LSS with Lagrangian perturbation theory. (including the stream crossing)

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with:

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&
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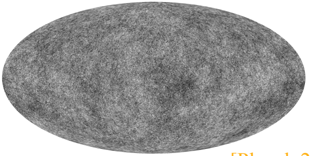
Structure Formation and Evolution

CMB: $\Delta\rho/\rho \sim 10^{-6}$ | $z=1100$

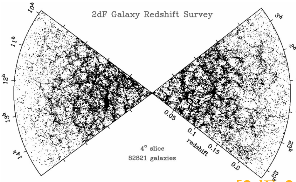
LSS: $\Delta\rho/\rho \sim 10^{0}$ | $z=2$

Galaxies: $\Delta\rho/\rho \sim 10^{6}$ | $z=0$

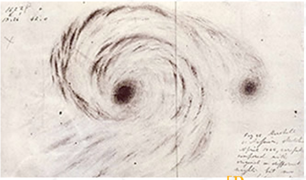
$$\boxed{z=1100} \Downarrow \boxed{z=2} \Downarrow \boxed{z=0}$$



[Planck,2013]



[2dF, 2002]



[Parsons, 1845]

LSS: motivations and observations

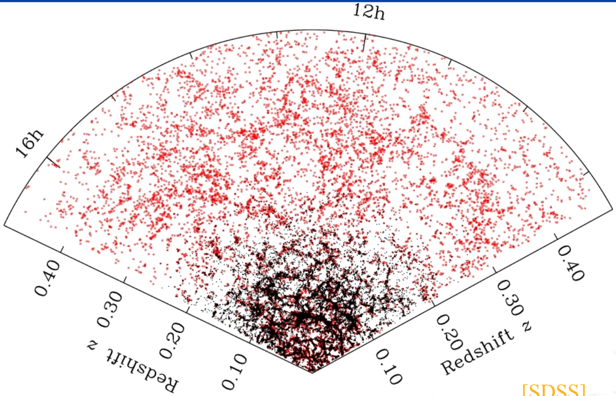
Theoretical motivations:

- Inflation - origin of structures
- Expansion history
- Composition of the universe
- Nature of dark energy and dark matter
- Neutrino mass and number of species
- Test of GR and modifications of gravity

Current and future observations:

- SDSS and SDSS3/4: Sloan Digital Sky Survey
- BOSS: the Baryon Oscillation Spectroscopic Survey
- DES: the Dark Energy Survey
- LSST: the large synoptic survey telescope.
- Euclid: the ESA mission to map the geometry of the dark Universe
- DESI: Dark Energy Spectroscopic Instrument
- SPHEREx: An All-Sky Spectral Survey (?)

Galaxy clustering



[SDSS]

- Measured $3D$ distribution $\Rightarrow$ much more modes than projected quantities (shear from weak lensing, etc.)
- Redshift surveys measure: θ, ϕ, redshift z

$$\text{overdensity:} \qquad \delta = (n - \bar{n})/\bar{n},$$
$$\text{power spectrum:} \quad P(k) \sim \langle \delta(\mathbf{k}) | \delta(\mathbf{k}) \rangle$$

Key messages:

- Nonlinear effects of dark matter clustering allow analytic investigation (UV complete) - including *shell crossing*.
- In a regime of largest possible scales (BAO) further simplifications can be achieved in the *EFT* framework.
- Nonlinear redshift distortions can be constructed out of the real space correlations.

Why perturbative approach?

- This problem is also amenable to direct simulation.
  - Though the combination of volume, mass and force resolution and numerical accuracy is very demanding - in scope of next gen. surveys.
  - PT is a viable alternative as well as a guide what range of k, M_h, scales are necessary and what statistics are needed.
  - N-body can be used to test PT for 'fiducial' models.
- However PT can be used to search a large parameter space efficiently, and find what kinds of effects are most important.
  - Can be much more flexible/inclusive, especially for biasing schemes.
  - It much easier to add new physics, especially if the effects are small (e.g. neutrinos, clustering dark energy, non-Gaussianity)
- Gaining insights!
- Goal is the highly precise at large scales (in scope of next gen. surveys), as well as to push to small scales.
- Complementarity reason; if we can, we should.

