QED corrections to $B_s \to \mu\bar{\mu}$

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With M.Beneke and R.Szafron, arXiv:1708.09157

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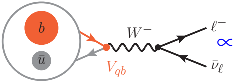
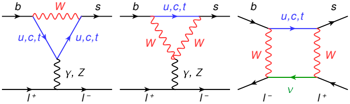
Motivation to study $B_q \to \ell\bar{\ell}$ and $B_u \to \ell\bar{\nu}_\ell$

- **Test CKM in SM** at tree- (CC) and loop-level (FCNC)



$\propto |V_{ub}|^2$



$\propto |V_{tb} V_{tq}^*|^2$

- **Helicity suppression of SM** $\Rightarrow$ sensitive to NP (pseudo-) scalar interactions

$$Br(B_q \to \ell\bar{\ell}) \propto |V_{tb} V_{tq}^*|^2 \; f_{B_q}^2 \times \left\{ \left| \frac{2m_\ell}{m_{B_q}} \left(C_{10} - C_{10}'\right) + \left(C_P - C_P'\right) \right|^2 + \beta_\ell^2 \left| C_S - C_S' \right|^2 \right\}$$

- **Hadronic uncertainty** $\Rightarrow$ from decay constant (at LO in QED)

$$\langle 0 | \bar{q} \gamma_\mu \gamma_5 b | \overline{B}_q(p) \rangle = i \, f_{B_q} p_\mu$$

$\Rightarrow$ from lattice in future $\delta f_{B_q} \lesssim 0.5\%$

$f_{B_u} = (189.4 \pm 1.4)$ MeV $\quad f_{B_s} = (230.7 \pm 1.2)$ MeV $\quad$ [FNAL/MILC 1712.09262]

$\Rightarrow$ **theoretical control of $\delta Br \sim 1\%$ possible**

!!! only other compareable precision in flavor: $Br(K^+ \to \pi^+ \nu\bar{\nu})$ (NA62), $Br(K_L \to \pi^0 \nu\bar{\nu})$ (KOTO), $\Delta M_{d,s}$ (lattice)

Need to include also small effects, like QED etc.

Analysing NP in $B_s \to \mu\bar{\mu}$ via time-dependence

3 CP asymmetries

$$\frac{\Gamma\left[B_s(t) \to \mu_\lambda \bar{\mu}_\lambda\right] - \Gamma\left[\overline{B}_s(t) \to \mu_\lambda \bar{\mu}_\lambda\right]}{\Gamma\left[B_s(t) \to \mu_\lambda \bar{\mu}_\lambda\right] + \Gamma\left[\overline{B}_s(t) \to \mu_\lambda \bar{\mu}_\lambda\right]} = \frac{\mathcal{C}^\lambda \cos(\Delta M_s t) + \mathcal{S}^\lambda \sin(\Delta M_s t)}{\cosh(y_s t/\tau_{B_s}) + \mathcal{A}^\lambda_{\Delta\Gamma} \sinh(y_s t/\tau_{B_s})}$$

- $\mathcal{A}_{\Delta\Gamma}$ ... without flavor tagging
  $\mathcal{S}$ ... requires flavor-tagging
  $\mathcal{C}^\lambda$ ... requires helicity of leptons

$$\left|\mathcal{C}^\lambda\right|^2 + \left|\mathcal{S}^\lambda\right|^2 + \left|\mathcal{A}^\lambda_{\Delta\Gamma}\right|^2 = 1$$

- in **SM "clean" observables** (at LO QED): $\mathcal{A}_{\Delta\Gamma} = 1$ $\quad \mathcal{S} = 0$ $\quad \mathcal{C}^\lambda = 0$
  $\Rightarrow$ QED corrections are SM background to NP contributions

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Distinguishing NP

[Fleischer/Galarraga Espinosa/Jaarsma/Tetlalmatzi-Xolocotzi 1709.04735]

