NExT Meeting at RAL

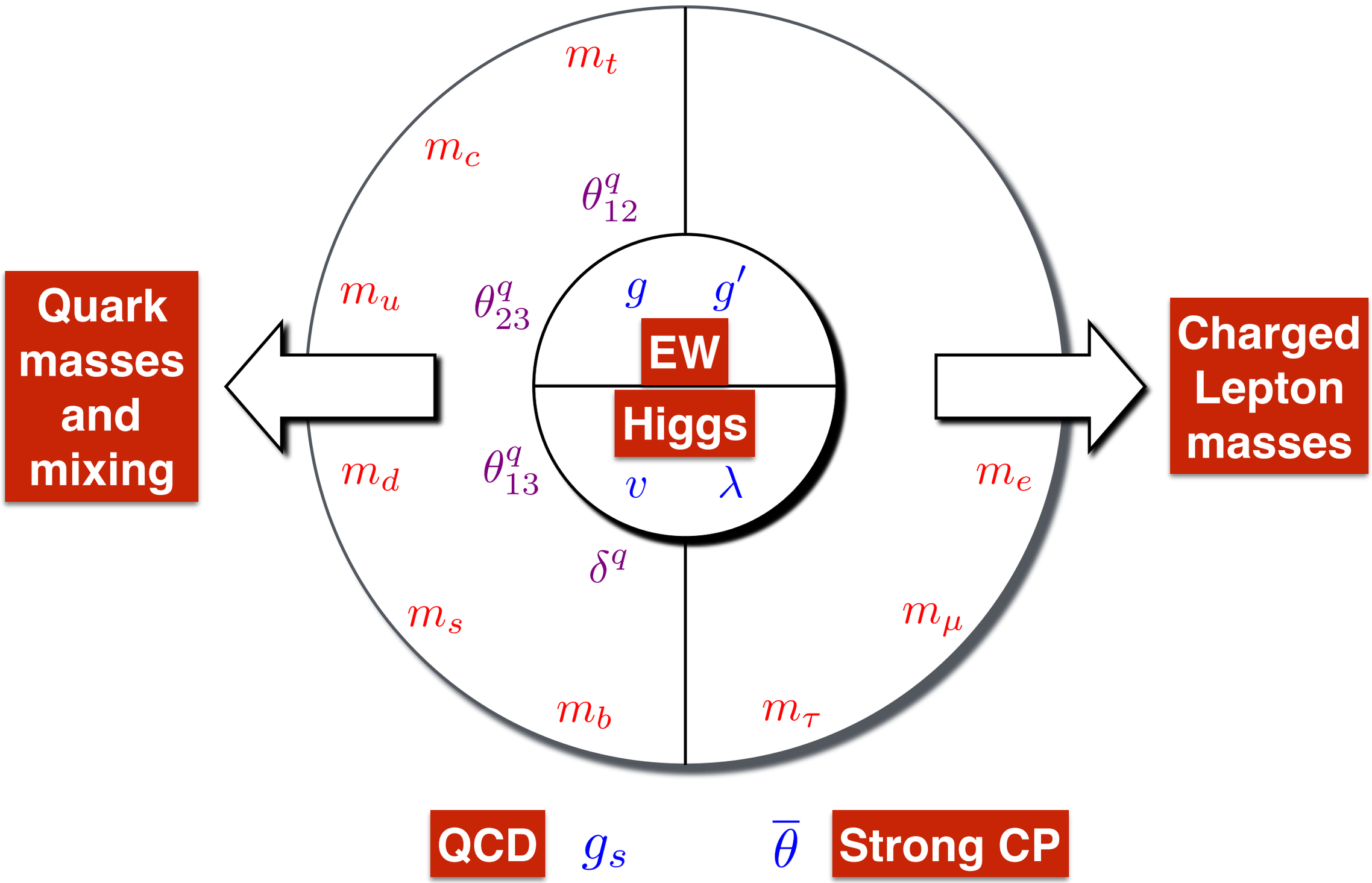
9 May 2018
Rutherford-Appleton Laboratory
Europe/London timezone

Flavour symmetries in the lepton sector

Ye-Ling Zhou, (IPPP, Durham), 9 May 2018

The Stardard Model (SM)

17+2 free parameters



Neutrino oscillations confirmed neutrino masses and lepton mixing

Neutrino masses and lepton mixing

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

Flavour eigenstates ← $|\nu_\alpha\rangle$ = $\sum_i U^*_{\alpha i} |\nu_i\rangle$ → **Mass eigenstates**

$$m_1, m_2, m_3$$

$$\Delta m^2_{21} = m^2_2 - m^2_1 \qquad \Delta m^2_{31} = m^2_3 - m^2_1$$

Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\frac{\alpha_{21}}{2}} & \\ & & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

