The new method of interference contributions accounting for inelastic scattering diagrams.

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First problem - Calculation of a multidimensional integral

$$\sigma_n = \frac{(2\pi)^4}{4n!I} \int \frac{d\vec{P}_3}{(2\pi)^3 2P_{30}} \frac{d\vec{P}_4}{(2\pi)^3 2P_{40}} \prod_{a=1}^{n} \frac{d\vec{p}_a}{(2\pi)^3 2p_{a0}} \times$$

$$\times \delta\left(P_{initial} - P_{final}\right) \left| A\left(P_1, P_2, P_3, P_4, p_1, p_2, \ldots, p_n\right) \right|^2 \quad (1)$$

This problem is solved with the Laplace method.

Laplace method. Phi3 model.

It is proved that the scattering amplitude $A(\sqrt{s}, X)$ *has a maximum point* X_0 at fixed energy $\sqrt{s}$.
So we can represent the scattering amplitude as

$$A(\sqrt{s}, X) = e^{lnA(\sqrt{s}, X)} \tag{2}$$

After Taylor expansion

$$A(\sqrt{s}, X) = A(\sqrt{s}, X_0) e^{\frac{1}{2} D_{ab}(X_a - X_{0a})(X_b - X_{0b})} \tag{3}$$

where

$$D_{ab} = \left. \frac{\partial^2 lnA(\sqrt{s}, X)}{\partial X_a \partial X_b} \right|_{X=X_0} \tag{4}$$

X - vector of variables $y_1, ...y_n, p_{1,x} ... p_{nx}, p_{1y} ... p_{ny}, P_{ax}, P_{ay}$

Second problem - A large number of integrals

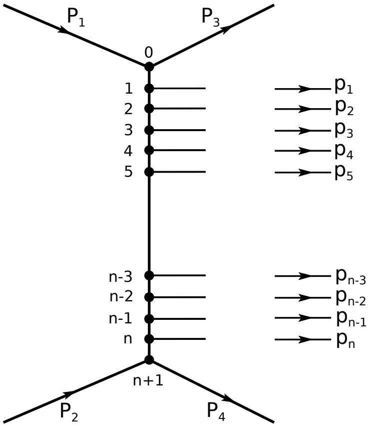


Figure: Disconnected Feynman diagram.

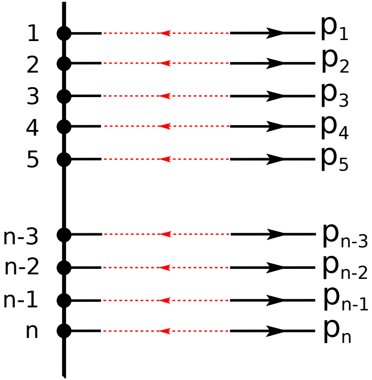
We have **n!** possible ways to connect secondary particle lines to the diagram vertices.

Particles number n	n! value
$n = 5$	120
$n = 7$	5040
$n = 15$	$13 \cdot 10^{11}$
$n = 60$	$8.32 \cdot 10^{81}$

For each connection way we have integral (1), *interference contribution* to the cross-section.

Normal order

Let connect first line to the first vertex, second line to the second vertex and so on... We call it *"normal order"* or "normal connection". So now we can account all possible connections as the permutations of seconrady particles.



Rapidity arithemic progression

At the maximum point of scattering amplitude we have arithmetic progression of secondary particle rapidities.

$$y_{max} = -y_{min}$$

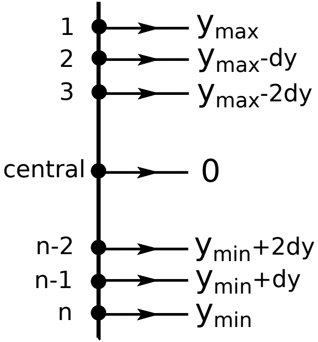


Figure: Central particle has a zero rapidity.

An essence of the new method

At low energies, permutations of the secondary particles (different connections) lead to the same maximum point of scattering amplitude.

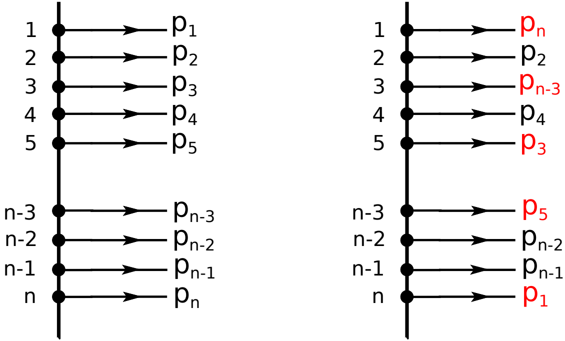


Figure: Particle permutations - same maximum point X_0.

Group the particles

But at high energies we can combine particles into groups in that way, when permutation of the particles inside the groups does not change the maximum point. It means, that the rapidities of particles from the same group are equal.

