Distribution function and shear viscosity in magnetic field

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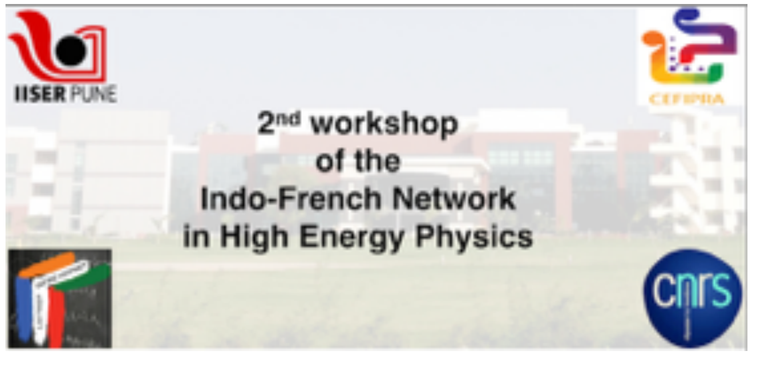
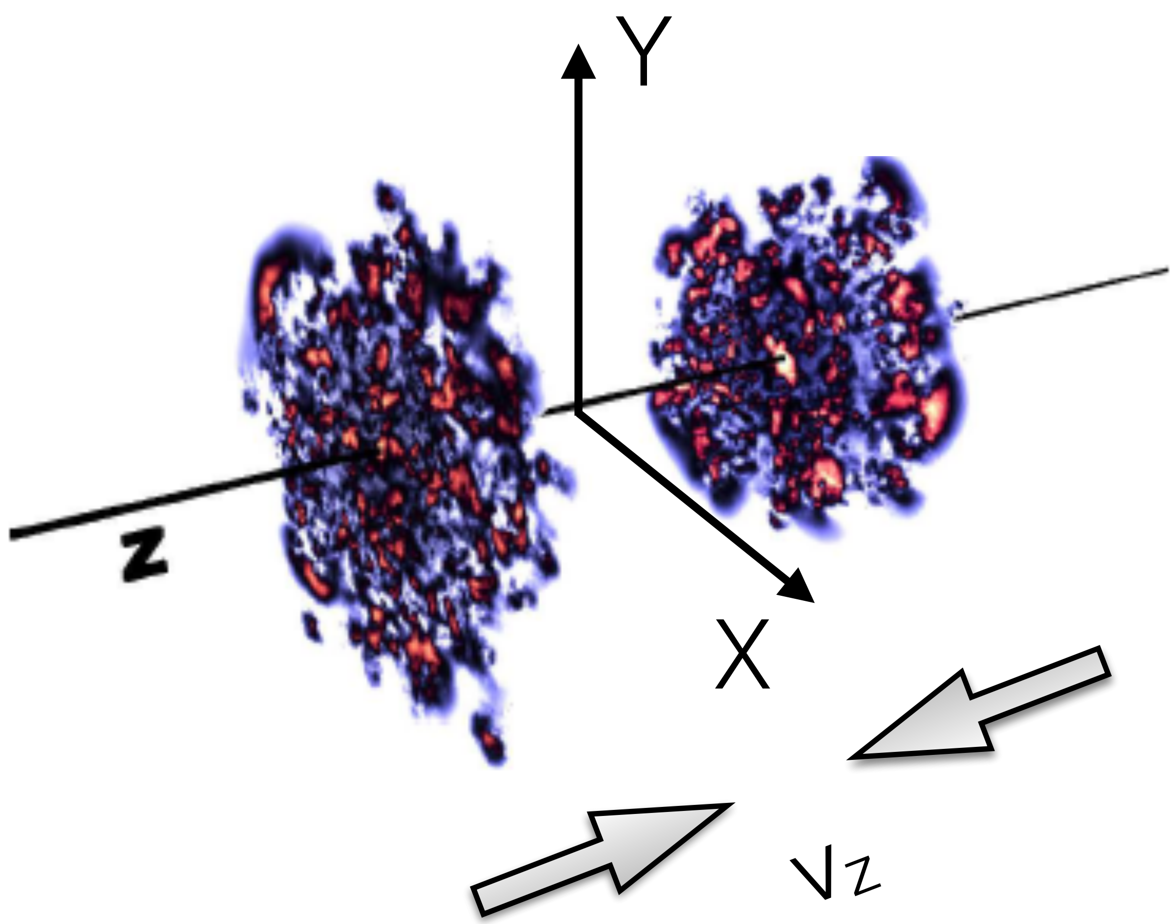


Outline

- Motivation
- δf correction and shear viscosity in magnetic field
- results
- Conclusion

Electromagnetic field in HIC

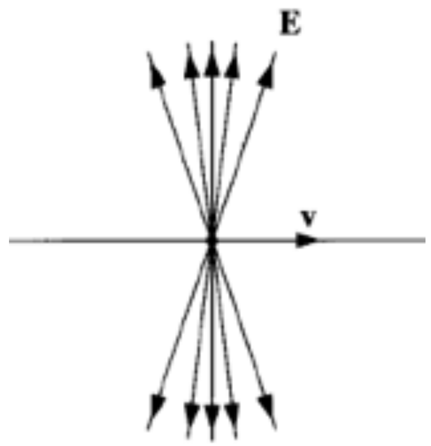
In high energy heavy ion collisions +ve charged nucleus collide at a speed~c -> produce intense magnetic fields