Atmospheric | **Reactor** | **Solar** | **Majorana**

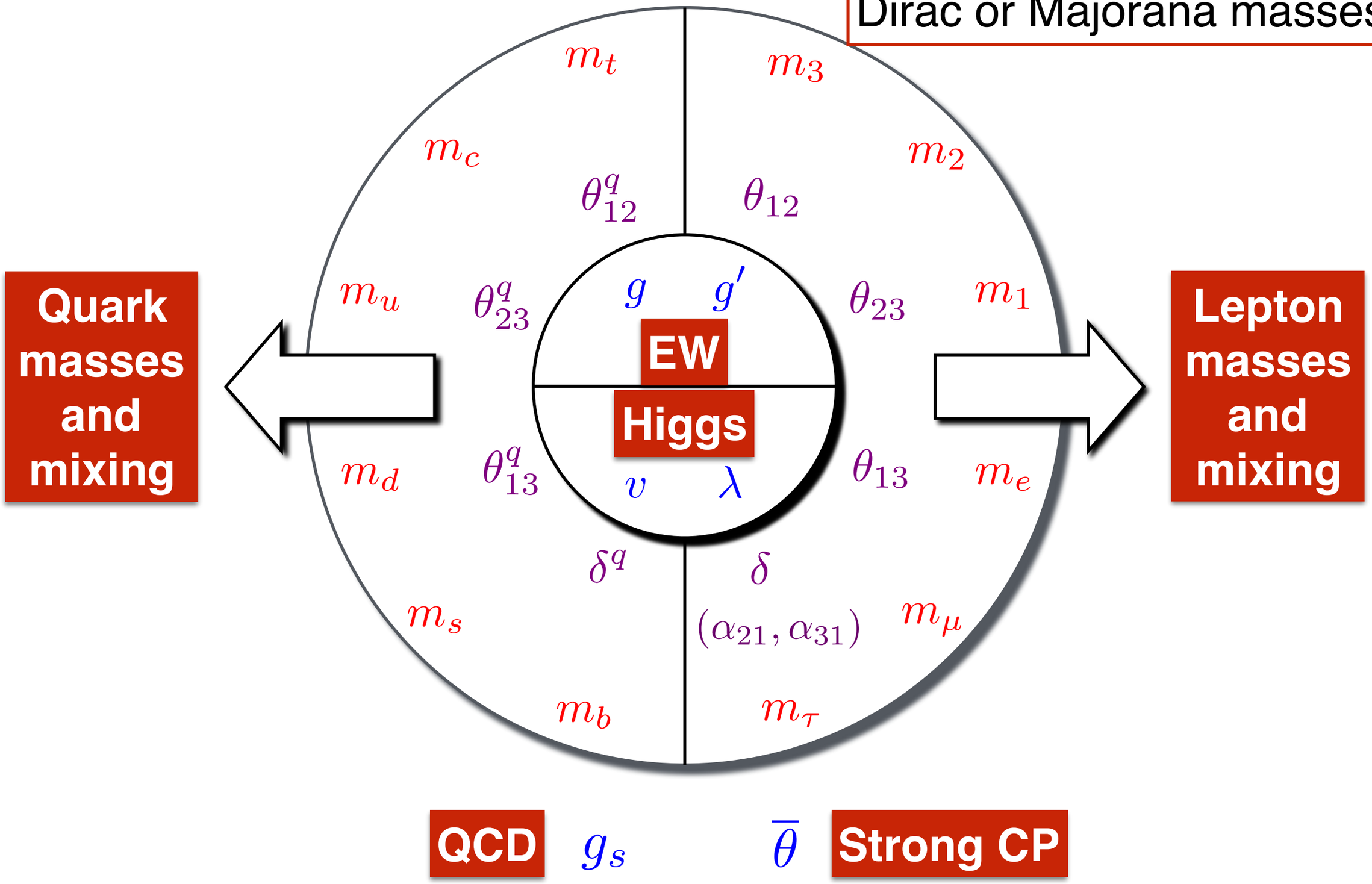
Mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ $\quad c_{ij} = \cos\theta_{ij}, \;\; s_{ij} = \sin\theta_{ij}$

CP-violating phases Dirac-type δ Majorana-type α_{21}, α_{31}

SM + massive neutrinos

24(26)+2 free parameters

Neutrino may take
Dirac or Majorana masses



Quark masses and mixing

Lepton masses and mixing

QCD g_s $\overline{\theta}$ **Strong CP**

Neutrino oscillations confirmed neutrino masses and lepton mixing

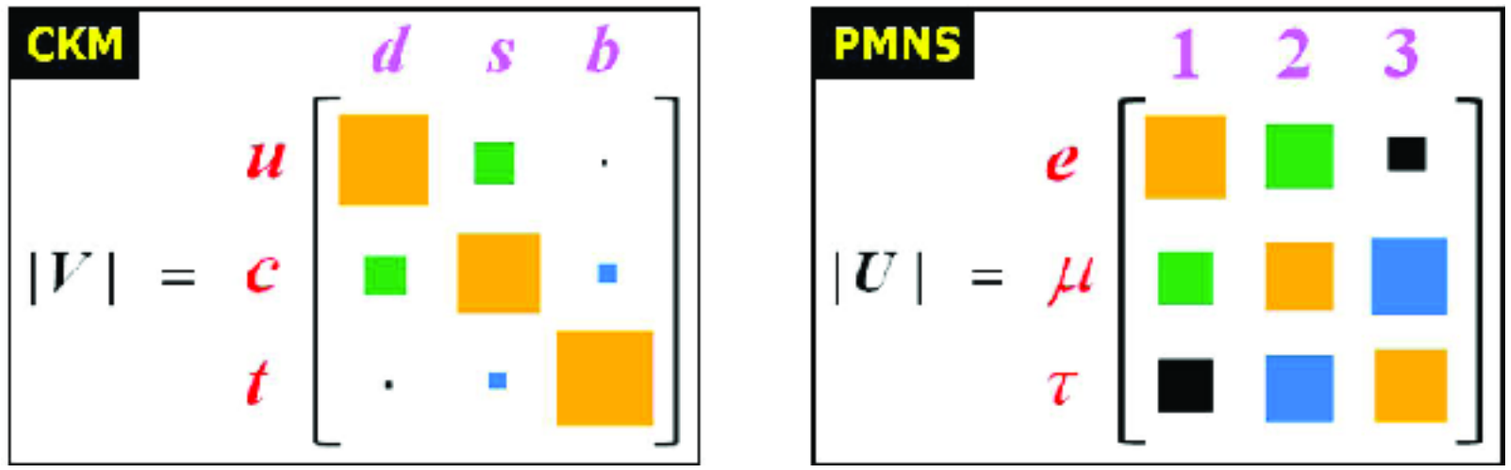
Esteban, Gonzalez-Garcia, Maltoni, Martinez-Soler, Schwetz, 1611.01514

