# The decays of the Higgs boson into two off-mass-shell $Z$ or $W$ bosons

Taras Zagoskin

Kharkov Institute of Physics and Technology,
Kharkov, Ukraine

August 19, 2015

# Introduction

- In 2012 the ATLAS and CMS Collaborations observed a boson $h$ with the mass around 126 GeV. We call this particle the Higgs boson. However, clarification of properties of the observed boson $h$ requires more data.

$$q_h = 0$$

$$S_h = 0 \; or \; S_h = 2\,(very\ unlikely)$$

$$CP_h = ?$$

- In the SM for the Higgs boson

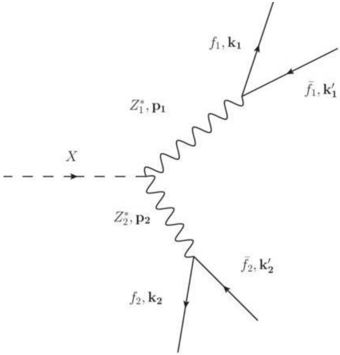
$$q = 0, S = 0, C = P = 1,$$

but some supersymmetric extensions of the SM assume the existence of neutral bosons with negative or indefinite $CP$ parity.

# Plan of the investigation

In order to clarify the $CP$ properties of $h$ the following way has been chosen.

- We consider the decay $X \to Z_1^* Z_2^* \to f_1 \bar{f}_1 f_2 \bar{f}_2$, where $X$ is a neutral particle with zero spin and arbitrary $CP$ parity, $f_1 \neq f_2$.



# Plan of the investigation

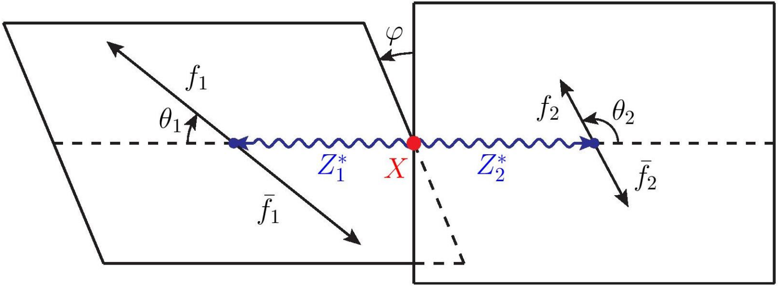
$$A_{X\to Z_1^* Z_2^*} \sim a(e_1^* \cdot e_2^*) + \frac{b}{m_X^2}(e_1^* \cdot p_2)(e_2^* \cdot p_1) + i\frac{c}{m_X^2}\varepsilon_{\mu\nu\rho\sigma}(p_1^\mu + p_2^\mu)(p_1^\nu - p_2^\nu)e_1^{*\rho} e_2^{*\sigma}$$

$e_1$ and $e_2$ are the polarization 4-vectors of $Z_1^*$ and $Z_2^*$ respectively.
$a$, $b$, $c$ are complex-valued functions of the masses of $Z_1^*$ and $Z_2^*$. These functions characterize the $CP$ properties of the boson $X$. At tree level

| $CP_X$ | $a$ | $b$ | $c$ |
|---|---|---|---|
| 1 | any | any | 0 |
| 1 (SM) | 1 | 0 | 0 |
| -1 | 0 | 0 | $\neq 0$ |
| indefinite | $\neq 0$ | any | $\neq 0$ |
| | any | $\neq 0$ | $\neq 0$ |

- We derive the full distribution of the decay $X \to Z_1^* Z_2^* \to f_1 \bar{f}_1 f_2 \bar{f}_2$.
- Experimentalists measure an experimental full distribution of this decay for $X = h$.
- Comparing the theoretical and experimental distributions, one can get constraints on the values of $a, b, c$ at various masses of $Z_1^*$ and $Z_2^*$.

# Definitions of $\theta_1$, $\theta_2$, $\varphi$




$\theta_1$ is the angle between the momentum of $Z_1^*$ in a rest frame of $X$ and the momentum of $f_1$ in a rest frame of $Z_1^*$,
$\theta_2$ is the angle between the momentum of $Z_2^*$ in a rest frame of $X$ and the momentum of $f_2$ in a rest frame of $Z_2^*$,
$\varphi$ is the azimuthal angle between the planes of the decays $Z_1^* \to f_1 \bar{f}_1$ and $Z_2^* \to f_2 \bar{f}_2$.