- even measurement of $\text{sgn}(\mathcal{C}^\lambda)$ can reduce degeneracy

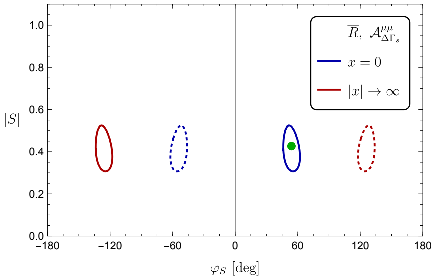
  Benchmark measurement

  $\mathcal{A}_{\Delta\Gamma} = +0.58 \pm 0.20$
  $\mathcal{S} = -0.80 \pm 0.20$

  $\to$ 4 solutions from Br and $\mathcal{A}_{\Delta\Gamma}$

  dashed: ruled out by $\mathcal{S}$

  blue: ruled out by $\text{sgn}(\mathcal{C}^\lambda)$



Measurement and experimental prospects

Experimental measurement

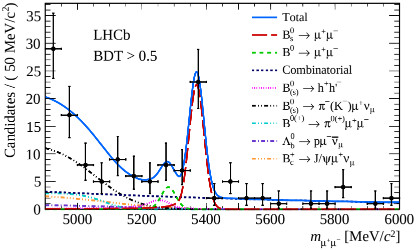
$$\overline{Br}(B_s \to \mu\bar{\mu}) = (3.0 \pm 0.5) \times 10^{-9}$$

$$\overline{Br}(B_d \to \mu\bar{\mu}) < 3.4 \times 10^{-10} \text{ @ } 95\%$$

$$\mathcal{A}_{\Delta\Gamma}(B_s \to \mu\bar{\mu}) = 8.24 \pm 10.72$$

[CMS 1307.5025, LHCb 1307.5024, 1703.05747]

LHCb → mass-eigenstate rate asymmetry



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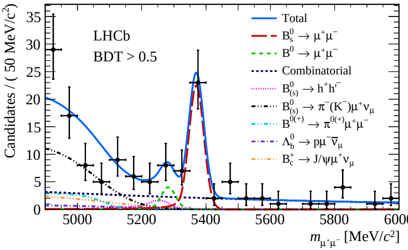


Experimental prospects

Events in channel	Run I	Run II	50/fb	300/fb
$B^0_s \to \mu\bar{\mu}$	15	60	500	2 700
$B^0_s \to \mu\bar{\mu}$ (3% tag-power)	—	—	—	80

[K. Petridis, @ Barchelona 2016; K. Alvarez Cartelle @ HL-LHC WS, CERN, Oct. 2017]

- for $B_s \to \mu\bar{\mu}$

  @ CMS with 100 fb^{-1} : $\delta(Br) \sim 15\%$ error of SM [Kai-Feng Chen, KEK Flavor Factory WS, 2014]

  @ LHCb with 50 fb^{-1} : $\sigma(Br) \sim 0.15 \times 10^{-9}$ ($\approx 4\%$ of SM) (only stat. err) [LHCb arXiv:1208.3355]

  @ LHCb with 300 fb^{-1} : $\sigma(Br) \sim 0.16 \times 10^{-9}$ ($\approx 4\%$ of SM)

  (with current syst. err = f_s/f_d (5.8%) and norm. mode (3%))

  $\sigma(Br) \sim 0.13 \times 10^{-9}$ ($\lesssim 4\%$ of SM)

  (with 3 % syst. err) [A. Puig @ LHCb Upgrade WS, LAPP, Annecy, 03/2018]

  $\delta(\tau_{\text{eff}}) \sim 2\%$, $\sigma(\mathcal{S}) \sim 0.2$

- for $B_d \to \mu\bar{\mu}$ $\quad \delta(R_{d/s}) \sim 10\%$ $\quad R_{d/s} \equiv Br(B_d \to \mu\bar{\mu})/Br(B_s \to \mu\bar{\mu})$

