



**Alexander von Humboldt**  
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TECHNISCHE  
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THEORETICAL STATUS OF  
**SINGLE TOP**  
AND  
**TOP ASSOCIATED**  
PREDICTIONS

**Rikkert Frederix**  
**Technische Universität München**

# NLO+RESUMMATION FOR TOP ASSOCIATED PRODUCTION

[A. Broggio et al., 2016-2017]

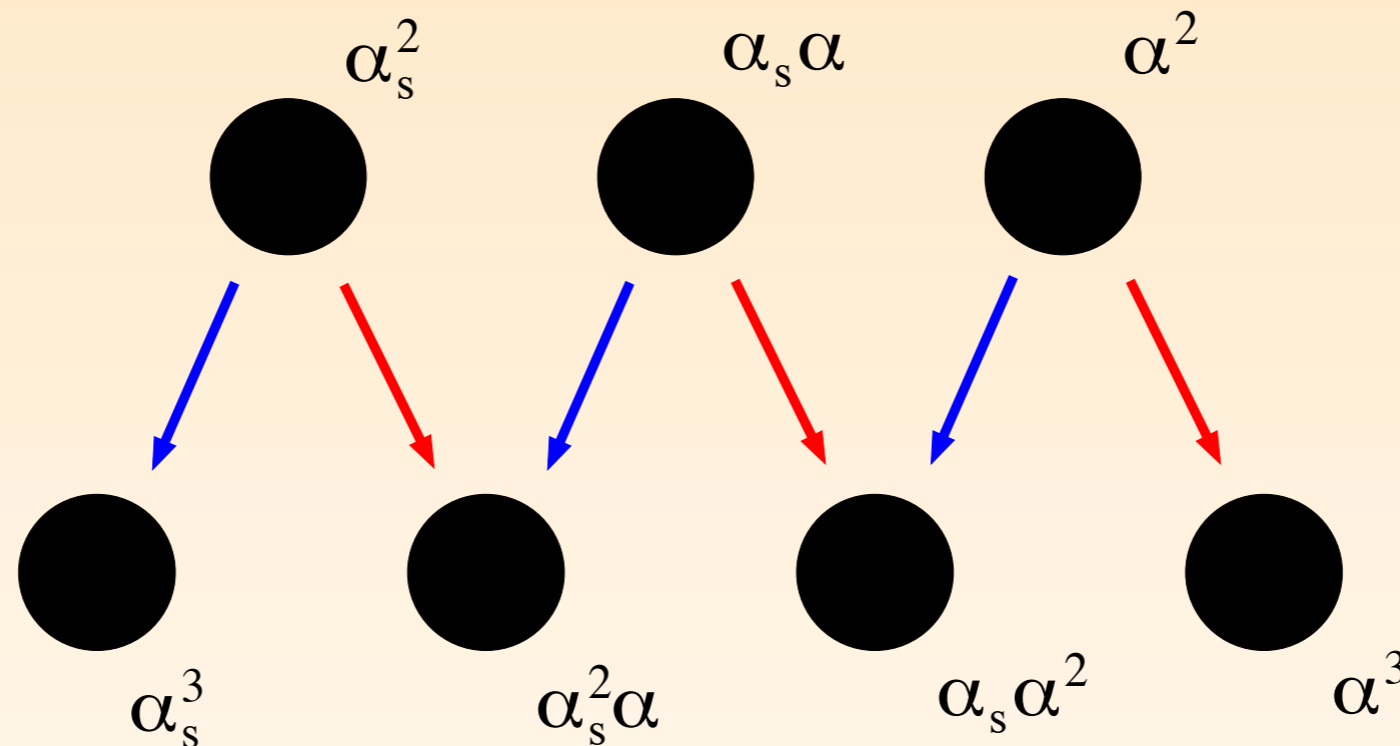
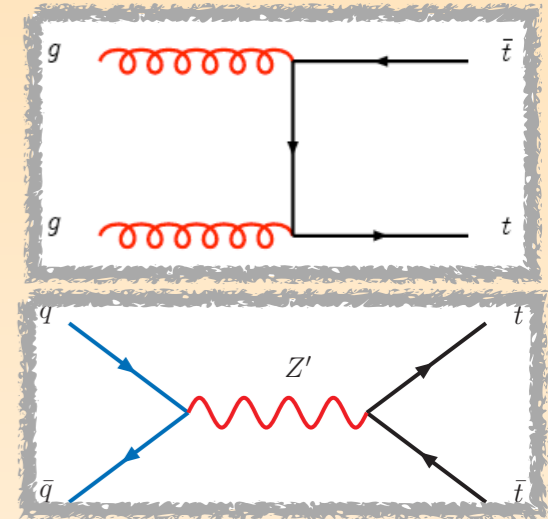
13 TeV

- ◆ Scale dependence for NLO top pair associated production is of the order of 10-15%
- ◆ NNLO is beyond what is currently possible
- ◆ The resummation of threshold logarithms in the limit  $z \rightarrow 1$  with  $z \equiv M^2/\hat{s}$  captures effects beyond the NLO
- ◆ This reduces the scale dependence by 30-50%, and leads to the most-accurate theoretical predictions
- ◆ Also differential distributions
- ◆ What about EW corrections?

pert. order	process	PDF order	$\sigma$ [fb]
NLO	$t\bar{t}H$	NLO	$474.8^{+47.2}_{-51.9}$
NLO+NNLL	$t\bar{t}H$	NNLO	$486.4^{+29.9}_{-24.5}$
NLO	$t\bar{t}W^+$	NLO	$356.3^{+43.7}_{-39.5}$
NLO+NNLL	$t\bar{t}W^+$	NNLO	$341.0^{+23.1}_{-13.6}$
NLO	$t\bar{t}W^-$	NLO	$182.2^{+23.1}_{-20.4}$
NLO+NNLL	$t\bar{t}W^-$	NNLO	$177.1^{+12.0}_{-6.9}$
NLO	$t\bar{t}Z$	NLO	$728.3^{+93.8}_{-90.3}$
NLO+NNLL	$t\bar{t}Z$	NNLO	$777.8^{+61.3}_{-65.2}$

# NLO DISSECTION

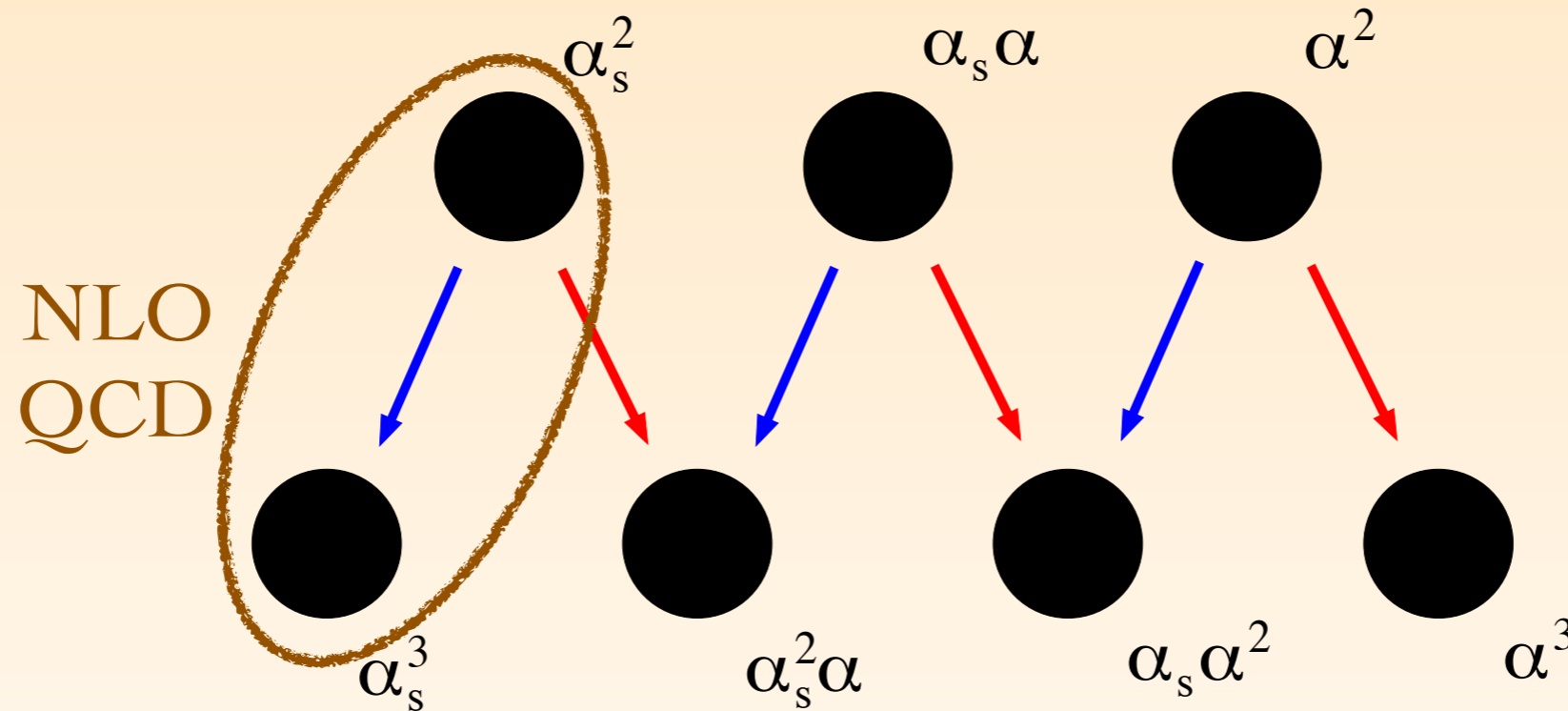
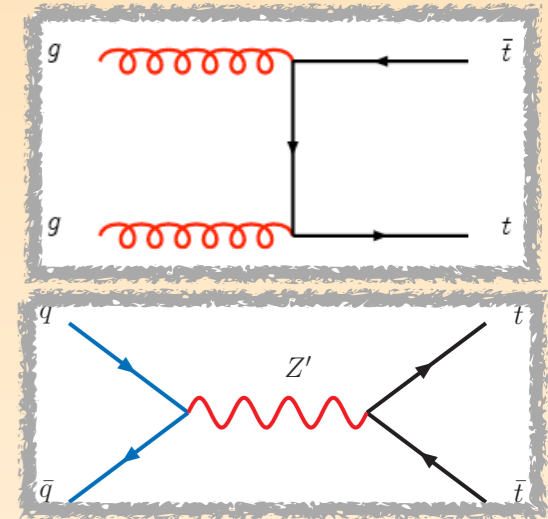
- ◆ For example: consider top-pair production



- ◆ "NLO EW" is a bit of a misnomer:  
NLO<sub>2</sub> and NLO<sub>3</sub> part of a "mixed" expansion
- ◆ "Complete-NLO" takes all the LO and NLO contributions in the mixed coupling expansion into account

