# Utilising $B \rightarrow \pi K$ decays at the High-Precision Frontier 

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# Introduction to 

$B \rightarrow \pi K$
decays

## Phenomenology

* Tree topologies suppressed by


CKM element $V_{u b}$

* Leading contribution from QCD penguins
* CA EW penguins at same level as tree topologies
* QCD flavour symmetry to link topologies



## $B \rightarrow \pi K$ decays

* Decays in the spotlight for over 2 decades
- Particular $B_{d}^{0} \rightarrow \pi^{0} K_{\mathrm{S}}$ interesting: only channel with

mixing-induced CP asymmetry
* Puzzling data in correlation between CP asymmetries
[R. Fleischer, S. Jäger, D. Pirjol, J. Zupan (2008)]
* Modified EWP sector?



## $B \rightarrow \pi K$ decays

* What is the status of these decays?
* Little attention in recent years:
* Neutral final state challenging for LHCb, good potential for upcoming Belle II experiment
* Difficult from theory side (QCD), but we can learn a lot!


## We shall provide the state-of-the-art picture

## $B \rightarrow \pi K$ decays in detail

## Amplitudes

## Neglect small

* General parametrization: [A. J. Buras, R. Fleischer, S. Recksiegel, F. Schwab (2004)]

$$
\begin{aligned}
A\left(B^{+} \rightarrow \pi^{+} K^{0}\right) & =-P^{\prime}\left[1+\rho_{\mathrm{c}} e^{i \theta_{\mathrm{c}}} e^{i \gamma}\right] \\
\sqrt{2} A\left(B^{+} \rightarrow \pi^{0} K^{+}\right) & =P^{\prime}\left[1+\rho_{\mathrm{c}} e^{i \theta_{\mathrm{c}}} e^{i \gamma}-\left(e^{i \gamma}-q e^{i \phi} e^{i \omega}\right) r_{\mathrm{c}} e^{i \delta_{\mathrm{c}}}\right] \\
A\left(B_{d}^{0} \rightarrow \pi^{-} K^{+}\right) & =P^{\prime}\left[1-r e^{i \delta} e^{i \gamma}\right] \\
\sqrt{2} A\left(B_{d}^{0} \rightarrow \pi^{0} K^{0}\right) & =-P^{\prime}\left[1+\rho_{\mathrm{n}} e^{i \theta_{\mathrm{n}}} e^{i \gamma}-q e^{i \phi} e^{i \omega} r_{\mathrm{c}} e^{i \delta_{\mathrm{c}}}\right]
\end{aligned}
$$

* CP-conserving strong amplitude $P^{\prime}=\left(1-\lambda^{2} / 2\right) A \lambda^{2}\left(P_{t}-P_{c}\right)$
* Amplitudes satisfy isospin relation (Wolfenstein parametrization)

$$
\begin{gathered}
\sqrt{2} A\left(B_{d}^{0} \rightarrow \pi^{0} K^{0}\right)+A\left(B_{d}^{0} \rightarrow \pi^{-} K^{+}\right)= \\
\sqrt{2} A\left(B^{+} \rightarrow \pi^{0} K^{+}\right)+A\left(B^{+} \rightarrow \pi^{+} K^{0}\right)=3 A_{3 / 2} \\
3 A_{3 / 2} \equiv 3\left|A_{3 / 2}\right| e^{i \phi_{3 / 2}}=-(\hat{T}+\hat{C})\left(e^{i \gamma}-q e^{i \phi} e^{i \omega}\right)
\end{gathered}
$$

## Amplitudes

* Hadronic parameters:

Reminder:
T: colour-allowed (CA) tree C: colour-suppressed (CS) tree $P$ : QCD penguin
$r e^{i \delta} \equiv\left(\frac{\lambda^{2} R_{b}}{1-\lambda^{2}}\right)\left[\frac{T-\left(P_{t}-P_{u}\right)}{P_{t}-P_{c}}\right], \quad \rho_{c} e^{i \theta_{c}} \equiv\left(\frac{\lambda^{2} R_{b}}{1-\lambda^{2}}\right)\left[\frac{P_{t}-P_{u}}{P_{t}-P_{c}}\right] \approx 0$,
$r_{\mathrm{c}} e^{i \delta_{\mathrm{c}}} \equiv\left(\frac{\lambda^{2} R_{b}}{1-\lambda^{2}}\right)\left[\frac{T+C}{P_{t}-P_{c}}\right], \quad \rho_{\mathrm{n}} e^{i \theta_{\mathrm{n}}} \equiv\left(\frac{\lambda^{2} R_{b}}{1-\lambda^{2}}\right)\left[\frac{C+\left(P_{t}-P_{u}\right)}{P_{t}-P_{c}}\right]=r_{\mathrm{c}} e^{i \delta_{\mathrm{c}}}-r e^{i \delta}$

* $r_{\mathrm{c}} e^{i \delta_{c}}, r e^{i \delta}$ are non-perturbative, challenging to calculate from first principles
* Use $B \rightarrow \pi \pi$ and $\operatorname{SU(3)}$ flavour symmetry [A. J. Buras, R. Fleischer, S. Recksiegel, F. Schwab (2004)]

$$
\begin{aligned}
r_{\mathrm{c}} e^{i \delta_{\mathrm{c}}} & =(0.17 \pm 0.06) e^{i(1.9 \pm 23.9)^{\circ}} \\
r e^{i \delta} & =(0.09 \pm 0.03) e^{i(28.6 \pm 21.4)^{\circ}}
\end{aligned}
$$

* Assumes 20\% non-factorizable SU(3)-breaking corrections (guided by data)


## Electroweak penguins

* The parameter $q e^{i \phi} e^{i \omega}$ describes EW penguin effects:

$$
\begin{gathered}
q e^{i \phi} e^{i \omega} \equiv-\left(\frac{\hat{P}_{E W}+\hat{P}_{E W}^{\mathrm{C}}}{\hat{T}+\hat{C}}\right) \\
\begin{array}{c}
\text { CP-violating phase } \\
\text { CP-conserving phase, } \\
\text { vanishes in } \operatorname{SU(}) \text { limit }
\end{array}
\end{gathered}
$$

Short-distance coefficients

# $B \rightarrow \pi K$ observables 

## Branching ratios

$$
\begin{aligned}
& \text { Experiment: } \\
& R_{\mathrm{c}}=1.09 \pm 0.06, \\
& R_{\mathrm{n}}=0.99 \pm 0.06, \\
& R=0.89 \pm 0.04 \\
& \quad[\operatorname{PDG}(2016)]
\end{aligned}
$$

* First observables:
(Ratios of) CP-averaged branching ratios [A. J. Buras, R. Fleischer, S. Recksiegel, F. Schwab (2004)]

$$
\begin{aligned}
& R_{\mathrm{c}} \equiv 2\left[\frac{\mathscr{B} r\left(B^{ \pm} \rightarrow \pi^{0} K^{ \pm}\right)}{\mathscr{B} r\left(B^{ \pm} \rightarrow \pi^{ \pm} K\right)}\right]=1-2 r_{\mathrm{c}} \cos \delta_{\mathrm{c}}(\cos \gamma-q \cos \phi)+\mathcal{O}\left(r_{\mathrm{c}}^{2}\right), \\
& R_{\mathrm{n}} \equiv \frac{1}{2}\left[\frac{\mathscr{B} r\left(B_{d} \rightarrow \pi^{\mp} K^{ \pm}\right)}{\mathscr{B} r\left(B_{d} \rightarrow \pi^{0} K\right)}\right]=1-2 r_{\mathrm{c}} \cos \delta_{\mathrm{c}}(\cos \gamma-q \cos \phi)+\mathcal{O}\left(r_{\mathrm{c}}^{2}\right), \\
& R \equiv\left[\frac{\mathscr{B} r\left(B_{d} \rightarrow \pi^{\mp} K^{ \pm}\right)}{\mathscr{B} r\left(B^{ \pm} \rightarrow \pi^{ \pm} K\right)}\right] \frac{\tau_{B^{ \pm}}}{\tau_{B_{d}}}=1-2 r \cos \delta \cos \gamma+2 r_{\mathrm{c}} \tilde{a}_{\mathrm{C}} q \cos \phi+\mathcal{O}\left(r_{(\mathrm{c})}^{2}\right)
\end{aligned}
$$

