Collectivity in small systems and in heavy-ion collisions: theoretical overview

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Little bangs in the laboratory
Fluid dynamics

- long distances, long times or strong enough interactions
- matter or quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - thermodynamic equation of state $p(T, \mu)$
  - shear viscosity $\eta(T, \mu)$
  - bulk viscosity $\zeta(T, \mu)$
  - heat conductivity $\kappa(T, \mu)$
  - relaxation times, ...

- **ab initio** calculation of fluid properties difficult but fixed by **microscopic** properties in $\mathcal{L}_{\text{QCD}}$
Thermodynamics of QCD

from lattice gauge theory

- thermodynamic equation of state \( p(T) \) rather well understood now
- also \( \mu \neq 0 \) is being explored
- progress in computing power
Transport coefficients

- from perturbation theory / effective kinetic theory at leading order
  [Arnold, Moore, Yaffe (2003)]
  \[ \eta(T) = k \frac{T^3}{g^4 \log(1/g)} , \]

- next-to-leading order also understood now
  [Ghiglieri, Moore, Teaney (2015-2018)]

- form AdS/CFT correspondence (very strong coupling)
  [Kovtun, Son, Starinets (2003)]

- more transport properties and intermediate coupling regime to be understood
\textbf{Fluid dynamics in heavy ion collisions}

\[ \text{Initial conditions matter} \]

\( p \) for pions in PbPb collisions well described by fluid dynamics.

\( v \)\(_n\)\( (p_T) \) for pions in PbPb collisions well described by fluid dynamics.

Initial conditions matter.
Fluid dynamics for smaller systems

- flow coefficients from higher order cumulants $v_2\{n\}$ agree:
  - collective behavior
- elliptic flow signals also in $pPb$ and $pp$!
- can fluid approximation work for pp collisions?
Fluid dynamics for smaller systems

rather good agreement between data and theory for large multiplicity

fluid approximation + initial state model works best for PbPb but still reasonable for pPb and pp
Questions and puzzles

- how universal are collective flow and fluid dynamics? or: when does it break down and how?
- what determines density distribution in a proton?
- role of multi-parton interactions
- more elementary systems such as ep or $e^+e^-$ [News at Quark Matter 2018!]
Idea behind relativistic fluid dynamics

- General principle: macroscopic physics governed by conservation laws
- **Energy-momentum tensor** and conserved current
  \[
  T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}
  \]
  \[
  N^\mu = n u^\mu + \nu^\mu
  \]

- Tensor decomposition using fluid velocity \(u^\mu\), \(\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu\)
- Thermodynamic equation of state \(p = p(T, \mu)\)
- Thermal equilibrium = ideal fluid approximation
  \[
  \pi_{\text{bulk}} = \pi^{\mu\nu} = \nu^\mu = 0.
  \]
Conservation laws

Covariant conservation laws $\nabla_\mu T^{\mu \nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply

- equation for energy density $\epsilon$
  \[ u^\mu \partial_\mu \epsilon + (\epsilon + p + \pi_{\text{bulk}}) \nabla_\mu u^\mu + \pi^{\mu \nu} \nabla_\mu u_\nu = 0 \]

- equation for fluid velocity $u^\mu$
  \[ (\epsilon + p + \pi_{\text{bulk}}) u^\mu \nabla_\mu u_\nu + \Delta^{\nu \mu} \partial_\mu (p + \pi_{\text{bulk}}) + \Delta^{\nu \alpha} \nabla_\mu \pi^{\mu \alpha} = 0 \]

- equation for particle number density $n$
  \[ u^\mu \partial_\mu n + n \nabla_\mu u^\mu + \nabla_\mu n^{\mu} = 0 \]

Relativistic dynamics

- covariance
- causality
Standard derivative or Chapman-Enskog expansion

- take fluid velocity $u^\mu$ and thermodynamic fields $T, \mu$ as degrees of freedom
- express "viscous stresses" in terms of derivatives
- bulk viscous pressure
  \[ \pi_{\text{bulk}} = -\zeta \nabla_\mu u^\mu + \ldots \]

- shear stress
  \[ \pi^{\mu\nu} = -\eta \left[ \Delta^{\mu\alpha} \nabla_\alpha u^\nu + \Delta^{\nu\alpha} \nabla_\alpha u^\mu - \frac{2}{3} \Delta^{\mu\nu} \nabla_\alpha u^\alpha \right] + \ldots \]

- diffusion current
  \[ \nu^\alpha = -\kappa \left[ \frac{nT}{\epsilon + p} \right]^2 \Delta^{\alpha\beta} \partial_\beta \left( \frac{\mu}{T} \right) + \ldots \]

