THEORY OF MIXING+CPV

Alexander Lenz (IPPP Durham)
LHCP 2018 Bologna 4.6.2018
$|M_{12}|$, $|\Gamma_{12}|$ and $\phi = \arg(-M_{12}/\Gamma_{12})$ can be related to three observables:

- **Mass difference**: $\Delta M := M_H - M_L \approx 2|M_{12}|$ (off-shell)
  $|M_{12}|$ : heavy internal particles: t, SUSY, ...

- **Decay rate difference**: $\Delta \Gamma := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}| \cos \phi$ (on-shell)
  $|\Gamma_{12}|$ : light internal particles: u, c, ... *(almost)* no NP!!!

- **Flavor specific/semi-leptonic CP asymmetries**: e.g. $B_q \to Xl\nu$ *(semi-leptonic)*

$$a_{sl} = a_{fs} = \frac{\Gamma(\bar{B}_q(t) \to f) - \Gamma(B_q(t) \to \bar{f})}{\Gamma(\bar{B}_q(t) \to f) + \Gamma(B_q(t) \to \bar{f})} = \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin \phi$$
Total decay rate can be expanded in inverse powers of $m_b$

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \ldots$$

Each term in the series can be further expanded in the strong coupling

$$\Gamma_j = \Gamma_j^{(0)} + \frac{\alpha_s(\mu)}{4\pi}\Gamma_j^{(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2}\Gamma_j^{(2)} + \ldots$$

Each term is a product of a perturbative function and the matrix element of $\Delta B = 0$ operators \(\text{(lattice, sum rules)}\)

Mixing obeys a similar HQE

$$\Gamma_{12}^q = \left(\frac{\Lambda}{m_b}\right)^3\Gamma_3 + \left(\frac{\Lambda}{m_b}\right)^4\Gamma_4 + \ldots$$

Now $\Delta B = 2$ operators appear \(\text{(lattice, sum rules)}\)
<table>
<thead>
<tr>
<th></th>
<th>$\Gamma_3^{(0)}$</th>
<th>$\Gamma_3^{(1)}$</th>
<th>$\Gamma_3^{(2)}$</th>
<th>$\Gamma_4^{(0)}$</th>
<th>$\Gamma_4^{(1)}$</th>
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<tr>
<td></td>
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<tr>
<td><strong>Bs</strong></td>
<td>1985-1996</td>
<td>2002</td>
<td>x</td>
<td>2001</td>
<td>2003</td>
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STATUS BEFORE 2017

*Large uncertainties due to outdated non-perturbative input
*Perfect cancellation in Bs lifetime - test of NP models

\[
\frac{\tau(B^+)}{\tau(B_d)}^{\text{HQE 2014}} = 1.04^{+0.07}_{-0.03}, \quad 1.076 \pm 0.004
\]

\[
\frac{\tau(B_s)}{\tau(B_d)}^{\text{HQE 2014}} = 1.001 \pm 0.002, \quad 0.993 \pm 0.004
\]

\[
\frac{\tau(\Lambda_b)}{\tau(B_d)}^{\text{HQE 2014}} = 0.935 \pm 0.054, \quad 0.967 \pm 0.007
\]

\[
\frac{\tau(\Xi_b^0)}{\tau(\Xi_b^+)}^{\text{HQE 2014}} = 0.95 \pm 0.06, \quad 0.929 \pm 0.028
\]

<table>
<thead>
<tr>
<th>Observable</th>
<th>SM – conservative</th>
<th>SM – aggressive</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta M_s)</td>
<td>(18.3 \pm 2.7) ps(^{-1})</td>
<td>(20.11 \pm 1.37) ps(^{-1})</td>
<td>(17.757 \pm 0.021) ps(^{-1})</td>
</tr>
<tr>
<td>(\Delta \Gamma_s)</td>
<td>(0.088 \pm 0.020) ps(^{-1})</td>
<td>(0.098 \pm 0.014) ps(^{-1})</td>
<td>(0.082 \pm 0.006) ps(^{-1})</td>
</tr>
<tr>
<td>(a_{sL}^s)</td>
<td>(2.22 \pm 0.27) \cdot 10^{-5}</td>
<td>(2.27 \pm 0.25) \cdot 10^{-5}</td>
<td>(−7.5 \pm 4.1) \cdot 10^{-3}</td>
</tr>
</tbody>
</table>

Ideal for NP searches - experimental precision > theory precision!

