Synergies between LHCb and BESIII

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On behalf of the BESIII collaboration

LHCP, 4 - 9 June 2018, Bologna Italy
A successful “Common Phase” workshop!
Further synergy between BESIII/LHCb planned.
Discussion might lead to shift in BESIII priorities!
A lot of material in this talk from the workshop
Outline

✧ Introduction
✧ Common interests
   ✧ \( \gamma \) measurement input from Charm threshold & CPV & Mixing in charm sector
   ✧ Absolute branching fraction measurements in \( D, D_s, \Lambda_c \) decays
   ✧ Spectroscopy and exotics
✧ Summary

Synergy and complementarity

Disclaimer: A BESIII-member point of view
LHCb vs Threshold Experiments

- **LHCb (Hadron Collider)**
  - Huge cross-section ⇒ statistics overwhelming
  - Energy boost

- **CLEO-c & BESIII (e⁺e⁻ Collider @ Charm threshold)**
  - Quantum Correlations (QC) and CP-tagging are unique ⇒ Strong Phases
  - Double tag technique ⇒ Systematic cancellation & Absolute Branching Fractions

- **BESIII data set** (ψ(3770) : 4x larger than CLEO-c, 25x larger possible; **DₛDₛ**⁺ : 5x larger than CLEO-c; existing **Λ_c** threshold data; more energy scans data)
BESIII data samples & Prospect

Data taking plan for the next 3 – 5 years

- **10B J/ψ**
- **τ mass scan & ψ(2S) scan**
- **high-statistics XYZ data taking (500 pb⁻¹ per point)**
- **10 fb⁻¹ on ψ(3770) (20 fb⁻¹ feasible?)**
Quantum Correlations at the $\psi(3770)$

- $e^+e^- \to \psi(3770) \to D\bar{D}$ : Pure $J^{PC} = 1^- -$ initial state
  - $\psi(3770)$ : spin=1, $c\bar{c}$ bound state; $D^0$ : spin=0
  - $\Rightarrow D\bar{D}$ orbit angular momentum $L=1$

- A typical entangled 2-state system
  - $L=1$ and Bose statistics $\Rightarrow D\bar{D}$ state anti-symmetric
  - $|\alpha\rangle = \frac{1}{\sqrt{2}} \left( |D^0(p)\rangle|\bar{D}^0(-p)\rangle - |\bar{D}^0(p)\rangle|D^0(-p)\rangle \right)$
  - $\Rightarrow D^0D^0$ and $\bar{D}^0\bar{D}^0$ are prohibited
  - At any time until one $D$ decays : one $D^0$ and one $\bar{D}^0$

- Similar to $CP$ eigenstates $|D_{CP\pm}\rangle = (|D^0\rangle \pm |\bar{D}^0\rangle) / \sqrt{2}$, assuming no CPV:
  - $|\alpha\rangle = \frac{1}{\sqrt{2}} \left( |D_{CP+}(p)\rangle|D_{CP-}(-p)\rangle - |D_{CP-}(p)\rangle|D_{CP+}(-p)\rangle \right)$
  - At any time until one $D$ decays : one $D_{CP+}$, one $D_{CP-}$
**Time-integrated decay rates**

- No time dependent information at CLEO-c & BESIII
- Anti-symmetric wavefunction:
  \[
  \Gamma_{ij} = \left| \langle i | D^0 \rangle \langle j | \bar{D}^0 \rangle - \langle j | D^0 \rangle \langle i | \bar{D}^0 \rangle \right|^2
  \]
- Double tag rates:
  \[
  A_i^2 A_j^2 [1 + r_i^2 r_j^2 - 2 r_i r_j \cos(\delta_i + \delta_j)]
  \]
- CP tag: \( r=1, \delta=0 \) or \( \pi; \) \( l^\pm \) tag: \( r=0 \)
- Single and Double tag rates
  \[
  z_f \equiv 2 \cos \delta_f, \quad r_f \equiv \frac{A_{DCS}}{A_{CF}}, \quad R_M \approx \frac{x^2 + y^2}{2}
  \]