NuFIT 3.2 (2018)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.14$)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.272 \to 0.346$
$\theta_{12}/^\circ$	$33.62^{+0.78}_{-0.76}$	$31.42 \to 36.05$	$33.62^{+0.78}_{-0.76}$	$31.43 \to 36.06$	$31.42 \to 36.05$
$\sin^2\theta_{23}$	$0.538^{+0.033}_{-0.069}$	$0.418 \to 0.613$	$0.554^{+0.023}_{-0.033}$	$0.435 \to 0.616$	$0.418 \to 0.613$
$\theta_{23}/^\circ$	$47.2^{+1.9}_{-3.9}$	$40.3 \to 51.5$	$48.1^{+1.4}_{-1.9}$	$41.3 \to 51.7$	$40.3 \to 51.5$
$\sin^2\theta_{13}$	$0.02206^{+0.00075}_{-0.00075}$	$0.01981 \to 0.02436$	$0.02227^{+0.00074}_{-0.00074}$	$0.02006 \to 0.02452$	$0.01981 \to 0.02436$
$\theta_{13}/^\circ$	$8.54^{+0.15}_{-0.15}$	$8.09 \to 8.98$	$8.58^{+0.14}_{-0.14}$	$8.14 \to 9.01$	$8.09 \to 8.98$
$\delta_{\rm CP}/^\circ$	234^{+43}_{-31}	$144 \to 374$	278^{+26}_{-29}	$192 \to 354$	$144 \to 374$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$6.80 \to 8.02$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.494^{+0.033}_{-0.031}$	$+2.399 \to +2.593$	$-2.465^{+0.032}_{-0.031}$	$-2.562 \to -2.369$	$\begin{bmatrix}+2.399 \to +2.593\\ -2.536 \to -2.395\end{bmatrix}$

Why we need a flavour symmetry (FS)?

- To explain large mixing in the lepton sector

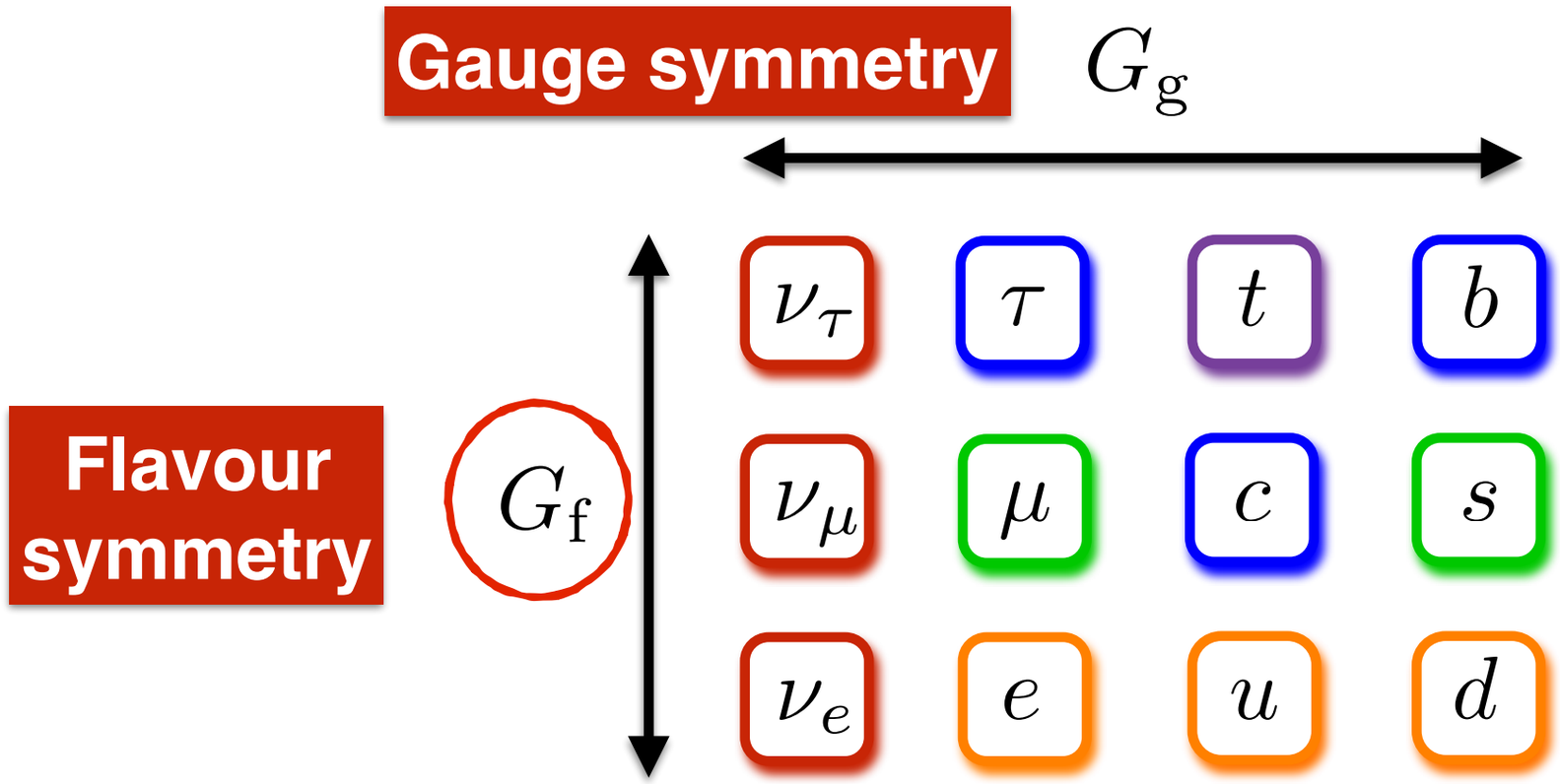


- Tri-bimaximal (TBM) mixing

$$|U| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad \begin{aligned} \sin^2\theta_{13} &= 0 \\ \sin^2\theta_{12} &= 1/3 \\ \sin^2\theta_{23} &= 1/2 \end{aligned}$$