- restricted to small gradients (large systems)
- does not lead to relativistically causal evolution equations
Israel-Stewart type theories

Evolution equations instead of constraints

- equation for **shear stress** $\pi^{\mu\nu}$

$$\tau_{\text{shear}} P_{\alpha\beta}^{\rho\sigma} u^{\mu} \nabla_{\mu} \pi_{\alpha\beta}^{\rho\sigma} + \pi^{\rho\sigma} + 2\eta P_{\beta}^{\rho\sigma} \nabla_{\alpha} u^{\beta} + \ldots = 0$$

with **shear viscosity** $\eta(T, \mu)$

- equation for **bulk viscous pressure** $\pi_{\text{bulk}}$

$$\tau_{\text{bulk}} u^{\mu} \partial_{\mu} \pi_{\text{bulk}} + \pi_{\text{bulk}} + \zeta \nabla_{\mu} u^{\mu} + \ldots = 0$$

with **bulk viscosity** $\zeta(T, \mu)$

- equation for **baryon diffusion current** $\nu^{\mu}$

$$\tau_{\text{heat}} \Delta_{\alpha \beta}^{\nu} u^{\mu} \nabla_{\mu} \nu^{\beta} + \nu^{\alpha} + \kappa \left[ \frac{nT}{\epsilon + p} \right]^{2} \Delta_{\alpha \beta}^{\nu} \partial_{\beta} \left( \frac{\mu}{T} \right) + \ldots = 0$$

with **heat conductivity** $\kappa(T, \mu)$
Transverse expansion

- for central collisions $\epsilon = \epsilon(\tau, r)$
- initial pressure gradient leads to radial flow
- fluid evolution equations for Israel-Stewart type theories

$$A_{ij}(\Phi, \tau, r) \frac{\partial}{\partial \tau} \Phi_j + B_{ij}(\Phi, \tau, r) \frac{\partial}{\partial r} \Phi_j + C_i(\Phi, \tau, r) = 0.$$  

- mathematically set of quasi-linear, first order partial differential equations
Characteristic velocities

[Floerchinger, Grossi (2017)]

- characteristic velocities $\lambda^{(n)}$ follow from
  \[
  \det \left( B - \lambda^{(n)} A \right) = 0
  \]

  \[
  \begin{align*}
  \lambda^{(1)} &= \frac{v + \tilde{c}}{1 + \tilde{c}v}, \\
  \lambda^{(2)} &= \frac{v - \tilde{c}}{1 - \tilde{c}v}, \\
  \lambda^{(3)} &= \lambda^{(4)} = \lambda^{(5)} = v
  \end{align*}
  \]

- equations hyperbolic if $\lambda^{(n)} \in \mathbb{R}$
- causal signal propagation for $\tilde{c} \leq 1$
- modified velocity of sound
  \[
  \tilde{c} = \sqrt{c_s^2 + d}
  \]

- ideal fluid velocity of sound
  \[
  \frac{c_s^2}{\partial} = \frac{\partial p}{\partial \epsilon}
  \]

- viscous correction
  \[
  d = \frac{4\eta}{3\tau_{\text{shear}}} + \frac{\zeta}{\tau_{\text{bulk}}} + \ldots \\
  \frac{\epsilon + p + \pi_{\text{bulk}} - \pi_{\phi} - \pi_{\eta}}
  \]
Domains of influence and dependence

Floerchinger, Grossi (2017)

- causality cones are space-, time- and state dependent!
- causality poses a bound to applicability of relativistic fluid dynamics
Causality as bound to fluid approximation

[Floerchinger, Grossi (2017)]

- Navier-Stokes initial conditions at $\tau_{\text{initial}} = 0.6 \text{ fm/c}$

- Navier-Stokes initial conditions at $\tau_{\text{initial}} = 0.1 \text{ fm/c}$

- causality violations for too large gradients!
The hydro “attractor”

- ratio of longitudinal to transverse “pressure” in Israel-Stewart theory

- approach to attractor governed by $\tau_{\text{shear}}$
- causality: non-hydrodynamic modes needed!
- also negative longitudinal “pressure” allowed by causality constraint
Mode-by-mode fluid dynamics

[Floerchinger, Wiedemann (2014), work in progress with E. Grossi, J. Lion]

- evolution of background & perturbations

- detailed understanding of perturbations
Conclusions

- high energy nuclear collisions produce a relativistic QCD fluid!
- fluid dynamics seems surprisingly universal
- experimental hints for collective flow also in pPb and pp collisions
- improved understanding of relativistic fluid dynamics
- causality: one must go beyond strict derivative expansion
- non-hydrodynamic modes needed
- bound on applicability posed by causality