<table>
<thead>
<tr>
<th>C-odd</th>
<th>( f )</th>
<th>( \bar{f} )</th>
<th>( l^+ )</th>
<th>( l )</th>
<th>( CP+ )</th>
<th>( CP- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>( R_M [1 + r_f^2 (2 - z_f^2)] )</td>
<td>( 1 + r_f^2 (2 - z_f^2) + r_f^4 )</td>
<td>( R_M [1 + r_f^2 (2 - z_f^2) + r_f^4] )</td>
<td>( R_M )</td>
<td>( r_f^2 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( \bar{f} )</td>
<td>( 1 + r_f^2 (2 - z_f^2) + r_f^4 )</td>
<td>( R_M [1 + r_f^2 (2 - z_f^2) + r_f^4] )</td>
<td>( R_M )</td>
<td>( 1 )</td>
<td>( R_M )</td>
<td>( r_f^2 )</td>
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<tr>
<td>( l^+ )</td>
<td>( r_f^2 )</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( R_M )</td>
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<td>( 1 )</td>
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<td>( l )</td>
<td>( 1 )</td>
<td>( r_f^2 )</td>
<td>( 1 )</td>
<td>( R_M )</td>
<td>( 1 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( CP+ )</td>
<td>( 1 + r_f (r_f + z_f) )</td>
<td>( 1 + r_f (r_f + z_f) )</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( 0 )</td>
<td></td>
</tr>
<tr>
<td>( CP- )</td>
<td>( 1 + r_f (r_f - z_f) )</td>
<td>( 1 + r_f (r_f - z_f) )</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( 4 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>Single Tag</td>
<td>( 1 + r_f^2 - r_f z_f (A - y) )</td>
<td>( 1 )</td>
<td>( 2 [1 \pm (A - y)] )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Quantum Correlations inputs

- Precision CKM test: $\gamma$ measurements
- Charm Mixing & CP violation

- Inputs from Quantum Correlated (QC) $\psi(3770) \rightarrow D\bar{D}$ decays
  - (Averaged)Strong phase difference: $\delta_D$
  - Coherent factors: $R_D$
  - (Averaged)Strong phase in Dalitz bins: $c_i, s_i$
- LHCb, Belle II are the customers

\[
c_i \equiv \int dp A_{12,13} A_{13,12} \cos(\delta_{12,13} - \delta_{13,12})
\]
\[
s_i \equiv \int dp A_{12,13} A_{13,12} \sin(\delta_{12,13} - \delta_{13,12})
\]
**Status of $\gamma$ measurements**

- LHCb results can be found in the Susan Haines & Giovanni Passaleva’s talk on June 4
- LHCb latest GGSZ method (RUN I): $\gamma = (80^{+10}_{-9})^0$
  - Most precise determination of $\gamma$ from a single channel!
- LHCb Combined and World Average: (other methods: GLW, ADS...)

<table>
<thead>
<tr>
<th>LHCb Average - [LHCb-CONF-2018-002]</th>
<th>World Average (HFLAV) - [Spring update]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = (74.0^{+5.0}_{-5.8})^0$</td>
<td>$\gamma = (73.5^{+4.2}_{-5.1})^0$</td>
</tr>
</tbody>
</table>

- Constrains from indirect measurements: $\gamma = (65.3^{+1.0}_{-2.5})^0$
- $\gamma$ from $B^0_S$ and $B^+$: $2\sigma$ discrepancy
$D^0 \rightarrow K_S h^+ h^-$ inputs (CLEO-c: 818 pb$^{-1}$)

**PRD82,112006(2010)**

$B^\pm \rightarrow DK^\pm$ yields

\[ N_{i}^{\pm} = h_B \left[ K_{i}^{\pm} + r_B^2 K_{\mp i} + 2\sqrt{K_i K_{-i}} \left( x_{\pm} c_i \pm y_{\pm} s_i \right) \right] \]

where $x_{\pm} = r_B \cos(\delta_B \pm \gamma)$ and $y_{\pm} = r_B \sin(\delta_B \pm \gamma)$

LHCb preliminary:

\[ x_- = (9.0 \pm 1.7 \pm 0.7 \pm 0.4) \times 10^{-2}, \]
\[ y_- = (2.1 \pm 2.2 \pm 0.5 \pm 1.1) \times 10^{-2}, \]
\[ x_+ = (-7.7 \pm 1.9 \pm 0.7 \pm 0.4) \times 10^{-2}, \]
\[ y_+ = (-1.0 \pm 1.9 \pm 0.4 \pm 0.9) \times 10^{-2}, \]