Mid central $\sqrt{s_{\mathrm{NN}}}$=200GeV Au+Au collisions -> B~10^{18}Gauss

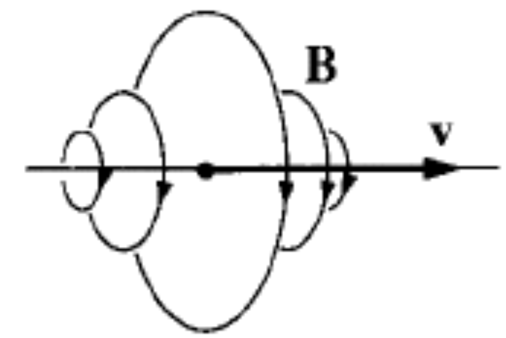
Field of a moving charge

Electric field



$$\vec{E}(\vec{r},t) = \frac{e}{4\pi}\frac{\vec{R}_i(1-v_i^2)}{R_i^3\left(1-\left[\vec{R}_i\times\vec{v_i}\right]^2/R_i^2\right)^{3/2}}$$

Magnetic field



$$\vec{B}\,(\vec{r},t) = \vec{v_i}\times\vec{E}_i$$

$$\hbar = c = \epsilon_0 = 1$$

$$[eB] = \mathrm{GeV}^2$$

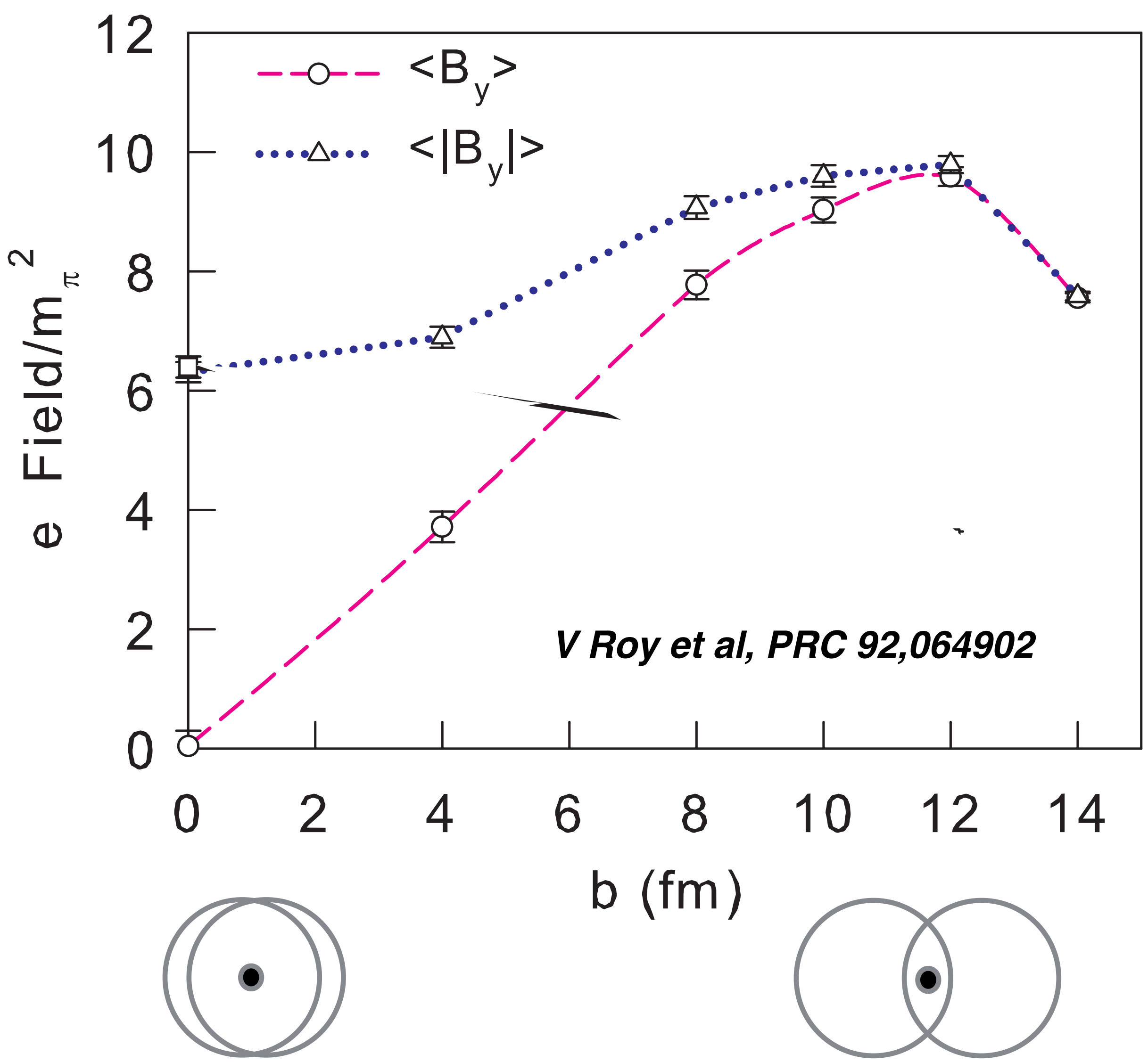
$$1\mathrm{GeV}^2 = 5.128\times10^{19}\mathrm{Gauss}$$