1. **Tri-bimaximal mixing and the neutrino oscillation data**
   P.F. Harrison (Queen Mary, U. of London), D.H. Perkins (Oxford U.), W.G. Scott (Rutherford). Feb 2002. 14 pp.
   Published in **Phys.Lett. B530 (2002) 167**
   RAL-TR-2002-002
   DOI: 10.1016/S0370-2693(02)01336-9
   e-Print: **hep-ph/0202074** | **PDF**
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   ADS Abstract Service
   Detailed record - Cited by 1347 records 1000+

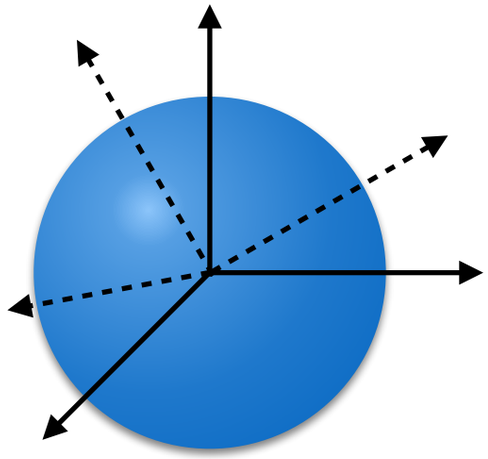
What is the FS?



	Continuous	Discrete
Abelian	U(1)	Z_n
Non-Abelian	SU(3), SO(3),…	S_3, A_4, S_4, T', A_5, Δ(48), …

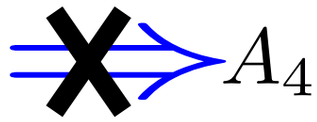
Where is the FS from?

- Breaking from a continuous symmetry



$$SO(3)$$

$$V = \mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 \qquad \varphi^\dagger \varphi = \sum_a \varphi_a^* \varphi_a$$