Colour-suppressed (CS) EWP parameter $\tilde{\mathrm{a}}_{\mathrm{C}} \equiv a_{\mathrm{C}} \cos \left(\Delta_{\mathrm{C}}+\delta_{\mathrm{c}}\right)$

* We obtain the relation: $R_{\mathrm{c}}-R_{\mathrm{n}}=0+\mathcal{O}\left(r_{\mathrm{c}}^{2}\right)$
* Is satisfied experimentally at the $1 \sigma$ level


## Direct CP asymmetries

* Interference of penguin and tree
$\Rightarrow$ direct $C P$ asymmetry $A_{\mathrm{CP}}^{f}$
* Proportional to $r_{(\mathrm{c})} \sin \delta_{(\mathrm{c})} \rightarrow$ values at $\mathcal{O}(\mathbf{1 0 \%})$ level

Experiment: [PDG (2016)]

$$
A_{\mathrm{CP}}^{\pi^{0} K^{0}}=0.00 \pm 0.13
$$

$$
A_{\mathrm{CP}}^{\pi^{+} K^{0}}=-0.017 \pm 0.016
$$

$$
A_{\mathrm{CP}}^{K^{+} \pi^{0}}=0.037 \pm 0.021
$$

$$
A_{\mathrm{CP}}^{\pi^{-K^{+}}}=-0.082 \pm 0.006
$$

* Direct CP asymmetries and branching ratios satisfy sum rule:
[M. Gronau (2005); M. Gronau, J. L. Rosner (2006)]

$$
\begin{aligned}
\Delta_{\mathrm{SR}} \equiv & {\left[A_{\mathrm{CP}}^{\pi^{+} K^{0}} \frac{\mathscr{B} r\left(\pi^{+} K^{0}\right)}{\mathscr{B} r\left(\pi^{-} K^{+}\right)}-A_{\mathrm{CP}}^{\pi^{0} K^{+}} \frac{2 \mathscr{B} r\left(\pi^{0} K^{+}\right)}{\mathscr{B} r\left(\pi^{-} K^{+}\right)}\right] \frac{\tau_{B_{d}}}{\tau_{B^{ \pm}}} } \\
& +A_{\mathrm{CP}}^{\pi^{-} K^{+}}-A_{\mathrm{CP}}^{\pi^{0} K^{0}} \frac{2 \mathscr{B} r\left(\pi^{0} K^{0}\right)}{\mathscr{B} r\left(\pi^{-} K^{+}\right)}=0+\mathscr{O}\left(r_{(c)}^{2}\right)
\end{aligned}
$$

Difficult for LHCb

* Satisfied experimentally at $1 \sigma$ level but uncertainty large due to $A_{\mathrm{CP}}^{\pi^{0} K^{0}}$
* Experimental uncertainty at Belle II $\rightarrow \pm \mathbf{0 . 0 4}$ [Belle-II Collaboration, arXiv:1011.0352]
* Prediction from sum rule: $A_{\mathrm{CP}}^{\pi^{0} K^{0}}=-0.14 \pm 0.03$


## Mixing-induced CP asymmetry

* $\quad B_{d}^{0} \rightarrow \pi^{0} K^{0}$ is special $\rightarrow$ only channel with mixing-induced CP asymmetry
- Arises from interference between $B_{d}^{0}-\bar{B}_{d}^{0}$ mixing and decay
- Just like $A_{\mathrm{CP}}^{\pi^{0} K^{0}}$, also difficult for $\mathrm{LHCb} \rightarrow$ large uncertainty
* Also great prospects for Belle II


$$
S_{\mathrm{CP}}^{\pi^{0} K_{S}}=0.58 \pm 0.17\left[{ }_{[P D G}^{(2006]}\right]
$$