On the ultimate precision of meson mixing observables
Thomas Jubb, Matthew Kirk, Alexander Lenz, Gilberto Tetlalmatzi-Xolocotzi
THEORY UNCERTAINTIES IN MIXING

3 dominant uncertainties:

\[ \langle R_2 \rangle = -\frac{2}{3} \frac{M_{B_s}^2}{m_b^{\text{pow2}}} - 1 \]
\[ R_2 = \frac{1}{m_b^2} s_\alpha D_\alpha \gamma^\mu (1 - \gamma_5) D^\beta b_\alpha \bar{s}_\beta \gamma^\mu (1 - \gamma_5) b_\beta \]

**Dim 7 has never been done**

\[ \langle Q \rangle = \langle \bar{B}_s^0 | Q | B_s^0 \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B(\mu) \]
\[ Q = \bar{s}^\alpha \gamma_{\mu} (1 - \gamma_5) b^\alpha \times \bar{s}^\beta \gamma_{\mu} (1 - \gamma_5) b^\beta \]

**Dim 6 is done on the lattice**

newest results (Fermilab MILC 1602:03560) indicate a small tension with experiment

**NNLO QCD has not been done**

CP violation in the Bs system

Marina Artuso, Guennadi Borissov, Alexander Lenz
Rev.Mod.Phys. 88 (2016) no.4,045002
### NEWS

|       | \( \Gamma_3^{(0)} \) | \( \Gamma_3^{(1)} \) | \( \Gamma_3^{(2)} \) | \( |\text{dim 6}| \) | \( \Gamma_4^{(0)} \) | \( \Gamma_4^{(1)} \) | \( |\text{dim 7}| \) |
|-------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| **B+** | 1985-1996            | 2002                | \( \times \)        | 2001                | 2003                | \( \times \)        | \( \times \)        |
| **Bs** | 1985-1996            | 2002                | \( \times \)        | 2001                | 2003                | \( \times \)        | \( \times \)        |
| **G12s** | 1985-1996          | 1998-2006            | \( \times \)        | -2016                | 1996                | \( \times \)        | \( \times \)        |
| **G12d** | 1985-1996          | 2003-2006            | \( \times \)        | -2016                | 2003                | \( \times \)        | \( \times \)        |

First steps: Asatrian et al 1709.02160
Sum rules: this talk, Kirk, Lenz, Rauh 1711.02100
HPQCD in progress, see LATTICE 2016, 2017
Sum rules: Kirk, Lenz, Rauh in progress
NEW RESULTS FOR NON-PERTURBATIVE PARAMETERS

all dim-6 Delta B = 0,2 operators

1 dim-6 Delta B =2 operator

PHYSICAL REVIEW D 94, 034024 (2016)

B^0 - \bar{B}^0 mixing at next-to-leading order

Andrey G. Grozin
Budker Institute of Nuclear Physics SB RAS, Novosibirsk 630090, Russia
and Novosibirsk State University, Novosibirsk 630090, Russia

Rebecca Klein, Thomas Mannel, and Alexei A. Pivovarov
Theoretische Elementarteilchenphysik, Naturwiss.- techn. Fakultät, Universität Siegen,
57068 Siegen, Germany
(Received 4 July 2016; published 11 August 2016)

We compute the perturbative corrections to the heavy quark effective theory sum rules for the
matrix element of the AB = 2 operator that determines the mass difference of B^0, \bar{B}^0 states. Technically,
we obtain analytically the nonfactorizable contributions at order \alpha_s to the bag parameter that first appear at the three-loop
level. Together with the known nonperturbative corrections due to vacuum condensates and 1/m_b
corrections, the full next-to-leading order result is now available. We present a numerical value for the
renormalization group invariant bag parameter that is phenomenologically relevant and compare it with
recent lattice determinations.