At relativistic speed

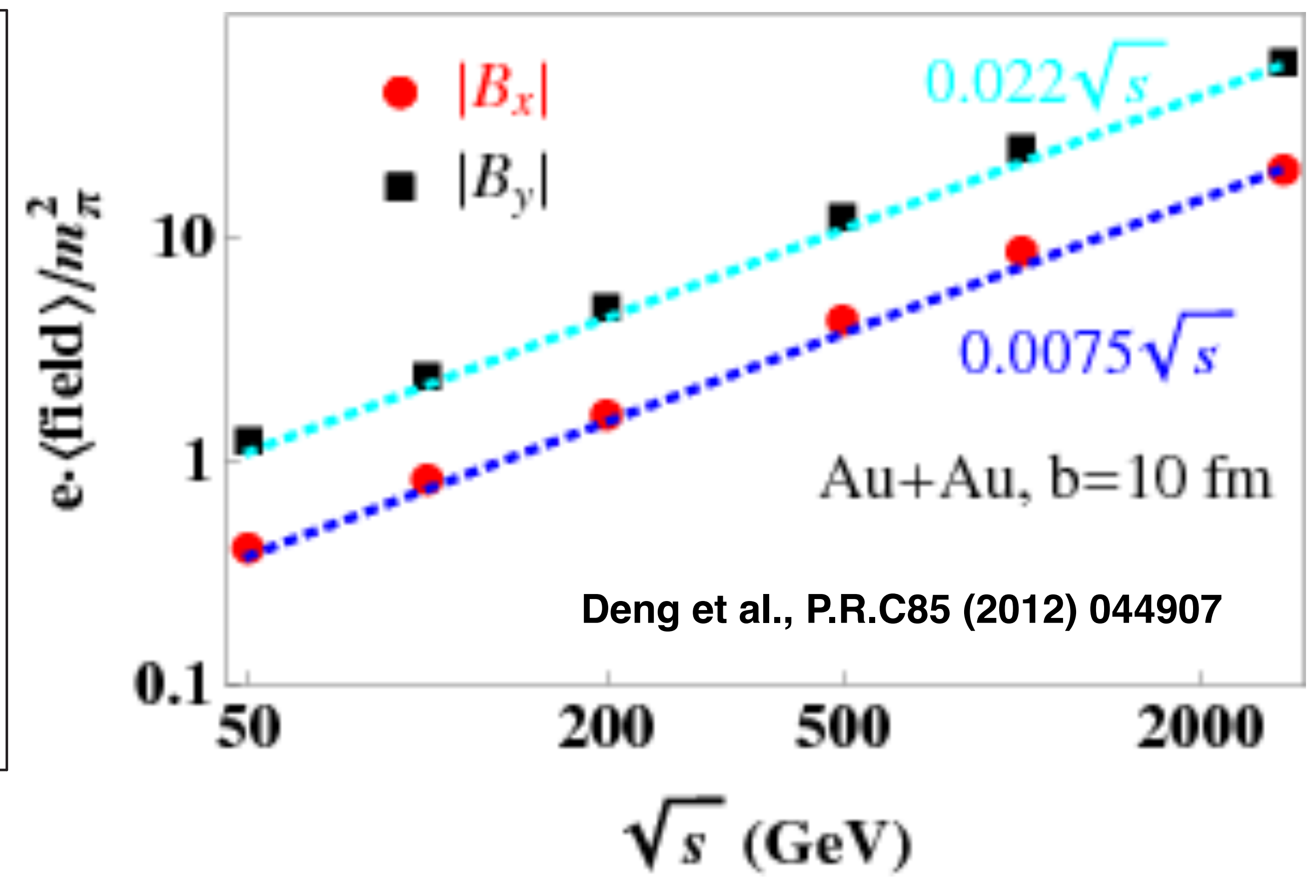
- Transverse components dominates
- Field rapidly fall off as we move away from charge along z