**Extracted from fit to the $B^\pm$ yields**

**Measured by CLEO [PRD82, 112006 (2010)]**

### Table: Tag vs $K^0_S \pi^+ \pi^-$

<table>
<thead>
<tr>
<th>Tag</th>
<th>$K^0_S \pi^+ \pi^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^- \pi^+$</td>
<td>1444</td>
</tr>
<tr>
<td>$K^- \pi^+ \pi^0$</td>
<td>2759</td>
</tr>
<tr>
<td>$K^- \pi^+ \pi^- \pi^+$</td>
<td>2240</td>
</tr>
<tr>
<td>$K^- \pi^- \pi^+$</td>
<td>1191</td>
</tr>
<tr>
<td>$K^+ K^-$</td>
<td>124</td>
</tr>
<tr>
<td>$\pi^+ \pi^-$</td>
<td>61</td>
</tr>
<tr>
<td>$K^0_S \pi^0 \pi^0$</td>
<td>(CP+) 56</td>
</tr>
<tr>
<td>$K^0_S \pi^0$</td>
<td>237</td>
</tr>
<tr>
<td>$K^0_S \pi^0$</td>
<td>189</td>
</tr>
<tr>
<td>$K^0_S \pi^0$</td>
<td>39</td>
</tr>
<tr>
<td>$K^0_S \pi^+ \pi^-$</td>
<td>83</td>
</tr>
</tbody>
</table>

**Signal to background 10-100 depending on tag mode**
**Sensitivities from LHCb data sets**

<table>
<thead>
<tr>
<th>Run Period ([E_{CM}])</th>
<th>Collected / Projected luminosity per run</th>
<th>Cumulative yield factor compared to Run 1</th>
<th>Year attained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1 ([7.8 \text{ TeV}])</td>
<td>3 fb(^{-1}) 5 fb(^{-1}) 50 fb(^{-1}) 300 fb(^{-1})</td>
<td>1 4 60 (~400)</td>
<td>2012 2018 2030 2035(?)</td>
</tr>
</tbody>
</table>
| Run 2 \([13 \text{ TeV}]\) | \(\gamma\) Sensitivities from LHCb data sets

➢ **Future sensitivities**
  (scaled according to statistical uncertainties)

- Run 1 \(5.5^\circ\)
- Run 2 \(2.8^\circ\)
- Upgrade 1 \(0.71^\circ\)
- Upgrade 2 \(0.28^\circ\)

CLEOc Q.C. inputs contribute \(~2^\circ\) [LHCb-PUB-2016-025]

Q.C. inputs from BESIII (contribute \(~1^\circ\) with existing data set, \(~0.4^\circ\) with 20 fb\(^{-1}\)) or even from Super \(\tau\)-charm factory will be crucial!
**K_s \pi^+ \pi^- (BESIII preliminary: 2.9 fb^{-1})**

- Extract \( c_i, s_i \) for “\( \gamma \) GGSZ method” (4x CLEO-c data)
- Preliminary results presented @ APS meeting, Apr. 2014

BESIII \( K_L \pi^+ \pi^- \), \( K_s K^+ K^- \) Analysis is on going. Also explore many more \( D \) decay modes: three-body decays, four-body decays, …
Other interesting channels for $\gamma$

- Other $D$ decay channels have been used for $\gamma$ measurements:
  - $K_SK\pi$, $4\pi$ global, $K3\pi$ global, $K\pi\pi^0$, $\pi\pi\pi^0$, $KK\pi^0$ etc.