Gravitational clustering of dark matter

Evolution of collisionless particles - Vlasov equation:

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{1}{m}\mathbf{p}\cdot\nabla f - am\nabla\phi\cdot\nabla_p f = 0,$$

and $\nabla^2\phi = 3/2\mathcal{H}\Omega_m\delta$.

EFT approach introduces a tress tensor for the long-distance fluid:

$$\frac{\partial \delta_L}{\partial \tau} + \nabla\cdot[(1+\delta_L)\mathbf{v}_L] = 0$$
$$\frac{\partial v_{i,L}}{\partial \tau} + \mathcal{H}v_{i,L} + \mathbf{v}_L\cdot\nabla v_{i,L} = -\nabla_i\phi_L - \frac{1}{\rho}\nabla_j(\tau_{ij,L}),$$

with given as $\tau_{ij} = p_0\delta_{ij} + c_s^2\delta\rho_L\delta_{ij} + O(\partial^2\delta_L,\dots)$
-derived by smoothing the short scales in the fluid with
the smoothing filter $W(\Lambda)$, where $\Lambda \propto 1/k_{\rm NL}$.

[Baumann et al 2010, Carrasco et al 2012]

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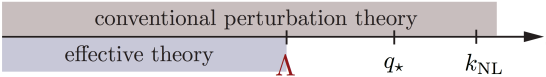

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Lagrangian vs Eulerian framework

Eulerian:

Lagrangian:

Coordinate of a (t)racer particle at a given moment in time **r**

$$\mathbf{r}(\mathbf{q},\tau) = \mathbf{q} + \psi(\mathbf{q},\tau),$$

is given in terms of Lagrangian displacement.

Continuity equation:

$$(1+\delta(\mathbf{r}))\, d^3 r = d^3 q \quad \text{vs.} \quad 1+\delta(\mathbf{r}) = \int_q \delta^D\left(\mathbf{r} - \mathbf{q} - \psi(\mathbf{q})\right),$$

Fourier space

$$(2\pi)^3 \delta^D(\mathbf{k}) + \delta(\mathbf{k}) = \int_q e^{i\mathbf{k}\cdot\mathbf{q}} \, \exp\left(i\mathbf{k}\cdot\psi\right),$$

Lagrangian dynamics and EFT

The correlation function and power spectrum can now be defined through the cumulants of the displacement, e.g.

$$P(k) = \int_q e^{i\boldsymbol{q}\cdot\boldsymbol{k}} \left[\left\langle e^{i\boldsymbol{k}\cdot\boldsymbol{\Delta}(\boldsymbol{q})} \right\rangle - 1 \right].$$

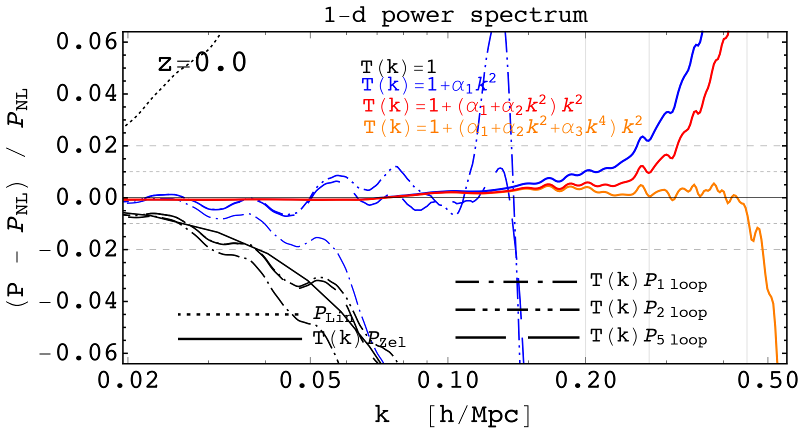
For one loop power spectrum results, keeping linear modes resumed:

$$P(k) = \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q}} \exp\left[-\frac{1}{2} k_i k_j \left\langle \Delta_i \Delta_j \right\rangle_c + \frac{i}{6} k_i k_j k_k \left\langle \Delta_i \Delta_j \Delta_k \right\rangle_c + \cdots \right]$$

Final results equivalent to the Eulerian scheme. [Sugiyama '14, Vlah et al, '14 & '15]
Allows for the insight in the counter term structure and IR resummation schemes (in particular one leads to the scheme in [Senatore&Zaldarriaga, '14]).
Simple IR scheme was suggested also in [Baldauf et al, '15].