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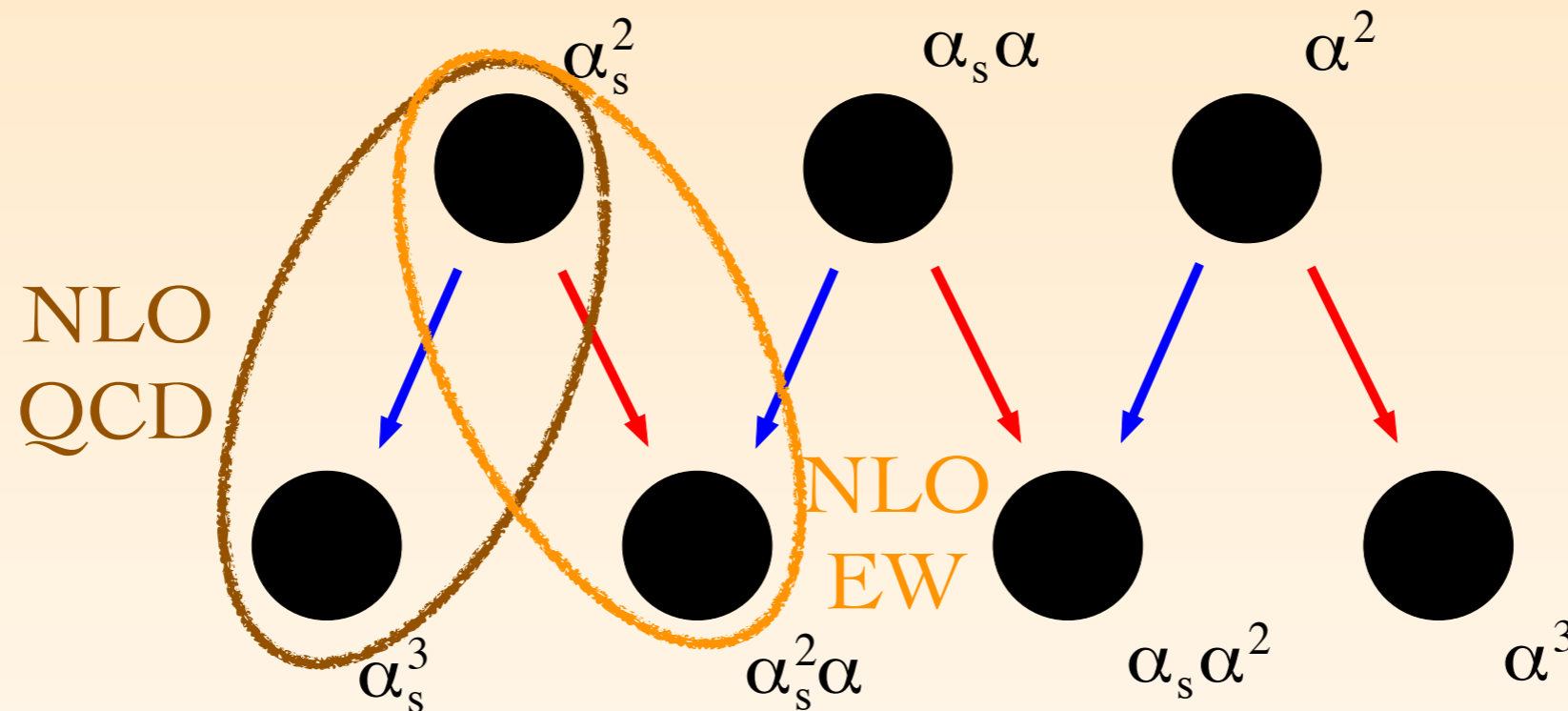
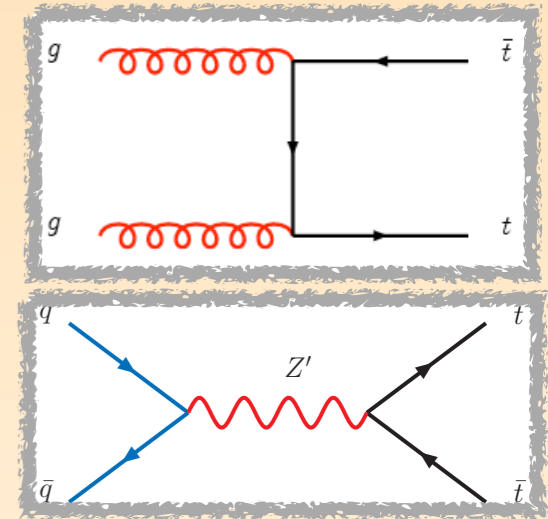
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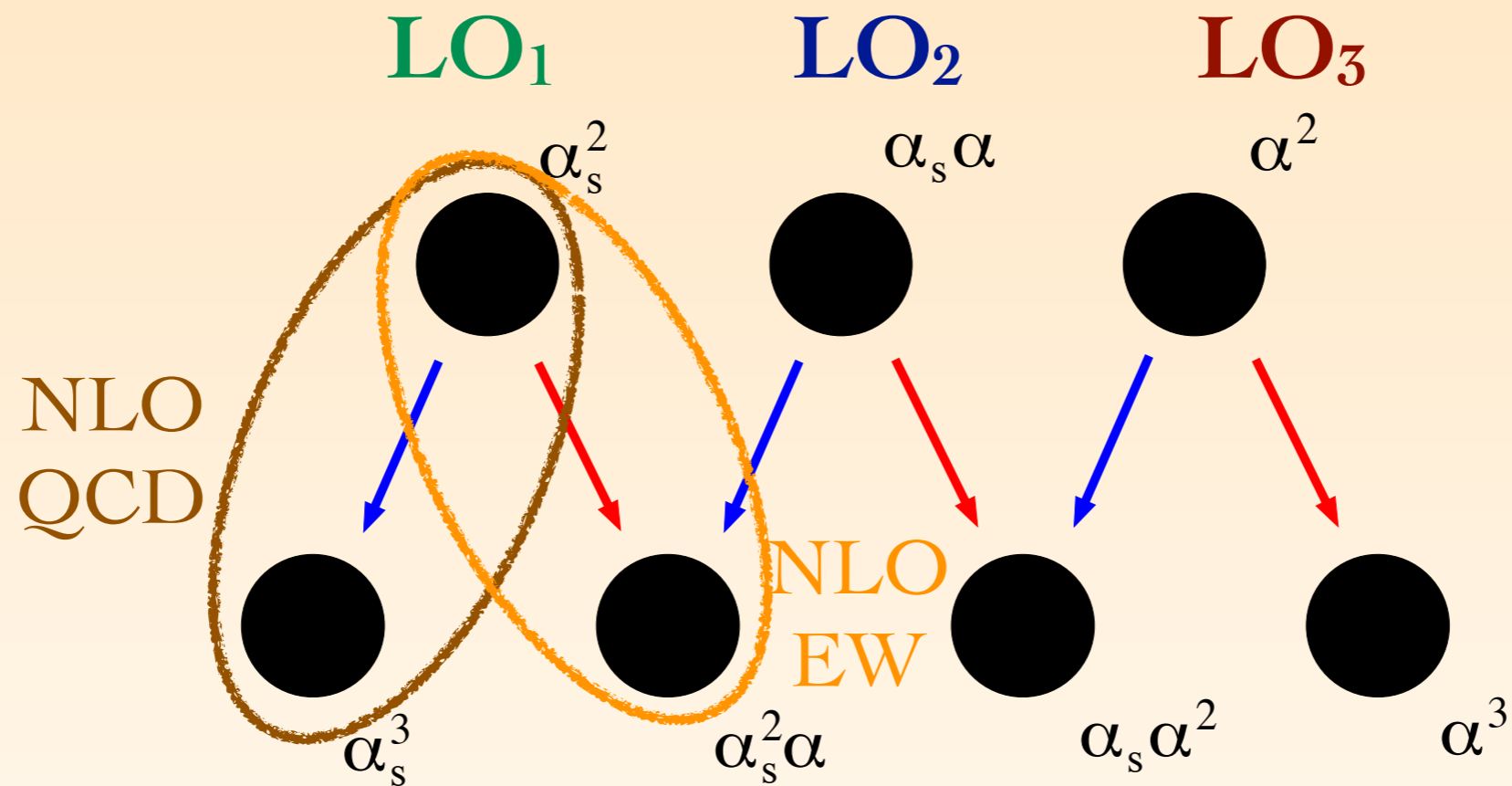
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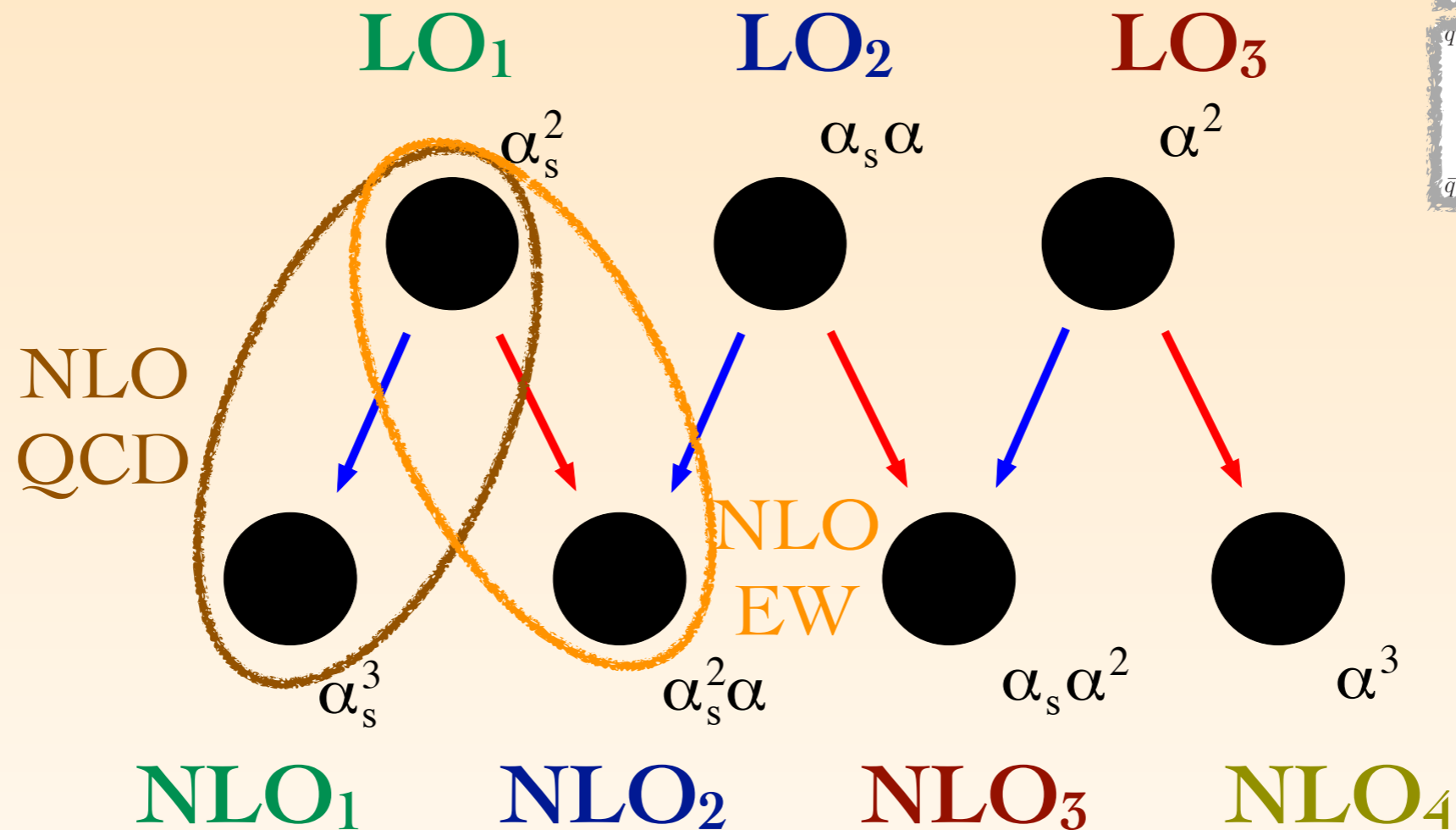
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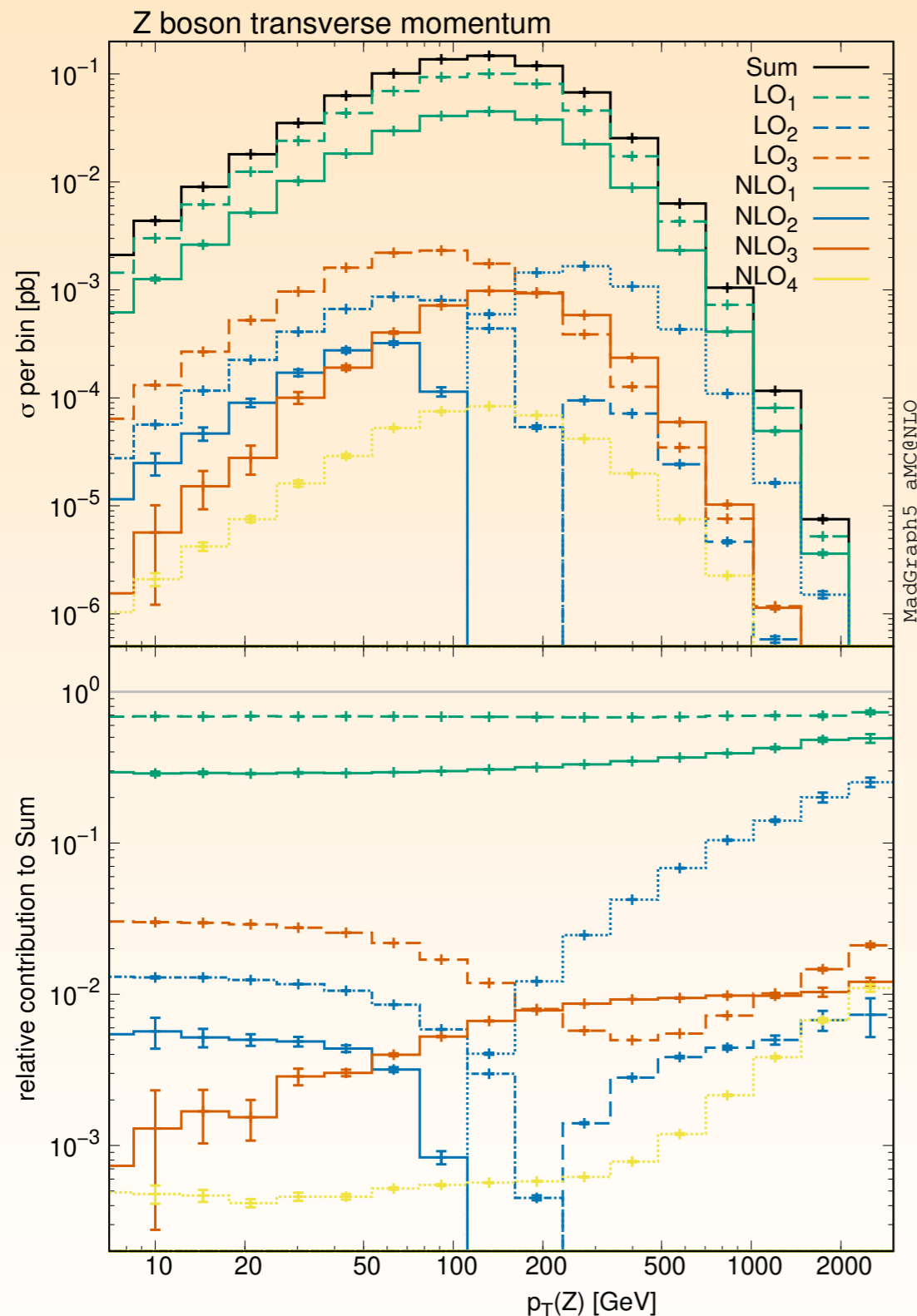
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# COMPLETE-NLO $TT\bar{B}\bar{A}R+Z$

[RF, S. Frixione, V. Hirschi, D. Pagani,  
H.-S. Shao, M. Zaro, 2018]



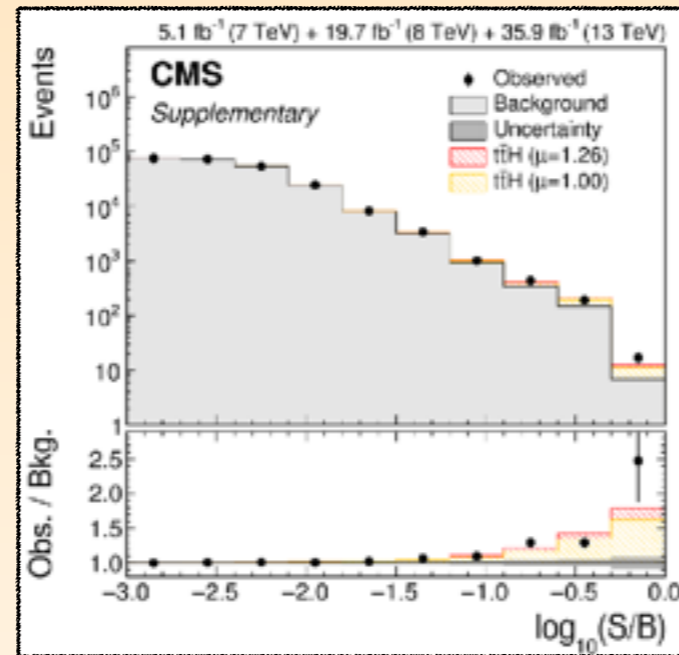
- ◆ Top pair production in association with a Z-boson
  - Transverse momentum of the vector boson
- ◆ Significant EW corrections (NLO<sub>2</sub>) at very large  $p_T$ s, where they can reach  $\sim 25\%$  of the total rate
  - Partly canceling the QCD corrections (NLO<sub>1</sub>), which grow with increasing  $p_T$
- ◆ (N)LO<sub>3</sub> and NLO<sub>4</sub> typically small and negligible for most practical purposes