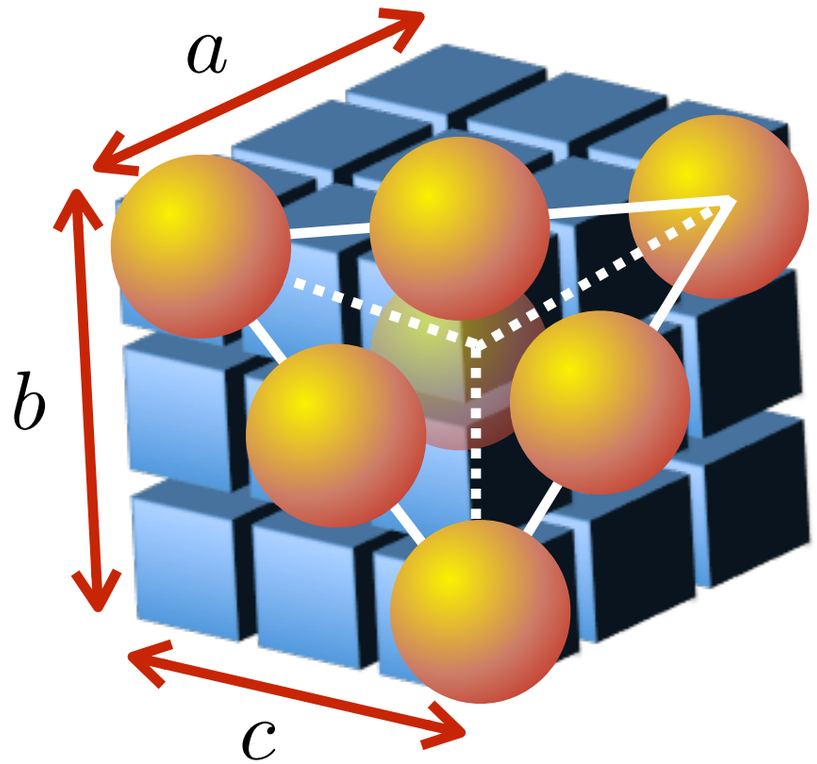


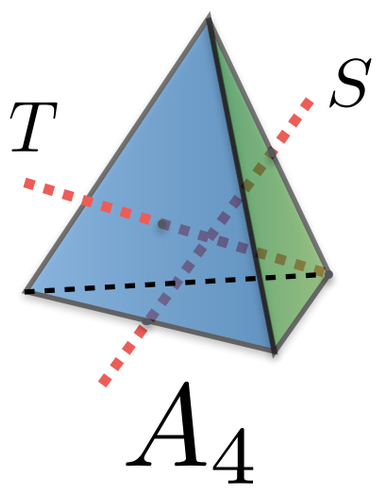
An example

$$T_{abc}$$

symmetric and traceless
3rd-order tensor

$$a, b, c = 1, 2, 3$$



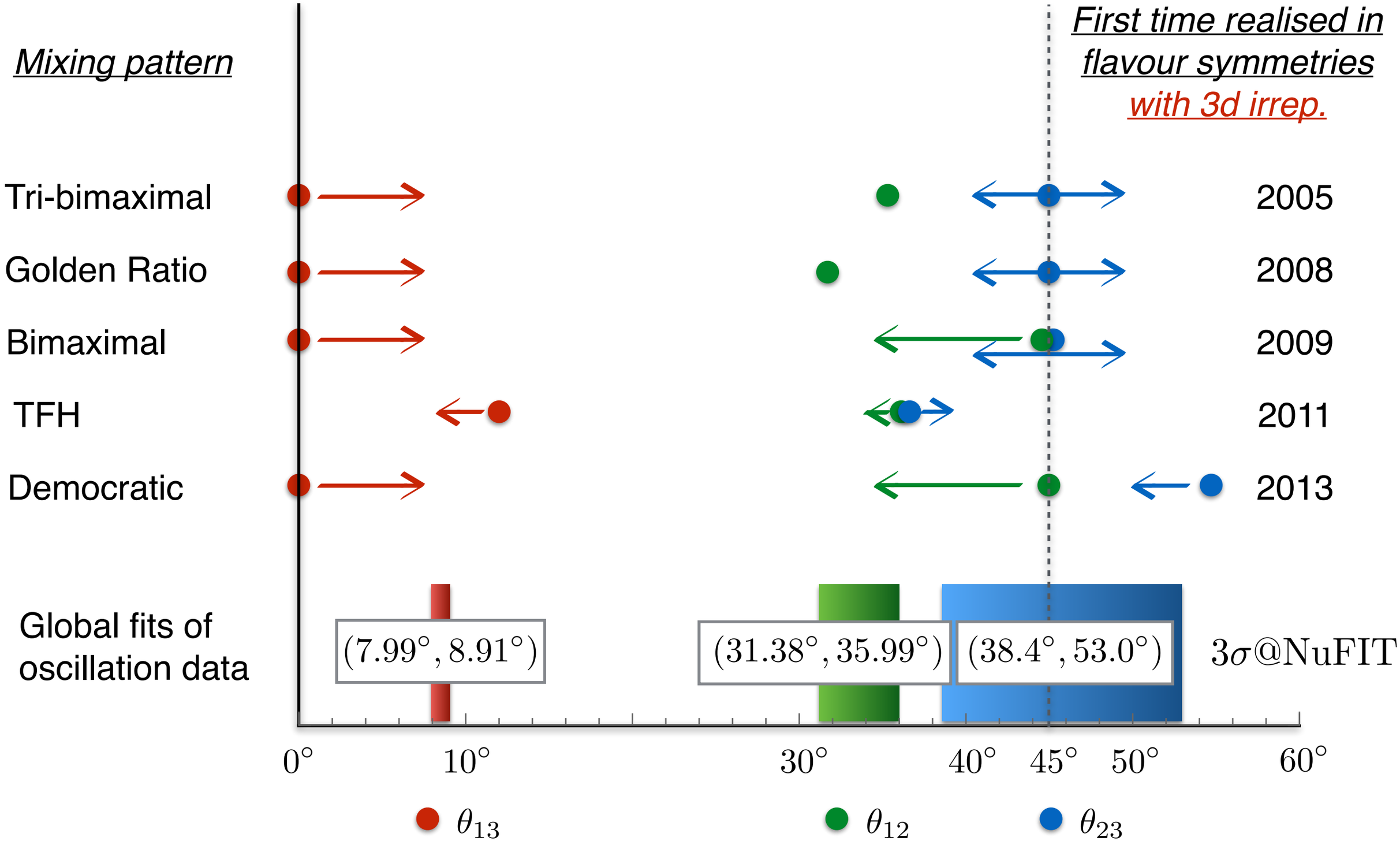


$$V = \mu^2 T_{abc} T_{abc} + \lambda (T_{abc} T_{abc})^2 + \lambda' T_{abc} T_{bcd} T_{def} T_{efa}$$

From the breaking of SO(3): Ovrut, 77; Berger, Grossman, 0910.4392

From the breaking of SU(3): Luhn, 1101.2417; Merle, Zwicky, 1110.4891

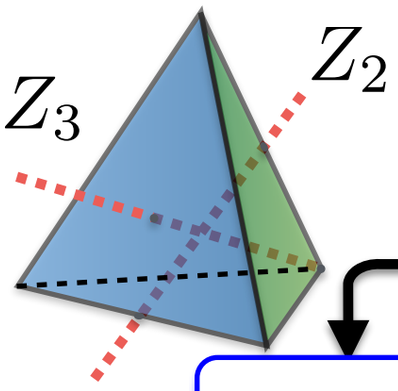
Powerful predictions of FS



Symmetries vs mixing patterns

Sketch between A_4 and TBM

$$|U_{\rm TBM}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{aligned} \sin\theta_{13} &= 0 \\ \sin\theta_{12} &= \frac{1}{\sqrt{3}} \\ \sin\theta_{23} &= \frac{1}{\sqrt{2}} \end{aligned}$$



A_4

Z_3 | Z_2

$M_l M_l^\dagger$ | M_ν

Tri-bimaximal

Altarelli, Feruglio,
hep-ph/0504165; 0512103

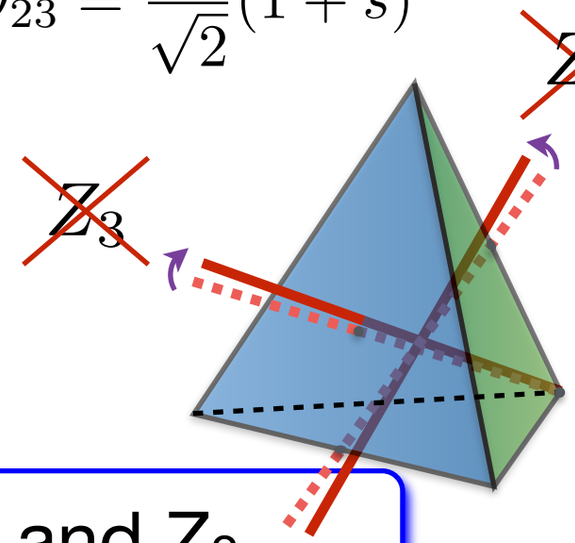
Necessary corrections

$$U = U_{\rm TBM} + \delta U$$

$$\sin\theta_{13} = \frac{r}{\sqrt{2}} \qquad \sin\theta_{12} = \frac{1}{\sqrt{3}}(1+a)$$

$$\sin\theta_{23} = \frac{1}{\sqrt{2}}(1+s)$$

Interference between Z_3 and Z_2



Break Z_3 and Z_2

Modify mass structures

Deviation from TBM,
sizable θ_{13} and δ

An economical model

S. Pascoli, YLZ, 1604.00925

Flavons $\varphi = (\varphi_1, \varphi_2, \varphi_3)^T \sim \mathbf{3}$ $\chi = (\chi_1, \chi_2, \chi_3)^T \sim \mathbf{3}$, $\eta \sim \mathbf{1}$