Previous SM prediction

- at $\mu_{\rm EW}$: NLO EW + NNLO QCD [CB/Gorbahn/Stamou 1311.1348, Hermann/Misiak/Steinhauser 1311.1347]
- RGE $\mu_{\rm EW} \to \mu_b$: NLO QED + NNLO QCD

$$\overline{Br}(B_s \to \mu\bar{\mu})_{\rm SM} = (3.65 \pm 0.23) \times 10^{-9} \quad \xrightarrow{\text{update 2017}} \quad = (3.59 \pm 0.17) \times 10^{-9}$$

[CB/Gorbahn/Hermann/Misiak/Stamou/Steinhauser 1311.0903] [2017: f_{B_s} from FLAG, CKM from CKMfitter/UTfit, τ_H^s HFLAV]

Error budget	f_{B_s}	CKM	τ_H^s	m_t	α_s	other param.	non-param.	Σ
2013	4.0%	4.3%	1.3%	1.6%	0.1%	$< 0.1\%$	1.5%	6.4%
2017	3.2%	3.1%	0.6%	1.6%	0.1%	$< 0.1\%$	1.5%	4.7%

Non-parametric uncertainties

- **0.3% from $\mathcal{O}(\alpha_e)$ corrections from $\mu_b \in [m_b/2, 2m_b]$**
- $2 \times 0.2\%$ from $\mathcal{O}(\alpha_s^3, \alpha_e^2, \alpha_s\alpha_e)$ matching corrections from $\mu_{\rm EW} \in [m_t/2, 2m_t]$
- 0.3% from top-mass conversion from on-shell to $\overline{\rm MS}$ scheme
- 0.5% further uncertainties (power corrections $\mathcal{O}(m_b^2/m_W^2), \ldots$)

!!! used $|V_{cb}|_{\rm incl}$ $\Rightarrow$ rescale $\overline{Br} \propto \left(|V_{cb}|_{\text{your favorite}} \,/\, |V_{cb}|_{\rm incl}\right)^2$

- lacking: QED corrections below $\mu_b \Rightarrow$ in principle nonperturbative

QED corrections below $\mu_b \sim m_b$

- b and s quarks: soft residual $\sim \Lambda_{\rm QCD}$
- energetic leptons $E_\ell \sim m_{B_s}/2$ (in B_s-RF)
- hierarchy of modes with virtualities:

$$m_b^2 \;\to\; m_b\Lambda_{\rm QCD} \;\to\; \Lambda_{\rm QCD}^2 \approx m_\mu^2$$

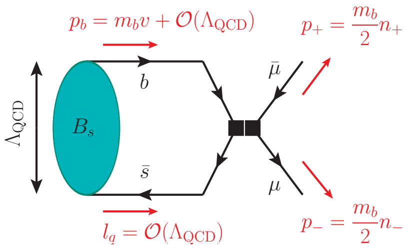
"hard" → "hard-collinear" → "collinear/soft"

full QED → SCET_I → SCET_II

$$\lambda \equiv \frac{\Lambda_{\rm QCD}}{m_b} \ll 1$$

⇒ **Soft Collinear EFT = SCET**, but only $\boldsymbol{\ell = \mu}$

Special **external kinematics**



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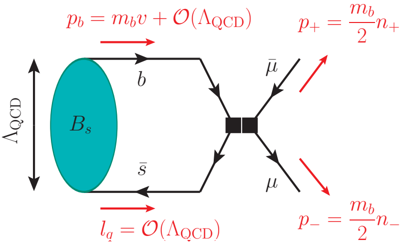
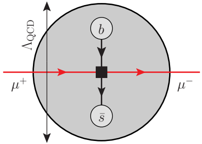


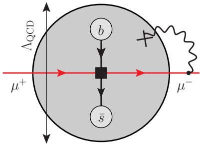
no QED



B_s decay constant

$\langle 0|\bar{s}\gamma_\mu\gamma_5|b|\overline{B}_s\rangle \propto f_{B_s}$

soft photon $\lesssim \Lambda_{\rm QCD}^2$

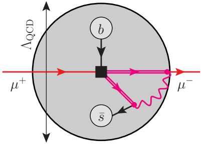


also $\sim f_{B_s}$

⇒ helicity flip &

local annihilation

hard-collinear photon $\sim m_b\Lambda_{\rm QCD}$



acts as weak probe

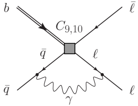
⇒ B-meson distribution

amplitude (DA)