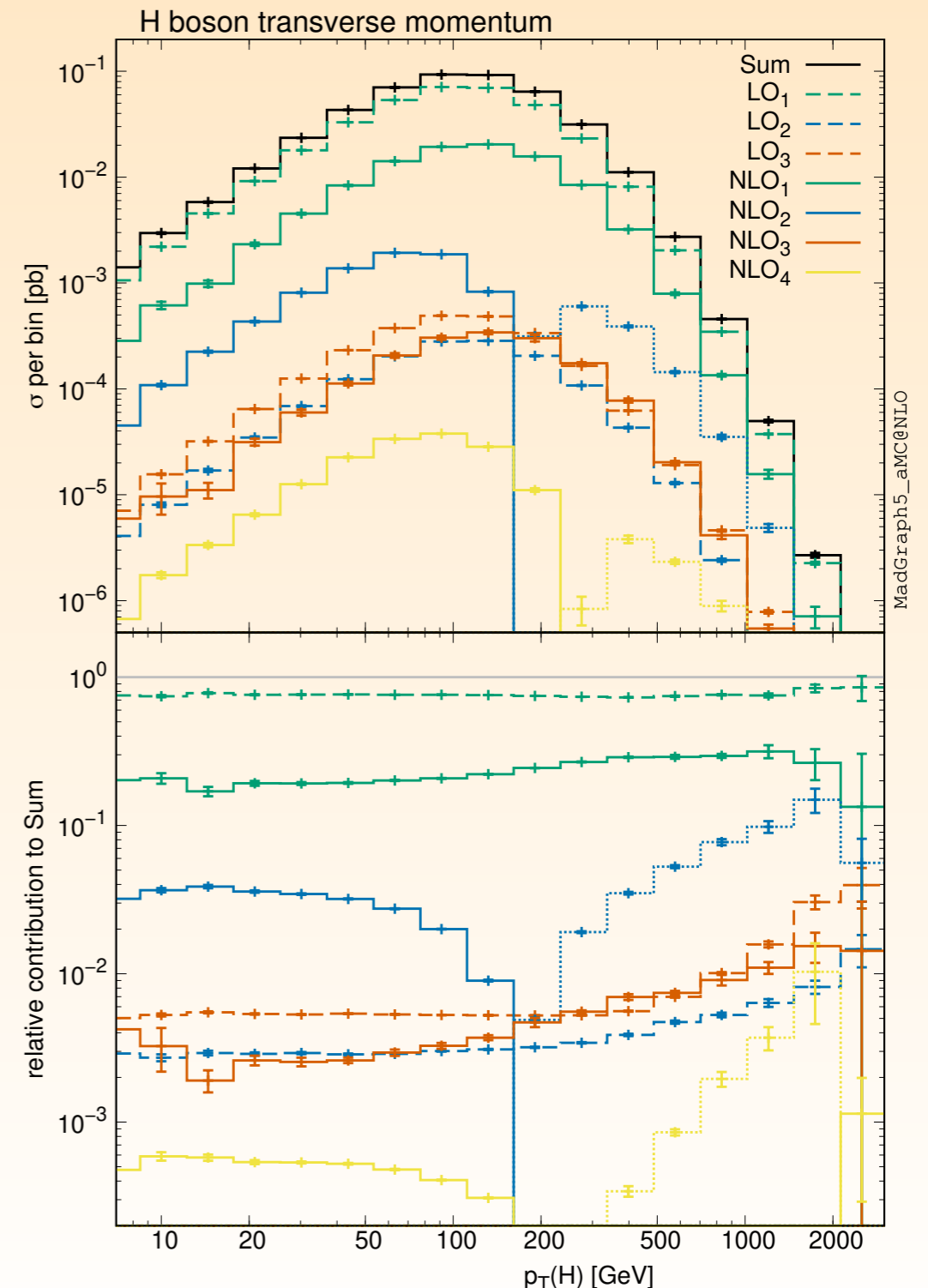
# COMPLETE-NLO $tt\bar{t}+H$

[RF, S. Frixione, V. Hirschi, D. Pagani,  
H.-S. Shao, M. Zaro, 2018]

- ◆ Higgs production in association with a top-quark pair now observed by both CMS and ATLAS



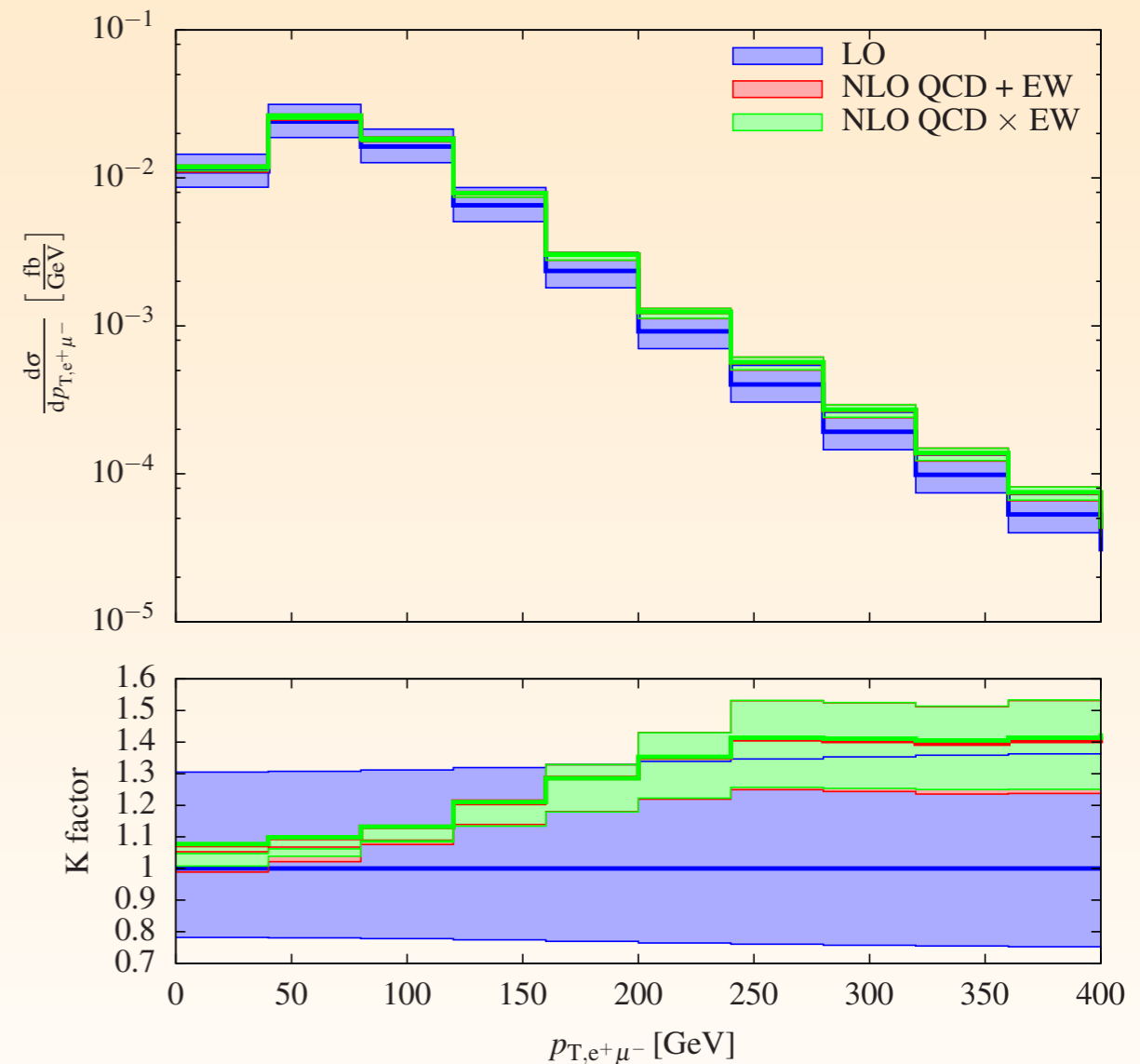
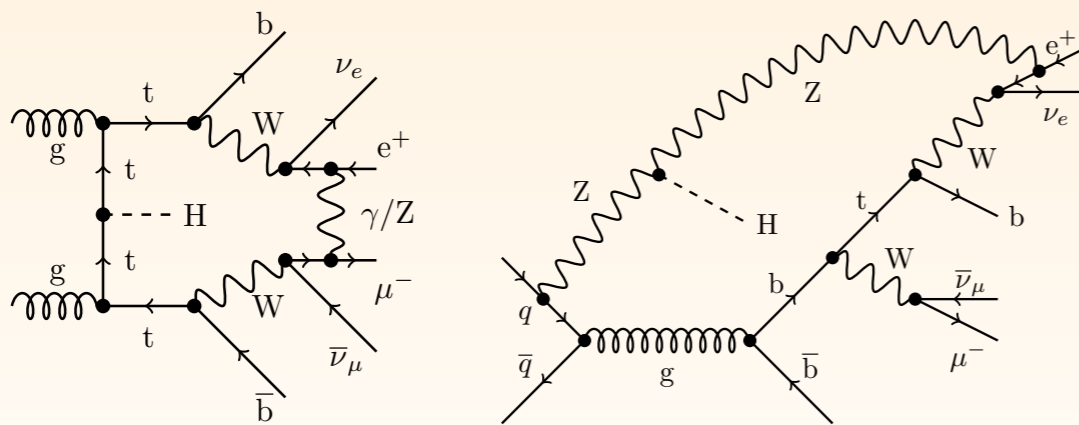
- ◆ Corrections (slightly) smaller than for  $tt\bar{t}+Z$ 
  - $NLO_2$  at the percent-level, apart from the far tail, where its effect is slightly larger
  - $(N)LO_3$  and  $NLO_4$  negligibly small



# NLO QCD+EW FOR TTH WITH OFF-SHELL TOPS

[Denner, Lang, Pellen, Uccirati, 2016]

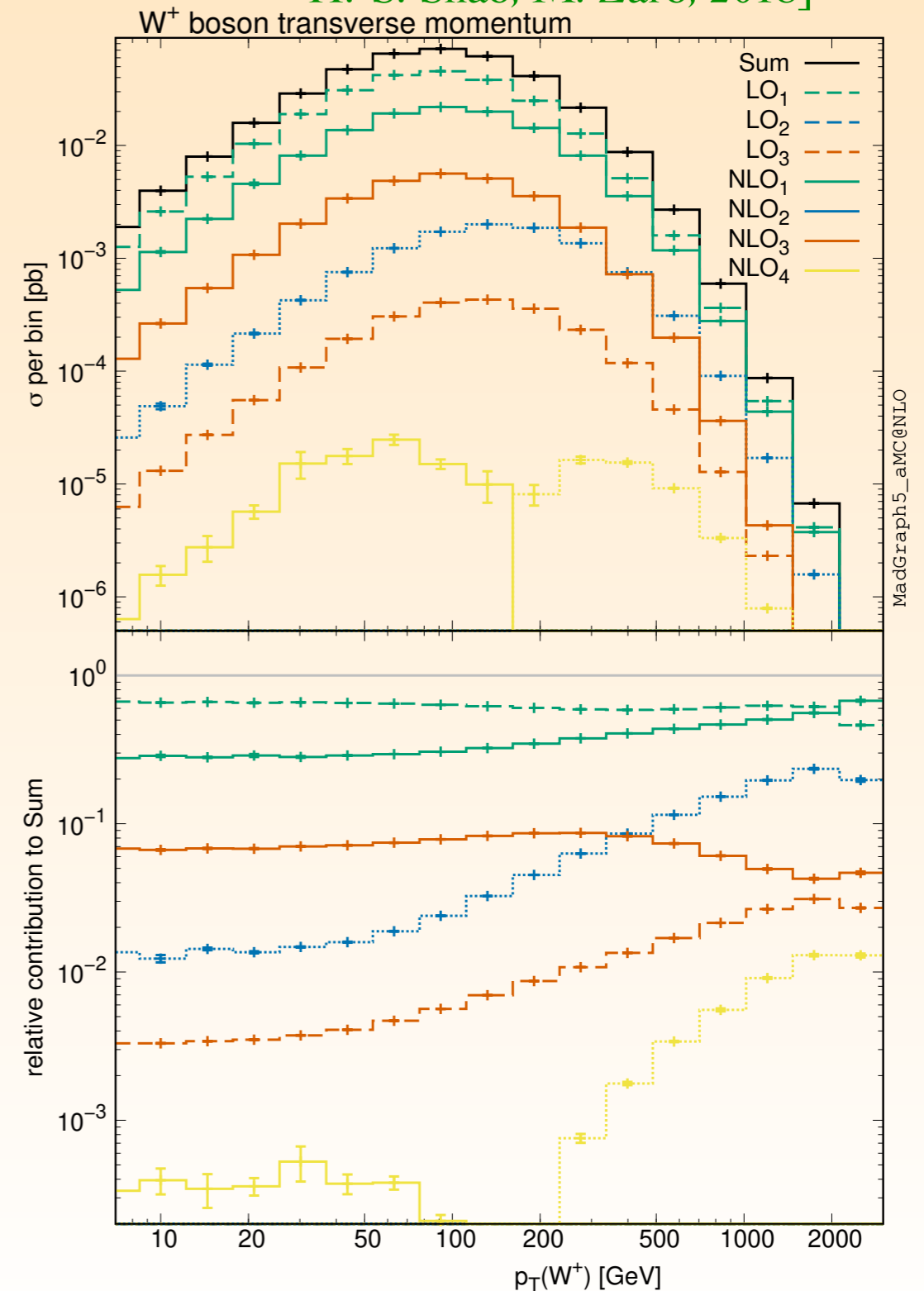
- ◆ NLO QCD+EW corrections to ttH with off-shell top quarks
- ◆ Amazingly complicated calculation
  - "octagons" and even "nonagons"



# COMPLETE-NLO $tt\bar{t} + W$

[RE, S. Frixione, V. Hirschi, D. Pagani,  
H.-S. Shao, M. Zaro, 2018]

- ◆ Top pair production in association with a W-boson
  - Transverse momentum of the boson
- ◆ Known: **NLO<sub>1</sub>** dominant at large  $p_T$  (larger than **LO<sub>1</sub>**); would be even more pronounced for  $p_T(tt)$  observable
  - can be avoided with a jet veto
- ◆ Surprise!: **NLO<sub>3</sub>** is the largest subleading NLO correction; begin close to 10% of the complete-NLO at small and medium transverse momenta
- ◆ Significant EW corrections (**NLO<sub>2</sub>**) at very large  $p_T$ s, where they can reach  $\sim 25\%$  of the total rate
  - **LO<sub>2</sub>** are exactly zero



