilson fermions



Inversion cost (for a lattice of $64^3 x128$):$\sim e^{(m_p - \frac{3}{2} m_\pi) t_s}$
Methods to reduce further the statistical error are being developed

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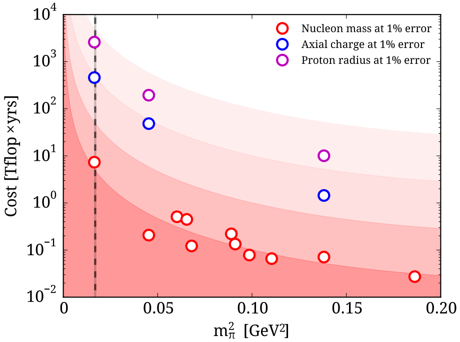


Inversion cost (for a lattice of $64^3 x128$):$\sim\ e^{(m_p - \frac{3}{2} m_\pi) t_s}$
Methods to reduce further the statistical error are being developed

Exascale computing is needed for lattice QCD

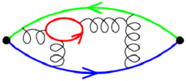
Hadron mass

First goal: reproduce the low-lying masses

- Use Euclidean correlation functions:

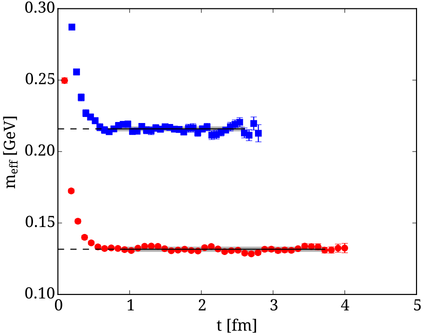
$$G(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s\cdot\vec{q}} \langle J(\vec{x}_s, t_s) J^\dagger(0)\rangle$$

$$= \sum_{n=0,\cdots,\infty} A_n e^{-E_n(\vec{q})t_s} \overset{t_s\to\infty}{\longrightarrow} A_0 e^{-E_0(\vec{q})t_s}$$



Interpolating field with the quantum numbers of π^+: $J(x) = \bar{d}(x)\gamma_5 u(x)$

- Large Euclidean time evolution gives ground state for given quantum numbers $\Longrightarrow$ enables determination of low-lying hadron properties
- $aE_{\rm eff}(\vec{q}, t_s) = \ln\left[G(\vec{q}, t_s)/G(\vec{q}, t_s + a)\right]$
  $= aE_0(\vec{q}) + \text{excited}\rangle\text{states}$
  $\to aE_0(\vec{q}) \overset{\vec{q}=0}{\to} am$

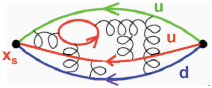


$N_f = 2+1+1$ TM fermions at $m_\pi = 210$ MeV
$N_f = 2$ TM plus clover fermions at physical pion mass

Hadron mass

First goal: reproduce the low-lying masses

- Use Euclidean correlation functions:



$$G(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s\cdot\vec{q}} \langle J(\vec{x}_s, t_s) J^\dagger(0)\rangle$$
$$= \sum_{n=0,\cdots,\infty} A_n e^{-E_n(\vec{q})t_s} \xrightarrow{t_s\to\infty} A_0 e^{-E_0(\vec{q})t_s}$$

Interpolating field with the quantum numbers of p: $J(x) = \epsilon^{abc}\left(u^{a\top}(x) C\gamma_5 d^b(x)\right) u^c(x)$

- Large Euclidean time evolution gives ground state for given quantum numbers $\Longrightarrow$ enables determination of low-lying hadron properties
- $aE_{\rm eff}(\vec{q}, t_s) = \ln\left[G(\vec{q}, t_s)/G(\vec{q}, t_s + a)\right]$
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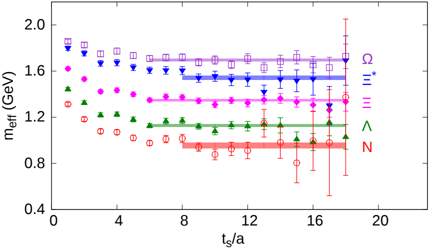


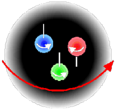
$N_f = 2$ TM plus clover fermions at physical pion mass

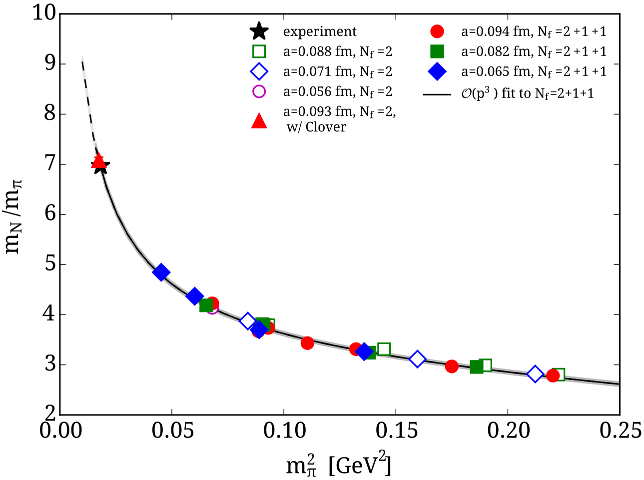
Noise to signal increases with t_s:$\sim e^{(m_h - \frac{3}{2} m_\pi) t_s}$

Simulations with physical quark masses

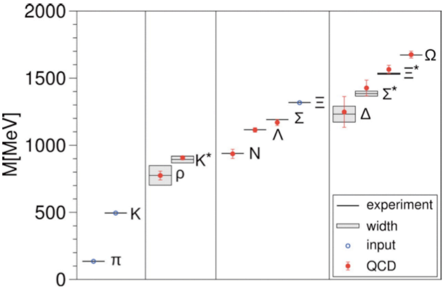
A number of collaborations are producing simulations with physical values of the quark mass

European Twisted Mass Collaboration (ETMC): The nucleon, A. Abdel-Rehim *et al.* (ETMC) arXiv:1507.04936





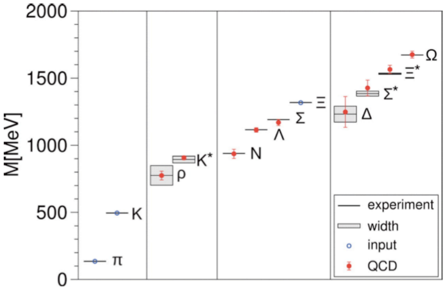
Hadron spectrum



$N_f = 2 + 1$ Clover, BMW, Science 322 (2008)

Milestone calculation for lattice QCD $\rightarrow$ agreement with experiment is a success for QCD & LQCD

