The HERA data for $F_2(x, Q^2)$ show a strong rise for $x \to 0$ at fixed Q^2 for larger Q^2.

Can this rise go on forever, or is there a "unitarity limit"?

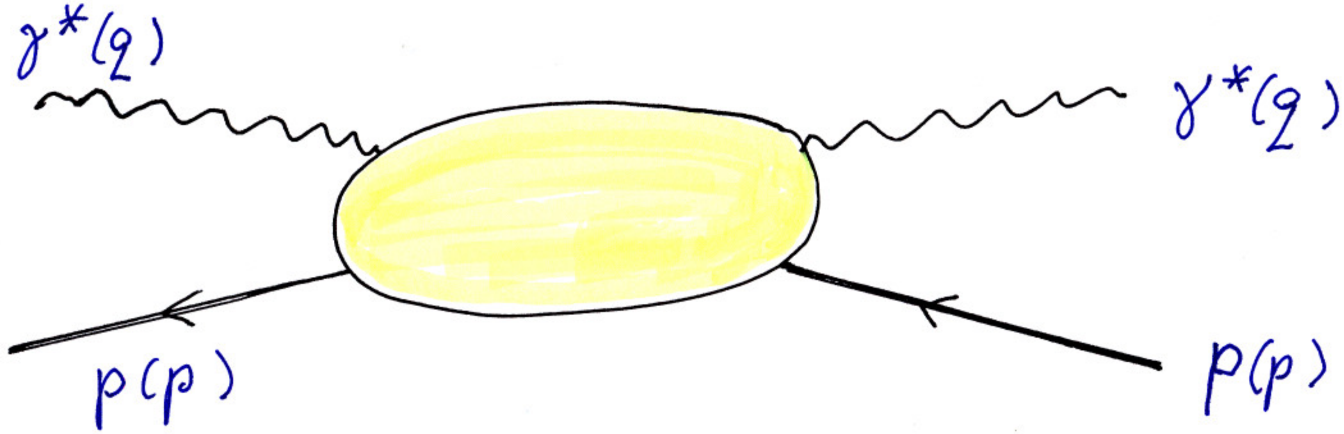
Answer from a rigorous theoretical point of view:

There is no unitarity limit!

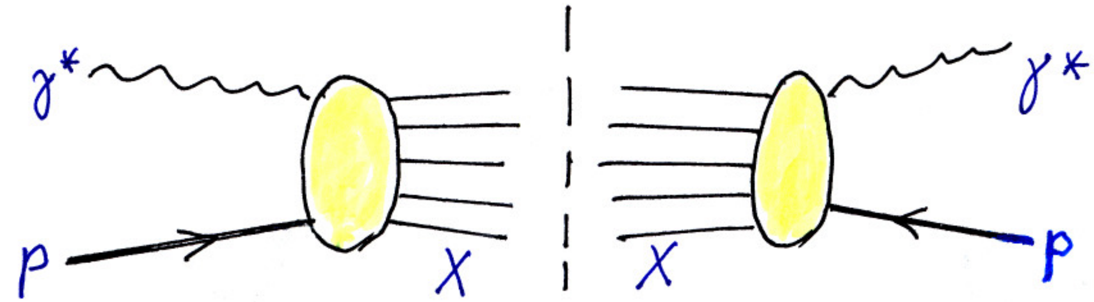
Forward virtual Compton scattering amplitude:

$$\gamma^*(q) + p(p) \longrightarrow \gamma^*(q) + p(p)$$

Im



$= \sum_X$



$\propto \quad \sigma_{T,L} \quad$ resp. $\quad W_{1,2}$

- The S matrix is a unitary operator

$$S^\dagger S = \mathbb{1}$$

This gives relations / restrictions for the scattering of asymptotic particles, that is, asymptotic states. The virtual photon γ^* is not an asymptotic state.

- The relation of the imaginary part of the forward virtual Compton amplitude to the structure functions follows from the definition of the T-product and is not a unitarity relation.

In a unitarity relation $\sum_X$ would have to include an asymptotic hadronic state $|X\rangle = |\gamma^* p\rangle$.

- Can we get unitarity restrictions for the structure functions by considering ep scattering?

  Again no! In a "Gedankenexperiment" we can make α and thus the ep amplitude as small as we wish without changing the structure functions.

- If for the physical value of α the lowest order ep cross section containing the structure functions becomes very very large, then higher order effects in α must come into play "taming" the cross section.

Conclusion

- Unitarity and, thus, the Froissart bound have nothing to say on $F_2(x, Q^2)$ for $x \to 0$.
- In our view $F_2(x, Q^2)$ may continue to rise for $x \to 0$. It may also saturate, but then, not because of unitarity.