# Definitions of $A_0$, $A_\parallel$, $A_\perp$

Moreover, it is convenient to write down the fully differential width by means of the following amplitudes:

$$A_0 \equiv -\left( a\frac{m_X^2 - a_1 - a_2}{2\sqrt{a_1 a_2}} + b\frac{\lambda(m_X^2, a_1, a_2)}{4m_X^2\sqrt{a_1 a_2}} \right),$$

$$A_\parallel \equiv \sqrt{2}a,$$

$$A_\perp \equiv \sqrt{2}c\frac{\lambda^{\frac{1}{2}}(m_X^2, a_1, a_2)}{m_X^2}.$$

$a_j$ is the mass squared of $Z_j^*$, or, in other words, $a_j$ is the invariant mass squared of the fermion pair $f_j\bar{f}_j$,
$\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

# The differential width with respect to $a_1$, $a_2$, $\theta_1$, $\theta_2$, $\varphi$

Using approximations $m_{f_1} = m_{f_2} = 0$, we have derived that

$$\frac{d^5\Gamma}{da_1 da_2 d\theta_1 d\theta_2 d\varphi} = |A_0|^2 f_1 + (|A_{||}|^2 + |A_\perp|^2) f_2 + (|A_{||}|^2 - |A_\perp|^2) f_3 + Re(A_0^* A_{||}) f_4 + Re(A_0^* A_\perp) f_5 + Re(A_{||}^* A_\perp) f_6 + Im(A_0^* A_{||}) f_7 + Im(A_0^* A_\perp) f_8 + Im(A_{||}^* A_\perp) f_9.$$

$f_1$, $f_2$, ..., $f_9$ depend on $a_1$, $a_2$, $\theta_1$, $\theta_2$, $\varphi$, but they are independent of $a$, $b$ and $c$.

The dependence of the fully differential width on the couplings $a$, $b$ and $c$ is concentrated in nine quadratic combinations of the amplitudes $A_0$, $A_{||}$, $A_\perp$.

*How many decays should be measured for obtaining a precise enough experimental full distribution of the decay?*

$$d^n\Gamma \leftrightarrow 10^{n+1} \text{ decays}$$
$$d^5\Gamma \leftrightarrow 10^6 \text{ decays}$$

*How many decays have been observed?*

$$h \to Z_1^* Z_2^* \to e^- e^+ \mu^- \mu^+$$

26 decays (ATLAS and CMS together after about 1.5 years of measurements)

# Distributions of four and less variables should be considered

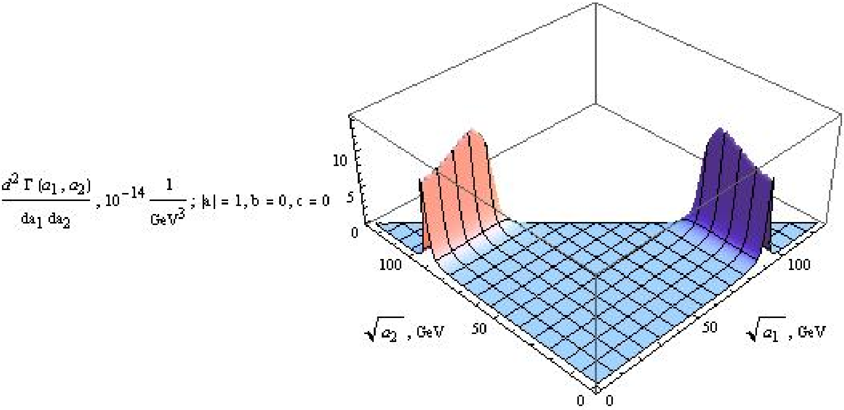
We will probably have a precise enough experimental full distribution

**in 60 000 years** (roughly).

That is why we should try to get constraints on $a$, $b$, $c$ by means of measuring distributions of as little a number of variables as possible.

# $a_1 a_2$-differential width

Figure: $\frac{d^2\Gamma}{da_1 da_2}$ of the decay $X \to Z_1^* Z_2^* \to l_1^- l_1^+ l_2^- l_2^+$ as a function of $\sqrt{a_1}$, $\sqrt{a_2}$ if $X$ is the SM Higgs boson and $m_X = 125.7$ GeV.
$l_1, l_2 = e, \mu, \tau, l_1 \neq l_2$.