<table>
<thead>
<tr>
<th>Double tag Rate</th>
<th>Sensitive to</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^{\pm}<em>{\pi^\pm}\pi^\mp\pi^\mp\pi^\mp$ vs $K^{\pm}</em>{\pi^\pm}\pi^\mp\pi^\mp\pi^\mp$</td>
<td>$(R_{K3\pi})^2$</td>
</tr>
<tr>
<td>$K^{\pm}<em>{\pi^\pm}\pi^0$ vs $K^{\pm}</em>{\pi^\pm}\pi^0$</td>
<td>$(R_{K\pi\pi^0})^2$</td>
</tr>
<tr>
<td>$K^{\pm}_{\pi^\pm}\pi^\mp\pi^\mp\pi^\mp$ vs CP</td>
<td>$R_{K3\pi}\cos(\delta_{K3\pi})$</td>
</tr>
<tr>
<td>$K^{\pm}_{\pi^\pm}\pi^0$ vs CP</td>
<td>$R_{K\pi\pi^0}\cos(\delta_{K\pi\pi^0})$</td>
</tr>
<tr>
<td>$K^{\pm}<em>{\pi^\pm}\pi^\mp\pi^\mp\pi^\mp$ vs $K^{\pm}</em>{\pi^\pm}\pi^\mp$</td>
<td>$R_{K3\pi}\cos(\delta_{K3\pi} - \delta_{K\pi})$</td>
</tr>
<tr>
<td>$K^{\pm}<em>{\pi^\pm}\pi^0$ vs $K^{\pm}</em>{\pi^\pm}\pi^0$</td>
<td>$R_{K\pi\pi^0}\cos(\delta_{K\pi\pi^0} - \delta_{K\pi})$</td>
</tr>
<tr>
<td>$K^{\pm}<em>{\pi^\pm}\pi^\mp\pi^\mp\pi^\mp$ vs $K^{\pm}</em>{\pi^\pm}\pi^0$</td>
<td>$R_{K3\pi}R_{K\pi\pi^0}\cos(\delta_{K3\pi} - \delta_{K\pi\pi^0})$</td>
</tr>
</tbody>
</table>

- Inputs from BESIII can definitely help a lot.

![Graph showing $D\to K3\pi$ decay]

Anton Poluektov
arXiv:1712.08326

![Graph showing $D\to 4\pi$ decay]

Run 2: $5^\circ$ stat.
8° sys. from CLEO-c

Run 2: $10^\circ$ stat.
7° sys. from CLEO-c
Charm branching fraction measurements

- LHCb measures relative branching fractions;

\[
\begin{align*}
\mathcal{B}(\Lambda_c^+ \to p\pi^-\pi^+) \times 10^3 &= 4.72 \pm 0.05 \pm 0.11 \pm 0.25 \\
\mathcal{B}(\Lambda_c^+ \to pK^-K^+) \times 10^3 &= 1.08 \pm 0.02 \pm 0.02 \pm 0.06 \\
\mathcal{B}(\Lambda_c^+ \to p\pi^-K^+) \times 10^4 &= 1.04 \pm 0.09 \pm 0.03 \pm 0.05 \\
\end{align*}
\]

(6.35 ± 0.33)%

- |V_{ub}| is one of the key CKM matrix elements

\[
\frac{\mathcal{B}(\Lambda_b^0 \to p\mu^+\nu_\mu)_{q^2>15\text{GeV}^2}}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \mu^-\bar{\nu}_\mu)_{q^2>7\text{GeV}^2}} = (1.471 \pm 0.095 \pm 0.109) \left| \frac{V_{ub}}{V_{cb}} \right|^2
\]

- Ratio between |V_{ub}| and |V_{cb}| has been measured by LHCb using baryon decays

\[
\frac{\mathcal{B}(\Lambda_b^0 \to p\mu^+\nu_\mu)_{q^2>15\text{GeV}^2}}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \mu^-\bar{\nu}_\mu)_{q^2>7\text{GeV}^2}} = \frac{N(\Lambda_b^0 \to p\mu^+\nu_\mu)}{N(\Lambda_b^0 \to \Lambda_c^+(pK^-\pi^+)) \mu^-\bar{\nu}_\mu} \cdot r_e \cdot \mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)
\]

(6.35 ± 0.33)%

- Clearly, uncertainties due to external inputs are dominant/important
Hadronization factors for $b$ hadrons uses charm branching fraction measurements unavoidably.

$$\frac{f_s}{f_d} = \frac{\mathcal{B}(B^0 \rightarrow D^- (K^+\pi^-\pi^-)K^+)}{\mathcal{B}(B^0_s \rightarrow D_s^- (K^+K^-\pi^-)\pi^+)} \frac{\epsilon_{D^-K^+}}{\epsilon_{D^-\pi^+}} \frac{N_{D_s^-\pi^+}}{N_{D^-K^+}}$$

$$= \Phi_{PS} \left| \frac{V_{us}}{V_{ud}} \right|^2 \left( \frac{f_K}{f_\pi} \right)^2 \frac{\tau_{B^0}}{\tau_{B^0_s} N_a N_F} \frac{1}{\mathcal{B}(D^- \rightarrow K^+\pi^-\pi^-)} \frac{\mathcal{B}(D_s^- \rightarrow K^+K^-\pi^-)}{\epsilon_{D^-\pi^+} N_{D^-K^+}} \frac{\epsilon_{D^-K^+}}{N_{D_s^-\pi^+}}$$

$\Phi_{PS}$ is a phase space factor, $N_a$ parameterizes nonfactorizable $SU(3)$-breaking, $N_F$ is the ratio of form factors.