Hadron spectrum



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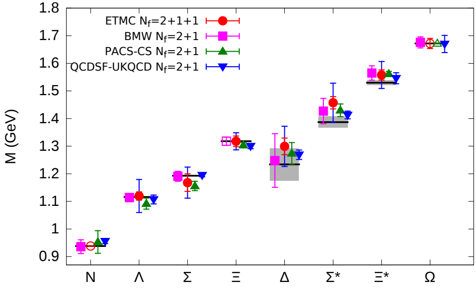
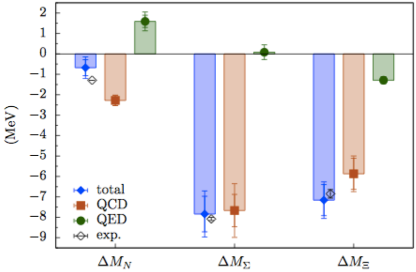


Several collaborations producing the hadron spectrum

Milestone calculation for lattice QCD $\rightarrow$ agreement with experiment is a success for QCD & LQCD

Isospin and electromagnetic mass splitting

RBC and BMW collaborations: Treat isospin and electromagnetic effects to LO

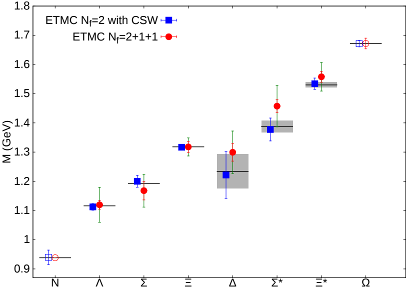


Baryon spectrum with mass splitting from BMW

- Nucleon mass: isospin and electromagnetic effects with opposite signs
- Physical splitting reproduced

Hyperons and Charmed baryons

- Spectrum using $N_f = 2 + 1 + 1$ for a range of pion masses from about 450 MeV to 210 MeV, 3 lattice spacings and different volumes
- Spectrum using an $N_f = 2$ ensemble with physical pion mass



- Continuum extrapolation using three lattice spacings, $a = 0.094$ fm, 0.082 fm and 0.065 fm
- Volume dependence - no observable effects within our statistics
- Chiral extrapolation - biggest systematic error
- Strange quark mass fixed using the Ω^- mass (open symbols)
- The lattice spacing was fixed using the nucleon mass (open symbols)

C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis, G. Koutsou, Phys.Rev. D90 (2014) 7, 074501; C. Alexandrou *et al.* (ETMC) to appear

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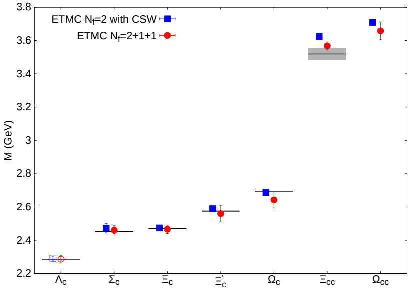
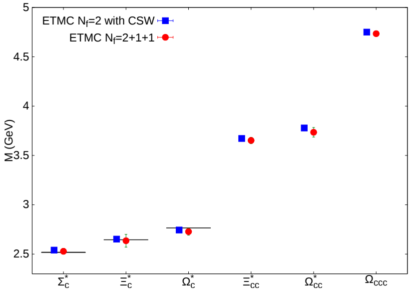


Spin-1/2 charmed baryons



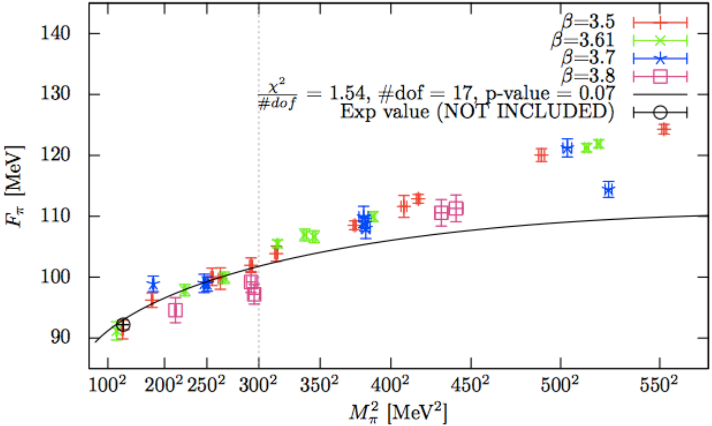
Spin-3/2 charmed baryons

- The charm quark mass was fixed using the Λ_c mass (open symbols)

C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis, G. Koutsou, Phys.Rev. D90 (2014) 7, 074501; C. Alexandrou *et al.* (ETMC) to appear

Pion decay constant by BMW

Budapest-Marseille-Wuppertal (BMW) Collaboration: $N_f = 2 + 1$ Clover improved Wilson fermions



NLO SU(2) chiral perturbation theory for $m_\pi < 300$ MeV, S. Durr *et al.*, 1310.3626

Excited states, resonances & exotics

Variational approach: Enlarge basis of interpolating fields $\to$ correlation matrix

$G_{jk}(\vec{q}, t_s) = \sum_{\vec{x}_s} e^{-i\vec{x}_s \cdot \vec{q}} \langle J_j(\vec{x}_s, t_s) J_k^\dagger(0) \rangle \rangle, j, k = 1, \ldots N$

Solve the generalized eigenvalue problem (GEVP)

$G(t)v_n(t; t_0) = \lambda_n(t; t_0)G(t_0)v_n(t; t_0) \to \lambda_n(t; t_0) = e^{-E_n(t-t_0)}$ yields N lowest eigenstates, M. Lüscher & U. Wolff (1990)

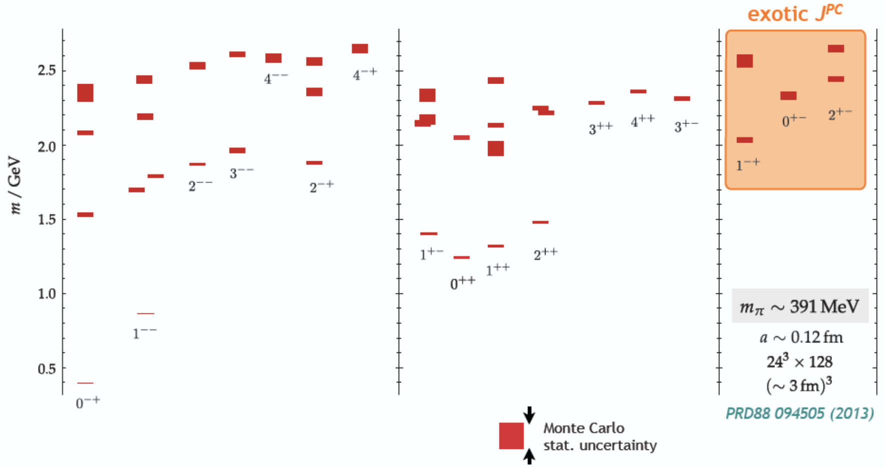
Large effort to construct the appropriate basis using lattice symmetries, Hadron Spectrum Collaboration