# COMPLETE-NLO TTBAR+W

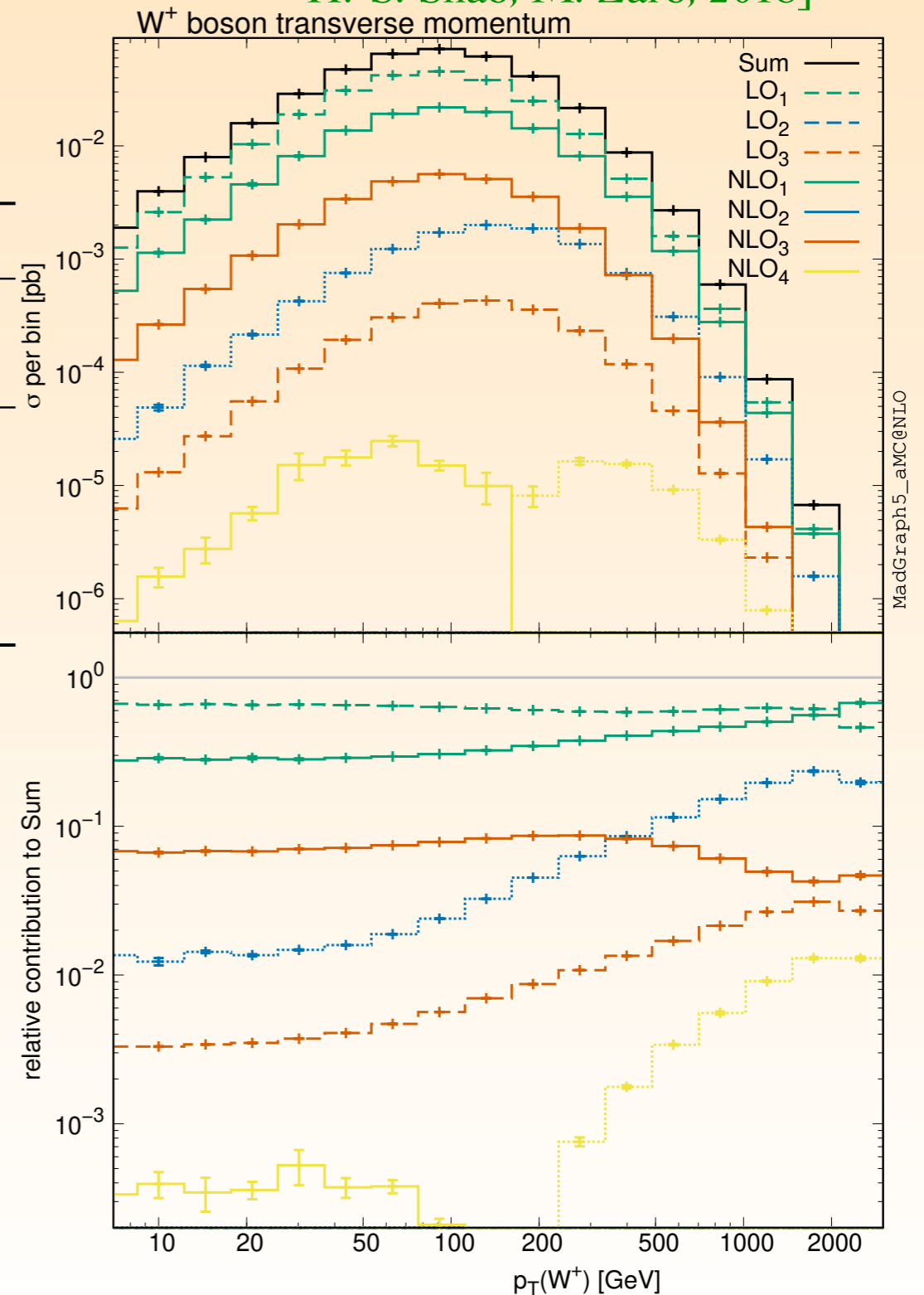
[RF, Pagani, Zaro, 2017]

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Slightly different normalisation  
as compared to ratio plot  $\delta_{(N)LO_i}(\mu) = \frac{\Sigma_{(N)LO_i}(\mu)}{\Sigma_{LO_{QCD}}(\mu)}$

Naive expectation	$\delta[\%]$	$\mu = H_T/4$	$\mu = H_T/2$	$\mu = H_T$
10%	LO <sub>2</sub>	-	-	-
1%	LO <sub>3</sub>	0.8	0.9	1.1
10%	NLO <sub>1</sub>	34.8 (7.0)	50.0 (25.7)	63.4 (42.0)
1%	NLO <sub>2</sub>	-4.4 (-4.8)	-4.2 (-4.6)	-4.0 (-4.4)
0.1%	NLO <sub>3</sub>	11.9 (8.9)	12.2 (9.1)	12.5 (9.3)
0.01%	NLO <sub>4</sub>	0.02 (-0.02)	0.04 (-0.02)	0.05 (-0.01)

- ◆ Numbers in brackets correspond to a 100 GeV jet-veto
  - This reduces the NLO<sub>1</sub> enormously
  - large scale dependence in NLO<sub>1</sub> to compensate scale dependence in LO<sub>1</sub>
  - But does not affect the large NLO<sub>3</sub>
- ◆ NLO<sub>3</sub> contribution due to *t*-W scattering



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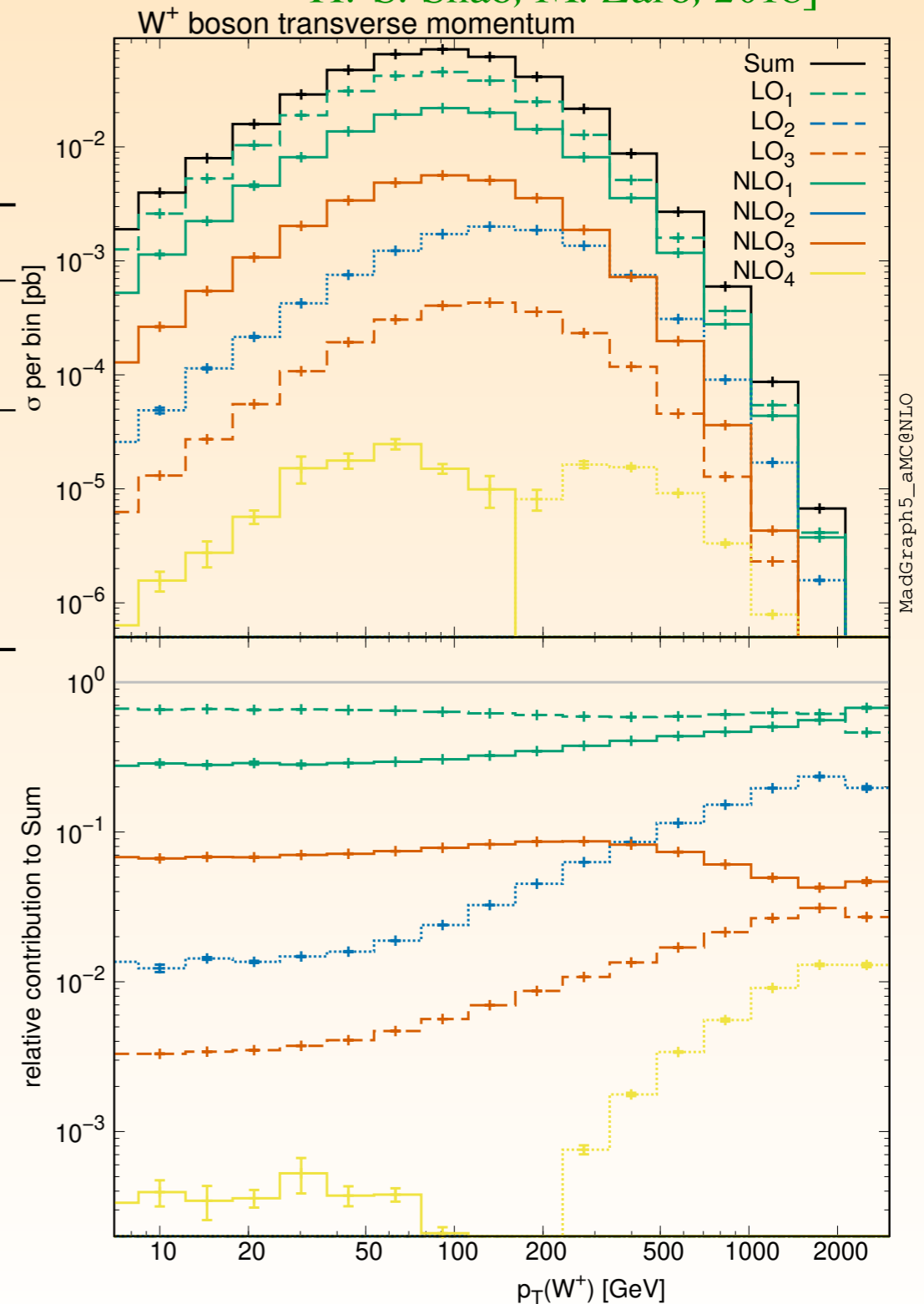
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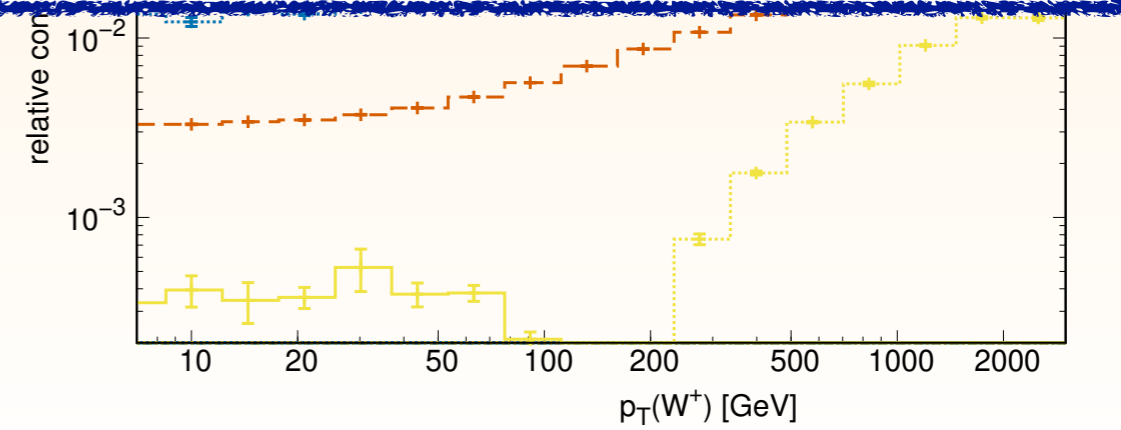
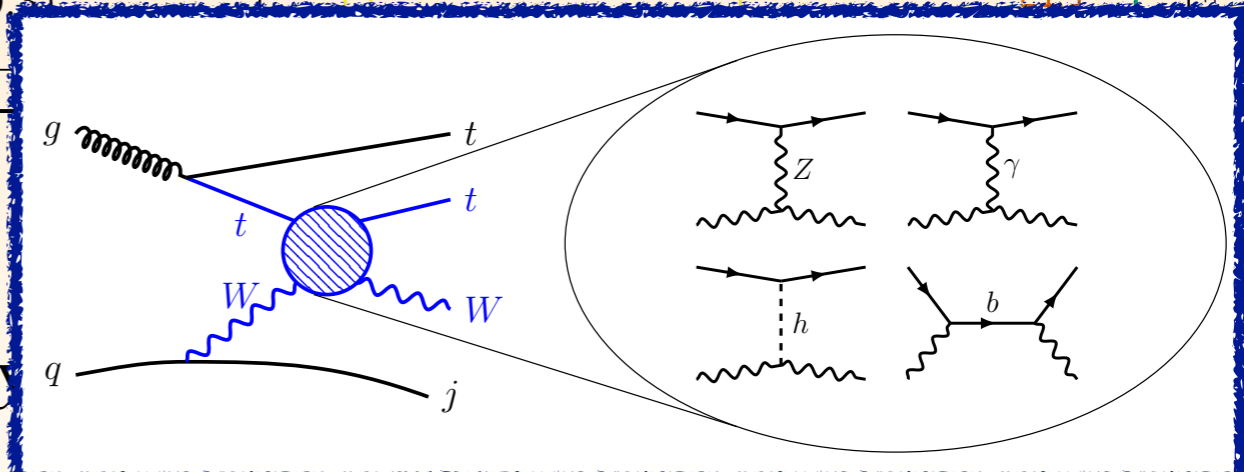
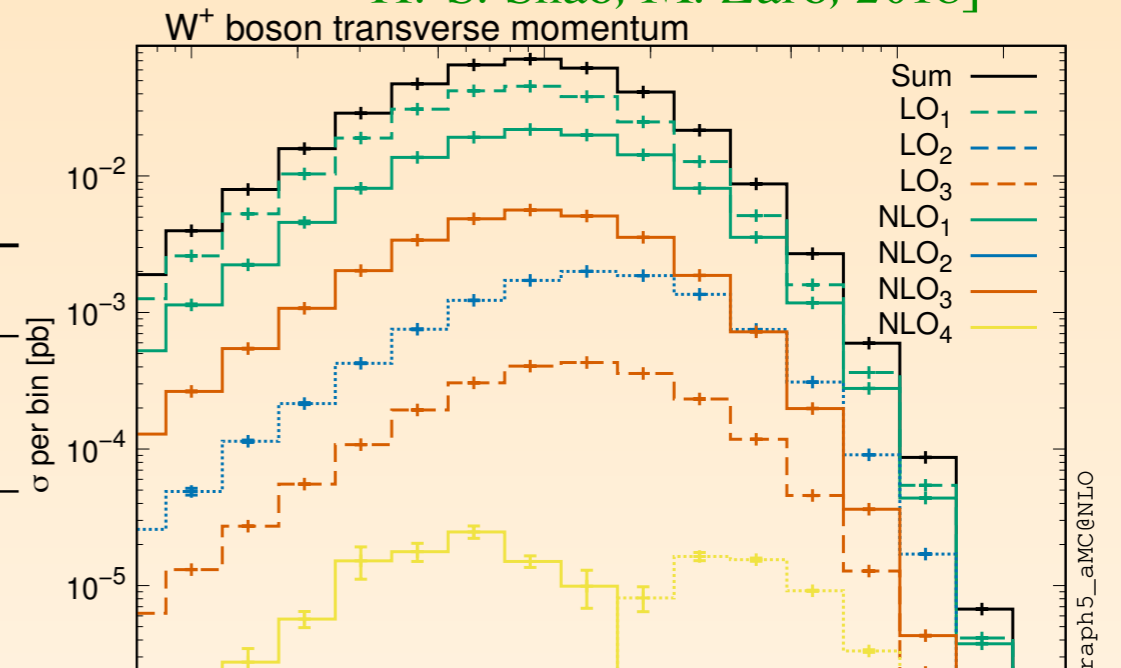
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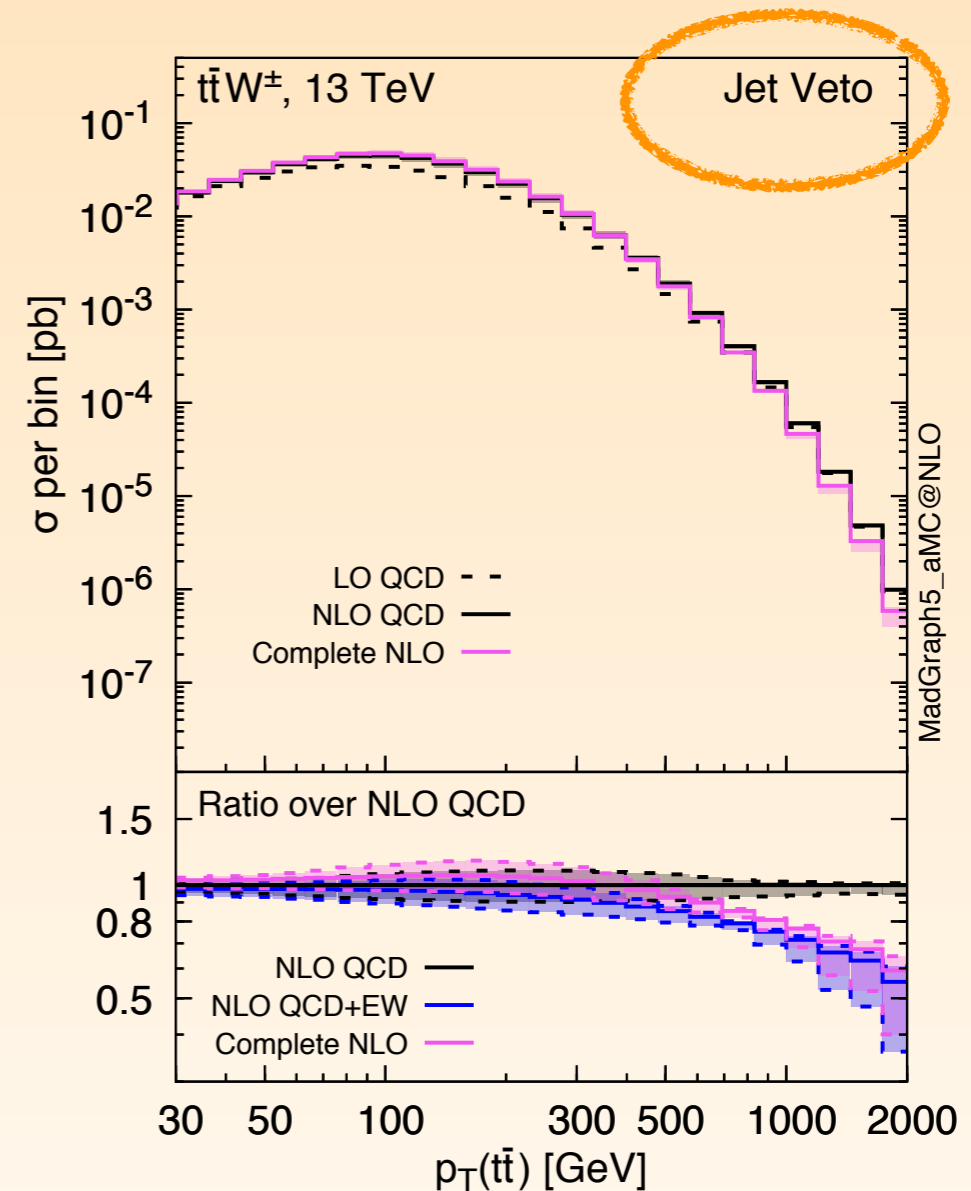
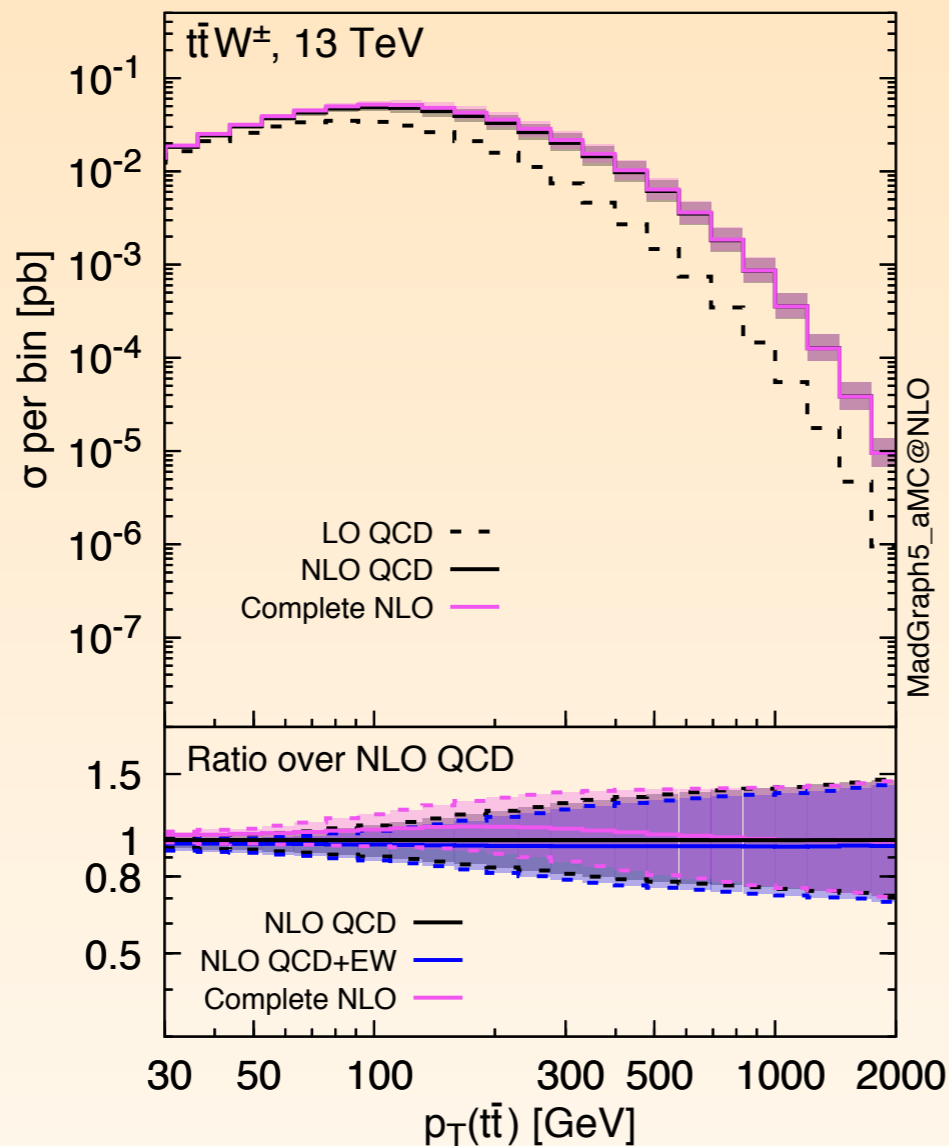
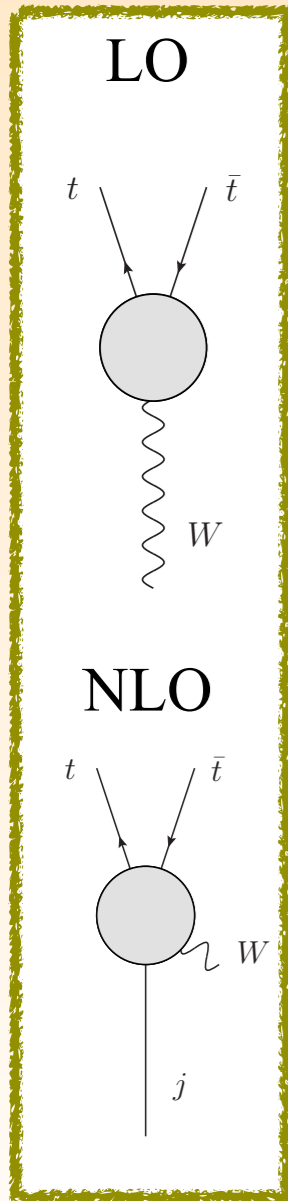