Systematic uncertainties from charm branching fractions normally dominate (similar situation also for $f_{Ab}/(f_d + f_u)$ etc)
Lepton flavor universality tests

➢ Lepton flavor universality test becomes a hot topic after many deviations seen from B-factories and LHCb measurements

➢ Charm inputs are very important for understanding backgrounds of R(D) and R(D*) measurements with $\tau \rightarrow 3\pi \nu$

Surprisingly, number of LHCb measurements would profit enormously from improved precision in some branching fractions of $D$, $Ds$ and $\Lambda_C$
Charmonium spectroscopy

- LHCb study of $B^+ \rightarrow K^+\mu^+\mu^-$ shows significant and larger-than-expected contributions from $\psi(4160)$

- Cross sections for $e^+e^- \rightarrow$ open charm above DD threshold is crucial input

- Obvious place of common interest: amplitude analysis techniques and tools
Summary

-> We seem to be in a unique situation for some very precise measurements in particle physics:
  -> Many important physics results need synergy between LHCb and BESIII
  -> No direct competition in several areas, clear case for synergy and closer collaboration
  -> Discussions might lead to shift in BESIII priorities!

-> Combined efforts from both experiments may lead to long-waited new physics

3 == New Physics !!!
Backup
LHCb upgrade plans

- **Upgrade I**: several detector replaced; 40 MHz readout with fully software trigger
- **Upgrade II**: new ideas under study on tracking, calorimeter, adding timing info etc.

**Timeline**

- **2011-2012 Run 1**: Belle2 starts
- **2015-2018 Run 2**: LHCb upgrade I
- **2021-2023 Run 3**: upgrade I consolidation
- **2026-2029 Run 4**: install LHCb upgrade II
- **2031-... Run 5/6**: Belle2 ends

**Production**

- inst $L_{\text{LHCb}}$: $4 \times 10^{32}$
- $\int dt L_{\text{LHCb}}$: $3 \text{ fb}^{-1}$
- visible int/bunch crossing: $\sim 1$
- $\sim 1$
- $\sim 6$
- $\sim 55$
- $L_{\text{LHC}}$: $2 \times 10^{33}$
- $L_{\text{LHC}}$: $1 - 2 \times 10^{34}$
- $50 \text{ fb}^{-1}$
- $300 \text{ fb}^{-1}$

**LHC phase 2 (HL-LHC)**

- **Hardware + software trigger**
- **Fully software trigger**
Methodology for $\gamma$ measurement

- Sensitive channels with small BFs: need to combine many channels
  - GLW: $D = \text{CP eigenstates, e.g. } KK, \pi\pi$
  - ADS: $D = \text{quasi-flavour-specific states e.g. } K\pi$
  - GGSZ: $D = \text{self-conjugate multi(3)-body states e.g. } K_s\pi\pi$
  - GLS: ADS variant with singly Cabbibo-suppressed decay $D \rightarrow K_sK\pi$
  - time-dependent $B_s \rightarrow D_sK$, $B^0 \rightarrow D\pi$ etc (not discussed here)
  - Dalitz (GW) method: $B^0 \rightarrow DK\pi$

- Global fit needed to extract $\gamma$ (also other nuisance parameters)

- Charm inputs crucial (BESIII)
GLW measurements

2 fb^{-1} (7 TeV) + 1 fb^{-1} (8 TeV) + 2 fb^{-1} (13 TeV)

Using both fully reco. B→DK and partially reco. B→D^*(Dγ/Dπ^0)K

\[ R_{CP^{\pm}} = \frac{2[\Gamma(B^- \to D_{CP^{\pm}}K^-) + \Gamma(B^+ \to D_{CP^{\pm}}K^+)]}{\Gamma(B^- \to D^0K^-) + \Gamma(B^+ \to \bar{D}^0K^+)} ,\]

\[ A_{CP^{\pm}} = \frac{\Gamma(B^- \to D_{CP^{\pm}}K^-) - \Gamma(B^+ \to D_{CP^{\pm}}K^+)}{\Gamma(B^- \to D_{CP^{\pm}}K^-) + \Gamma(B^+ \to D_{CP^{\pm}}K^+)} .\]

\[ R_{CP^{\pm}} = 1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma \]
\[ A_{CP^{\pm}} = \pm 2r_B \sin \delta_B \sin \gamma / R_{CP^{\pm}} .\]