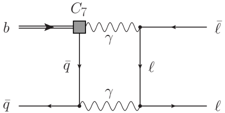
Power-enhanced contribution

Leading QED corrections **in λ-expansion** to $b\bar{s} \to \mu\bar{\mu}$

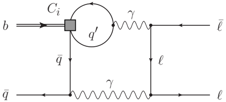
$C_{10} \approx -4$, $C_9 \approx +4$, $C_7 \approx -0.3$



$\mathcal{O}_{9(10)} \propto [\bar{s}\gamma_\mu P_L b][\bar{\ell}\gamma^\mu(\gamma_5)\ell]$



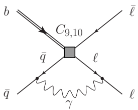
$\mathcal{O}_7 \propto m_b[\bar{s}\sigma_{\mu\nu}P_R b]F^{\mu\nu}$

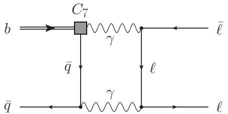


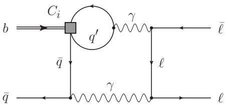
$\mathcal{O}_i \propto [\bar{s}\Gamma_i P_L b]\sum_q[\bar{q}'\Gamma_i q']$

Power-enhanced contribution

Leading QED corrections **in λ-expansion** to $b\bar{s} \to \mu\bar{\mu}$ $\qquad C_{10} \approx -4,\ C_9 \approx +4,\ C_7 \approx -0.3$







$\mathcal{O}_{9(10)} \propto [\bar{s}\gamma_\mu P_L b][\bar{\ell}\gamma^\mu(\gamma_5)\ell]$ $\qquad \mathcal{O}_7 \propto m_b[\bar{s}\sigma_{\mu\nu}P_R b]F^{\mu\nu}$ $\qquad \mathcal{O}_i \propto [\bar{s}\Gamma_i P_L b]\sum_q[\bar{q}'\Gamma_i q']$

$$\frac{i\mathcal{A}}{\mathcal{N}} = \overbrace{m_\ell f_{B_q} C_{10}\,[\bar{\ell}\,\gamma_5\,\ell]}^{\text{LO}} + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q\, m_\ell f_{B_q} \overbrace{\frac{m_B}{\lambda_B}}^{\text{power-enh.}} [\bar{\ell}\,(1+\gamma_5)\,\ell] \times \Bigg\{$$

$$\int_0^1 du\,\bar{u}\,C_9^{\rm eff}(u m_b^2)\left[L + \ln\frac{u}{\bar{u}} - \sigma_1\right] - Q_\ell C_7^{\rm eff}\left[\underbrace{L^2}_{\text{large (Log)}^2} - 2L(\sigma_1+1) + 2\sigma_1 + \sigma_2 + \frac{2\pi^2}{3}\right]\Bigg\} + \dots$$

- power enhancement: $m_B \approx 5$ GeV $\leftrightarrow$ $\lambda_B \approx (0.27 \pm 0.08)$ GeV $\Rightarrow$ $m_B/\lambda_B \approx 18$
- $L \equiv \ln \dfrac{m_b \mu_0}{m_\mu^2}$ with $\mu_0 = 1$ GeV — log (hard-collinear)^2/(collinear)^2 $\Rightarrow L \approx \ln 500 \approx 6$
- only limited knowledge of B-meson DA: $\sigma_1 \approx (1.5 \pm 1.0)$, $\sigma_2 \approx (3 \pm 2)$ $\Rightarrow$ large uncert.