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[RF, Pagani,  
Zaro, 2017]

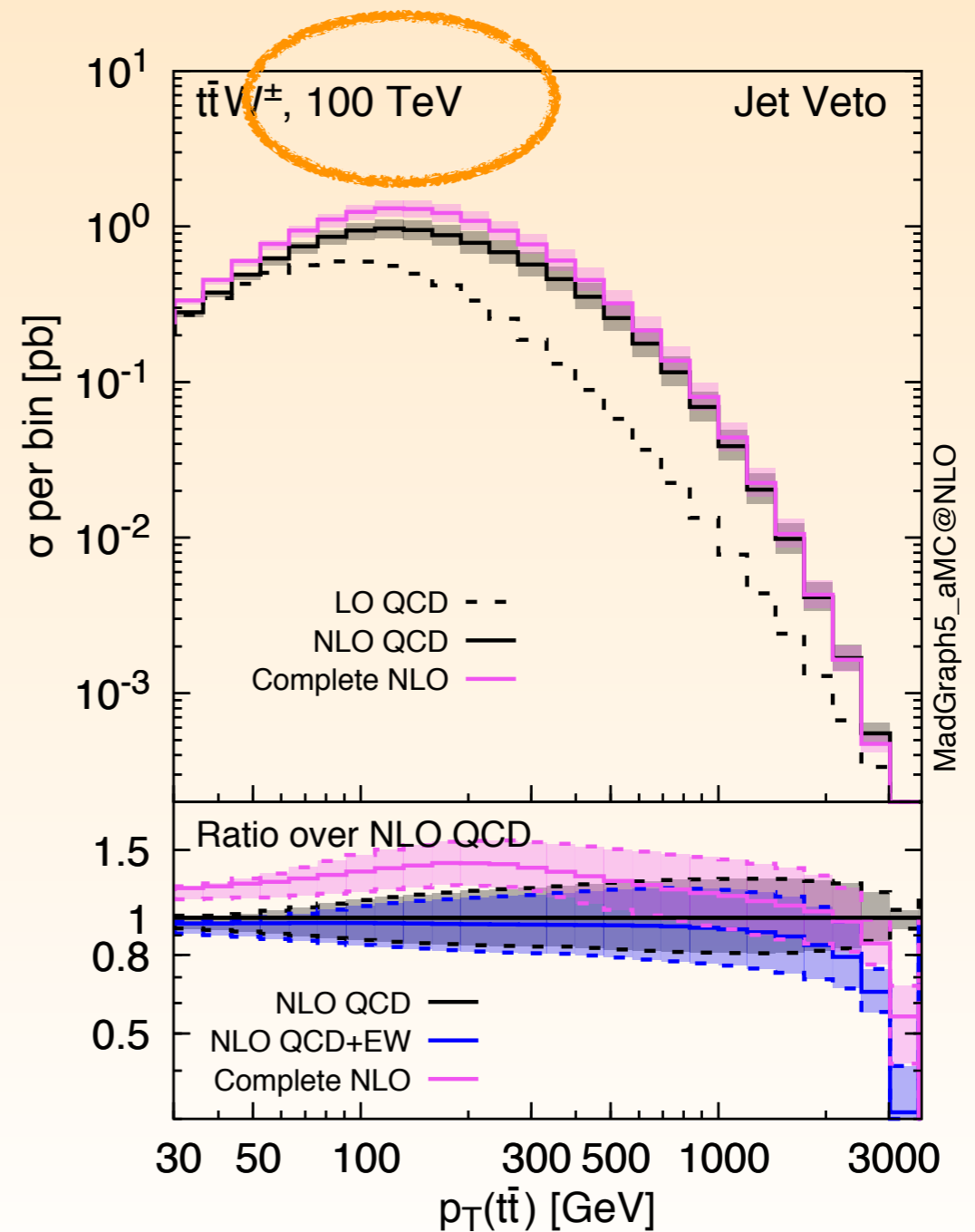
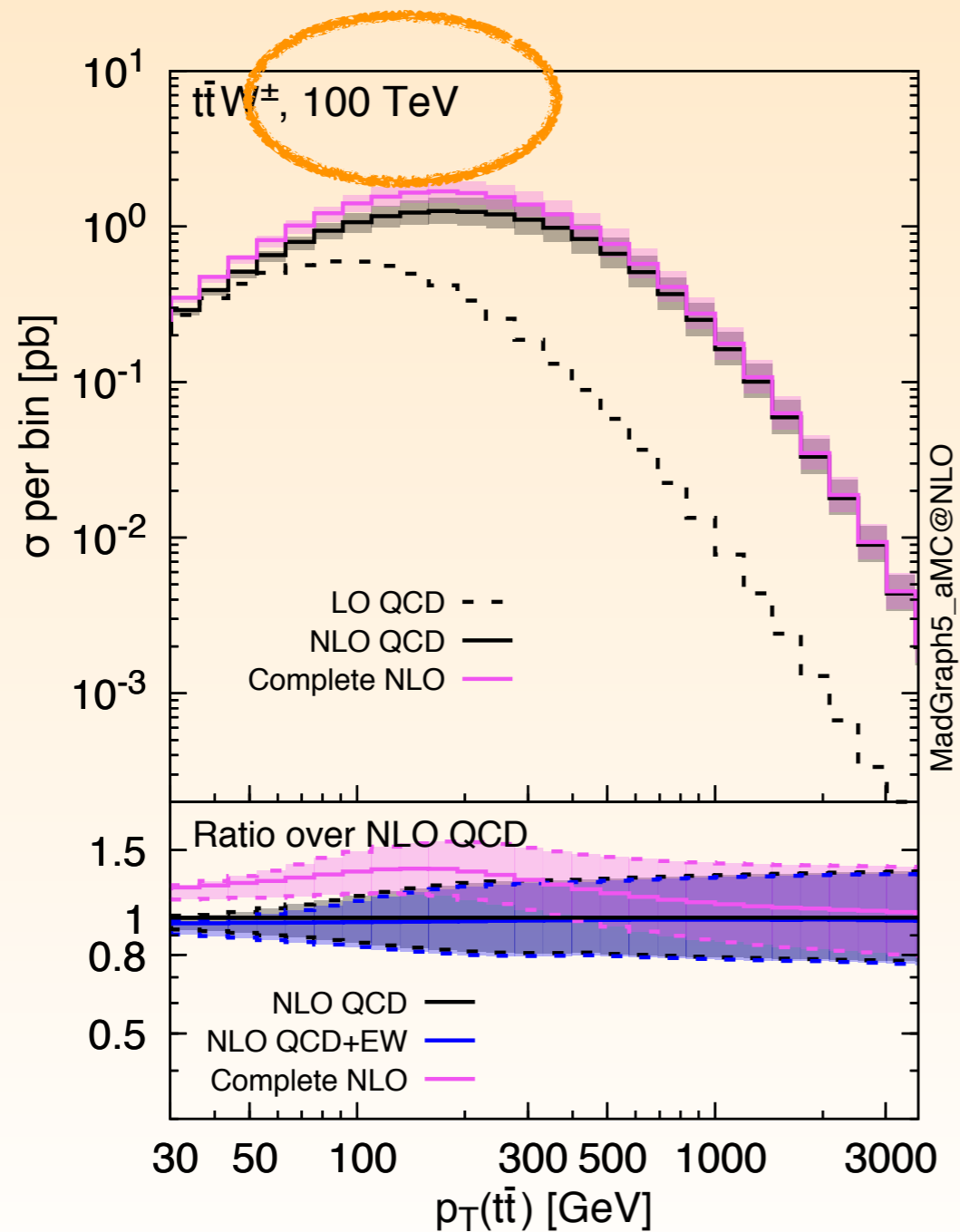


- ◆ Large NLO corrections, and large scale dependence without the jet veto
- ← ⊙  $t\bar{t}bar$  recoiling predominantly against jet (instead of W-boson) at NLO
- ◆ EW corrections are large when the results are not dominated by  $NLO_1$
- ◆ Subleading corrections ( $NLO_3$ ) are larger than expected, but uncertainty bands overlap

# COMPLETE-NLO TTBAR+W

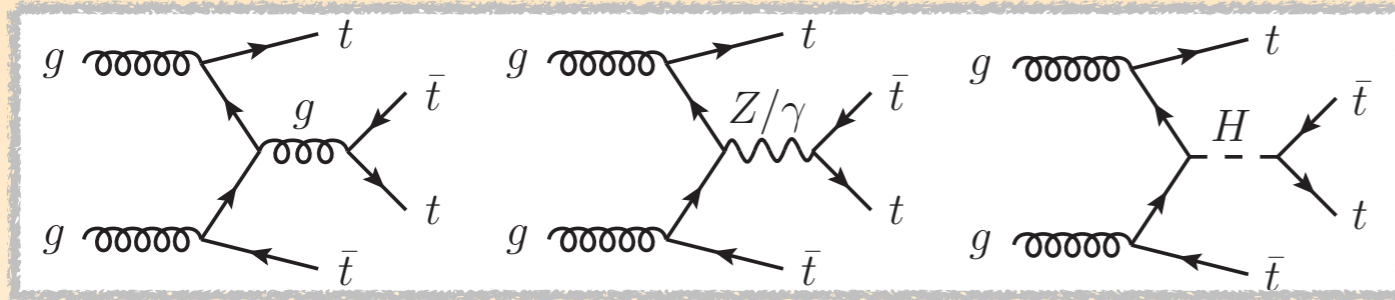
[RF, Pagani,  
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◆ Effects much more extreme at 100 TeV!