## Mixing-induced CP asymmetry

- Follows from time-dependent rate asymmetry:

$$
\frac{\Gamma\left(\bar{B}_{d}^{0}(t) \rightarrow \pi^{0} K_{\mathrm{S}}\right)-\Gamma\left(B_{d}^{0}(t) \rightarrow \pi^{0} K_{\mathrm{S}}\right)}{\Gamma\left(\bar{B}_{d}^{0}(t) \rightarrow \pi^{0} K_{\mathrm{S}}\right)+\Gamma\left(B_{d}^{0}(t) \rightarrow \pi^{0} K_{\mathrm{S}}\right)}=A_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}} \cos \left(\Delta M_{d} t\right)+S_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}} \sin \left(\Delta M_{d} t\right)
$$

with $\Delta M_{d}$ mass difference $B_{d}$ mass eigenstates
[A. J. Buras, R. Fleischer (1999); R. Fleischer, S. Jäger, D. Pirjol, J. Zupan (2008)]

$$
\underbrace{S_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}} \sin \left(\phi_{d}-\phi_{00}\right) \sqrt{1-\left(A_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}}\right)^{2}}}_{\text {spred in } B_{d}^{0} \rightarrow J / \psi K_{\mathrm{S}} \quad \phi_{00} \equiv \arg \left(\overline{\mathrm{~A}}_{00} \mathrm{~A}_{00}^{*}\right)}
$$

* Angle given by

CS EWP parameter

$$
\tan \phi_{00}=2\left(r \cos \delta-r_{\mathrm{c}} \cos \delta_{\mathrm{c}}\right) \sin \gamma+2 r_{\mathrm{c}}\left(\cos \delta_{\mathrm{c}}-2 \tilde{a}_{\mathrm{C}} / 3\right) q \sin \phi+\mathcal{O}\left(r_{(\mathrm{c})}^{2}\right)
$$

What is the best way to calculate $\phi_{00}$ ?

## Isospin relation

* We may use the isospin relation:

$$
\begin{gathered}
\sqrt{2} A\left(B_{d}^{0} \rightarrow \pi^{0} K^{0}\right)+A\left(B_{d}^{0} \rightarrow \pi^{-} K^{+}\right) \equiv 3 A_{3 / 2} \\
3 A_{3 / 2} \equiv 3\left|A_{3 / 2}\right| e^{i \phi_{3 / 2}}=-(\hat{T}+\hat{C})\left(e^{i \gamma}-q e^{i \phi} e^{i \omega}\right)
\end{gathered}
$$

* $\phi_{00}$ follows from amplitude triangles
- If $q$ and $\phi$ are known, only $\operatorname{SU}(3)$ input for:

$$
\begin{gathered}
|\hat{T}+\hat{C}|=R_{T+C}\left|\frac{V_{u s}}{V_{u d}}\right| \sqrt{2}\left|A\left(B^{+} \rightarrow \pi^{+} \pi^{0}\right)\right| \\
R_{T+C} \approx f_{K} \mid f_{\pi}=1.2 \pm 0.2
\end{gathered}
$$

* Minimal hadronic input

Unitarity triangle angle $\gamma$ as input


## Correlation between CP asymmetries

* We may now use

$$
S_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}}=\sin \left(\phi_{d}-\phi_{00}\right) \sqrt{1-\left(A_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}}\right)^{2}}
$$

to obtain a correlation between $S_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}}$ and $A_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}}$

* Discrepancy with SM in 2008 [R. Fleischer, S. Jäger, D. Pirjol, J. Zupan (2008)]


## What is the current status?



## Correlation between CP asymmetries



Sharper inputs $(\gamma) \rightarrow$ discrepancy stronger!

## Puzzling patterns

$$
\begin{gathered}
\text { New aspect: } \phi_{ \pm}=\arg \left(\overline{\mathrm{A}}_{ \pm} \mathrm{A}_{ \pm}^{*}\right) \\
\left.\phi_{ \pm}\right|_{\phi=0}=2 r \cos \delta \sin \gamma+\mathcal{O}\left(r^{2}\right)=(8.7 \pm 3.5)^{\circ}
\end{gathered}
$$

Also the correlation is inconsistent!