Dimension-six matrix elements for meson mixing
and lifetimes from sum rules

M. KIRK, A. LENZ and T. RAUH
IPPP, Department of Physics, University of Durham,
DH1 3LE, United Kingdom

Abstract

The hadronic matrix elements of dimension-six \Delta F = 0,2 operators are
-crucial inputs for the theory predictions of mixing observables and lifetime
-ratios in the B and D system. We determine them using HQET sum rules
for three-point correlators. The results of the required three-loop compu-
tation of the correlators and the one-loop computation of the QCD-HQET
matching are given in analytic form. For mixing matrix elements we find
very good agreement with recent lattice results and comparable theoretical
-uncertainties. For lifetime matrix elements we present the first ever deter-
mination in the D meson sector and the first determination of \Delta B = 0
-matrix elements with uncertainties under control - supereeding preliminary
lattice studies stemming from 2001 and earlier. With our state-of-the-art
determination of the bag parameters we predict: \tau(B^+) / \tau(B_0^+) = 1.082_{-0.026}^{+0.026},
\tau(B_s^+) / \tau(B_0^+) = 0.9994 \pm 0.0028, \tau(D^+) / \tau(D^0) = 2.7_{-0.5}^{+0.7}, and the mixing-
observables in the B_s and B_d system, in good agreement with the most
recent experimental averages.

Three-loop HQET vertex diagrams for B^0 - \bar{B}^0 mixing

Andrey G. Grozin and Roman N. Lee
Budker Institute of Nuclear Physics,
Novosibirsk 630090, Russia
E-mail: a.g.grozin@inp.nsk.su, r.n.lee@inp.nsk.su

ABSTRACT: Three-loop vertex diagrams in HQET needed for sum rules for B^0 - \bar{B}^0 mixing
are considered. They depend on two residual energies. An algorithm of reduction of these
diagrams to master integrals has been constructed. All master integrals are calculated
exactly in d dimensions; their \epsilon expansions are also obtained.

KEYWORDS: NLO Computations, B-Physics.
HQET SUM RULES

- Do all dim 6 and dim 7 operators for mixing AND lifetimes
- 3 loop diagrams with FIRE reduced (2 external momenta)
- Master integrals known: Grozin, Lee; hep-ph/0812.4522
- HQET running to scale $m_b$
- HQET-QCD matching at scale $m_b$

\[ \alpha_s \left( \Lambda_{\text{QCD}} \right) \sim O(1) \]
\[ \Rightarrow \text{non-perturbative} \]

\[ \begin{align*}
\text{Hadronic matrix element} & \\
\text{Characteristic scale: } \Lambda_{\text{QCD}} \\
\Rightarrow \text{non-perturbative} \\
\end{align*} \]

\[ \begin{align*}
\text{Sum rule} & \\
\text{Quark-hadron duality} \\
\text{Analyticity} \\
\end{align*} \]

\[ \begin{align*}
\text{Correlation function} & \\
\text{Characteristic scale: } \text{‘virtuality’ } \omega \\
\text{Choose } \omega \text{ s.t. } \alpha_s (\omega) \ll 1 \\
\Rightarrow \text{perturbatively calculable} \\
\end{align*} \]

1 mixing operator $Q$ done by Grozin, Klein, Mannel, Pivovarov
hep-ph/1606.06054

all Delta $B=0$ and 2 dim 6 operators
Kirk, Lenz, Rauh; 1711.02100

\[ \text{dominant uncertainty: today 2-loop} \ 1806.00253 \text{ Grozin, Mannel, Pivovarov} \]
NEW RESULTS 1: B-MIXING

- Very good agreement with lattice
- Comparable uncertainties as lattice: B-1 vs B
- Independent confirmation of FNAL/MILC vs ETM desirable
NEW RESULTS 2: B LIFETIMES

• Only modern determination - else: 2001

• Independent confirmation from lattice urgently needed!!!
NEW RESULTS 3: D MIXING

- Very good agreement with lattice
- Larger uncertainties than lattice
- First ever determination of D lifetimes!!!
**FINAL RESULTS: LIFETIMES**

- **HQE works for D lifetimes!** (roughly 30% precision)
  
  $$\frac{\tau(D^+)}{\tau(D^0)} = 2.7 = 1 + 16\pi^2 (0.25)^3 (1 - 0.34)$$

- **B+ and Bs lifetime ratios agree perfectly with experiment**

- **Confirmation from lattice urgently needed**
CONSEQUENCES FOR BSM MODELS
A popular BSM model for solving the anomalies related to loop-level (semi) leptonic decays are $Z'$ models:

Such a new tree-level transition will also affect many other observables, most notably **B-mixing at tree-level**, but also many loop processes.