Clustering in 1D

1D case studied recently in: [McQuinn&White, '15, Vlah et al, '15]



Path integrals and going beyond shell crossing

- as we saw the Lagrangian framework includes shell crossing
- Lagrangian dynamics can be compactly written using

$$\boldsymbol{L}_0 \phi + \boldsymbol{\Delta}_0(\phi) = \boldsymbol{\epsilon},$$

where:

$$\phi \equiv (\psi, \upsilon)\,, \quad [\boldsymbol{L}_0]_{i_2 i_1} = \begin{pmatrix} \frac{\partial}{\partial \eta_2} & -1 \\ -\frac{3}{2} & \frac{\partial}{\partial \eta_2} + \frac{1}{2} \end{pmatrix}, \quad \boldsymbol{\Delta}_0(\phi) = \frac{3}{2}\left(0, \partial_{\boldsymbol{x}} \partial_{\boldsymbol{x}}^{-2} \delta + \psi\right).$$

Statistics of interest given by generating function

$$Z(\boldsymbol{j}) \equiv \int d\boldsymbol{\epsilon}\; e^{-\frac{1}{2}\boldsymbol{\epsilon} \boldsymbol{N}^{-1} \boldsymbol{\epsilon} + \boldsymbol{j}\phi[\boldsymbol{\epsilon}]} \;\text{ and }\; \langle \phi_{i_1} \phi_{i_2} \rangle = \frac{\partial^2}{\partial j_{i_1} \partial j_{i_2}} Z(\boldsymbol{j}) \Big|_{\boldsymbol{j}=0},$$

which after the variable change becomes

$$Z(\boldsymbol{j}) \equiv \int d\phi\; e^{-S(\phi) + \boldsymbol{j}\phi},$$

with $S(\phi) = 1/2\,[\boldsymbol{L}_0 \phi + \boldsymbol{\Delta}_0(\phi)]\, \boldsymbol{N}^{-1}\, [\boldsymbol{L}_0 \phi + \boldsymbol{\Delta}_0(\phi)]\,.$ [McDonald&Vlah, '17]

Path integrals and going beyond shell crossing

We can organize our perturbation theory as:

$$S = S_g + S_p, \quad \text{where then we do} \quad \exp(-S) = \exp(-S_g)(1 - S_p + S_p^2/2 + \ldots)$$

where we can choose what the "Gaussian part" will be, i.e.

$$S_g \equiv 1/2\chi \boldsymbol{N}\chi + i\chi[W^{-1}\boldsymbol{L}_0]\phi \equiv 1/2\chi\boldsymbol{N}\chi + i\chi\boldsymbol{L}\phi$$

and

$$S_p \equiv i\chi\boldsymbol{\Delta}_0(\phi) + i\chi[(1 - W^{-1})\boldsymbol{L}_0]\phi \equiv i\chi\boldsymbol{\Delta}(\phi),$$

where χ is the auxiliary field from the Hubbard-Stratonovich transformation.

Perturbation theory result : $Z(\boldsymbol{j}) = Z_0(\boldsymbol{j}) + Z_1(\boldsymbol{j}) + \ldots$

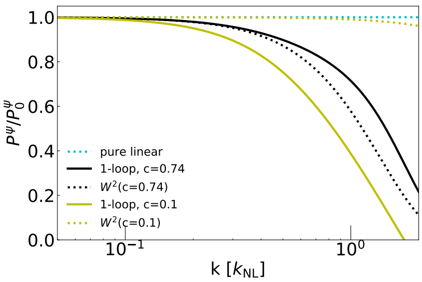
Leading order result: truncate Zel'dovich dynamics!!!