## Current status

$$
\text { State-of-the-art analysis of } S_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}} \text { : }
$$

* Problem with measurements? Discrepancy could be solved if
- CP asymmetries $B_{d}^{0} \rightarrow \pi^{0} K_{\mathrm{S}}$ move by $\sim 1 \sigma$
* $\mathscr{B} r\left(B_{d} \rightarrow \pi^{0} K^{0}\right)$ moves by $\sim 2.5 \sigma$
* Or is it New Physics? $\rightarrow$ Study possibility of a modified EWP sector

With future data from LHCb (upgrade) and Belle II the situation should be resolved

## Determination of $q$ and $\phi$

* Use the amplitude triangles in a different way: convert $S_{\mathrm{CP}}^{\pi^{0} K_{s}}$ into $q$ and $\phi$
* The isospin relation holds also for neutral as well as charged decays:

$$
\begin{gathered}
\sqrt{2} A\left(B_{d}^{0} \rightarrow \pi^{0} K^{0}\right)+A\left(B_{d}^{0} \rightarrow \pi^{-} K^{+}\right)= \\
\sqrt{2} A\left(B^{+} \rightarrow \pi^{0} K^{+}\right)+A\left(B^{+} \rightarrow \pi^{+} K^{0}\right)=3 A_{3 / 2} \\
3 A_{3 / 2} \equiv 3\left|A_{3 / 2}\right| e^{i \phi_{3 / 2}}=-(\hat{T}+\hat{C})\left(e^{i \gamma}-q e^{i \phi} e^{i \omega}\right)
\end{gathered}
$$

* Current data is better for charged decays, but the method works for both.
* Derive a set of equations for contours in $q, \phi$-plane

$$
\begin{aligned}
q & =\sqrt{N^{2}-2 c \cos \gamma-2 s \sin \gamma+1} \\
\tan \phi & =\frac{\sin \gamma-s}{\cos \gamma-c}, \quad q \sin \phi=\sin \gamma-s
\end{aligned}
$$

* where

$$
\begin{gathered}
c \equiv \pm N \cos \left(\Delta \phi_{3 / 2} / 2\right), \quad s \equiv \pm N \sin \left(\Delta \phi_{3 / 2} / 2\right) \\
N \equiv 3\left|A_{3 / 2}\right| /|\hat{T}+\hat{C}|, \quad \Delta \phi_{3 / 2} \equiv \phi_{3 / 2}-\bar{\phi}_{3 / 2}
\end{gathered}
$$

## Determination of $q$ and $\phi$

$$
\text { assume } \omega=0^{\circ}
$$

(robust assumption)

* This method requires minimal $S U(3)$ input, only from

$$
\begin{gathered}
|\hat{T}+\hat{C}|=R_{T+C}\left|\frac{V_{u s}}{V_{u d}}\right| \sqrt{2}\left|A\left(B^{+} \rightarrow \pi^{+} \pi^{0}\right)\right| \\
R_{T+C} \approx f_{K} \mid f_{\pi}=1.2 \pm 0.2
\end{gathered}
$$

## No topologies have to be neglected

* Need to fix relative orientation triangles:

$$
\phi_{+0} \equiv \arg \left(\bar{A}_{+0} A_{+0}^{*}\right) \approx 0(\text { charged }) \text { or } S_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}}(\text { neutral })
$$

# Results of the new strategy 

## Results for current data

Apply method to charged data as current uncertainty $S_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}}$ still large
$\Rightarrow$ Potential to implement also for neutral data in the future!


## Results for current data

Complement analysis with:

$$
R_{\mathrm{c}}=1-2 r_{\mathrm{c}} \cos \delta_{\mathrm{c}}(\cos \gamma-q \cos \phi)+\mathcal{O}\left(r_{\mathrm{c}}^{2}\right)
$$

CS EWPs only at $\mathcal{O}\left(r_{\mathrm{c}}^{2}\right)$
$\rightarrow$ contour in $q, \phi$-plane
Excellent agreement
Further input needed to determine the value of $q$ and $\phi$