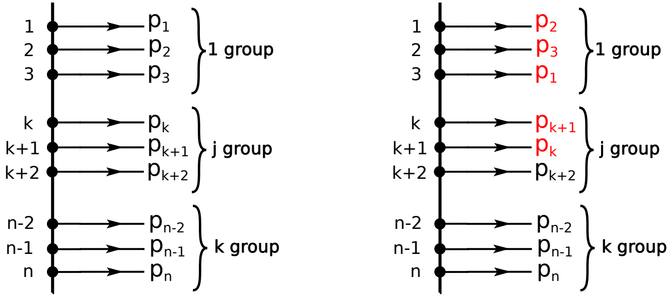


Figure: Permutations inside groups - same maximum point X_0.

To account all possible *n!* contributions we need to:

1. At first, consider all possible permutations inside the groups - *inner permutations*
2. Then consider all possible ways (methods) to fill the groups - *external permutations*.

$$A_{2\to 2+n}(\sqrt{s}, P, p) = \sum_{external\ perm.} \sum_{inner\ perm.} a(\sqrt{s}, P_3, P_4, p_1, p_2, ..., p_n) \quad (5)$$

Now we use Laplace method not to each contribution like (1), but to the partial summ of interference contributions

$$\sum_{inner\ perm.} a(\sqrt{s}, P_3, P_4, p_1, p_2, ..., p_n)$$

Inner permutations

Inner permutations can be accounted by multipliyng the scattering amplitude by coefficient (6):

$$Z = \prod_{i=1}^{N} k_i! \tag{6}$$

N - groups number;
k_i - size of i-group.

External permutations

Each external permutation can be represented by $N \times N$ dimentional matrix.

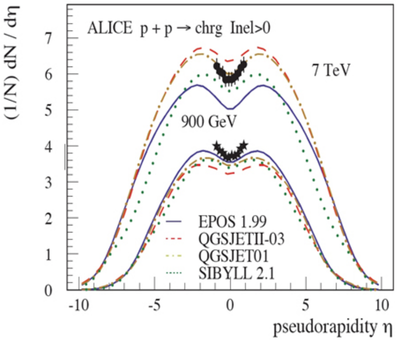
$$M_{ik} \tag{7}$$

Matrix element M_{ik} *shows, how many patricles is taken from i-group of normal order and then pushed to the k-group of new permutation. Then we calculate interference contribution (1) for this permutation.*
Each such contribution has weighting factor P_M:

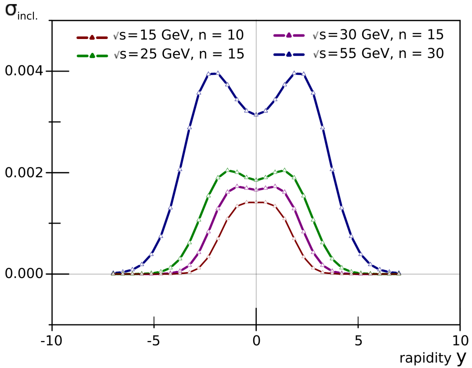
$$P_M = \prod_{i=1}^{N} \prod_{j=1}^{k} C^{M_{ij}}_{n_i - \sum_l^{j-1} M_{il}}$$

Results

The current result is the **qualitative** description of inclusive rapidity distribution.



a) Experimental data



b) Theory prediction

Figure: Inclusive rapidity distribution.

An explanation of peaks behaviour

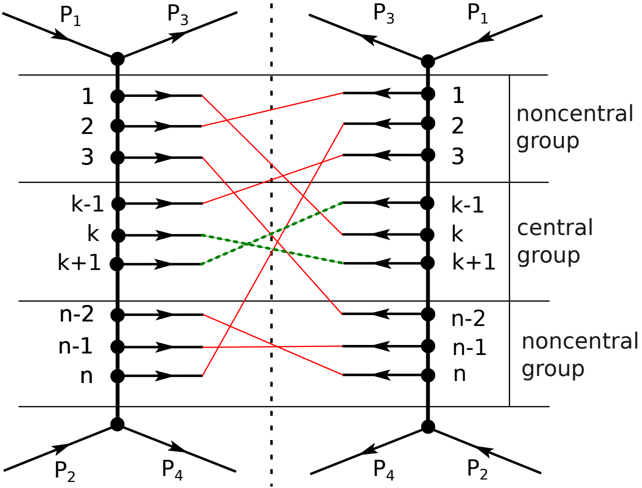


Figure: Cut diagram

- *Green lines* provide maximum at zero point.
- *Red lines* shift maximum from zero.

What is happening with energy $\sqrt{s}$ growth?

- Increases the possible number of secondary particles (n).
- Increases a difference of rapidities arithmetic progression.

But with n increasing:

Number of red lines is growing faster than green lines number.

So, exactly this fact provides the separation of maximum.

Conclusions

- Proposed a new method of interfrerence contributions accounting
- Obtained qualitative description of experimental data (5).
- Proposed an explanation of peaks behaviour if inclusive rapidity corss sections.