$\frac{1}{\lambda_B(\mu)} \equiv \int_0^\infty \frac{d\omega}{\omega}\phi_{B+}(\omega,\mu) \qquad \frac{\sigma_n(\mu)}{\lambda_B(\mu)} \equiv \int_0^\infty \frac{d\omega}{\omega}\ln^n\frac{\mu_0}{\omega}\phi_{B+}(\omega,\mu)$

Numerical impact

- power-enhanced NLO QED correction yield **“shift” in C_{10}**

$$C_{10} \to C_{10} + \frac{\alpha_e}{4\pi} Q_\ell Q_q \, \Delta_{\text{QED}} \qquad \Delta_{\text{QED}} = (33 \ldots 119) + i(9 \ldots 23)$$

$\Rightarrow$ large uncertainties from B-meson LCDA: λ_B, $\sigma_{1,2}$

$\Rightarrow$ large cancellation between C_9- and C_7-terms in Δ_{QED}

- **new SM prediction:** about $(0.3 - 1.1)\%$ reduction   without NLO QED $(3.59 \pm 0.17) \times 10^{-9}$

$$\overline{Br}(B_s \to \mu\bar{\mu})_{\text{SM}} = (3.57 \pm 0.17) \times 10^{-9}$$

Parametric: ± 0.167,   Non-parametric/non-QED: ± 0.043,   QED: ${}^{+0.022}_{-0.030}$ (B-meson LCDA)

- **QED in CP asymmetries** = SM background to NP searches

$\Rightarrow$ is tiny

$$(\mathcal{A}_{\Delta\Gamma} - 1) = r^2 \, |\Delta_{\text{QED}}|^2 \approx 10^{-5}$$

$$\mathcal{S} = 2r \, \text{Im}\Delta_{\text{QED}} \approx -0.1\,\%$$

$$r \equiv \frac{\alpha_e}{4\pi} \frac{Q_\ell Q_q}{C_{10}}$$

$\Rightarrow$ good prospects to unveil potential NP

- $B_u \to \mu\bar{\nu}_\mu$: NO power-enhanced contributions here due to different chiral structure

Summary

Presented fixed-order **power-enhanced NLO QED corrections for $B_s \to \mu\bar{\mu}$** in SM

- decrease $Br(B_s \to \mu\bar{\mu})$ by about $(0.3 - 1.1)\,\%$,
  $\Rightarrow$ similar size of other non-parametric uncertainties
- QED effects on CP asymmetries in $B_s \to \mu\bar{\mu}$ tiny
  $\Rightarrow$ ideal for NP searches
- no power-enhanced effect in $B_u \to \mu\bar{\nu}_\mu$
- important step for better understanding of QED factorization theorems

$Br(B_s \to \mu\bar{\mu})$ one of few flavor observables with $\approx 2\%$ long-distance theory control

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Next steps:

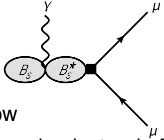
- resummation of large logarithms for power-enhanced contributions using SCET
- factorization theorem at amplitude level for power-enhanced contribution
- calculate non-enhanced contributions and attempt factorization theorem for observable
- consistent combination with soft-photon approximation / PHOTOS
- modify for application to $\ell = \tau$ or e and $B_d \to \ell\bar{\ell}$ decays
- modify for application to other exclusive $b \to q\ell\bar{\ell}$ decays

Backup Slides

Real QED corrections below $\mu_b \sim m_b$ for $B_q \to \ell\bar{\ell}$

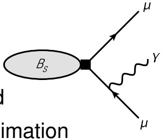
... are accounted for in experimental analysis

Initial state radiation



- tiny in signal window
- phase-space suppression instead of helicity suppression
- can be avoided with cuts

Final state radiation



- helicity suppressed
- soft-photon approximation
- extrapolated from signal window over all $m^2_{\mu\bar{\mu}}$ via PHOTOS by LHCb and CMS

ISR [Aditya/Healey/Petrov arXiv:1212.4166]

FSR [Buras et al. arXiv:1208.0934]

experimental signal windows (LHCb, CMS)

[LHCb arXiv:1307.5024, CMS arXiv:1307.5025]