Impact parameter and beam energy dependence of magnetic field

Au+Au @ 200 GeV



Au+Au ; b=10 fm



Effect of electromagnetic field

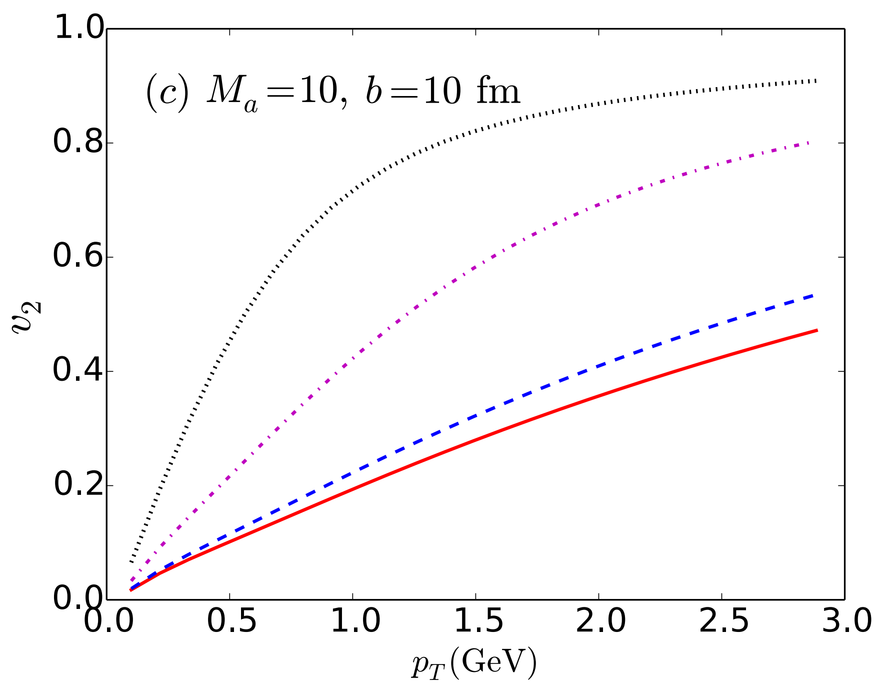
What is the effect of the magnetic field on QGP evolution ?
How electromagnetic field evolves in conducting QGP?

- Electric charge separation in a chiral imbalanced medium. a.k.a Chiral Magnetic Effect (CME) kharzeev et al Nucl Phys. A803

- Di-lepton and photon productions are changed K Tuchin PRC 88.024910 PRC 83,017901

Reduced Ideal MHD simulation

V Roy et al PRC



Elliptic flow @ 200 GeV Au+Au

$$eB = 10m_\pi^2$$

Red line —> without magnetic field

No δf correction in Cooper-Frye

Distribution function in magnetic field

$$f_m(p) = \frac{1}{(2\pi)^3} exp\left(-\beta\left[\left(p^\mu + qA^\mu\right)u_\mu - \mu\right]\right)$$

De Groot Relativistic kinetic theory

Gauge freedom for choosing A^μ

Cooper-Frey formula is ill defined for this case

$$\mathrm{E}\,\frac{d^3N}{d^3p} = \int f_0(p)p^\mu d\Sigma_\mu$$

Alternative way to find $f_m(p)$

Use Boltzmann transport equation to find

$$f(p) = f_0 + \delta f$$

Assume the correction δf to be small

$$\frac{\delta f}{f_0} << 1$$

Here we use simplest yet effective method Relaxation time approximation to calculate the δf

We also assume the fluid has non-zero shear viscosity

Boltzmann equation in RTA

Linearised Boltzmann equation in relaxation time approximation

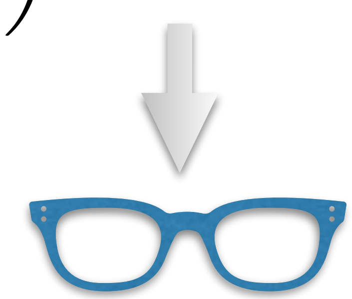
$$\left(v_i p_j \frac{\partial V_i}{\partial x_j} - \frac{1}{3} v_l p^l \nabla \cdot \mathbf{V}\right)\left(\frac{\partial f_0}{\partial \epsilon}\right) = -\frac{\delta f}{\tau} + q \varepsilon_{ijk} v_j B_k \frac{\partial \delta f}{\partial p_i}$$

Lifshitz-Pitaevski vol-10,
Offengeim et al, EPL, 112 (2015) 59001

$p^l =$ l-th component of momentum

$\tau =$ relaxation time

$\delta f(p)$ can be written as $\delta f(p) = -\left(\frac{\partial f_0}{\partial \epsilon}\right) C_{ijkl}(\epsilon) v^i p^j V^{kl},$



$$V^{kl} = \frac{1}{2}\left(\frac{\partial V^k}{\partial x^l} + \frac{\partial V^l}{\partial x^k}\right)$$

The shear viscosity in magnetic field

- Shear stress : $\sigma^{ij} = -\frac{g}{(2\pi)^3} \int \delta f(\mathbf{p}) v^i p^j d^3p = \eta^{ijkl} V_{kl}$
- Using the δf in definition of σ^{ij}

$$\eta^{ijkl} = \frac{g}{(2\pi)^3} \int \left(\frac{\partial f_0}{\partial \epsilon} \right) C^{klmn}(\epsilon) v_m p_n v^i p^j d^3p$$

- Further $\sigma^{ij} = \sum_{n=0}^{4} \eta_{(n)} S^{ij}_{(n)}$

→ 2nd rank symmetric traceless tensor
Combination of norm mag field, Kroneckar delta and V_{ij}

Using above equations and by taking appropriate tensor contraction we obtain the five viscosity coefficients $\eta_{(0)}, \cdots, \eta_{(4)}$