Extension like D→πππ^0 can be used => need inputs from charm friends
ADS measurements

\[ R_{\text{ADS}} = \frac{\Gamma(B^- \rightarrow D[\rightarrow \pi^- K^+] K^-) + \Gamma(B^+ \rightarrow D[\rightarrow \pi^+ K^-] K^+)}{\Gamma(B^- \rightarrow D[\rightarrow K^- \pi^+] K^-) + \Gamma(B^+ \rightarrow D[\rightarrow K^+ \pi^-] K^+)} \]

\[ A_{\text{ADS}} = \frac{\Gamma(B^- \rightarrow D[\rightarrow \pi^- K^+] K^-) - \Gamma(B^+ \rightarrow D[\rightarrow \pi^+ K^-] K^+)}{\Gamma(B^- \rightarrow D[\rightarrow \pi^- K^+] K^-) + \Gamma(B^+ \rightarrow D[\rightarrow \pi^+ K^-] K^+)} \]

\[ R_{\text{ADS}} = (r_B^K)^2 + r_{K\pi}^2 + 2r_B^K r_{K\pi} \cos \gamma \cos (\delta_B^K + \delta_{K\pi}) \]

\[ A_{\text{ADS}} = 2r_B^K r_{K\pi} \sin \gamma \sin (\delta_B^K + \delta_{K\pi}) / R_{\text{ADS}} \]

\[ 2 \text{ fb}^{-1} (7 \, \text{TeV}) + 1 \text{ fb}^{-1} (8 \, \text{TeV}) \]

Inputs from charm basic for the analysis
GGSZ measurements

Similar to many ADS analyses over Dalitz plane

\[ A_{B^-} \sim A_f + r_B e^{i(\delta_B - \gamma)} \tilde{A}_f, \]
\[ A_{B^+} \sim \tilde{A}_f + r_B e^{i(\delta_B + \gamma)} A_f \]

\[ A_f = \langle f \mid \mathcal{H} \mid D^0 \rangle \]
\[ \tilde{A}_f = \langle f \mid \mathcal{H} \mid \overline{D}^0 \rangle \]

- 2 fb\(^{-1}\) (7 TeV) + 1 fb\(^{-1}\) (8 TeV)

\[ x_{\pm} = r_B \cos(\delta_B \pm \gamma) \] and \[ y_{\pm} = r_B \sin(\delta_B \pm \gamma). \]

\[ N_{+i}^+ = h_{B^+} \left[ F_{\mp i} + (x_{\pm}^2 + y_{\mp}^2) F_{\mp i} + 2\sqrt{F_{\mp i} F_{\pm i}} (x_{+c_{\pm i}} - y_{-s_{\pm i}}) \right], \]
\[ N_{-i}^- = h_{B^-} \left[ F_{\mp i} + (x_{\mp}^2 + y_{\pm}^2) F_{\mp i} + 2\sqrt{F_{\mp i} F_{\pm i}} (x_{-c_{\pm i}} + y_{+s_{\pm i}}) \right], \]

Charm inputs of \( c_i, s_i \) crucial for the measurements
**GW measurements**

- $B^0 \rightarrow D K \pi$ are the decay channel of interest
- Sensitivity to $\gamma$ from $K^*$ resonances while $D^*$ as reference point
- Currently GLW type analysis has been performed by LHCb
- Starts to offer constraints to $\gamma$ and related parameters

![Graphs and diagrams representing LHCb data for $B^0 \rightarrow D K^+ \pi^-$ and $B^0 \rightarrow D K^- \pi^+$ decay channels.](image-url)
GW-GGSZ measurements

▷ $B^0 \rightarrow D K \pi$ are the decay channel of interest

▷ Studies also performed on GGSZ-type of analysis: two Dalitz plots!

$$|A_{dbl,Dlz}|^2 = |A_B|^2 |A_D|^2 + |A_B|^2 |A_D|^2$$

$$+ 2 |A_B| |A_D| |A_B| |A_D| \left[ (\chi c - \sigma s) \cos \gamma - (\chi s + \sigma c) \sin \gamma \right] ,$$