- must extract all states lying below the state of interest
- as $m_\pi \to m_{\rm physical}$ need to consider multi-hadron states
- must include disconnected diagrams
- most excited states are unstable (resonances)

Meson spectrum

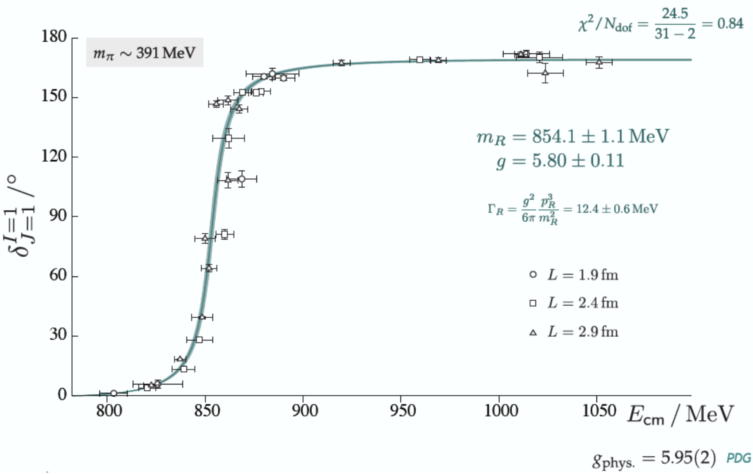
JLab analysis using anisotropic lattice at $m_\pi \sim 391$ MeV, J. Dudek, Presentation at St. Goar, March 2015

- meson spectrum for a range of J^{PC}



ρ-meson width

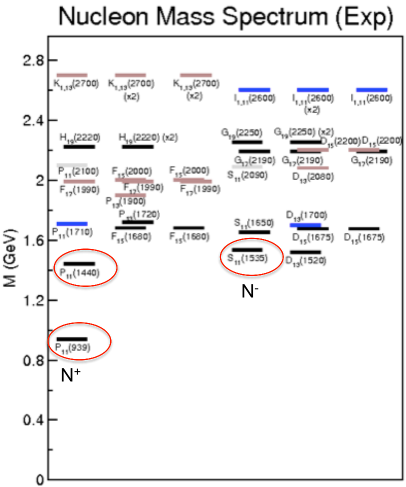
- Consider $\pi^+\pi^-$ in the $I = 1$-channel
- Estimate P-wave scattering phase shift $\delta_{11}(k)$ using finite size methods
- Use Lüscher's relation between energy in a finite box and the phase in infinite volume
- Use Center of Mass frame and Moving frame



Impressive results using $N_f = 2 + 1$ clover fermions and 3 asymmetric lattices for pion mass about 400 MeV,

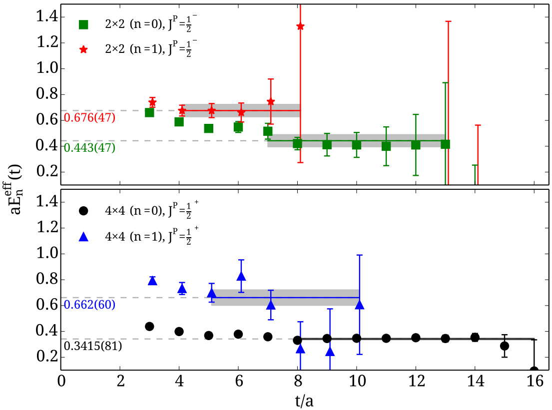
J. J. Dudek, R. G. Edwards and C.E. Thomas, Phys. Rev. D 87 (2013) 034505

Low-lying nucleon resonances



- R.G. Edwards, J. J. Dudek, D. G. Richards, S. J. Wallace. Phys. Rev. D 84 (2011) 074508
- M. Mahbub *et al.*, Phys. Lett. B679 (2009) 418
- G. P. Engel, C. Lang, D. Mohler, A. Schäfer, 1301.4318

Low-lying nucleon resonances



$N_f = 2$ Clover fermions, $m_\pi = 156$ MeV - configurations provided by QCDSF, C.A., T. Korzec, G. Koutsou, T. Leontiou, arXiv:1302.4410

- R.G. Edwards, J. J. Dudek, D. G. Richards, S. J. Wallace. Phys. Rev. D 84 (2011) 074508
- M. Mahbub *et al.*, Phys. Lett. B679 (2009) 418
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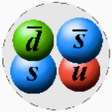
Scalar mesons

Start with $a_0(980)$ and $\kappa(800)$

In our study: 4(+2) operators with the quantum numbers of $a_0(980)$.

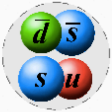
$$\mathcal{O}^{q\bar{q}} = \sum_{\mathbf{x}} \left(\bar{d}_{\mathbf{x}} u_{\mathbf{x}}\right)$$

$$\mathcal{O}^{K\bar{K},\,\text{point}} = \sum_{\mathbf{x}} \left(\bar{s}_{\mathbf{x}}\gamma_5 u_{\mathbf{x}}\right)\left(\bar{d}_{\mathbf{x}}\gamma_5 s_{\mathbf{x}}\right)$$

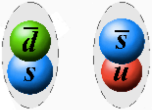


$$\mathcal{O}^{\eta_s\pi,\,\text{point}} = \sum_{\mathbf{x}} \left(\bar{s}_{\mathbf{x}}\gamma_5 s_{\mathbf{x}}\right)\left(\bar{d}_{\mathbf{x}}\gamma_5 u_{\mathbf{x}}\right)$$

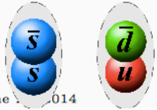
$$\mathcal{O}^{Q\bar{Q}} = \sum_{\mathbf{x}} \epsilon_{abc}\left(\bar{s}_{\mathbf{x},b}(C\gamma_5)\bar{d}^{T}_{\mathbf{x},c}\right)\epsilon_{ade}\left(u^{T}_{\mathbf{x},d}(C\gamma_5)s_{\mathbf{x},e}\right)$$



$$\mathcal{O}^{K\bar{K},\,\text{2-part}} = \sum_{\mathbf{x},\mathbf{y}} \left(\bar{s}_{\mathbf{x}}\gamma_5 u_{\mathbf{x}}\right)\left(\bar{d}_{\mathbf{y}}\gamma_5 s_{\mathbf{y}}\right)$$