Evaluation of C_{ijkl} and δf

- Symmetry $C_{ijkl} = C_{jikl} = C_{ijlk}$
- Decomposition $C_{ijkl}(\epsilon) = \sum_{n=1}^{8} c^{(n)}(\epsilon)\xi^{(n)}_{ijkl}$

$$\xi^{(1)}_{ijkl} = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk},$$

$$\xi^{(2)}_{ijkl} = \delta_{ij}\delta_{kl},$$

$$\xi^{(3)}_{ijkl} = \delta_{ik}b_j b_l + \delta_{jk}b_i b_l + \delta_{il}b_j b_k + \delta_{jl}b_i b_k,$$

$$\xi^{(4)}_{ijkl} = \delta_{ij}b_k b_l,$$

$$\xi^{(5)}_{ijkl} = b_i b_j \delta_{kl},$$

$$\xi^{(6)}_{ijkl} = b_i b_j b_k b_l,$$

$$\xi^{(7)}_{ijkl} = b_{ik}\delta_{jl} + b_{jk}\delta_{il} + b_{il}\delta_{jk} + b_{jl}\delta_{ik},$$

$$\xi^{(8)}_{ijkl} = b_{ik}b_j b_l + b_{jk}b_i b_l + b_{il}b_j b_k + b_{jl}b_i b_k.$$

Evaluation of C_{ijkl}

$$\left(v_i p_j \frac{\partial V_i}{\partial x_j} - \frac{1}{3} v_l p^l \nabla \cdot \mathbf{V}\right)\left(\frac{\partial f_0}{\partial \epsilon}\right) = -\frac{\delta f}{\tau} + q\varepsilon_{ijk} v_j B_k \frac{\partial \delta f}{\partial p_i}$$

Using $C_{ijkl}(\epsilon) = \sum_{n=1}^{8} c^{(n)}(\epsilon)\xi^{(n)}_{ijkl}$ in Boltzmann eq^n

$$\left(\xi^{(1)}_{ijkl} + \chi_H \xi^{(7)}_{ijkl}\right) C_{klmn} = \tau \left(\xi^{(1)}_{ijmn} - \frac{2}{3}\xi^{(2)}_{ijmn}\right).$$

$\chi_H = \frac{qB}{m}\tau$ Dimensionless Hall parameter

Taking proper contraction on both side yields $c^{(n)}$

δf correction due to magnetic field

$$\delta f(p) = -\sum_{n=1}^{8} c_{(n)} \xi^{(n)}_{ijkl} \left(\frac{\partial f_0}{\partial \epsilon} \right) v^i p^j V^{kl}$$

$$c_1 = \frac{1}{2(1+\chi_H^2)}$$

$$c_2 = -\frac{(1-\chi_H^2)}{3(1+\chi_H^2)}$$

$$c_3 = \frac{3\chi_H^2}{2(1+\chi_H^2)^2}$$

$$c_4 = \frac{-4\chi_H^2}{1+4\chi_H^2}$$

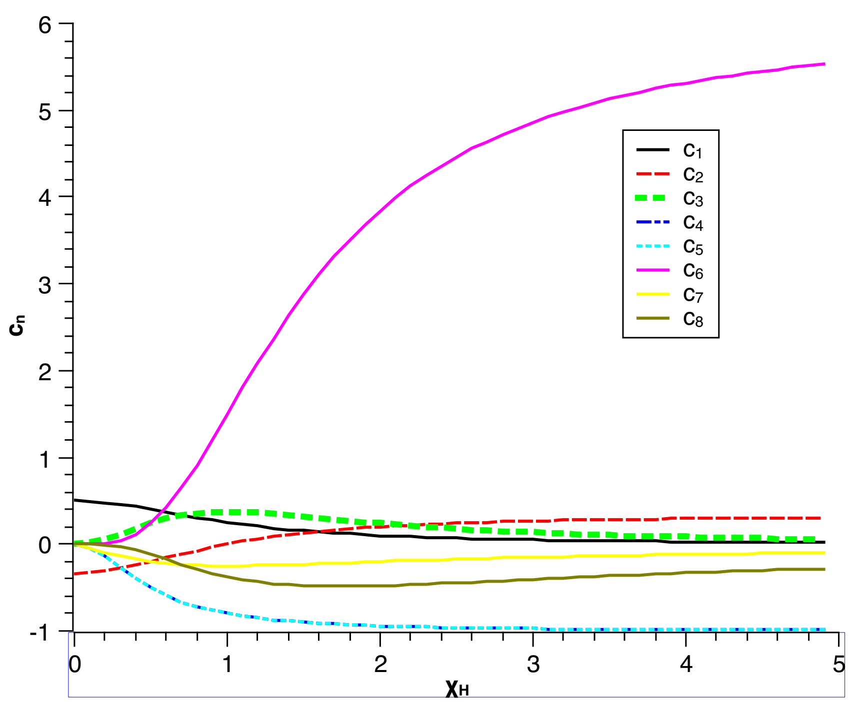
$$c_5 = \frac{-4\chi_H^2}{1+4\chi_H^2}$$