# FOUR-TOP PRODUCTION AND TOP YUKAWA COUPLING



$$\begin{aligned}\sigma^{\text{SM}}(t\bar{t}t\bar{t})_{g+Z/\gamma} &\propto |\mathcal{M}_g + \mathcal{M}_{Z/\gamma}|^2, \\ \sigma^{\text{SM}}(t\bar{t}t\bar{t})_H &\propto |\mathcal{M}_H|^2, \\ \sigma^{\text{SM}}(t\bar{t}t\bar{t})_{\text{int}} &\propto \mathcal{M}_{g+Z/\gamma} \mathcal{M}_H^\dagger + \mathcal{M}_{g+Z/\gamma}^\dagger \mathcal{M}_H\end{aligned}$$

$$\sigma(t\bar{t}t\bar{t}) = \sigma^{\text{SM}}(t\bar{t}t\bar{t})_{g+Z/\gamma} + \kappa_t^2 \sigma_{\text{int}}^{\text{SM}} + \kappa_t^4 \sigma^{\text{SM}}(t\bar{t}t\bar{t})_H$$

[Cao, Chen, Liu, 2016]

◆ Four-top production can be used together with ttH to constrain/measure an **anomalous top Yukawa coupling**

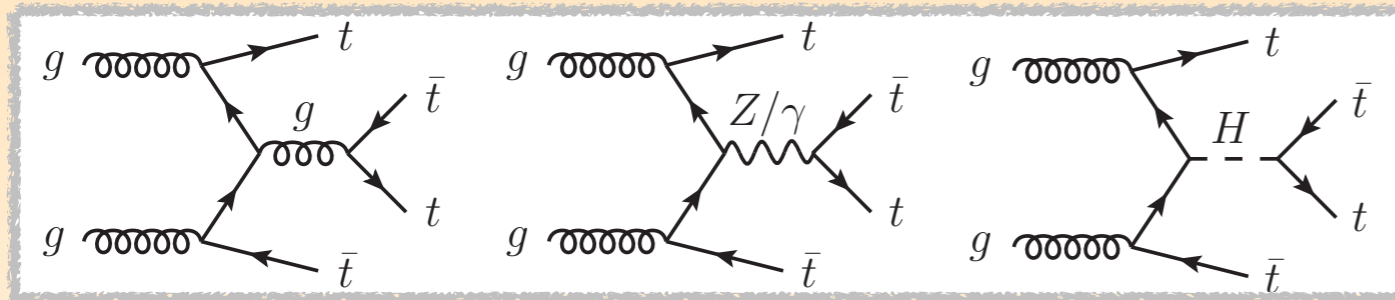
○ kappa-framework

◆ Large contributions from subleading **LO<sub>i</sub>**, with large cancelations

○ How do NLO corrections affect these?

	8 TeV	14 TeV
$\sigma^{\text{SM}}(t\bar{t}t\bar{t})_{g+Z/\gamma}$ :	1.193 fb,	12.390 fb,
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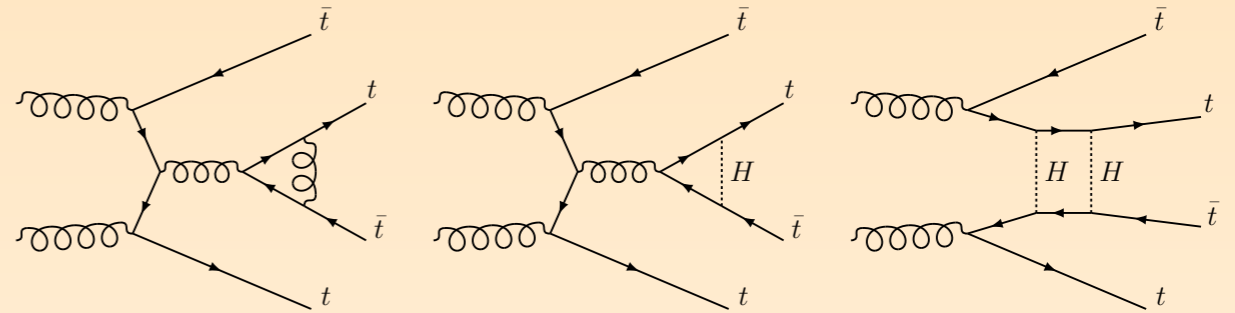
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# NLO KAPPA FRAMEWORK...?



- ◆ Kappa-framework: replace all SM Higgs couplings  $y_{\text{sm},i}$  with "anomalous" couplings, with strength  $y_i = \kappa_i \times y_{\text{sm},i}$
- ◆ When computing  $\text{NLO}_i$  (with  $i > 1$ ) corrections, e.g. **NLO EW**, top Yukawa coupling and top mass are not independent parameters
  - Cannot use kappa-framework
- ◆ Need complete **Effective Field Theory** framework
  - Currently beyond capabilities for four-top production: through renormalisation too many operators need to be considered together
- ◆ Still, NLO four-top in the SM will tell us about possible cancelations among various contributions

# NLO FOUR-TOP PRODUCTION

[RF, Pagani,  
Zaro, 2017]

- ◆  $LO_2$  and  $LO_3$  have large cancelations
- ◆  $NLO_2$  and  $NLO_3$  mainly given by QCD corrections on top of them
  - large and strongly dependent on the scale choice
- ◆ However, the sum of  $NLO_2+NLO_3$  very stable and small
- ◆ Different scale choices have even more extreme cancelations between  $NLO_2$  and  $NLO_3$

$\delta[\%]$	$\mu = H_T/8$	$\mu = H_T/4$	$\mu = H_T/2$	Naive expectation
$LO_2$	-18.7	-20.7	-22.8	10%
$LO_3$	26.3	31.8	37.8	1%
$LO_4$	0.05	0.07	0.09	0.1%
$LO_5$	0.03	0.05	0.08	0.01%
$NLO_1$	33.9	68.2	98.0	10%
$NLO_2$	-0.3	-5.7	-11.6	1%
$NLO_3$	-3.9	1.7	8.9	0.1%
$NLO_4$	0.7	0.9	1.2	0.01%
$NLO_5$	0.12	0.14	0.16	0.001%
$NLO_6$	< 0.01	< 0.01	< 0.01	0.0001%
$NLO_2 + NLO_3$	-4.2	-4.0	2.7	

- ◆  $LO_4$ ,  $(N)LO_5$  and  $NLO_6$  only qqbar initial state. Hence, very small

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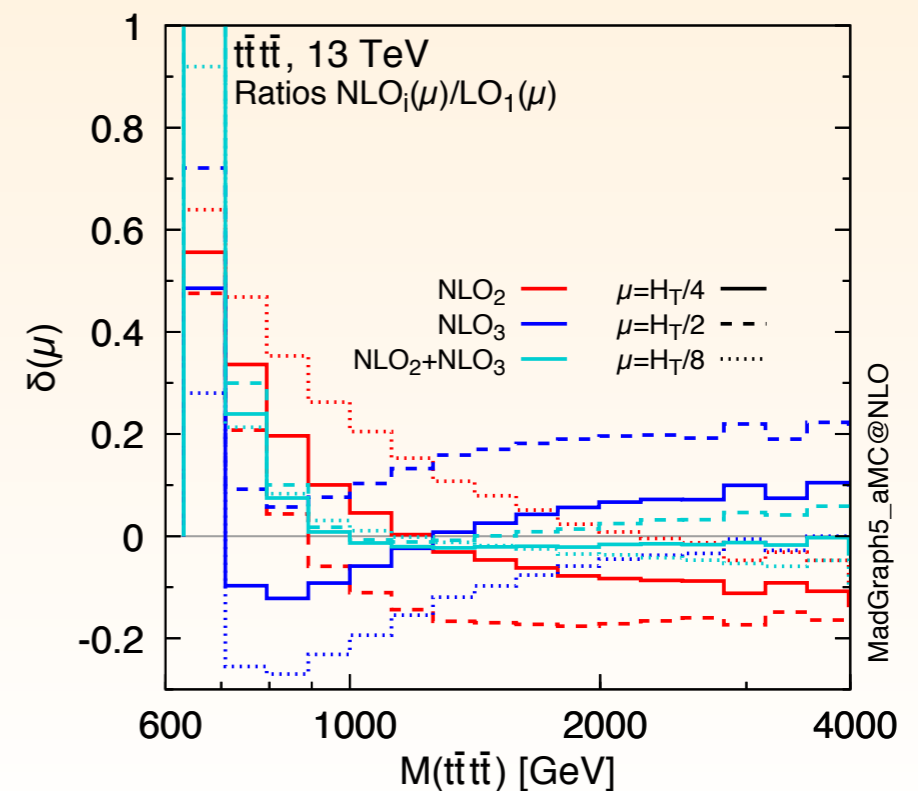
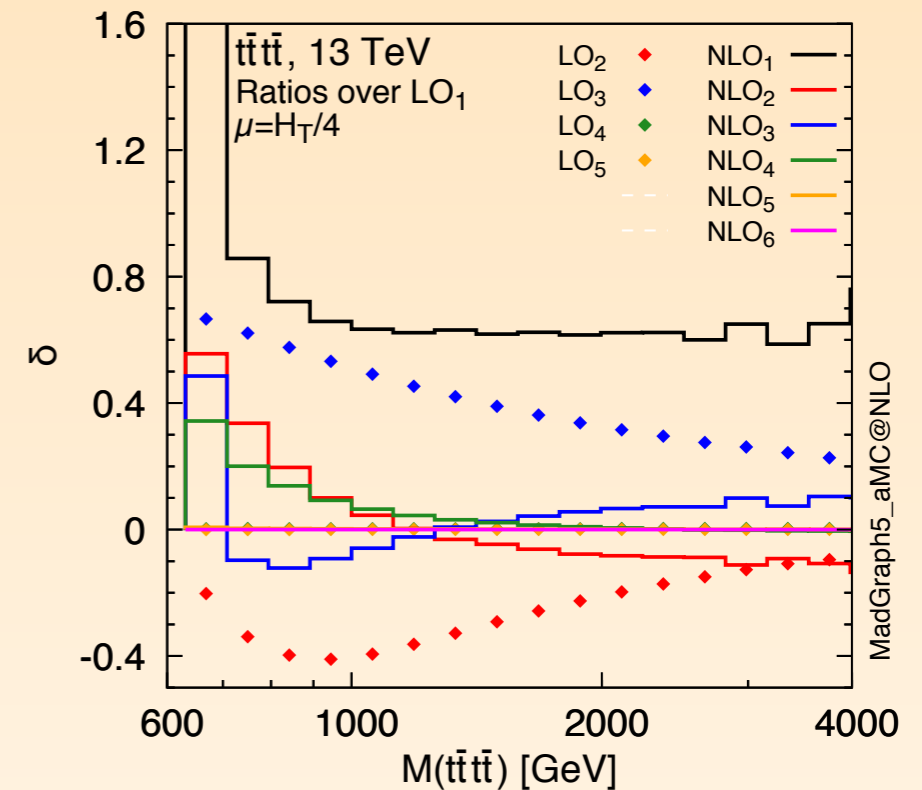
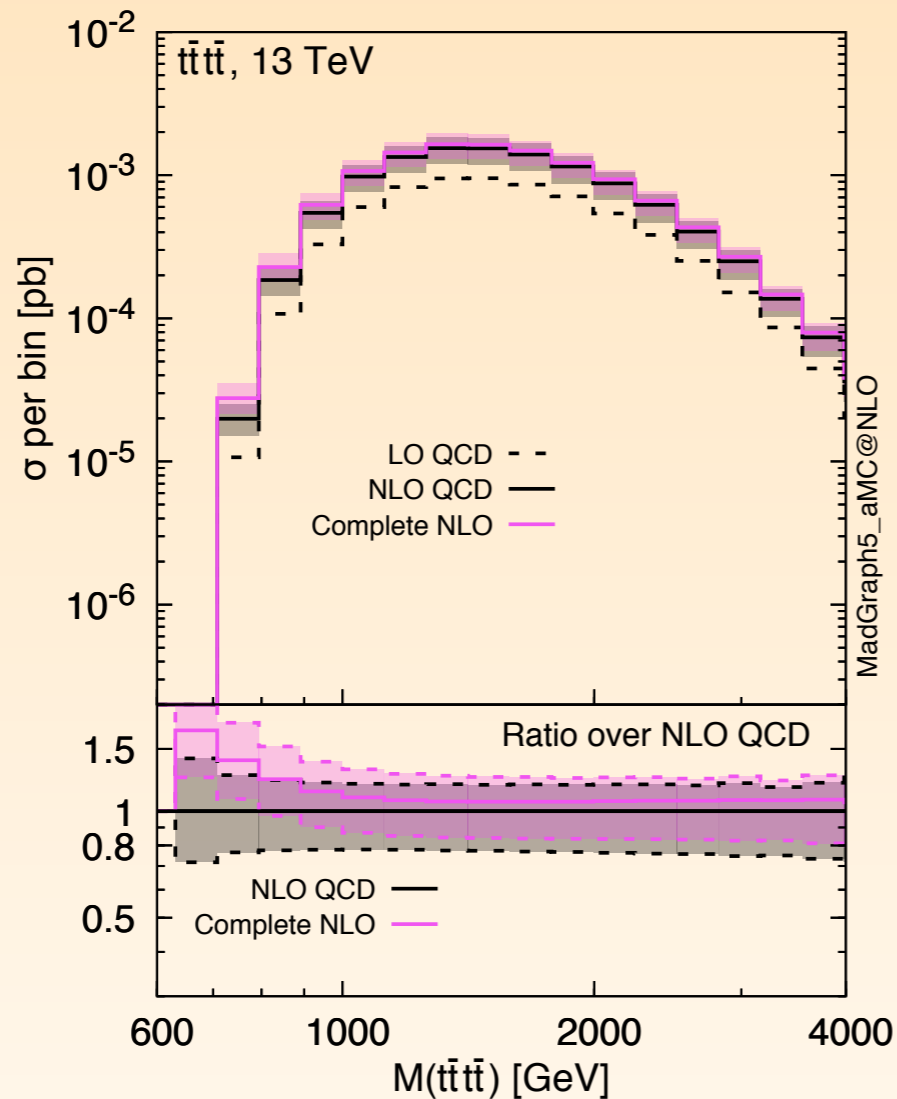
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$NLO_2$	-0.3	-5.7	-11.6	1%
$NLO_3$	-3.9	1.7	8.9	0.1%
$NLO_4$	0.7	0.9	1.2	0.01%
$NLO_5$	0.12	0.14	0.16	0.001%
$NLO_6$	< 0.01	< 0.01	< 0.01	0.0001%
$NLO_2 + NLO_3$	-4.2	-4.0	2.7	