Essential to break the flavour symmetry

- Cross couplings V(φ, χ) break Z_3 & Z_2 $V(\varphi, \chi) \ll V(\varphi), V(\chi)$

Cross couplings $\Longrightarrow$ Shifts of the flavon VEVs $\Longrightarrow$ Corrections to mixing

$$V(\varphi,\chi) = \frac{1}{2}\epsilon_1(\varphi\varphi)_{\mathbf{1}}(\chi\chi)_{\mathbf{1}} + \frac{1}{4}\left[\epsilon_2(\varphi\varphi)_{\mathbf{1}''}(\chi\chi)_{\mathbf{1}'} + \text{h.c.}\right] + \frac{1}{2}\epsilon_3\left((\varphi\varphi)_{\mathbf{3}_S}(\chi\chi)_{\mathbf{3}_S}\right)_{\mathbf{1}}$$

the only complex parameter, CP origin

φ and χ are pseudo-real triplets

$$\langle\varphi\rangle \approx \begin{pmatrix} 1 \\ \epsilon_\varphi \\ \epsilon_\varphi^* \end{pmatrix} v_\varphi \Rightarrow \text{Corrections of TBM} \Leftarrow \langle\chi\rangle \approx \begin{pmatrix} 1-2\epsilon_\chi \\ 1+\epsilon_\chi \\ 1+\epsilon_\chi \end{pmatrix} \frac{v_\chi}{\sqrt{3}}$$