# $a_2$-differential width

Integrating $\frac{d^2\Gamma}{da_1 da_2}$ approximately, we derive that

$$\frac{d\Gamma}{da_2} \approx \frac{\sqrt{2} G_F^3 m_Z^9}{288\pi^4 m_X^3 \Gamma_Z} (a_{f_1}^2 + v_{f_1}^2)(a_{f_2}^2 + v_{f_2}^2) \frac{\lambda^{\frac{1}{2}}(m_X^2, m_Z^2, a_2) a_2}{(a_2 - m_Z^2)^2 + (m_Z \Gamma_Z)^2} \sum_{\lambda=0,\|,\perp} |A'_\lambda|^2$$

$$\forall a_2 \mid 2m_{f_2} < \sqrt{a_2} \le m_X - \sqrt{m_Z^2 + 3 m_Z \Gamma_Z}.$$

$a_f$ and $v_f$ are constants depending on a fermion $f$, $A'_\lambda \equiv A_\lambda|_{a_1 = m_Z^2}$.
In several articles the formula for $\frac{d\Gamma}{da_2}$ has been used in the narrow-$Z$-width approximation when

$$\sqrt{a_2} \le m_X - m_Z,$$

and their approach is inaccurate.

$m_h - \sqrt{m_Z^2 + 3 m_Z \Gamma_Z} \approx 30.8$ GeV
$m_h - m_Z \approx 34.5$ GeV

# Constraints on $hZZ$ couplings

CMS data:

$$\frac{\sigma(pp \to h)\,\mathrm{BR}(h \to ZZ \to 4l)}{\sigma_{SM}(pp \to h)\,\mathrm{BR}_{SM}(h \to ZZ \to 4l)} = 0.93^{+0.26}_{-0.23}(\mathrm{stat})^{+0.13}_{-0.09}(\mathrm{syst}).$$

CMS and ATLAS have already constrained couplings of the Higgs boson to $ZZ$. ATLAS use the $hZZ$ couplings $\alpha$, $\kappa_{SM}$, $\tilde{\kappa}_{HZZ}$, $\tilde{\kappa}_{AZZ}$, connected with our ones via

$$a = \cos\alpha\left(\kappa_{SM} + \frac{m_X^2 - a_1 - a_2}{m_Z^2}\tilde{\kappa}_{HZZ}\right),$$

$$b = -2\frac{m_X^2}{m_Z^2}\tilde{\kappa}_{HZZ}\cos\alpha, \qquad c = -i\frac{m_X^2}{m_Z^2}\tilde{\kappa}_{AZZ}\sin\alpha.$$

# Constraints on $hZZ$ couplings

Table: ATLAS and CMS 95% CL allowed regions and our estimations for various $hZZ$ coupling ratios.

| | $\tilde{\kappa}_{HZZ}/\kappa_{SM}$ | $(\tilde{\kappa}_{AZZ}/\kappa_{SM})\tan\alpha$ | $\mathrm{Im}\,\tilde{\kappa}_{HZZ}/\mathrm{Re}\,\kappa_{SM}$ |
|---|---|---|---|
| ATLAS | (-0.75, 2.45) | (-2.85, 0.95) | n/a |
| CMS | $[-2.28, -1.88] \cup [-0.69, \infty)$ | [-2.05, 2.19] | n/a |
| Our est. | $[-2.38, -1.89] \cup [-0.24, 1.13]$ | [-1.28, 1.28] | [-1.01, 1.01] |

# Relations between the helicity coefficients and observables

$$\frac{d^5\Gamma}{da_1 da_2 d\theta_1 d\theta_2 d\varphi} = |A_0|^2 f_1 + (|A_\parallel|^2 + |A_\perp|^2) f_2 + (|A_\parallel|^2 - |A_\perp|^2) f_3$$
$$+ Re(A_0^* A_\parallel) f_4 + Re(A_0^* A_\perp) f_5 + Re(A_\parallel^* A_\perp) f_6$$
$$+ Im(A_0^* A_\parallel) f_7 + Im(A_0^* A_\perp) f_8 + Im(A_\parallel^* A_\perp) f_9 .$$

We call the ratios of the nine quadratic combinations of the fully differential width to $\sum_{\lambda=0,\parallel,\perp} |A_\lambda|^2$ 'the helicity coefficients'. Integrating $\frac{d^5\Gamma}{da_1 da_2 d\theta_1 d\theta_2 d\varphi}$, we can relate all the helicity coefficients to observables. For example,