- ◆  $LO_4$ ,  $(N)LO_5$  and  $NLO_6$  only qqbar initial state. Hence, very small

# FOUR-TOP INVARIANT MASS

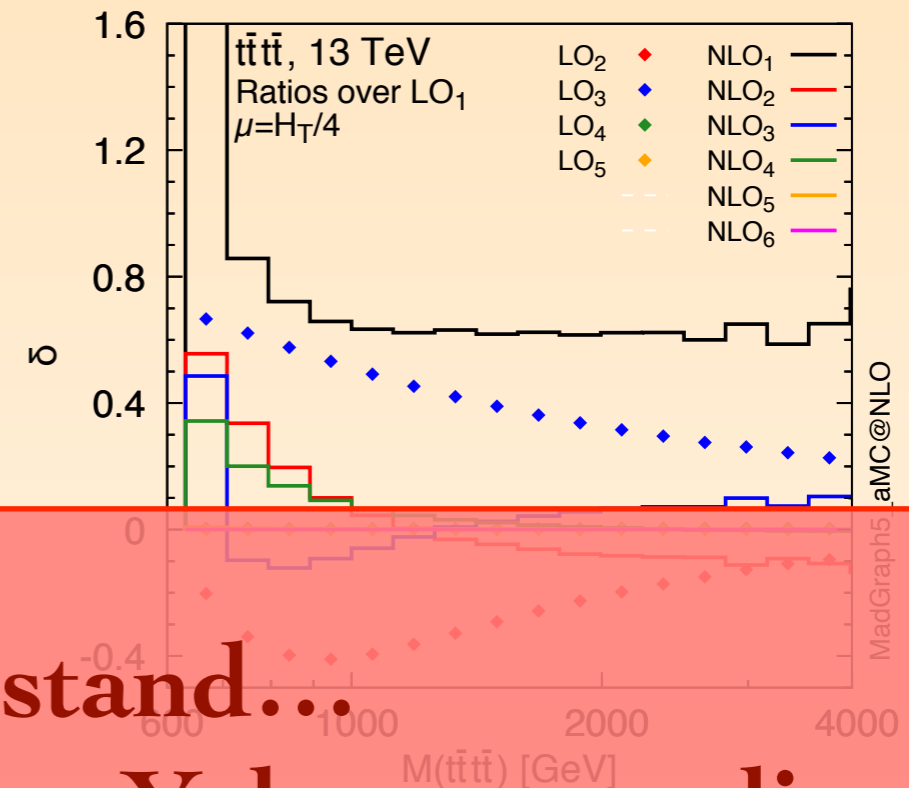
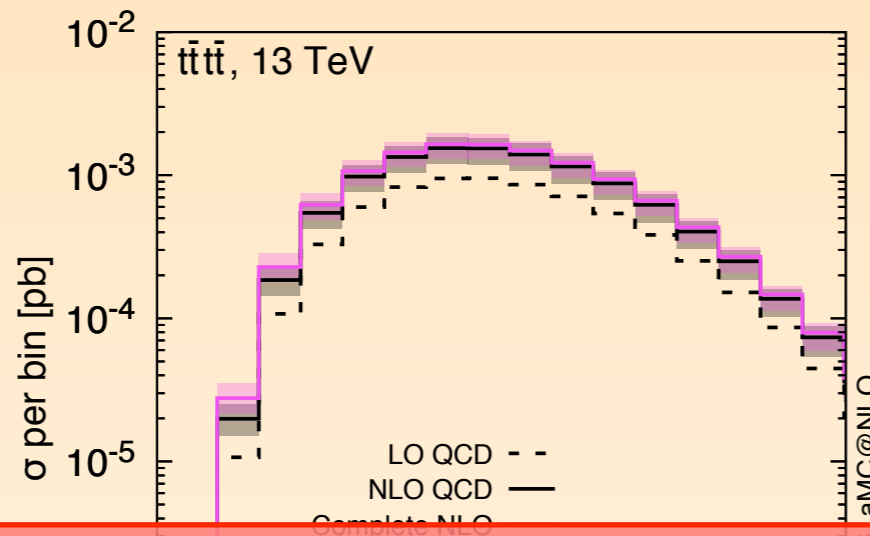
[RF, Pagani,  
Zaro, 2017]



- ◆ Large cancelations between **NLO<sub>2</sub>** and **NLO<sub>3</sub>** also at the differential level
- ◆ **NLO<sub>4</sub>** large at threshold

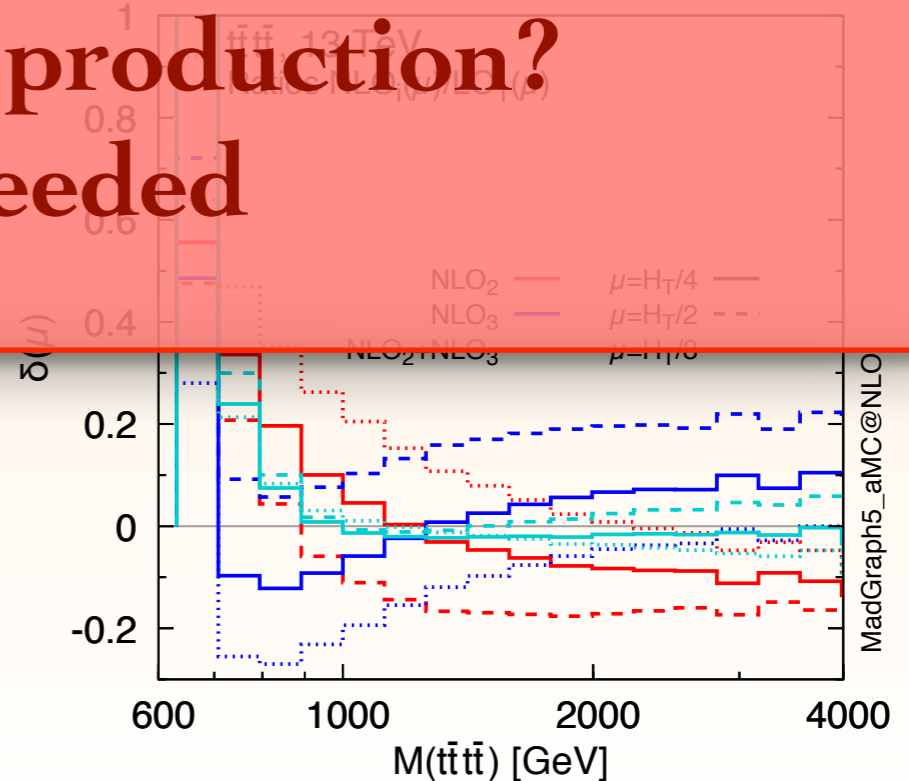
# FOUR-TOP INVARIANT MASS

[RF, Pagani,  
Zaro, 2017]



Difficult to understand...  
 What would this mean for a top Yukawa coupling  
 extraction from 4-top production?  
 More studies needed

- ◆ Large cancelations between  $NLO_2$  and  $NLO_3$  also at the differential level
- ◆  $NLO_4$  large at threshold

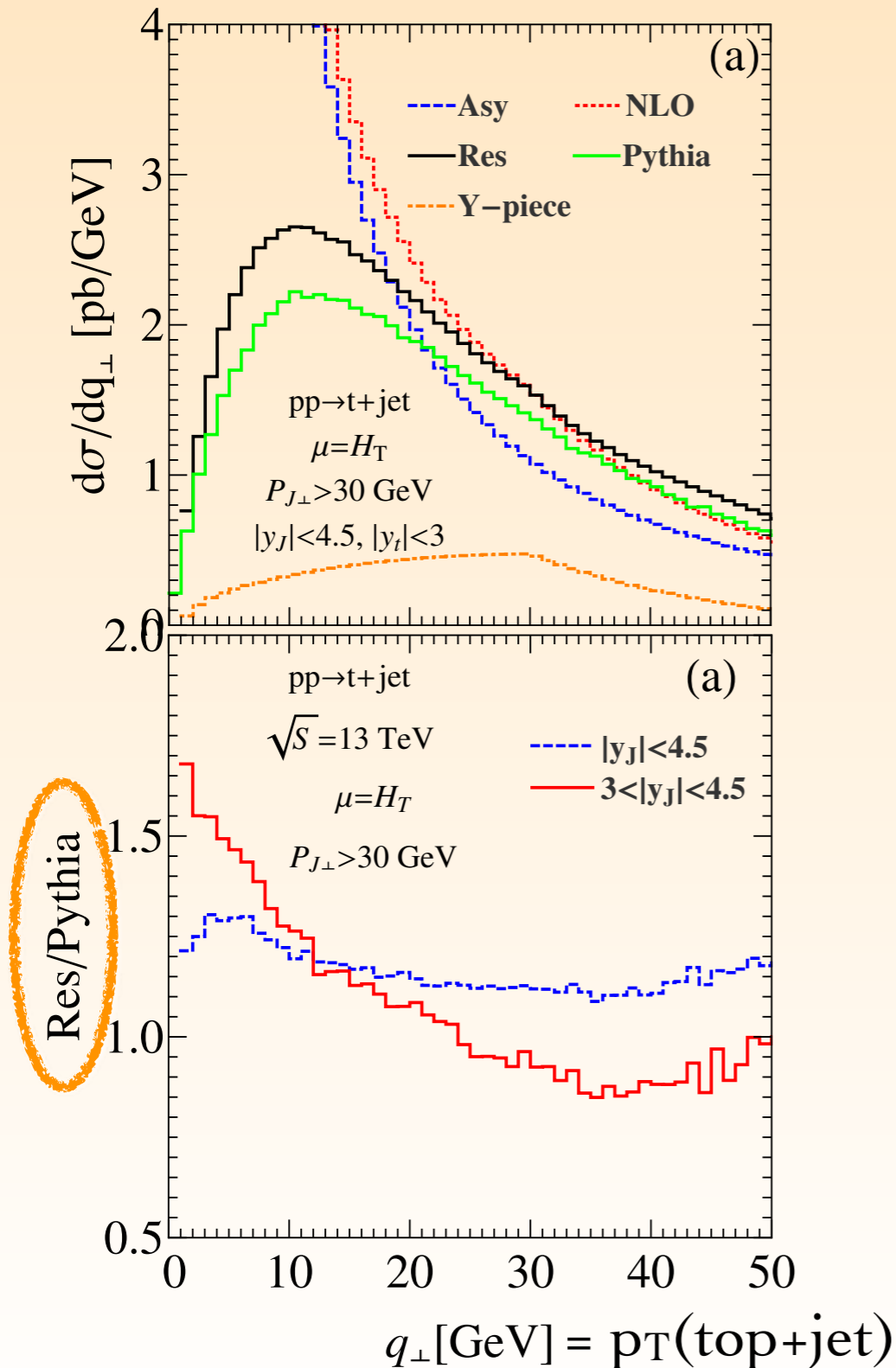


**SINGLE-TOP**



# TRANSVERSE MOMENTUM RESUMMATION

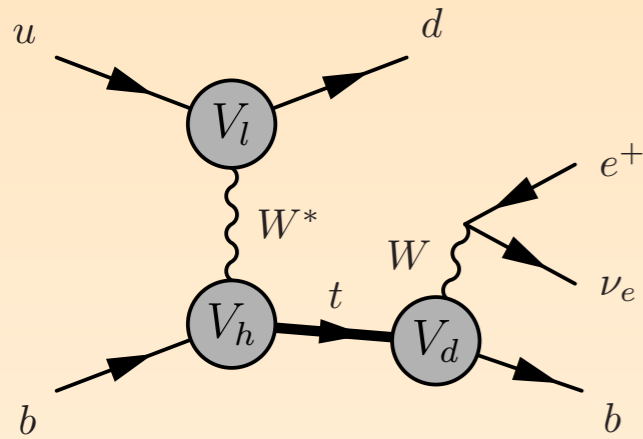
[Cao, Sun, Yan, Yuan, Yuan, 2018]