$$c_6 = \frac{6\chi_H^4}{(1+\chi_H^2)^2}$$

$$c_7 = \frac{-\chi_H}{2(1+\chi_H^2)}$$

$$c_8 = \frac{-3\chi_H^3}{2(1+\chi_H^2)^2}.$$

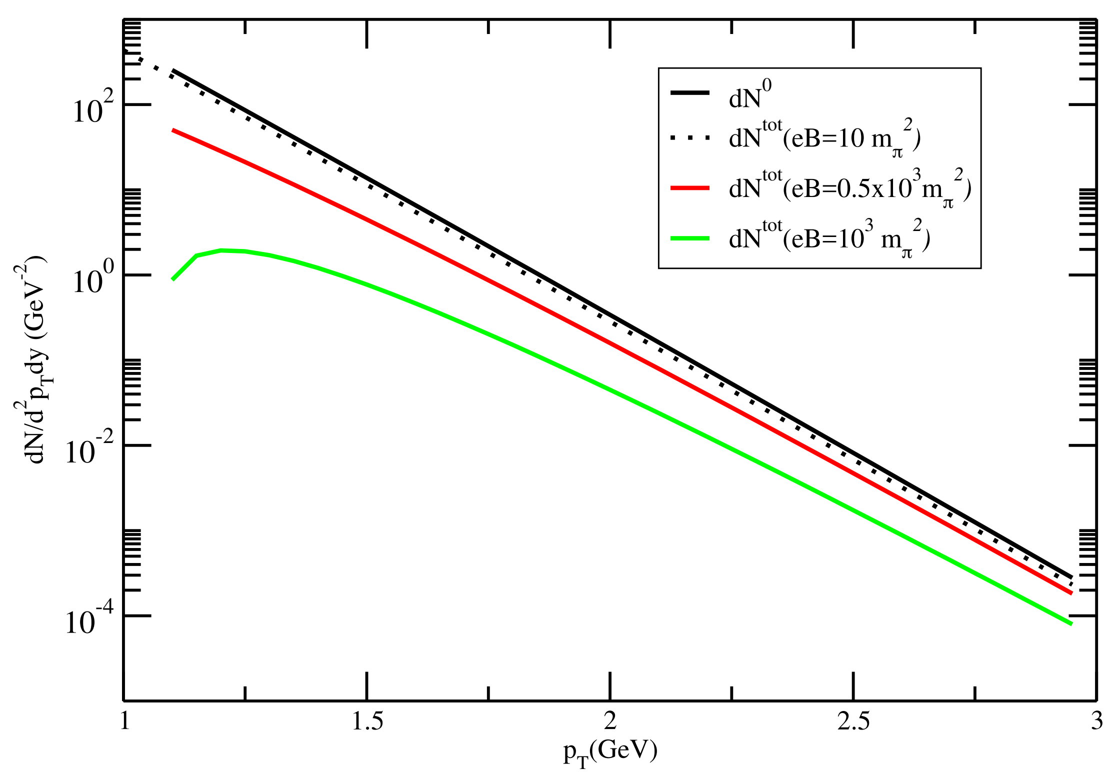
Charge dependent c_7 and c_8

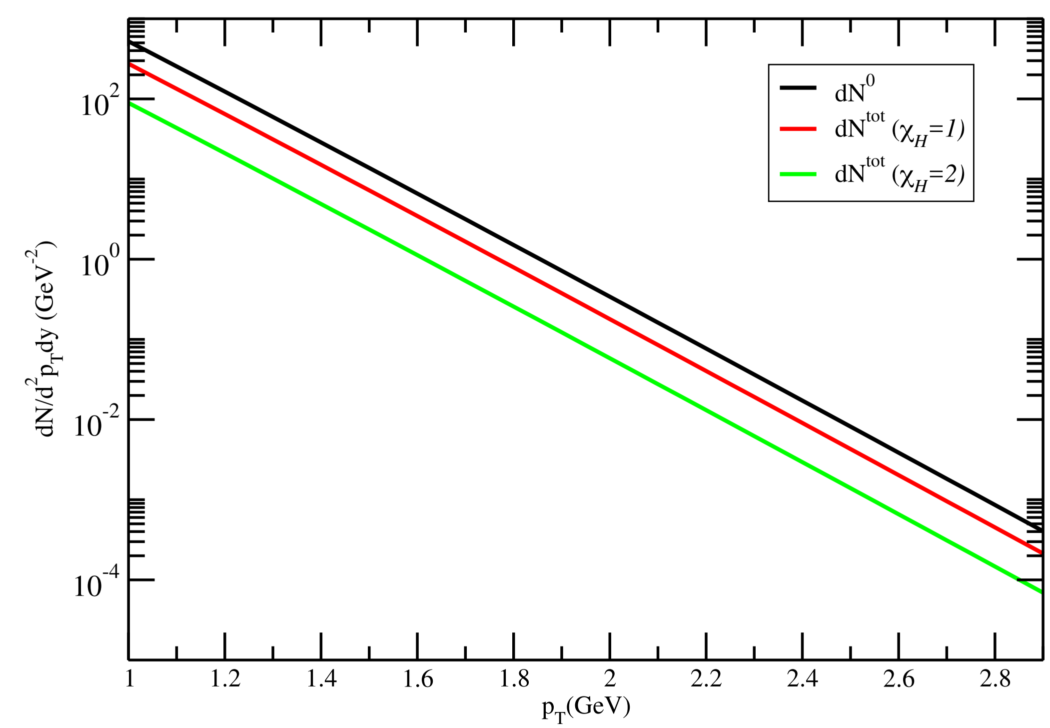


Correction to spectra in Bjorken expansion

$$u^{\tau} = 1 \quad u^{\eta} = u^{r} = u^{\phi} = 0$$

$$\frac{d^2N}{d^2p_T dy} = \frac{1}{(2\pi)^3}\int_0^{R_0} r dr \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} \tau d\eta_s m_T cosh(y-\eta_s) f(p)$$





Shear viscosities in magnetic field

- Using δf $\quad \eta^{ijkl} = \frac{g}{(2\pi)^3}\int\left(\frac{\partial f_0}{\partial\epsilon}\right)C^{klmn}(\epsilon)v_m p_n v^i p^j d^3p$

$$\eta^{ijkl} = \frac{1}{15}\frac{g}{(2\pi)^3}\int\left(\frac{\partial f_0}{\partial\epsilon}\right)\frac{p^4}{\epsilon^2}D^{ijkl}d^3p,$$

where $D^{ijkl} = \left(\xi^{ijmn}_{(1)} + \xi^{ijmn}_{(2)}\right)C^{mnkl}(\epsilon).$

$$\boxed{\left(\xi^{(1)}_{ijkl} - \frac{2}{5}\xi^{(2)}_{ijkl} + \chi_H\xi^{(7)}_{ijkl}\right)D_{klmn} = 2\tau\left(\xi^{(1)}_{ijmn} - \frac{2}{3}\xi^{(2)}_{ijmn}\right).}$$

$$D_{ijkl}(\epsilon) = \tau\sum_{n=1}^{8} d^{(n)}(\epsilon)\xi^{(n)}_{ijkl}.$$

Shear viscosities in magnetic field

$$d_1 = \frac{1}{1+4\chi_H^2}$$

$$d_2 = -\frac{2}{3}\frac{(1-2\chi_H^2)}{(1+4\chi_H^2)}$$

$$d_3 = \frac{3\chi_H^2}{(1+4\chi_H^2)(1+\chi_H^2)}$$