Z_3 ✗ Z_2 ✗

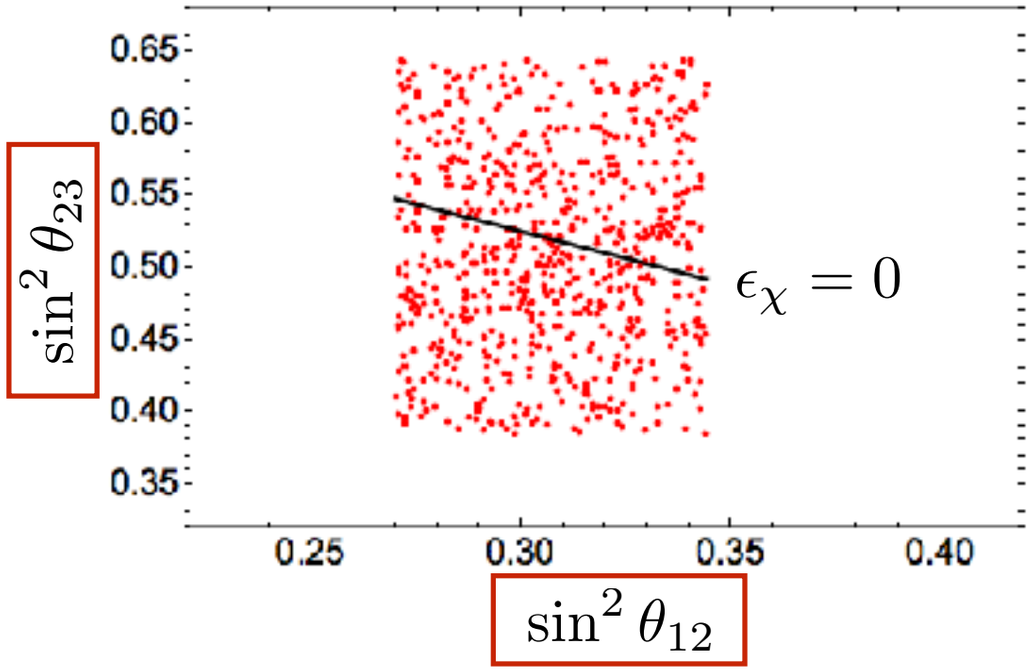
An economical model

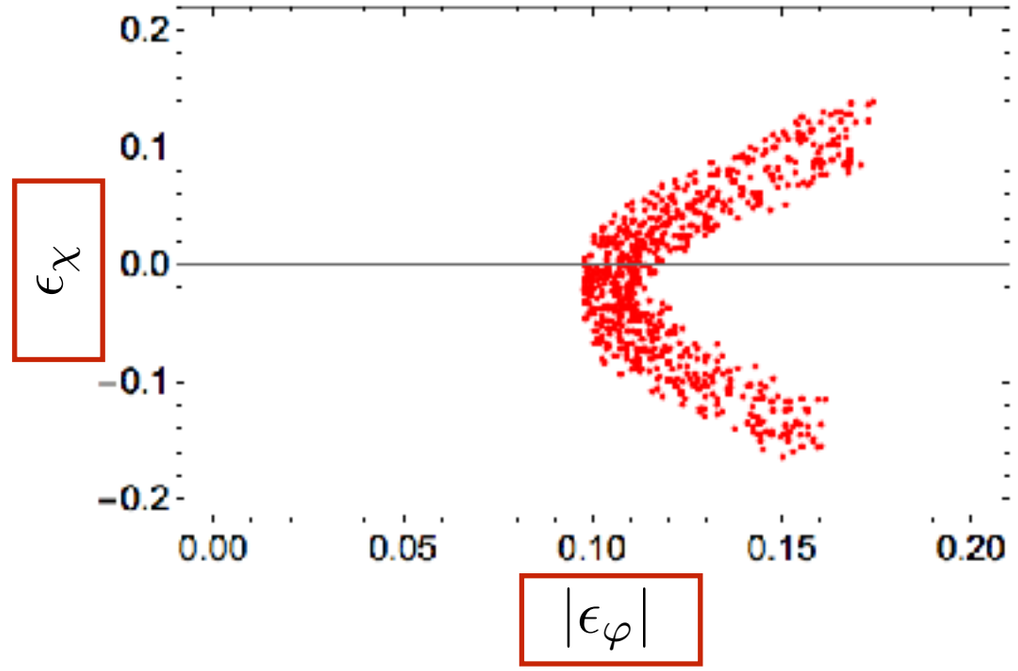
S. Pascoli, YLZ, 1604.00925

- Predictions of mixing parameters

$$\epsilon_\varphi = |\epsilon_\varphi| e^{i\theta_\varphi}$$

$$r = 2|\epsilon_\varphi \sin\theta_\varphi|\,, \quad s = -2|\epsilon_\varphi|\cos\theta_\varphi + 2\epsilon_\chi\,, \quad a = |\epsilon_\varphi|\cos\theta_\varphi\,.$$





Sum rule

$$\delta \approx \begin{cases} 270^\circ - \sqrt{2}\theta_{13} \approx 258^\circ & \theta_\varphi > 0\,, \\ 90^\circ + \sqrt{2}\theta_{13} \approx 102^\circ & \theta_\varphi < 0\,. \end{cases}$$

Almost maximal CP violation

- Simplicity — 7 real degrees of freedom: 3(φ) + 3(χ) + 1(η)

FS at high scale: leptogenesis via phase transition

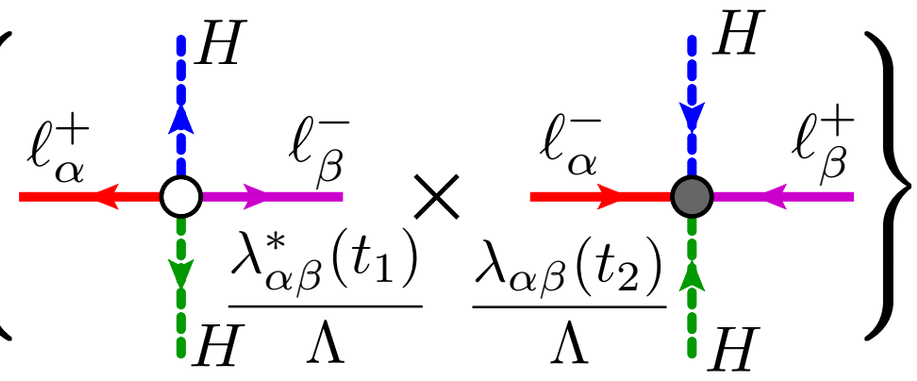
Pascoli, Turner, YLZ, 1609.07969

- Flavour symmetry breaking leads to cosmological CP-violating phase transition. It may explain matter-antimatter asymmetry.

- Assuming neutrino masses are generated by the Weinberg operator. During phase transition, the flavon has time-dependent VEV, and the Weinberg operator is time-dependent

$$\lambda(t) = \lambda^0 + \frac{\langle\phi(t)\rangle}{v_\phi} \qquad \mathcal{L}_W = \frac{\lambda_{\alpha\beta}(t)}{\Lambda}\ell_{\alpha L}HC\ell_{\beta L}H + \text{h.c.}$$