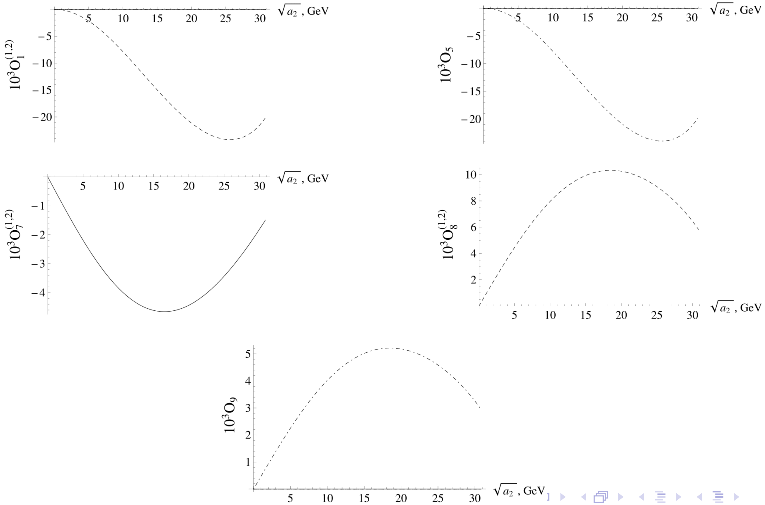
$$O_1^{(1,2)}(a_2) \equiv \left(\frac{d\Gamma}{da_2}\right)^{-1} \left( \int_0^{\frac{\pi}{2}} d\theta_{1,2} \frac{d^2\Gamma}{da_2 d\theta_{1,2}} - \int_{\frac{\pi}{2}}^{\pi} d\theta_{1,2} \frac{d^2\Gamma}{da_2 d\theta_{1,2}} \right) \sim \frac{\mathrm{Re}(A_\parallel'^* A_\perp')}{\sum_\lambda |A_\lambda'|^2}$$
$$= \mathrm{Re}(F_\parallel'^* F_\perp').$$

$$F_\lambda' \equiv \frac{A_\lambda'}{\sqrt{\sum_\lambda |A_\lambda|^2}}, \ \lambda = 0, \parallel, \perp .$$

# Relations between the helicity coefficients and observables

$$\left.\begin{aligned}
O_1^{(1,2)}(a_2) &\sim \mathrm{Re}(F_{\|}'^{*}F_{\perp}') \\
O_2^{(1,2)}(a_2) &\sim |F_0'|^2 \\
O_3(a_2) &\sim |F_{\|}'|^2 + |F_{\perp}'|^2 \\
O_4(a_2) &\sim |F_{\|}'|^2 - |F_{\perp}'|^2 \\
O_5(a_2) &\sim \mathrm{Im}(F_{\|}'^{*}F_{\perp}') \\
O_6(a_2) &\sim \mathrm{Re}(F_0'^{*}F_{\|}') \\
O_7^{(1,2)}(a_2) &\sim \mathrm{Im}(F_0'^{*}F_{\|}') \\
O_8^{(1,2)}(a_2) &\sim \mathrm{Re}(F_0'^{*}F_{\perp}') \\
O_9(a_2) &\sim \mathrm{Im}(F_0'^{*}F_{\perp}')
\end{aligned}\right\} \rightarrow \textit{Constraints on } a,\ b,\ c$$

Figure: Plots of observables for the decay $h \to Z_1^* Z_2^* \to l_1^- l_1^+ l_2^- l_2^+$ $(l_1 \neq l_2)$. Solid line: $a = 1, b = -0.5i, c = 0$, dashed lines: $a = 1, b = 0, c = 0.5$, dash-dotted lines: $a = 1, b = 0, c = 0.5i$.



# Conclusions

- In order to clarify the $CP$ properties of the Higgs boson we have considered the decay $X \to Z_1^* Z_2^* \to f_1 \bar{f}_1 f_2 \bar{f}_2$, where $X$ is a neutral particle with zero spin and arbitrary $CP$ parity, $f_1 \neq f_2$.
- We have constrained couplings of the Higgs boson to $ZZ$. Our constraints are in good agreement with those of CMS and ATLAS and provide an allowed interval for a coupling ratio unconstrained by CMS and ATLAS.
- Observables are defined measurement of which will allow one to obtain information on the $hZZ$ vertex.
- An analogous analysis has been carried out for the decay $X \to W^{-*} W^{+*} \to f_{1-} \bar{f}_{2-} \bar{f}_{1+} f_{2+}$.

The presentation is based on the paper *T.V. Zagoskin and A.Yu. Korchin, arXiv:1504.07187v2 [hep-ph]*.