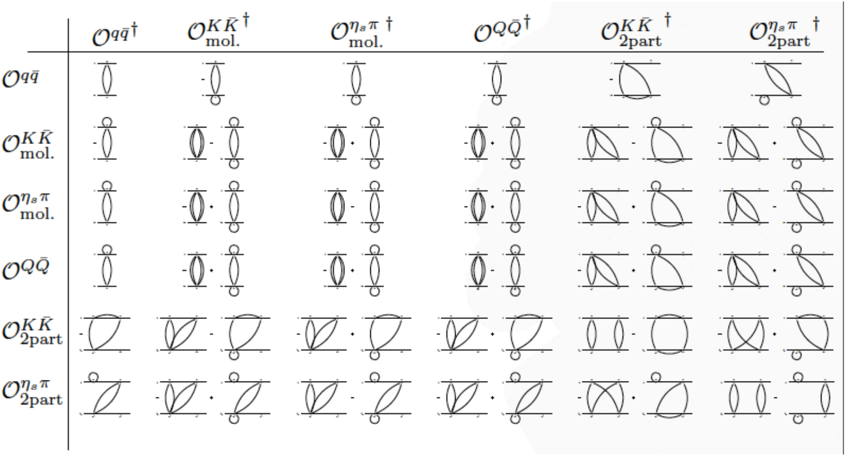
$$\mathcal{O}^{\eta_s\pi,\,\text{2-part}} = \sum_{\mathbf{x},\mathbf{y}} \left(\bar{s}_{\mathbf{x}}\gamma_5 s_{\mathbf{x}}\right)\left(\bar{d}_{\mathbf{y}}\gamma_5 u_{\mathbf{y}}\right)$$



Investigation of the tetraquark candidate $a_0(980)$: technical aspects - Joshua Berlin, June 2014

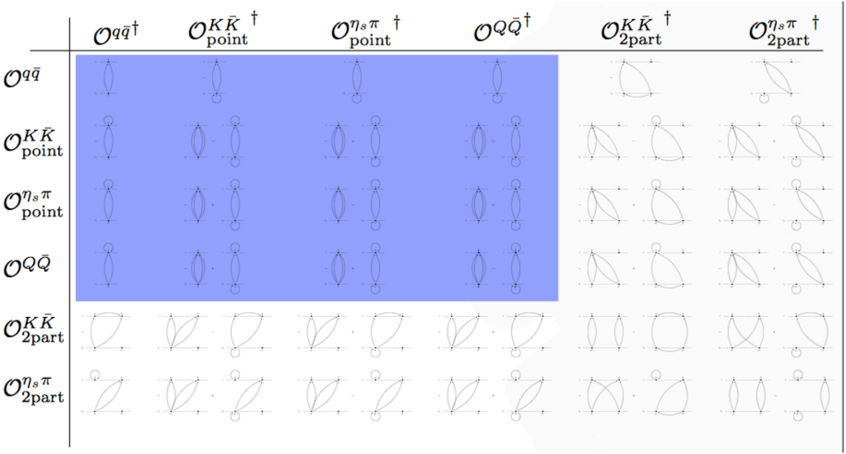
Scalar mesons

Start with $a_0(980)$ and $\kappa(800)$



Scalar mesons

Start with $a_0(980)$ and $\kappa(800)$



Preliminary results show weak overlap with a $q\bar{q}$ state

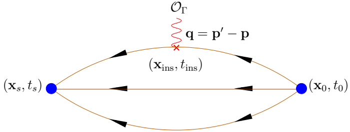
Need to compute disconnected diagrams - very challenging!

Nucleon charges

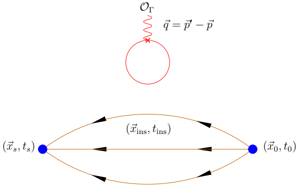
Evaluation of matrix elements

Evaluation of three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s)\mathcal{O}^{\mu\nu}_\Gamma(\vec{x}_{\rm ins}, t_{\rm ins})\bar{J}_\beta(\vec{x}_0, t_0)\rangle$$



connected contribution



disconnected contribution

Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

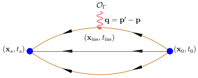
$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins} - t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$

- $\mathcal{M}$ the desired matrix element
- $t_s, t_{\rm ins}, t_0$ the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

Evaluation of matrix elements

Evaluation of three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s)\mathcal{O}^{\mu\nu}_\Gamma(\vec{x}_{\rm ins}, t_{\rm ins})\bar{J}_\beta(\vec{x}_0, t_0)\rangle$$



connected contribution

Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins} - t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$

- $\mathcal{M}$ the desired matrix element
- t_s, $t_{\rm ins}$, t_0 the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

Summing over $t_{\rm ins}$:

$$\sum_{t_{\rm ins}=t_0}^{t_s} R(t_s, t_{\rm ins}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s - t_0)})].$$

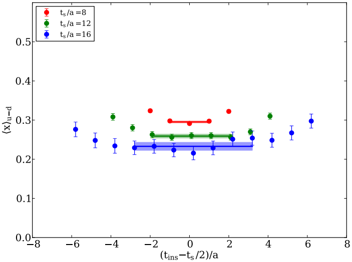
So the excited state contributions are suppressed by exponentials decaying with $t_s - t_0$, rather than $t_s - t_{\rm ins}$ and/or $t_{\rm ins} - t_0$.

However, one needs to fit the slope rather than to a constant

Connect lattice results to measurements:

$\mathcal{O}_{\overline{\rm MS}}(\mu) = Z(\mu, a)\mathcal{O}_{\rm latt}(a)$

$\Longrightarrow$ evaluate $Z(\mu, a)$ non-perturbatively



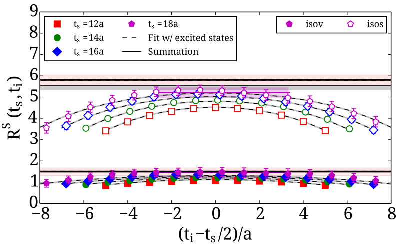
Nucleon charges and axial charge of hyperons

- axial-vector operator: $\mathcal{O}_A^a = \bar{\psi}(x)\gamma^\mu\gamma_5 \frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}_T^a = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$
- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$

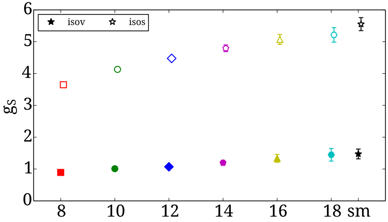
$\Longrightarrow \langle N(\vec{p}')\mathcal{O}_\Gamma N(\vec{p})\rangle|_{q^2=0}$ yields $g_s,\ g_A,\ g_T$