$$d_4 = \frac{-4\chi_H^2}{1+4\chi_H^2}$$

$$d_5 = \frac{-4\chi_H^2}{1+4\chi_H^2}$$

$$d_6 = \frac{12\chi_H^4}{(1+4\chi_H^2)(1+\chi_H^2)}$$

$$d_7 = \frac{-\chi_H}{1+4\chi_H^2}$$

$$d_8 = \frac{-3\chi_H^3}{(1+4\chi_H^2)(1+\chi_H^2)}$$

$$\begin{aligned}\eta^{ijkl} &= \eta_{(0)}\left(3\xi_{(6)} - \xi_{(5)} - \xi_{(4)} + \xi_{(2)}/3\right)^{ijkl} \\ &+\eta_{(1)}\left(\xi_{(1)} - \xi_{(2)} - \xi_{(3)} + \xi_{(4)} + \xi_{(5)} + \xi_{(6)}\right)^{ijkl} \\ &+\eta_{(2)}\left(\xi_{(3)} - 4\xi_{(6)}\right)^{ijkl} \\ &-\eta_{(3)}\left(\xi_{(7)}/2 - \xi_{(8)}/2\right)^{ijkl} \\ &-\eta_{(4)}\xi_{(8)}^{ijkl}.\end{aligned}$$

$$\eta^{ijkl} = \frac{1}{15}\frac{g}{(2\pi)^3}\int\left(\frac{\partial f_0}{\partial \epsilon}\right)\frac{p^4}{\epsilon^2}D^{ijkl}d^3p,$$

$$\eta_{(0)} = Shear\ \ viscosity\ \ without\ \ magnetic\ \ field$$

$$\eta_{(1)} = \frac{1}{1+4\chi_H^2}\eta_{(0)}$$

$$\eta_{(2)} = \frac{1}{1+\chi_H^2}\eta_{(0)}$$

$$\eta_{(3)} = \frac{2\chi_H}{1+4\chi_H^2}\eta_{(0)}$$

$$\eta_{(4)} = \frac{\chi_H}{1+\chi_H^2}\eta_{(0)}$$

Same as Dima et al

Conclusion & outlook

- The correction to the distribution function contain electric charge dependence factors

- Simple model calculation based on Bjorken evolution shows no significant effect

- In future the correction can be included in a numerical simulation with magnetic field