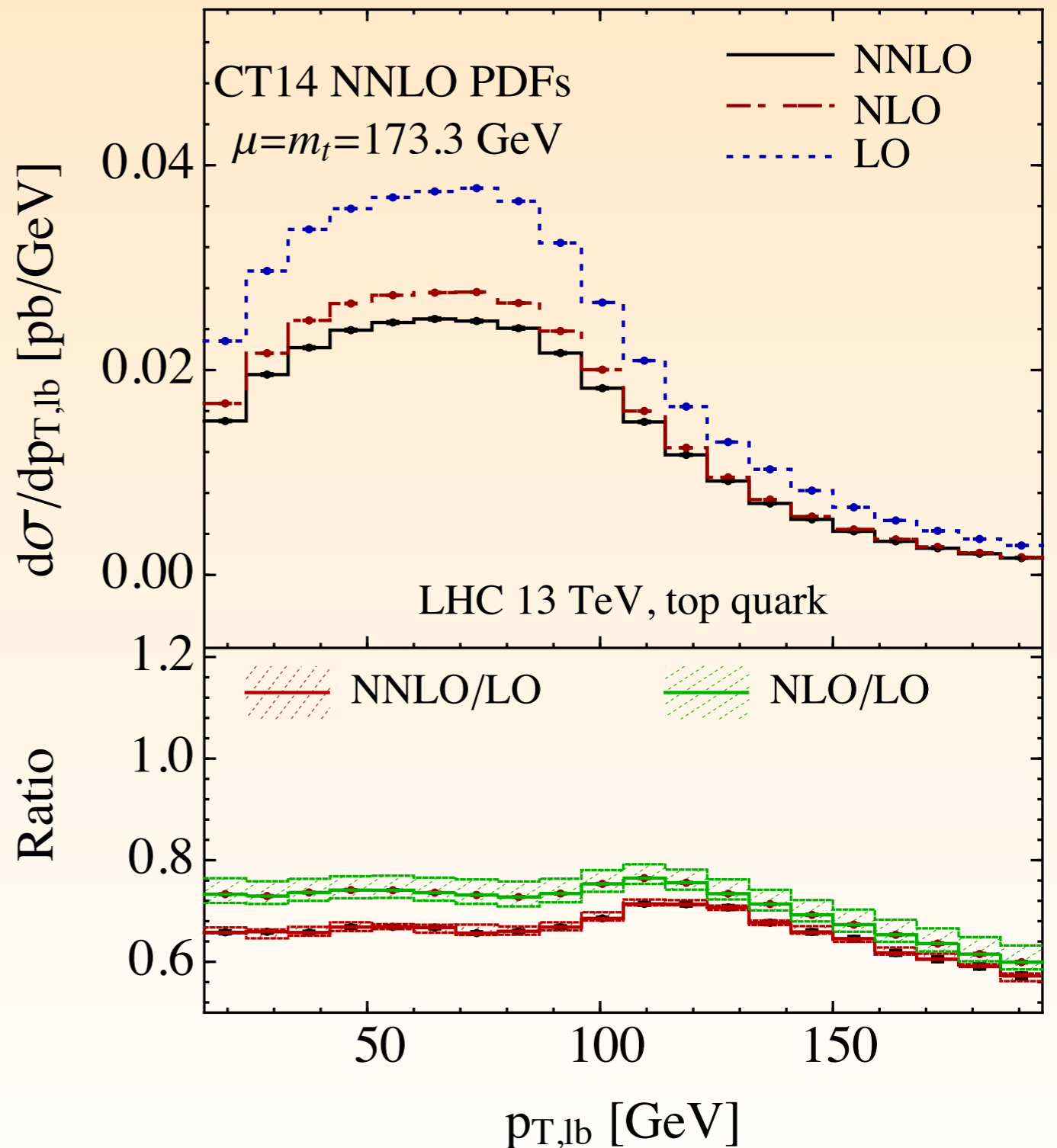
- ◆ When the top+jet system has small transverse momentum, large logs appear:  $\log[Q/q_T]$ 
  - These can be resummed to all orders in perturbation theory [Collins, Soper, Sterman, 1985]
- ◆ Resummed results similar to parton shower (Pythia8), except when jet is forward
  - However, default Pythia8 suboptimal for DIS like configurations; improved dipole recoil scheme is currently being considered [Cabouat & Sjöstrand, 2018]
  - Might have an effect on these conclusions?

# NNLO FOR T-CHANNEL SINGLE TOP

[Berger, Gao, Zhu, 2017, 2018]



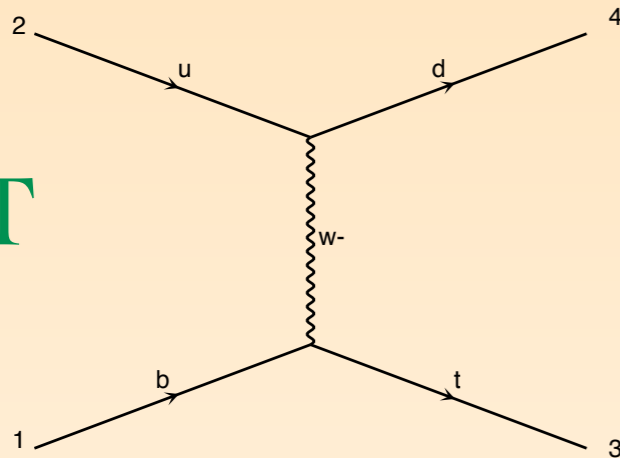
- ◆ NNLO corrections to factorised approach
- ◆ Fiducial region: requiring exactly 2 jets, of which one is b-tagged
- ◆ Effects larger than expected; outside of theoretical uncertainty bands
  - Origin is not obvious
  - Might the parton shower capture a part of these effects?  
... more investigation needed



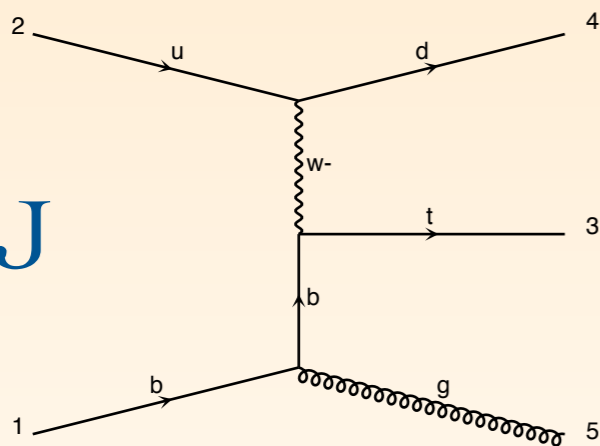
# SINGLE TOP MINLO

[Carazza, RF, Hamilton, Zanderighi, 2018]

ST



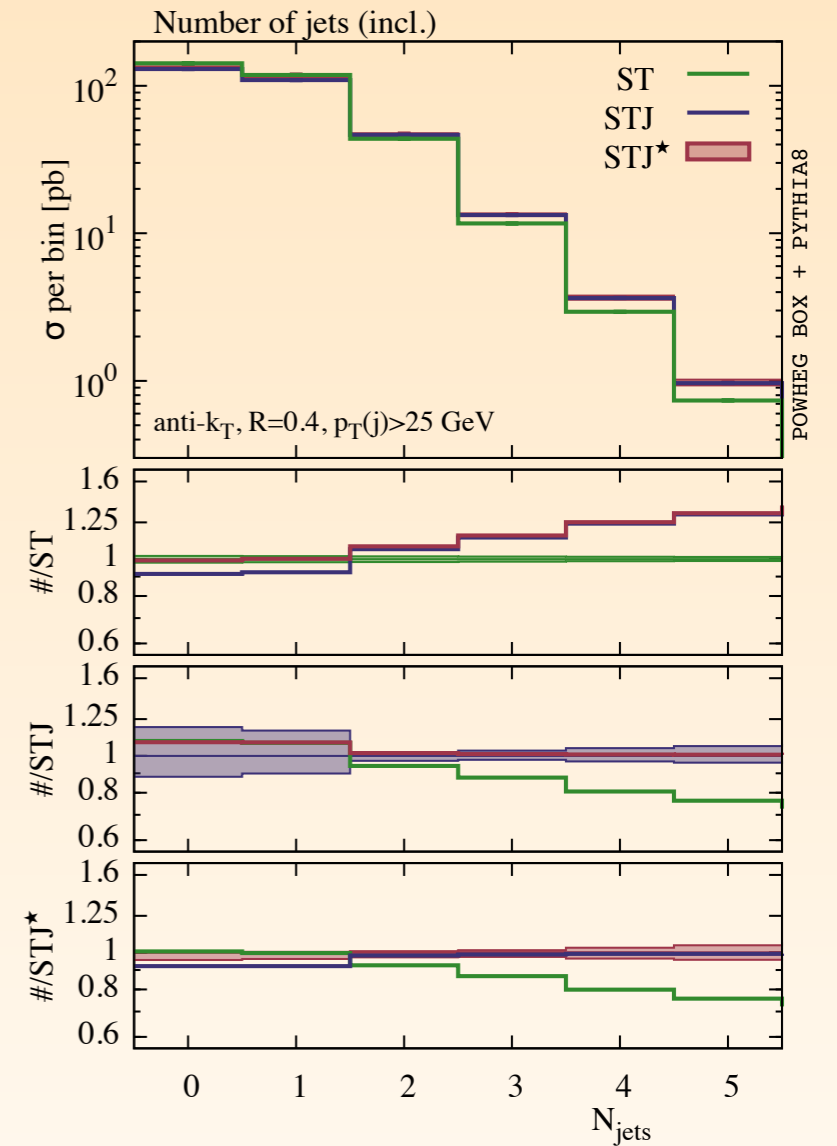
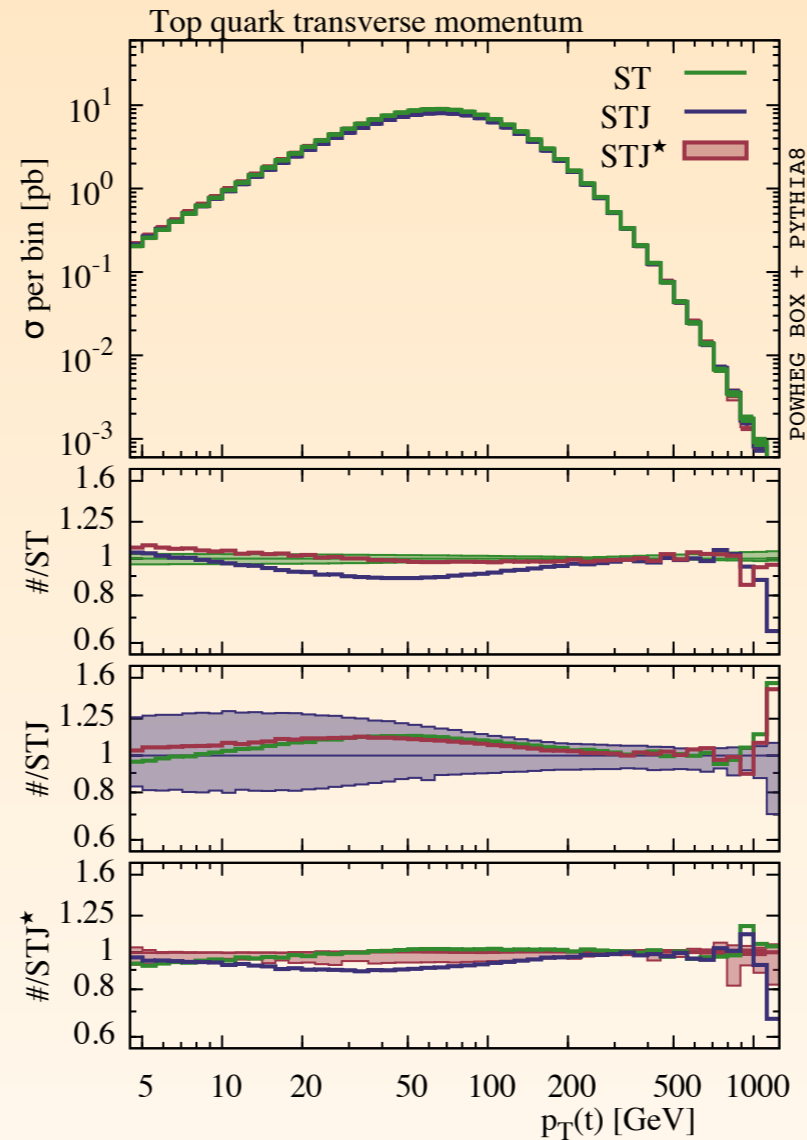
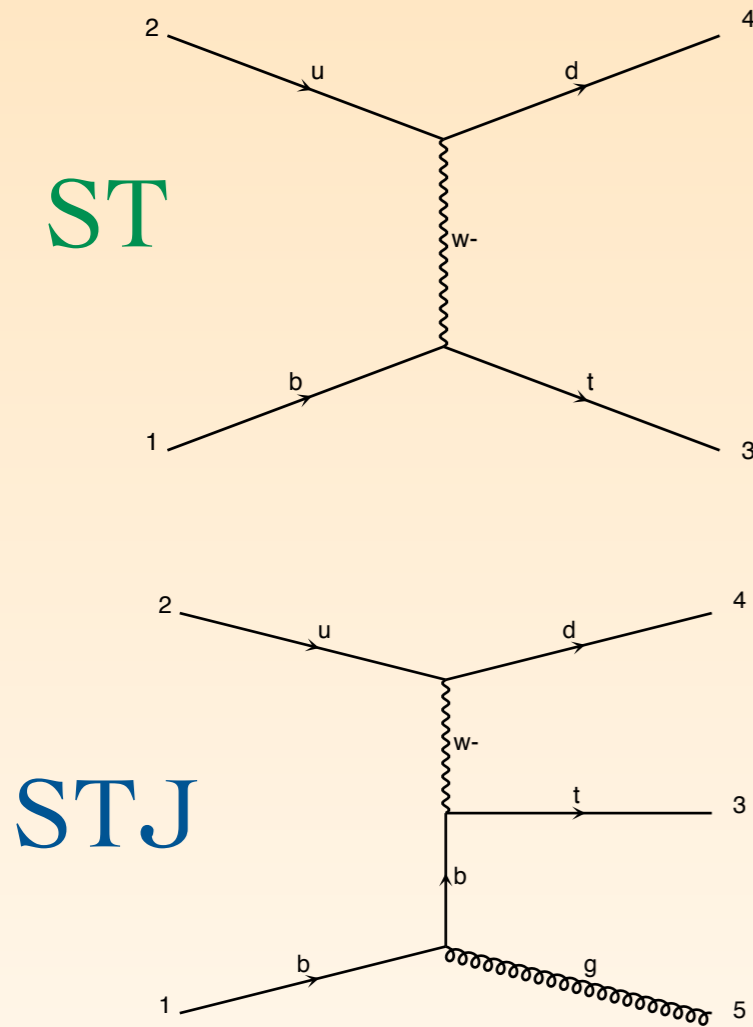
STJ



- ◆ t-channel single top MiNLO' merging, within POWHEG framework
- ◆ Start from NLO **STJ**, apply MiNLO algorithm to get LO correct in **ST**
- ◆ Use novel Artificial Neural Network techniques to reweight the MiNLO **STJ** to NLO **ST** for inclusive observables
- ◆ Hence:
  - **STJ**★ NLO correct in both the **ST** and **STJ** phase-spaces
  - No merging scale. Negligible merging ambiguities/uncertainties

# SINGLE TOP MINLO

[Carazza, RF, Hamilton, Zanderighi, 2018]



- ◆ **STJ\*** is NLO correct in both the **ST** and **STJ** phase-space
- ◆ Top transverse momentum: **ST** is NLO, **STJ** is LO, **STJ\*** NLO
- ◆ 0,1-jet bins: **ST** is NLO; 2-jet bin **STJ** is NLO; **STJ\*** is NLO in 0,1,2-jet bins

Top is kept stable;  
no hadronisation/  
underlying event

# CONCLUSIONS

- ◆ **NLO+NNLL resummation** available for  $t\bar{t}$ +heavy boson production. Uncertainties from QCD scale dependence well below 10%
- ◆ **Complete-NLO** available for  $t\bar{t}$ +X production processes
  - Some surprises: in particular for  $t\bar{t}W$  and **4-top** where **NLO<sub>3</sub>** effects are much larger than expected
- ◆ **NNLO for single-top production+decay in the NWA**: large effects for fiducial cross sections (jet veto). Missing logarithms?
- ◆ **MiNLO'** merging for **single-top** production within POWHEG
  - **STJ★** NLO correct in both the **ST** and **STJ** phase-space. No merging scale!
  - Opens a road to include NNLO corrections