Scalar charge

- High statistics analysis with $N_f = 2+1+1$ TMF, $a = 0.082$ fm, $m_\pi = 373$ MeV
- Connected part with 1200 statistics



Agreement of summation, plateau and two-states fits give confidence to the correctness of the final result



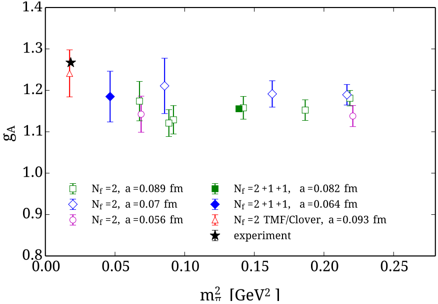
- g_A: No detectable excited states
- g_T: similar to g_A
- g_s: severe contamination from excited states

Nucleon charges: g_A, g_s, g_T

The good news:

Axial-vector FFs: $A^3_\mu = \bar\psi\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi(x) \Longrightarrow \frac{1}{2}\bar u_N(\vec{p}')\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]u_N(\vec{p})|_{q^2=0}$
$\rightarrow$ yields $G_A(0) \equiv g_A$: i) well known experimentally, & ii) no quark loop contributions

Isovector



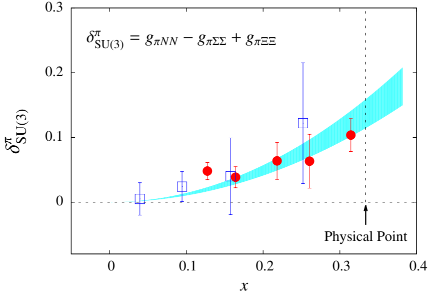
A. Abdel-Rehim *et al.* (ETMC) arXiv:1507.04936

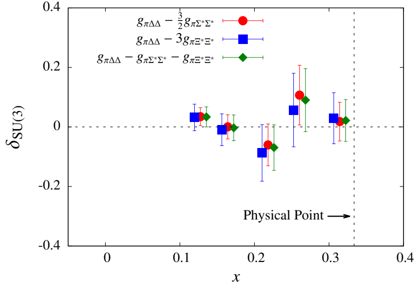
- g_A at the physical point using $\sim$ 1500 measurements indicates agreement with the physical value $\rightarrow$ important to reduce error
- many results from other collaborations, e.g.
  - $N_f = 2+1$ Clover, J. R. Green *et al.*, arXiv:1209.1687
  - $N_f = 2$ Clover, R.Hosley *et al.*, arXiv:1302.2233
  - $N_f = 2$ Clover, S. Capitani *et al.* arXiv:1205.0180
  - $N_f = 2+1$ Clover, B. J. Owen *et al.*, arXiv:1212.4668
  - $N_f = 2+1+1$ Mixed action (HISQ/Clover), T. Bhattacharya *et al.*, arXiv:1306.5435

Hyperon axial charges

- Hyperon axial charges: $g_{\Lambda\Sigma} \sim 0.60$, $g_{\Sigma\Sigma}$, $g_{\Xi\Xi}$ not known experimentally
- Calculation equivalent to g_A of the nucleon: $\langle h|\bar{\psi}\gamma_\mu\gamma_5\psi|h\rangle|_{q^2=0}$ - Efficient to calculate with fixed current method
- $SU(3)$ breaking can be checked systematically

Preliminary





Also results from H.- W. Lin and K. Orginos, PRD 79, (2009)

Probe deviation:

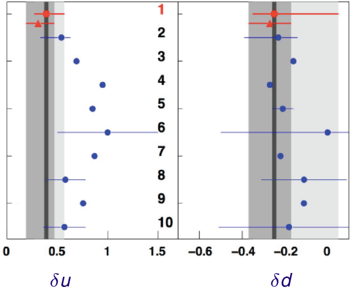
- Octet: $\delta_{SU(3)} = g_A^N - g_A^\Sigma + g_A^\Xi$ versus $x = (m_K^2 - m_\pi^2)/4\pi^2 f_\pi^2$
- Decuplet: Three relations one can check

Nucleon charges: g_S, g_T

- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- axial-vector operator: $\mathcal{O}_A^a = \bar{\psi}(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}_T^a = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

$\Longrightarrow$ extract from ratio: $\langle N(\vec{p'})\mathcal{O}_X N(\vec{p})\rangle|_{q^2=0}$ to obtain $g_s,\ g_A,\ g_T$
(i) isovector combination has no disconnect contributions; (ii) g_A well known experimentally, g_T to be measured at JLab

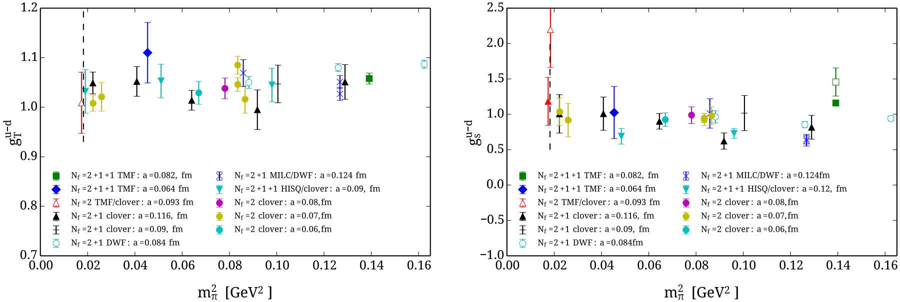
Planned experiment at JLab, SIDIS on ^{3}He/Proton at 11 GeV:



Experimental values: $\delta u = 0.39^{+0.18}_{-0.12}$ and $\delta d = -0.25^{+0.3}_{-0.1}$

Nucleon charges: g_S, g_T

Isovector

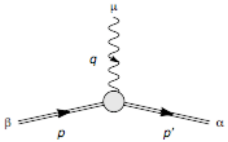


- Experimental value of $g_T \sim 0.54^{+0.30}_{-0.13}$ from global analysis of HERMES, COMPASS and Belle e^+e^- data, M. Anselmino *et al.* (2013)
- For g_s increasing the sink-source time separation to ~ 1.5 fm is crucial

Electromagnetic form factors

Electromagnetic form factors

$$\langle N(p', s')|j^\mu(0)|N(p, s)\rangle = \bar{u}_N(p', s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m} F_2(q^2)\right] u_N(p, s)$$