- Lepton asymmetry $\Delta f_{\ell_\alpha} \propto \text{Im}$



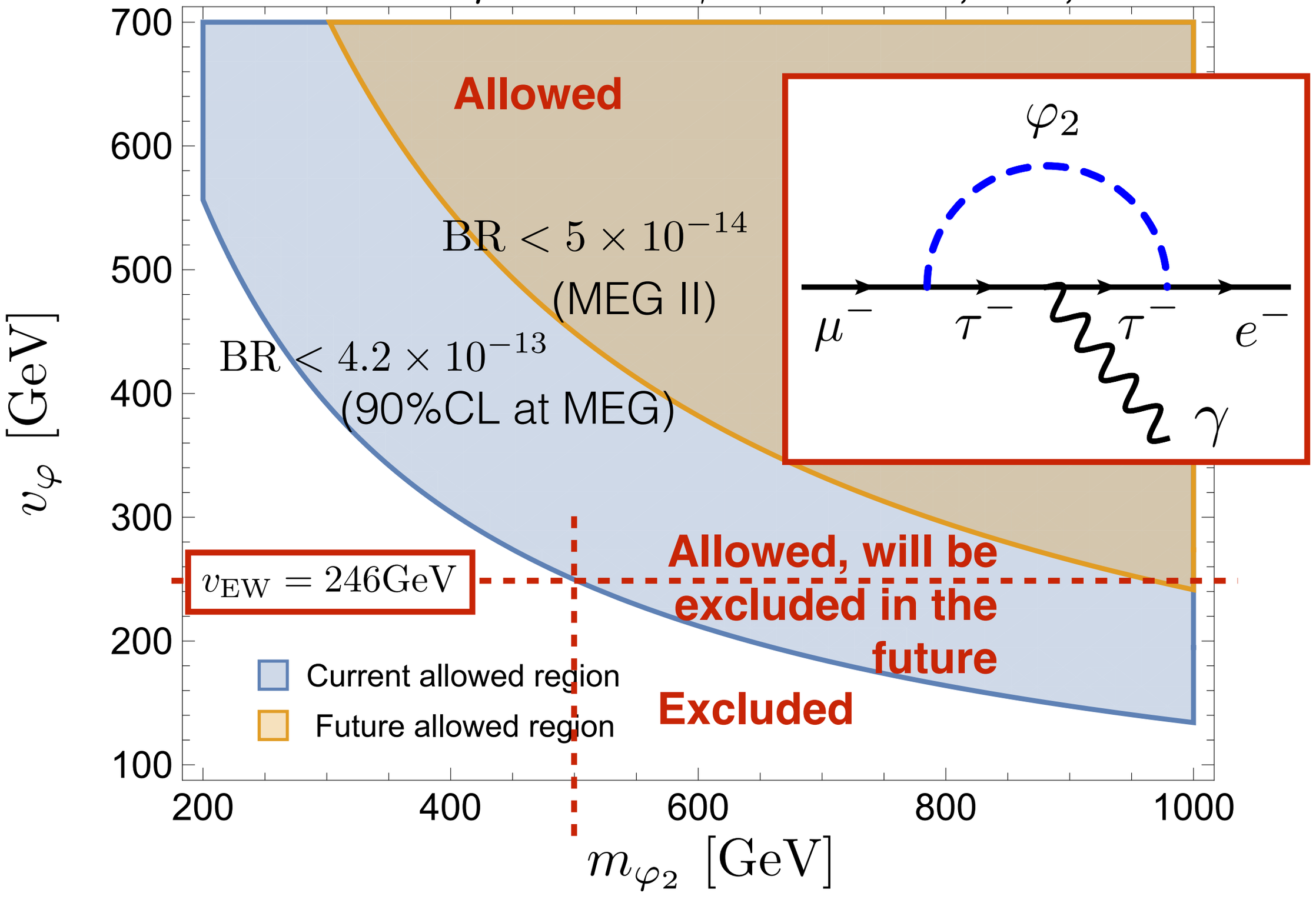
- Scale of the flavour symmetry

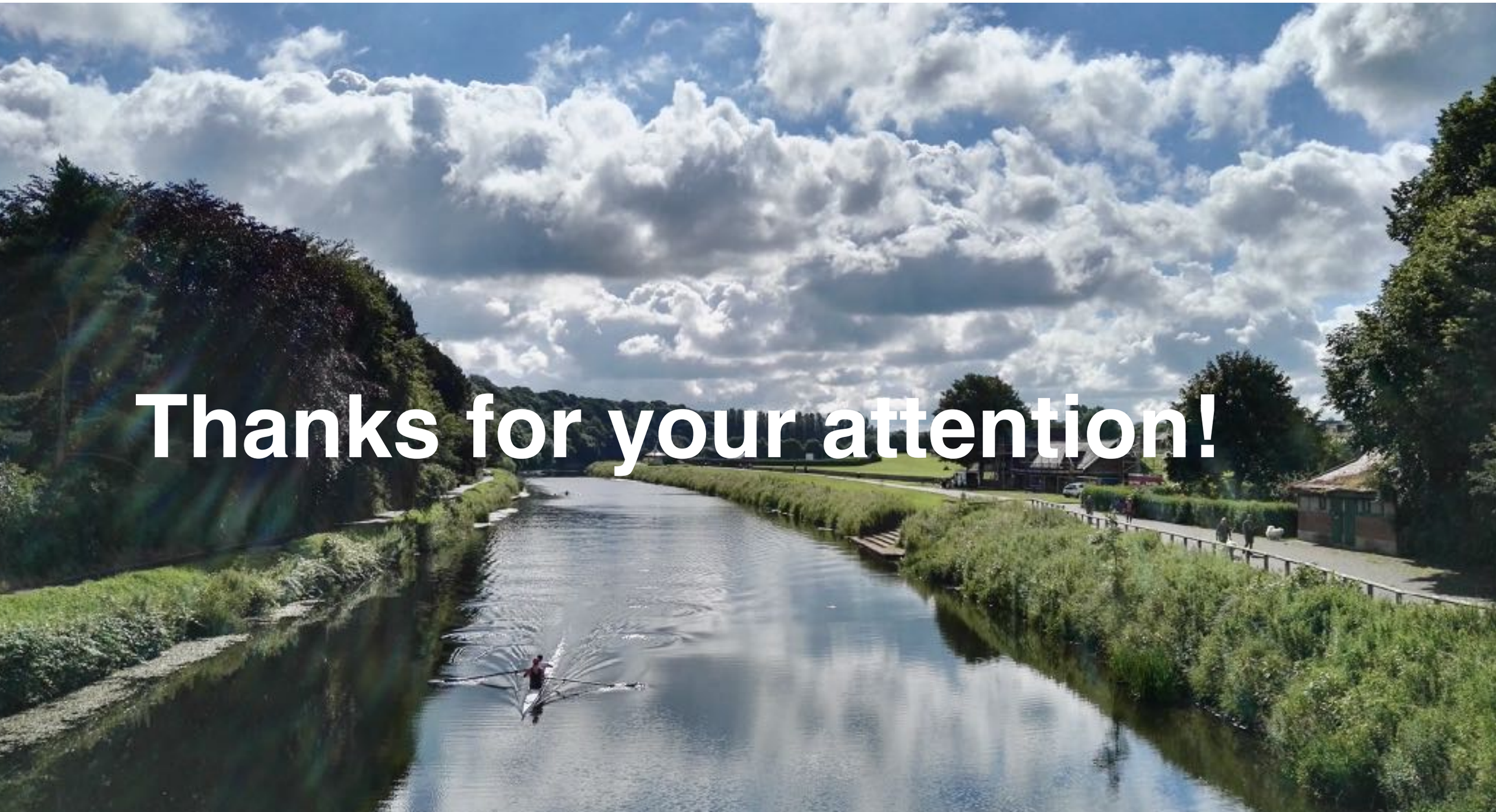
$$\Delta f_\ell \gtrsim \eta_B \quad m_\nu = 0.1\text{eV} \Longrightarrow \Lambda_{\text{FS}} \gtrsim 10^{11}\,\text{GeV}$$

- For leptogenesis via phase transitions involving right-handed neutrinos, see Long, Tesi, Wang, 1703.04902

FS at low scale: charged LFV

- Constraint from $\mu^- \to e^- \gamma$ Pascoli, YLZ,1604.00925



Summary

- Brief review of FS: mixing patterns, origins of FS, A4, models
- Scale of FS: Leptogenesis via phase transition, charged LFV