Make sure all relevant bounds are included, e.g. electro-weak precision bounds.
Text-book: Bs mixing agrees with the SM

$$\Delta M_{s}^{\text{SM}, 2011} = (17.3 \pm 2.6) \text{ ps}^{-1}$$

$$\Delta M_{s}^{\text{SM}, 2015} = (18.3 \pm 2.7) \text{ ps}^{-1}$$

$$\Delta M_{s}^{\text{Exp}} = (17.757 \pm 0.021) \text{ ps}^{-1}$$

- BSM contributions have to be within the large theory uncertainties
- they can be both positive and negative
- relatively stringent bound on BSM models that explain the b$\rightarrow$s mu mu anomalies
NEW: Bs mixing “disagrees” with the SM

using most recent input, in particular most recent lattice values for fBs^2 B from FLAG (dominated by Fermilab/MILC)

\[ \Delta M_{s}^{\text{SM, 2017}} = (20.01 \pm 1.25) \text{ ps} \]

\[ \Delta M_{s}^{\text{Exp}} = (17.757 \pm 0.021) \text{ ps}^{-1} \]

BSM contributions should be **negative**

very stringent bound on many BSM models that explain the b-> s mu mu anomalies

\[ \frac{\Delta M_{s}^{\text{Exp}}}{\Delta M_{s}^{\text{SM}}} = 1 + \frac{\kappa}{\Lambda_{\text{NP}}^2} \]

\[ \frac{\Lambda_{\text{NP}}^{2017}}{\Lambda_{\text{NP}}^{2015}} = \sqrt{\frac{\Delta M_{s}^{\text{Exp}}}{(\Delta M_{s}^{\text{SM}} - 2\delta \Delta M_{s}^{\text{SM}})^{2015}}} - 1 \approx 5.2 \]
### Range of Mixing Predictions

**Bag parameter: SR**

**Decay constant: SR**

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<table>
<thead>
<tr>
<th>Source</th>
<th>$f_{B_s} \sqrt{\hat{B}}$</th>
<th>$\Delta M_s^{SM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPQCD14 [132]</td>
<td>$(247 \pm 12)$ MeV</td>
<td>$(16.2 \pm 1.7)$ ps$^{-1}$</td>
</tr>
<tr>
<td>ETMCI3 [133]</td>
<td>$(262 \pm 10)$ MeV</td>
<td>$(18.3 \pm 1.5)$ ps$^{-1}$</td>
</tr>
<tr>
<td>HPQCD09 [134] = FLAG13 [135]</td>
<td>$(266 \pm 18)$ MeV</td>
<td>$(18.9 \pm 2.6)$ ps$^{-1}$</td>
</tr>
<tr>
<td><strong>FLAG17</strong> [70]</td>
<td><strong>$(274 \pm 8)$ MeV</strong></td>
<td><strong>$(20.01 \pm 1.25)$ ps$^{-1}$</strong></td>
</tr>
<tr>
<td>Fermilab16 [72]</td>
<td>$(274.6 \pm 8.8)$ MeV</td>
<td>$(20.1 \pm 1.5)$ ps$^{-1}$</td>
</tr>
<tr>
<td>HQET-SR [77, 136]</td>
<td>$(278^{+28}_{-24})$ MeV</td>
<td>$(20.6^{+4.4}_{-3.4})$ ps$^{-1}$</td>
</tr>
<tr>
<td>HPQCD06 [137]</td>
<td>$(281 \pm 20)$ MeV</td>
<td>$(21.0 \pm 3.0)$ ps$^{-1}$</td>
</tr>
<tr>
<td>RBC/UKQCD14 [138]</td>
<td>$(290 \pm 20)$ MeV</td>
<td>$(22.4 \pm 3.4)$ ps$^{-1}$</td>
</tr>
<tr>
<td>Fermilab11 [139]</td>
<td>$(291 \pm 18)$ MeV</td>
<td>$(22.6 \pm 2.8)$ ps$^{-1}$</td>
</tr>
</tbody>
</table>