Analogy to $c_i$, $s_i$ but for $B$ Dalitz plot

▷ Charm inputs of $c_i$, $s_i$ will be important for Run 1+2 data; LHCb will have self-constraints on $c_i$, $s_i$ with more data but external inputs are still important
New GGSZ analysis

➤ Use Fourier transformation over strong phase obtained from a model to extract information

\[ \phi(m_+^2, m_-^2) = \text{arg} A_D^{(\text{model})}(m_+^2, m_-^2) - \text{arg} A_D^{(\text{model})}(m_-^2, m_+^2) \]

➤ Fourier transformation applied to both quantum-correlated data and to LHCb B data

\[ \bar{p}_B(\phi) \propto p_D(\phi) + r_B^2 \bar{p}_D(\phi) + 2[x_+ C(\phi) - y_+ S(\phi)], \]

➤ All orders of Fourier transformations give constraints on \( x_\pm \) and \( y_\pm \)

➤ Binning of Dalitz plot according to amplitude ratio can help

<table>
<thead>
<tr>
<th>Sample size</th>
<th>( \gamma ) resolution, °</th>
<th>Ultimate precision: ( \sigma(\gamma) = 2.91 \pm 0.07° )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2 \times 10^4 \ B^\pm \rightarrow D K^\pm, \ 10^3 \ D^0 \bar{D}^0 )</td>
<td>Binned optimal: 4.33 ± 0.10</td>
<td>Fourier non-split: 4.54 ± 0.10</td>
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<td>Fourier split: 3.73 ± 0.08</td>
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<td>( 2 \times 10^4 \ B^\pm \rightarrow D K^\pm, \ 10^4 \ D^0 \bar{D}^0 )</td>
<td>3.60 ± 0.08</td>
<td>4.51 ± 0.10</td>
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<td>3.43 ± 0.08</td>
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<tr>
<td>( 2 \times 10^4 \ B^\pm \rightarrow D K^\pm, \ 10^5 \ D^0 \bar{D}^0 )</td>
<td>3.49 ± 0.10</td>
<td>4.47 ± 0.10</td>
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<td>3.32 ± 0.08</td>
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</table>

➤ New approaches requiring efforts from LHCb and BESIII to make it possible
Other similar measurements

➤ $B^0 \rightarrow D \pi \pi$ with $D \rightarrow hh$ was proposed to measure CKM angle $\beta$, where not only $\sin 2\beta$ but also $\cos 2\beta$ can be accessed;

➤ Same decay channel but with $D \rightarrow K_{sh}h$ is proposed and with similar ideas as GGSZ $B \rightarrow DK$ analysis;

➤ Charm inputs from $c_i, s_i$ required as for GGSZ analysis

\[
g_{ij}(\Delta t) \propto U_{ij} + q_B [D_{ij} \cos(\Delta m \Delta t) - F_{ij} \sin(\Delta m \Delta t)]
\]

\[
U_{ij} = K_i k_j + K_{-i} k_{-j}, \quad D_{ij} = K_i k_j - K_{-i} k_{-j},
\]

\[
F_{ij} = 2 \sqrt{K_i K_{-i} k_j k_{-j}} \left[(C_i s_j - S_i c_j) \cos 2\beta - (C_i c_j + S_i s_j) \sin 2\beta\right]
\]

Analogy to those from $B^0 \rightarrow DK\pi$ Dalitz

➤ $c_i, s_i$ inputs from quantum-correlated charm mesons will be important when statistic is low; The system can also offer self-constraints on $c_i, s_i$ with large enough dataset (i.e. LHCb 50 fb$^{-1}$)

➤ $c_i, s_i$ inputs are also used in charm mixing parameter measurements (model independent approach)
$r_D$, $\delta_D$ and $\gamma$ input

- $\gamma$ measurements from $B \rightarrow D^0 K$
  - $b \rightarrow u : \gamma = \text{arg}V^{*}_{ub}$
  - most sensitive method to constrain $\gamma$ at present
  - $r_D$, $\delta_D$ : QC measurements from Charm Threshold
  - GGSZ method (Giri, Grossman, Soffer & Zupan, PRD68, 054018 (2003))
  - $c_\psi, s_i$ : QC measurements from Charm Threshold

- Charm mixing parameters: $\chi = \frac{\Delta M}{\Gamma}$, $\gamma = \frac{\Delta \Gamma}{2\Gamma}$