## Additional contour from $S^{\pi^{0} K_{\mathrm{S}}}$ CP

* Convert measurement of $S_{\mathrm{CP}}^{\pi^{0} K_{s}}$ in value of $\phi_{00}$
* Obtain contour from cosines of small phases $\rightarrow$
low sensitivity to variations

$$
\tan \phi_{00}=2\left(r \cos \delta-r_{\mathrm{c}} \cos \delta_{\mathrm{c}}\right) \sin \gamma+2 r_{\mathrm{c}}\left(\cos \delta_{\mathrm{c}}-2 \tilde{a}_{\mathrm{C}} / 3\right) q \sin \phi+\mathcal{O}\left(r_{(\mathrm{c})}^{2}\right)
$$

* CS EWP parameter $\tilde{a}_{\mathrm{C}} \equiv a_{\mathrm{C}} \cos \left(\Delta_{\mathrm{C}}+\delta_{\mathrm{c}}\right)$ is determined from

$$
R=1-2 r \cos \delta \cos \gamma+2 r_{\mathrm{c}} \tilde{a}_{\mathrm{C}} q \cos \phi+\mathcal{O}\left(r_{(\mathrm{c})}^{2}\right)
$$

* Consider 3 different scenarios for measurements of $S_{\mathrm{CP}}^{\pi^{0} K_{S}}$ at Belle II


## Future scenarios



## Future scenarios

* Precision depends on region in parameter space
* Potential for discovery of NP with future data!




## Resolution of $B \rightarrow \pi K$ puzzle

* Can we now resolve the $B \rightarrow \pi K$ puzzle?
- Consider 2 scenarios:

Theory prediction $S_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{s}}}$



## Conclusions

* Data from $B \rightarrow \pi K$ decays have shown puzzling patterns in the past
* We have performed a state-of-the-art analysis:


## Discrepancy became stronger $\rightarrow$ something has to happen

* Data move to eventually confirm the Standard Model?
* Is it New Physics?
* We have presented a new strategy to pin down the EWP parameters
* We look forward to data from Belle II and LHCb!


## Backup slides

## What about the sum rule?

* Belle II performed feasibility study of the sum rule [Belle-II Collaboration, arXiv:1011.0352]

$$
\begin{aligned}
\Delta_{\mathrm{SR}} \equiv & {\left[A_{\mathrm{CP}}^{\pi^{+} K^{0}} \frac{\mathscr{B} r\left(\pi^{+} K^{0}\right)}{\mathscr{B} r\left(\pi^{-} K^{+}\right)}-A_{\mathrm{CP}}^{\pi^{0} K^{+}} \frac{2 \mathscr{B} r\left(\pi^{0} K^{+}\right)}{\mathscr{B} r\left(\pi^{-} K^{+}\right)}\right] \frac{\tau_{B_{d}}}{\tau_{B^{ \pm}}} } \\
& +A_{\mathrm{CP}}^{\pi^{-} K^{+}}-A_{\mathrm{CP}}^{\pi^{0} K^{0}} \frac{2 \mathscr{B} r\left(\pi^{0} K^{0}\right)}{\mathscr{B} r\left(\pi^{-} K^{+}\right)}=0+\mathscr{O}\left(r_{(c)}^{2}\right)
\end{aligned}
$$

* Could it reveal $q$ and $\phi$ ?


The resolution is not sufficient for $q<3$
[R. Fleischer, RJ, E. Malami, K. K. Vos; to appear]

## Prediction for $\phi=0$

* We can define

$$
(\sin 2 \beta)_{\pi^{0} K_{\mathrm{S}}} \equiv \frac{S_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}}}{\sqrt{1-\left(A_{\mathrm{CP}}^{\pi^{0} K_{\mathrm{S}}}\right)^{2}}}=\sin \left(\phi_{d}-\phi_{00}\right)
$$

Only CS EWPs in

* In the SM we have $\phi=0$, yielding

$$
\tan \phi_{00}=2\left(r \cos \delta-r_{\mathrm{c}} \cos \delta_{\mathrm{c}}\right) \sin \gamma+\mathcal{O}\left(r_{(\mathrm{c})}^{2}\right)
$$

* From the $B \rightarrow \pi \pi$ data we then find

$$
(\sin 2 \beta)_{\pi^{0} K_{\mathrm{S}}}=0.80 \pm 0.06
$$