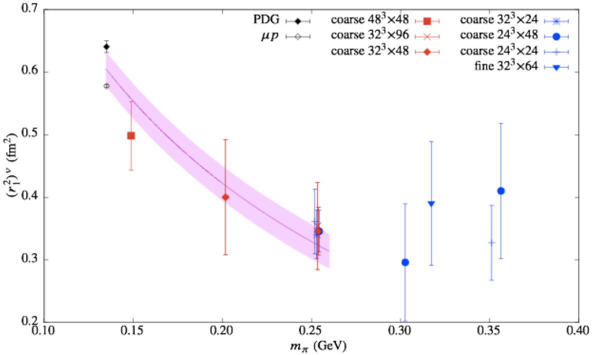


- Proton radius extracted from muonic hydrogen is 7.7 σ different from the one extracted from electron scattering, R. Pohl *et al.*, Nature 466 (2010) 213
- Muonic measurement is ten times more accurate

Dirac and Pauli radii

Dipole fits: $\frac{G_0}{(1+Q^2/M^2)^2} \Rightarrow \langle r_i^2 \rangle = -\frac{6}{F_i}\frac{dF_i}{dQ^2}\Big|_{Q^2=0} = \frac{12}{M_i^2}$

Need better accuracy at the physical point



Using results from summation method, J. M. Green *et al.*, 1404.4029

Position methods

- Avoid model dependence-fits
- Application to Sachs form factors $\rightarrow$ nucleon isovector magnetic moment $G_M^{\rm iso}(0)$
- Isovector rms charge radius of the nucleon
- Neutron electric dipole moment

As a first step we calculated $G_M(0)$ (equivalently $F_2(0)$) at $m_\pi = 373$ MeV.

C.A., G. Koutsou, K. Ottnad, M. Petschlies, PoS(Lattice2014), 144

Position methods for $G_M(0)$

- Plateau for large Euclidean times t, $t_s - t$

$$\lim_{t\to\infty}\ \lim_{t_s-t\to\infty} R^{\mu}(t_s, t, \vec{q}, \Gamma_\nu) = \Pi^{\mu}\left(\vec{q}, \Gamma_\nu\right),$$

- Extract Sachs form factors from

$$\Pi_0\left(\vec{q}, \Gamma_0\right) = -C\frac{E\left(\vec{q}\right) + m_N}{2m_N} G_E\left(Q^2\right),$$

$$\Pi_i\left(\vec{q}, \Gamma_0\right) = -C\frac{i}{2m_N} q_i G_E\left(Q^2\right),$$

$$\Pi_i\left(\vec{q}, \Gamma_k\right) = -C\frac{1}{4m_N}\epsilon_{ijk} q_j G_M\left(Q^2\right),$$

where $\Gamma_0 = \frac{1}{2}(1+\gamma_0)$, $\Gamma_k = \frac{1}{4} iS\Gamma_0\gamma_5\gamma_k$ and $C = \sqrt{\frac{2m_N^2}{E(\vec{q})(E(\vec{q})+m_N)}}$.

$\Rightarrow$ **Due to the factor q_j the magnetic moment $G_M(0)$ cannot be extracted directly!**

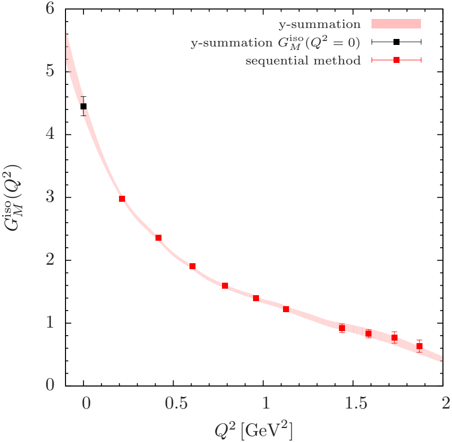
Apply a partial derivative to remove momentum dependence $\sim q_j$

$$\lim_{q^2\to 0}\frac{\partial}{\partial q_j}\Pi_i(t, \vec{q}, \Gamma_k) = \frac{1}{2m_N}\epsilon_{ijk} G_M(0).$$

... and calculate $G_M(0)$ directly.

Magnetic moment $G_M^{\rm iso}(0)$

- In principle, values at larger Q^2 have very little influence
- Value for $G_M^{\rm iso} = 4.45(15)_{\rm stat}$ larger than result from dipole fit $3.99(9)_{\rm stat}$
- Closer to exp. value (4.71)



$G_M^{\rm iso}(0)$ from $\mathcal{O}(4700)$ gauge confs of B55; $t_s/a = 14$

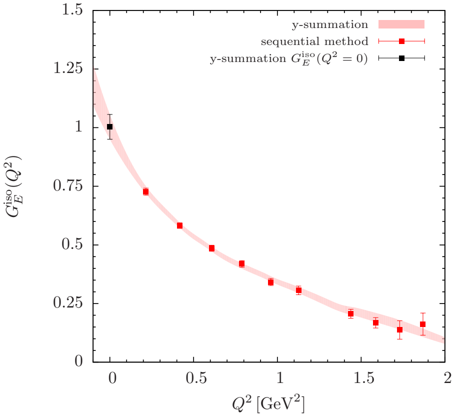
Check $G_E^{\rm iso}(0)$

- $G_E^{\rm iso}(Q^2=0)=1$ by definition
- Can use

$$\Pi_i\left(\vec{q},\Gamma_0\right) = -C\frac{i}{2m_N}q_i G_E\left(Q^2\right)\ ,$$

and apply same method as for $G_M^{\rm iso}(Q^2)$...

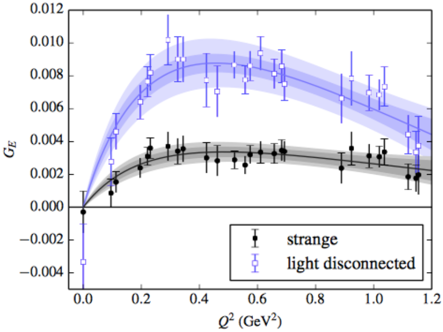
- Value at zero momentum nicely reproduced $G_E^{\rm iso}(0)=1.00(5)$.