- Time-dependent WS $D^0 \rightarrow K^+ \pi^-$ rate
  - $y' = y \cos \delta_{K\pi} - x \sin \delta_{K\pi} = (0.528 \pm 0.045 \pm 0.027)\%$ (LHCb 2017)
  - $\delta_{K\pi}$ : QC measurements from Charm Threshold
### Status of $\gamma$ at LHCb

#### Highest Statistics

<table>
<thead>
<tr>
<th>Method</th>
<th>B Decay</th>
<th>$B^- \to D^0 K^-$</th>
<th>$B^- \to D^0 K^{*-}$</th>
<th>$B^0 \to D^0 K^+$</th>
<th>$B^- \to D^0 K^-$</th>
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<td>$D^0 \to K^+ K^-$</td>
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<td>$D^0 \to \pi^+ \pi^-$</td>
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<td>$D^0 \to K^+ K^- \pi^0$</td>
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</table>

#### Poorer sensitivity at LHCb

- Requires input from Charm sector ($r_D, \delta_D, \kappa_D$)

#### High potential

- Dalitz structure of $B$

#### Low stats

- Multibody $B$

#### KEY:

- ●: (update) in progress
- ●: requires input from Charm sector ($r_D, \delta_D, \kappa_D$)

**NOTE:**

- LHCb has a 3 fb$^{-1}$ (●) TD result with $B^0 \to D^- K^+$
- LHCb has a 3 fb$^{-1}$ (●) GLS result from $B^- \to D^0 K^-$ with $D^0 \to K^0_s K^+ \pi^-$
$x_+ = (-7.7 \pm 2.4 \pm 1.0 \pm 0.4) \times 10^{-2}$,

$y_+ = (-2.2 \pm 2.5 \pm 0.4 \pm 1.0) \times 10^{-2}$,

$x_- = (2.5 \pm 2.5 \pm 1.0 \pm 0.5) \times 10^{-2}$,

$y_- = (7.5 \pm 2.9 \pm 0.5 \pm 1.4) \times 10^{-2}$,

- Effects from uncertainties on $c_i$, $s_i$ becomes important with more LHCb data (Run 3 and later)
- With large enough data, LHCb has sensitivity to $c_i$, $s_i$
- Dalitz model obtained from Dalitz analyses on charm decays ($K_{s}hh$ or similar channels) important for optimizing binning scheme for $c_i$, $s_i$
\[ \delta_{K\pi} \text{ in } D \rightarrow K\pi \text{ (BESIII: 2.9 fb}^{-1}) \]

A simple picture:

\[
\frac{\langle K\pi | D^0 \rangle}{\langle K\pi | D^0 \rangle} \equiv \frac{A_{K\pi}}{A_{K\pi}} \equiv r_{K\pi} e^{i\delta_{K\pi}}
\]

\[
\langle K\pi | D_{CP\pm} \rangle = (\langle K\pi | D^0 \rangle \pm \langle K\pi | D^0 \rangle) / \sqrt{2} \Rightarrow \sqrt{2} A_{CP\pm} = A_{K\pi} \pm \overline{A_{K\pi}}
\]

\[
2r_{K\pi} \cdot \cos \delta_{K\pi} \approx A_{CP \rightarrow K\pi} \equiv \frac{|A_{CP-}|^2 - |A_{CP+}|^2}{|A_{CP-}|^2 + |A_{CP+}|^2} = \frac{\text{Br}(D_{CP- \rightarrow K\pi}) - \text{Br}(D_{CP+ \rightarrow K\pi})}{\text{Br}(D_{CP- \rightarrow K\pi}) + \text{Br}(D_{CP+ \rightarrow K\pi})}
\]

Direct result:

\[ A_{CP \rightarrow K\pi} = (12.7 \pm 1.3 \text{(Stat.)} \pm 0.7 \text{(sys.)}) \] %

\[ 2r_{K\pi} \cos \delta_{K\pi} + y = (1 + R_{WS}) \cdot A_{CP \rightarrow K\pi} \]

Using external input for \( r_{K\pi}^2, y, R_{WS} \) we extract:

\[ \cos \delta_{K\pi} = 1.02 \pm 0.11 \pm 0.06 \pm 0.01 \]

If BESIII accumulate \( 10 \text{ fb}^{-1} \) on threshold \( D \) data:

sensitivity of \( \cos \delta_{K\pi} \approx 0.06 \)