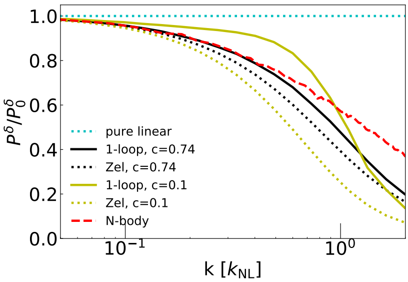
$$Z_0 = e^{\frac{1}{2}\boldsymbol{j}.\boldsymbol{C}.\boldsymbol{j}} \quad \text{and} \quad P(k) = \int d^3q \; e^{i\boldsymbol{q}\cdot\boldsymbol{k}} e^{-\frac{1}{2}k_i k_j A^W_{ij}}$$

higher orders more complicated, build in renormalization! [McDonald&Vlah, '17]

Path integrals and going beyond shell crossing

$W = \exp(-ck^2),\ n = 0.5$



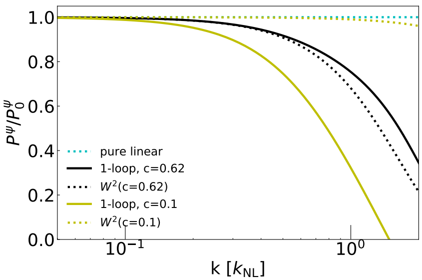


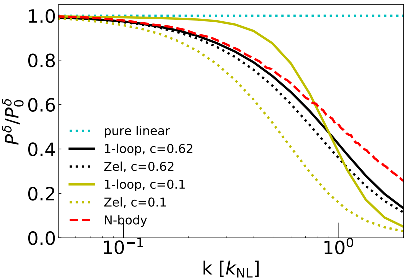
Significance and connection EFT formalism:

- no need of EFT free parameters, i.e. counter terms are predicted
- CMB lensing: direct information on baryonic and neutrinos physics
- reduction of degeneracy in galaxy bias coefficients
- possible connection to the EFT formalism by matching the $k \to 0$ limit

Path integrals and going beyond shell crossing

$W = \exp(-ck^2),\ n = 1.0$





Significance and connection EFT formalism: - goal is 3D!

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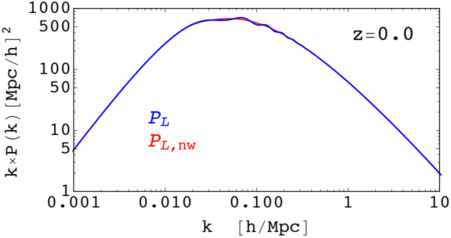
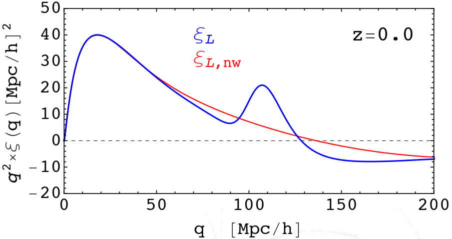
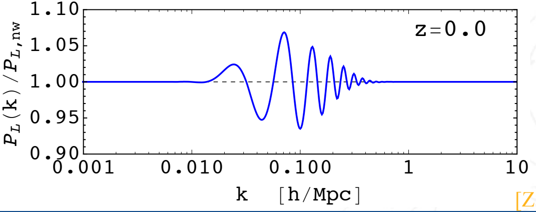
Linear power spectrum, correlation function & BAO

Linear power spectrum $P_{\rm L}$: obtained form Boltzmann codes (CAMB, Class). Formally we can divide it into smooth part $P_{\rm L,nw}$ and wiggle part $P_{\rm L,w}$ so that:
$$P_{\rm L} = P_{\rm L,nw} + P_{\rm L,w}$$





Wiggle power spectrum: $P_{\rm L,w} \quad \rightarrow \quad \sigma_n = \int_q q^{-n} P_{\rm L,w}(q) = 0$ for $n = \{0, 2\}$.