$G_E^{\rm iso}(0)$ from $\mathcal{O}(4700)$ gauge confs of B55; $t_s/a=14$

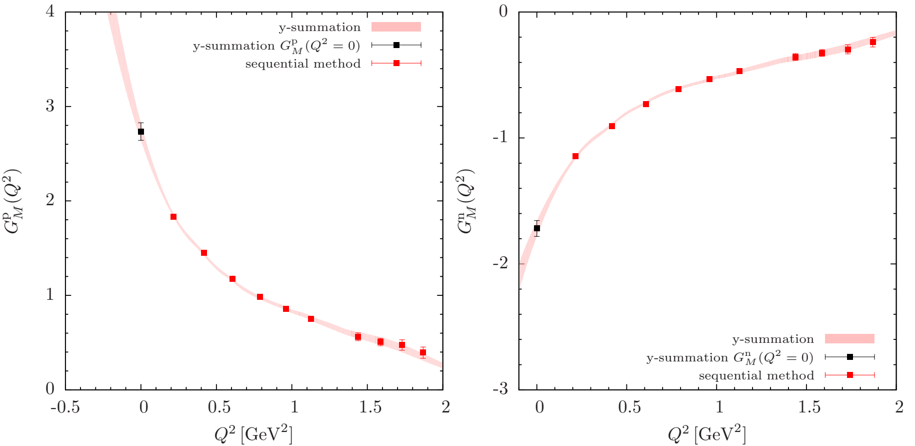
Magnetic moment for proton and neutron

Disconnected diagrams yield only small contribution



J. Green *et al.*, arXiv:1505.01803

Magnetic moment for proton and neutron



- Magnetic moment for proton (l.h.s.) and neutron (r.h.s.) on B55 (neglecting disc. contributions)
- $G_M^p(0) = 2.73(9)_{\rm stat}$ and $G_M^n(0) = -1.72(6)_{\rm stat}$ close to exp. values 2.79 and -1.91, respectively.
- Dipole fit gives again smaller values $G_M^p(0) = 2.47(6)$ and $G_M^n(0) = -1.54(4)$)

Isovector charge radius $r_{E,\mathrm{iso}}$

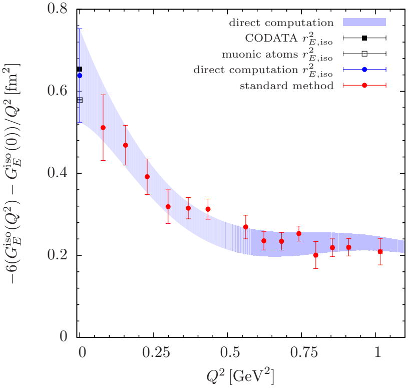
Consider isovector rms charge radius of the nucleon:

$$r^2_{E,\mathrm{iso}} = -6\frac{d}{dQ^2}G_E(Q^2)\Big|_{Q^2=0} .$$

- Start from most simple relation for $G_E(Q^2)$:

$$\Pi_0\left(\vec{q},\Gamma_0\right) = -\sqrt{\frac{E_N+m_N}{2E_N}}G_E\left(Q^2\right) .$$

Results for $r_E^{\rm iso}$



- We use an ETMC $48^3 \times 96$, $N_f = 2$ ensemble with **physical pion mass**
- Data shown in plot are for $O(1400)$ confs $\rightarrow$ very expensive!
- $t_s/a = 14$ **compatible with experiment!**
- Unfortunately errors are still not small enough to distinguish the two experimental values

Parton Distribution Functions

Generalized Parton Distributions

Factorization leads to matrix elements of local operators:

- vector operator

$$\mathcal{O}_{V^a}^{\mu_1\cdots\mu_n} = \bar{\psi}(x)\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}} \frac{\tau^a}{2}\psi(x)$$

- axial-vector operator

$$\mathcal{O}_{A^a}^{\mu_1\cdots\mu_n} = \bar{\psi}(x)\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}} \gamma_5 \frac{\tau^a}{2}\psi(x)$$

- tensor operator

$$\mathcal{O}_{T^a}^{\mu_1\cdots\mu_n} = \bar{\psi}(x)\sigma^{\{\mu_1,\mu_2} i \overleftrightarrow{D}^{\mu_3} \ldots i \overleftrightarrow{D}^{\mu_n\}} \frac{\tau^a}{2}\psi(x)$$

Special cases:

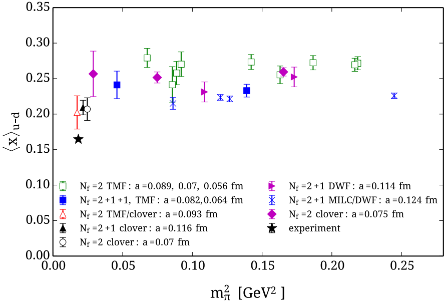
- no-derivative $\rightarrow$ nucleon form factors
- For $Q^2 = 0 \rightarrow$ parton distribution functions
  one-derivative $\rightarrow$ first moments e.g. average momentum fraction $\langle x \rangle$
  Generalized form factor decomposition:

$$\langle N(p',s')|\mathcal{O}_{V3}^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]\frac{1}{2}u_N(p,s)$$

Nucleon spin $J^q = \frac{1}{2}\left[A_{20}(0) + B_{20}(0)\right]$ and $\langle x \rangle = A_{20}(0)$

Momentum fraction and the nucleon spin

What is the distribution of the nucleon momentum among the nucleon constituents?
$\langle x \rangle$ obtained in the $\overline{MS}$ scheme at $\mu = 2$ GeV.



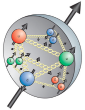
- $\langle x \rangle_{u-d}$ approach physical value for bigger source-sink separations $\rightarrow$ need an equivalent high statistics study
- Can provide a prediction for $\langle x \rangle_{\delta u - \delta d}$

Experimental values:
- $\langle x \rangle_{u-d}$ from S. Alekhin *et al.* arXiv:1202.2281

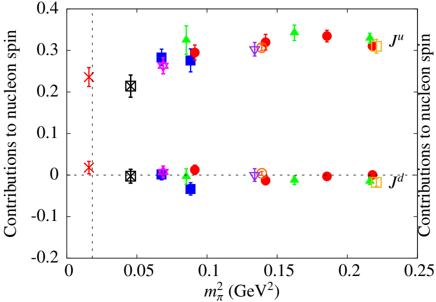
Where is the nucleon spin?

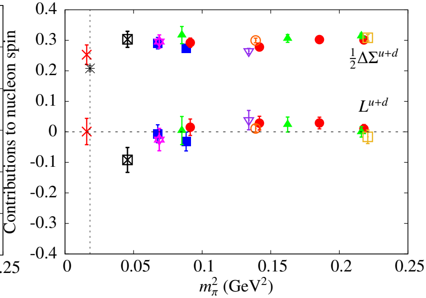
Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^G$

$J^q = A^q_{20}(0) + B^q_{20}(0)$ and $\Delta\Sigma^q = g^q_A$



For one ensemble at $m_\pi = 373$ MeV we have the disconnected contribution $\rightarrow$ we can check the effect on the observables, $\mathcal{O}(150,000)$ statistics





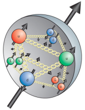
$\Longrightarrow$ Total spin for u-quarks $J^u \lesssim 0.25$ and for d-quark $J^d \sim 0$

- $L^{u+d} \sim 0$ at physical point
- $\Delta\Sigma^{u+d}$ in agreement with experimental value at physical point
- The total spin $J^{u+d} \sim 0.25$

Where is the nucleon spin?

Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^G$

$J^q = A_{20}^q(0) + B_{20}^q(0)$ and $\Delta\Sigma^q = g_A^q$



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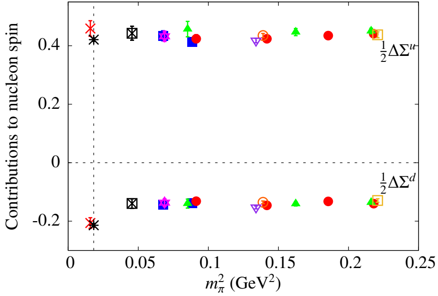
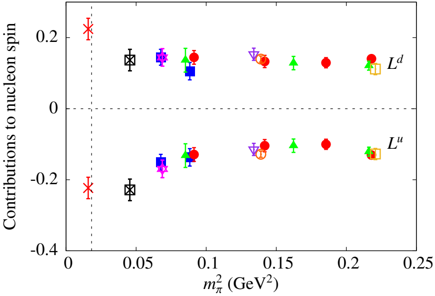




- $\Delta\Sigma^{u,d}$ consistent with experimental values
- $L^d \sim -L^u$

Parton distribution functions

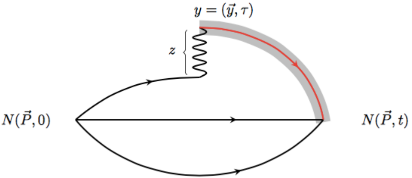
$$\tilde{a}_n(x, \Lambda, P_3) = \int_{-\infty}^{+\infty} dx\, x^{n-1}\, \tilde{q}(x, \Lambda, P_3)\rangle,$$

$$\tilde{q}(x, \Lambda, P_3) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-izxP_3} \underbrace{\langle P|\bar{\psi}(z,0)\rangle\gamma_3\, W(z)\psi(0,0)|P\rangle}_{h(P_3,z)}$$

is the quasi-distribution defined by X. Ji Phys.Rev.Lett. 110 (2013) 262002, arXiv:1305.1539

$h(P_3, z)$ can be computed in lattice QCD

- We use our test B55 ensemble
- Perform HYP-smearing on the gauge links
- Use the stochastic all-to-all propagator in the three-point function
- Extract quasi-distribution for $\frac{2\pi}{L}$, $\frac{4\pi}{L}$, $\frac{6\pi}{L}$



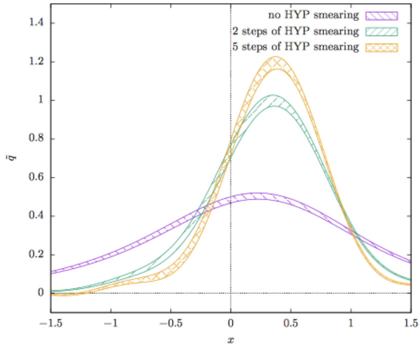
Relate the quasi-distribution to the measured PDF

Our starting point is

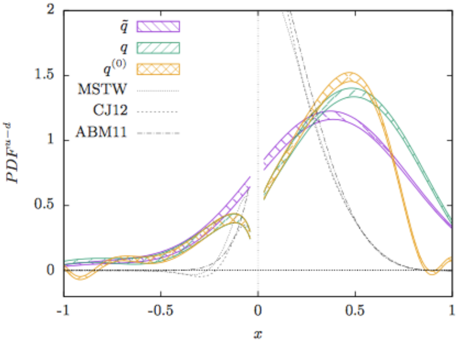
$$q(x,\mu) = \tilde{q}(x,\Lambda,P_3) - \frac{\alpha_s}{2\pi}\tilde{q}(x,\Lambda,P_3)\delta Z_F^{(1)}\left(\frac{\mu}{P_3},\frac{\Lambda}{P_3}\right) - \frac{\alpha_s}{2\pi}\int_{-1}^{1}\frac{dy}{y}Z^{(1)}\left(\frac{x}{y},\frac{\mu}{P_3},\frac{\Lambda}{P_3}\right)\tilde{q}(y,\Lambda,P_3) + \mathcal{O}(\alpha_s^2)$$

- The calculation of the leading UV divergences to $\tilde{q}$ in PT are done keeping P_3 fixed while taking $\Lambda \to \infty$ (in contrast to first taking $P_3 \to \infty$ for the renormalization of q)
- We still do not have a renormalization procedure
  $\to$ identify the UV regulator as μ for q and as Λ for the case of the quasi-distribution.
- The dependence on the UV regulator Λ will be translated, in the end, into a renormalization scale μ after proper renormalization
- Single pole terms cancel when combining the vertex and wave function corrections, and double poles are reduced to a single pole that are taken care via the principal value prescription
- A divergent term remains in $\delta Z^{(1)}$ that depends on the cut-off x_c

Preliminary results



Isovector quasi-distribution for $P_3 = 4\pi/L$



Results for 5-HYP steps

Renormalization still has to be done to remove the cut-off x_c and the remaining divergent term $\sim \ln(x_c^2 - 1)$

Conclusions

Future Perspectives

- Confirm g_A, $\langle x \rangle_{u-d}$, etc, at the physical point using $N_f = 2$ and $N_f = 2+1+1$
- Provide predictions for g_s, g_T, tensor moment, sigma-terms, etc.
- Improve the accuracy of the results on proton radius using position methods
- Use the developed methods for calculating the neutron electric dipole moment, PDFs, etc at the physical point
- Develop methods for resonances

European Twisted Mass Collaboration

European Twisted Mass Collaboration (ETMC)

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

Collaborators:
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Conference on *Electromagnetic Interactions with Nucleons and Nuclei*, 1-7 Nov. 2015

EINN2015

11th European Research Conference on
"Electromagnetic Interactions with Nucleons and Nuclei"
1-7 November 2015
Annabelle Hotel, Paphos, Cyprus

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Pre-conference:
Or Hen and Charlotte Van Hulse

IMPORTANT DEADLINE

15TH SEP 2015

- Registration
- Abstract submission

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OVERVIEW

Pre-conference: 1-2 November 2015
- Frontiers and Careers in Photonuclear Physics - skill development and talks for students
- Introductory talks

Main conference: 3-7 November 2015

Conference Topics

- Nucleon form factors and low-energy hadron structure
- Partonic structure of nucleons and nuclei
- Precision electroweak physics and new physics searches
- Meson structure
- Baryon and light-meson spectroscopy
- Nuclear effects and few-body physics

Parallel Workshops

I. Spin structure of nucleons and nuclei from low to large energy scales
II. Spectroscopy – status and future prospects

Poster Session

We invite you to submit abstracts for talks at the workshops and for the poster session. Contributions not selected for talks will be given the option of a poster presentation.

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Join us!

Deadline for abstracts: 31 Aug. 2015

Paphos, Cyprus

Acknowledgments

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