**C. Tarantino, UTfit**

\[
\Delta M_s^{\text{exp}} = (17.757 \pm 0.021) \text{ ps}\,^{-1}, \\
\Delta M_s^{SM} = (18.3 \pm 1.2 \text{ (had.)} \\
\pm 0.1 \text{ (scale)} \\
\pm 0.2 \text{ (param.)}) \text{ ps}\,^{-1},
\]

\[
\Delta \Gamma_s^{\text{exp}} = (0.090 \pm 0.005) \text{ ps}\,^{-1}, \\
\Delta \Gamma_s^{PS} = (0.087 \pm 0.020 \text{ (had.)} \\
+0.008 \text{ (scale)} \\
+0.001 \text{ (param.)}) \text{ ps}\,^{-1},
\]

\[
a_{s_{l_{s_{PS}}}^{\text{exp}}} = (-60 \pm 280) \cdot 10^{-5}, \\
a_{s_{l_{s}}} = (1.8 \pm 0.0 \text{ (had.)} \\
\pm 0.1 \text{ (scale)} \\
\pm 0.1 \text{ (param.)}) \cdot 10^{-5},
\]

---

**No mixing hypothesis**
One constraint to kill them all?

Luca Di Luzio, Matthew Kirk, Alexander Lenz

Institute for Particle Physics Phenomenology, Durham University,
DH1 3LE Durham, United Kingdom
luca.di-luzio@durham.ac.uk, m.j.kirk@durham.ac.uk, alexander.lenz@durham.ac.uk

Abstract

Many BSM models that explain the intriguing anomalies in the quark flavour sector are severely constrained by $B_s$-mixing, for which the SM prediction and experiment agreed well until recently. New non-perturbative calculations point, however, in the direction of a tiny discrepancy in this observable. Using this new input we find a considerable shift of the bounds on BSM models stemming from $B_s$-mixing.

\[
\frac{\Delta M_s^{\text{Exp}}}{\Delta M_s^{\text{SM}}} = 1 + \frac{C_{bs}^{LL}}{R_{\text{SM}}^{\text{loop}}} 
\]

\[
C_{bs}^{LL} = \frac{\eta_{LL}^{(M_{Z'})} \left( \frac{\lambda_{23}^Q}{V_{tb} V_{ts}^*} \right)^2}{4 \sqrt{2} G_F M_{Z'}^2}
\]

**FIG. 2.** Bounds from $B_s$-mixing on the parameter space of the simplified $Z'$ model of Eq. (20), for real $\lambda_{23}^Q$ and $\lambda_{22}^Q = 1$. The blue and red shaded areas correspond respectively to the $2\sigma$ exclusions from $\Delta M_{s,2015}^{\text{SM}}$ and $\Delta M_{s,2017}^{\text{SM}}$, while the solid (dashed) black curves encompass the $1\sigma$ ($2\sigma$) best-fit region from $R_{K(\ast)}$.

**FIG. 3.** Bounds from $B_s$-mixing on the parameter space of the scalar leptoquark model of Eq. (24), for real $y_{32}^{QL}$, $y_{22}^{QL}$ couplings. Meaning of shaded areas and curves as in Fig. 2.
BSM PHYSICS IS ON THE HORIZON?

* Look for Z’ models with complex couplings

First idea to avoid positive contributions to M_{12}:
Look for CP violation couplings of a Z’
strong constraints from the phase for Bs mixing

\[ B_s \rightarrow J/\psi \phi \]
TAKE HOME MESSAGES

Status Quo:

➤ Shape of HQE is getting better and better
   Lifetimes and mixing confirm HQE - no sign of duality violation

➤ Even a convergence in the D system seems to be plausible
   If confirmed, then next goal: understand D-mixing
   Remember: 20% of duality violation are sufficient to explain discrepancy in HQE approach

➤ Latest lattice results point towards a slight discrepancy in Bs mixing -> severe BSM constraint

Next steps:

➤ Lifetime of Bs should be known even more precisely from experiment

➤ Need lattice/SR results for dim 6, 7 operators for Delta B,C = 0,2

➤ NNLO calculations will soon be necessary

➤ Do baryon lifetimes
SINCE YEARS OF BEGGING DID NOT HELP – IT’S TIME TO PROVOKE

Lifetimes are too heavy for lattice physicists!

The strongest lattice researcher alive

Arbitrary sum rule researcher

Matrix elements for lifetimes of HEAVY mesons