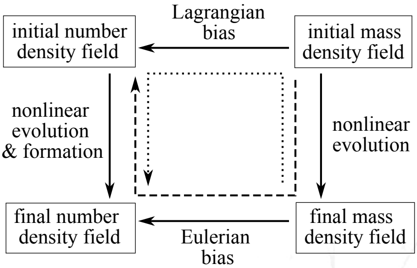


[Z.V. et al, '14 & '15]

Bias in Lagrangian space

- Eulerian bias: relation between the final mass density field and the final halo density field
- Lagrangian bias: relation between the initial mass density field and the initial halo density field

[Matsubara, 2011]



$$\delta_X(\boldsymbol{q}) = c_\delta \delta_L(\boldsymbol{q}) + c_{\delta^2}\delta_L^2(\boldsymbol{q}) + c_{s^2} s_L^2(\boldsymbol{q}) + c_{\delta^3}\delta_L^3(\boldsymbol{q}) + c_{\delta s^2}\delta_L s_L^2(\boldsymbol{q}) + c_{s^3} s_L^3(\boldsymbol{q}) + c_{\partial^2\delta}\frac{\partial_q^2}{k_L^2}\delta_L(\boldsymbol{q}) + \text{``stochastic''} + \ldots,$$

[with White & Castorina, 2016]

- Tracer defined in Lagrangian space need to be displaced to the final time.

Bias in Lagrangian space in redshift space

Results for a sample of DM halo multipoles in configurations space

○ ○ ○ $\ell = 0,\ 12.5 < \mathrm{lg}M < 13.0$ □ □ □ $\ell = 2,\ 12.5 < \mathrm{lg}M < 13.0$

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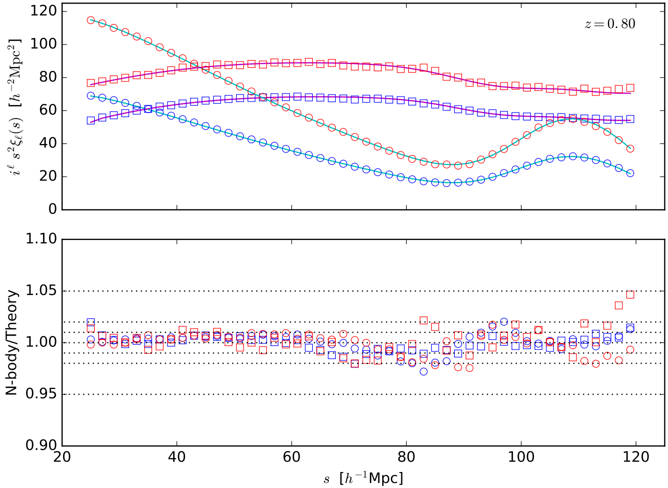


Wiggles for halos in redshift rpace

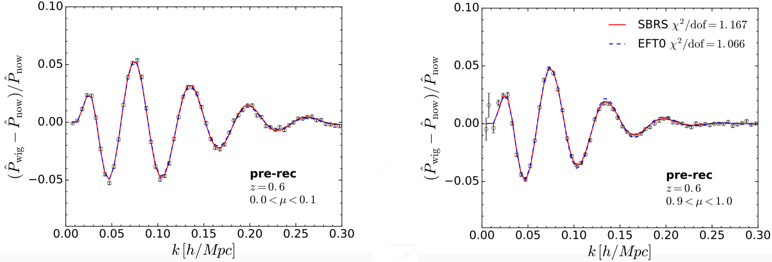
$$P(\boldsymbol{k}) = \int_q e^{-i\boldsymbol{q}\cdot\boldsymbol{k}}\Big(1 - \text{bias}\Big)\exp\left(-\frac{1}{2}A^s(\boldsymbol{k},\boldsymbol{q})\right)\Bigg|_{\lambda_1=\lambda_2=0} + \text{h.o.} + \text{``stochastic''},$$

where we e.g. $A^s(\boldsymbol{k},\boldsymbol{q}) = \left\langle\Big(\lambda_1\delta_L(\boldsymbol{q}_1) + \lambda_2\delta_L(\boldsymbol{q}_2) + \boldsymbol{k}\cdot\Delta^s(\boldsymbol{q})\Big)^2\right\rangle_c$, gives [with Ding, Seo, et. al.]

$$\delta P(k,\nu) = e^{-k^2(1+f(2+f)\nu^2)\Sigma^2(q_{\max})}\left(b_1^2 + 2fb_1\nu^2 + f^2\nu^4 + b_\partial\left(b_1 + f\nu^2\right)\frac{k^2}{k_L^2}\right)\delta P_L(k,\tau) + \text{h.o.}$$

where $q_{\max}$ implicitly given by $\frac{\partial}{\partial q}\left[\Big(1 - i\hat{c}_q(\partial_{\lambda_1} + \partial_{\lambda_2}) - \hat{c}_q^2\partial_{\lambda_1}\partial_{\lambda_2}\Big)\delta\mathcal{A}^s(\boldsymbol{k},\boldsymbol{q})\right]_{\substack{\lambda_1=\lambda_2=0\\ q=q_{\max}}} = 0.$

depends on k,ν as well as bias parameters $c_\delta, c_{\partial^2\delta},\ldots$ simplest $\Sigma^2 = \int\frac{dp}{3\pi^2}(1 - j_0(qk))P_L(p)$.

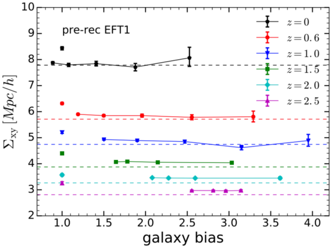


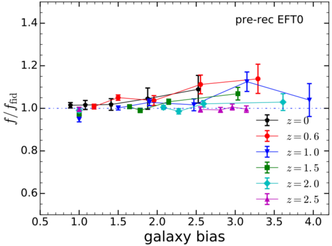
Wiggles for halos in redshift rpace

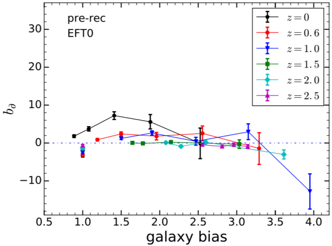
Results and parameters estimate:

[with Ding, Seo, et. al.]

	Pre-reconstruction
EFT0 model	Free: $\alpha_\perp$, $\alpha_\parallel$, f, b_1, b_∂ Fixed: Σ_{xy}, $\Sigma_z (= (1 + f_{\rm fid})\Sigma_{xy})$. For matter, $b_1 = 1$.
EFT1 model	Free: $\alpha_\perp$, $\alpha_\parallel$, Σ_{xy}, f, b_1, b_∂. Note* $\Sigma_z = (1 + f_{\rm fid})\Sigma_{xy}$ Fixed: for matter, $b_1 = 1$.







Redshift space distortions (RSD)

Power spectrum in RSD [with M. White, in prep.]

$$P_s(\boldsymbol{k}) = \int d^3r\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \mathcal{M}\big(k_{\parallel}\hat{z}, \boldsymbol{r}\big) = \int d^3r\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \mathcal{M}\big(\boldsymbol{J} = \boldsymbol{k}.\boldsymbol{R}, \boldsymbol{r}\big),$$

where $\boldsymbol{r} = \boldsymbol{x}_2 - \boldsymbol{x}_1$ and

$$1 + \mathcal{M}\big(\boldsymbol{J}, \boldsymbol{r}\big) = \Big\langle \big(1 + \delta(\boldsymbol{x})\big)\big(1 + \delta(\boldsymbol{x}')\big) e^{i\boldsymbol{J}\cdot\Delta\boldsymbol{u}} \Big\rangle\,,$$

is the generating funciton for the moments of pairwise velocity, where $\Delta\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}_2) - \boldsymbol{u}(\boldsymbol{x}_1)$. RSD families :

- Streaming approach: cumulant expansion theorem, $\mathcal{M}$ to $\mathcal{Z}$. [Peebles, White, ...]
- Distribution function approach: $\mathcal{M}$ is expanded moments that can be individually evaluated. [SPT, Seljak&McDonald, ...]
- Scoccimarro approach: Cumulant expansion theorem on individual contributions. [Scoccimarro, TNS ...]
- Direct Lagrangian approach: $\mathcal{M}$ is transformed into Lagrangian coordinates [Matsubara, White ...]

Summary

Key points:

- Shell crossing can be consistently added to the perturbative Lagrangian scheme.

- Perturbative Lagrangian approach is a viable and UV-complete approach in the study of structure formation.

- EFT framework offers further simplifications on largest scales & Lagrangian setting is a natural for the study of BAO effects in LSS statistics..