- Chapman-Enskog and Grads moments for calculating δf

Expressions for S_{ij}

$$S^{ij}_{(0)} = \left(3b^i b^j - \delta^{ij}\right)\left(b^j b^k V_{jk} - \frac{1}{3}\nabla\cdot\mathbf{V}\right)$$

$$S^{ij}_{(1)} = 2V^{ij} - \delta^{ij}\nabla\cdot\mathbf{V} - 2V^{ik}b_k b^j - 2b^i V^{jk} b_k + \delta^{ij}V^{kl}b_k b_l + b^i b^j \nabla\cdot\mathbf{V} + b^i b^j V^{kl} b_k b_l,$$

$$S^{ij}_{(2)} = 2\left(V^{ik}b_k b^j + b^i V^{jk} b_k - 2b^i b^j V^{kl} b_k b_l\right),$$

$$S^{ij}_{(3)} = -b^{ik}V^j_k - V^{ik}b^j_k + b^{ik}V_{kl}b^l b^j + b^i b^{jk} V_{kl} b^l,$$

$$S^{ij}_{(4)} = -2\left(b^{ik}V_{kl}b^l b^j + b^i b^{jk} V_{kl} b^l\right).$$

Electrical conductivity of QGP

Lattice:

S Gupta (2004)
PLB 597,57-62

$\frac{\sigma}{T}|_{(T=2T_c)} \sim 6$ ······▶ $\sigma \sim 90\text{MeV}$

taking $e^2 = 0.09$

G Arts et al (2007)
arXiv lat/0703008

$\frac{\sigma}{T} \sim 0.4$ ······▶ $\sigma|_{T=2T_c} \sim 12\text{MeV}$

A Amato et al (2013)
PRL 111,17,172001

$\frac{\sigma}{T} \sim 0.3$ ······▶ $\sigma|_{T=2T_c} \sim 9\text{MeV}$

PQCD(NLL):

P Arnold et al (2013)
PHKKb

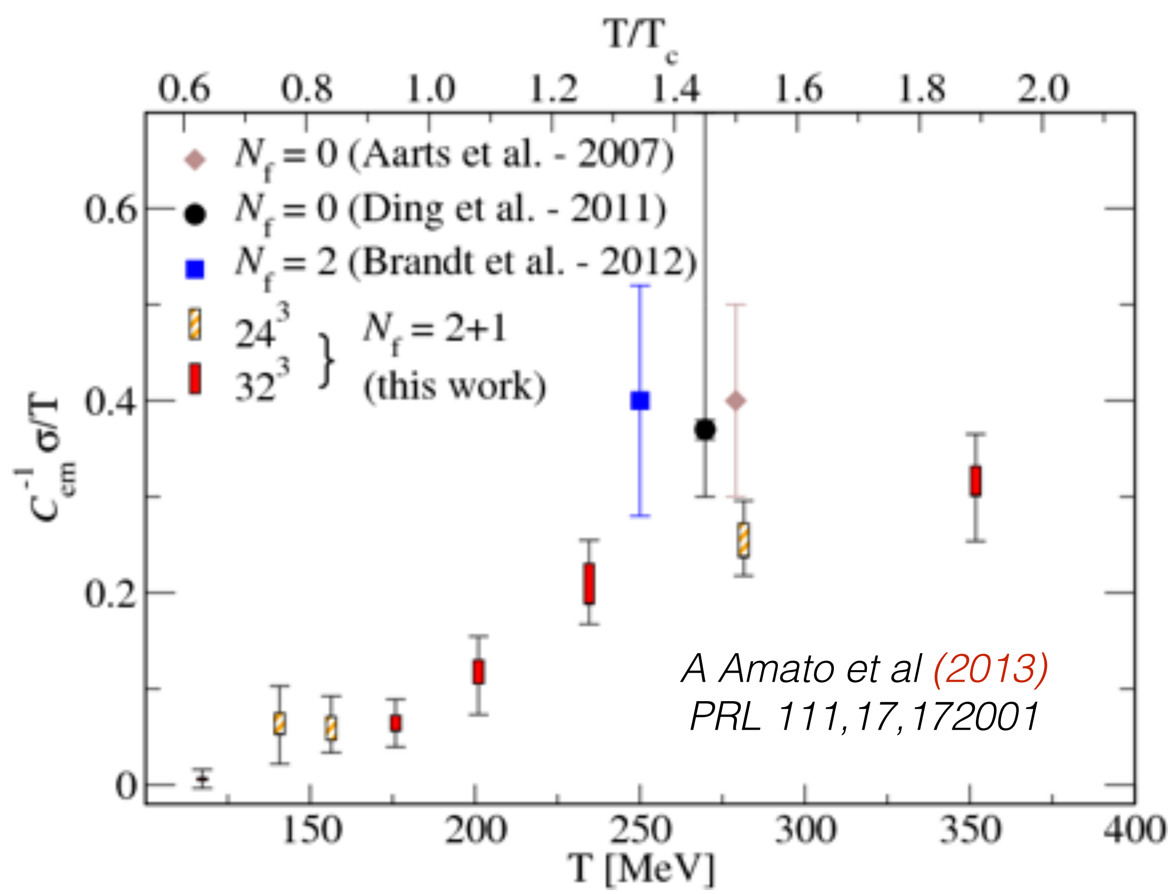
$\frac{\sigma}{T} \sim \frac{5}{e^2}$ ······▶ $\sigma|_{T=2T_c} \sim 1.7 \times 10^4\text{MeV}$

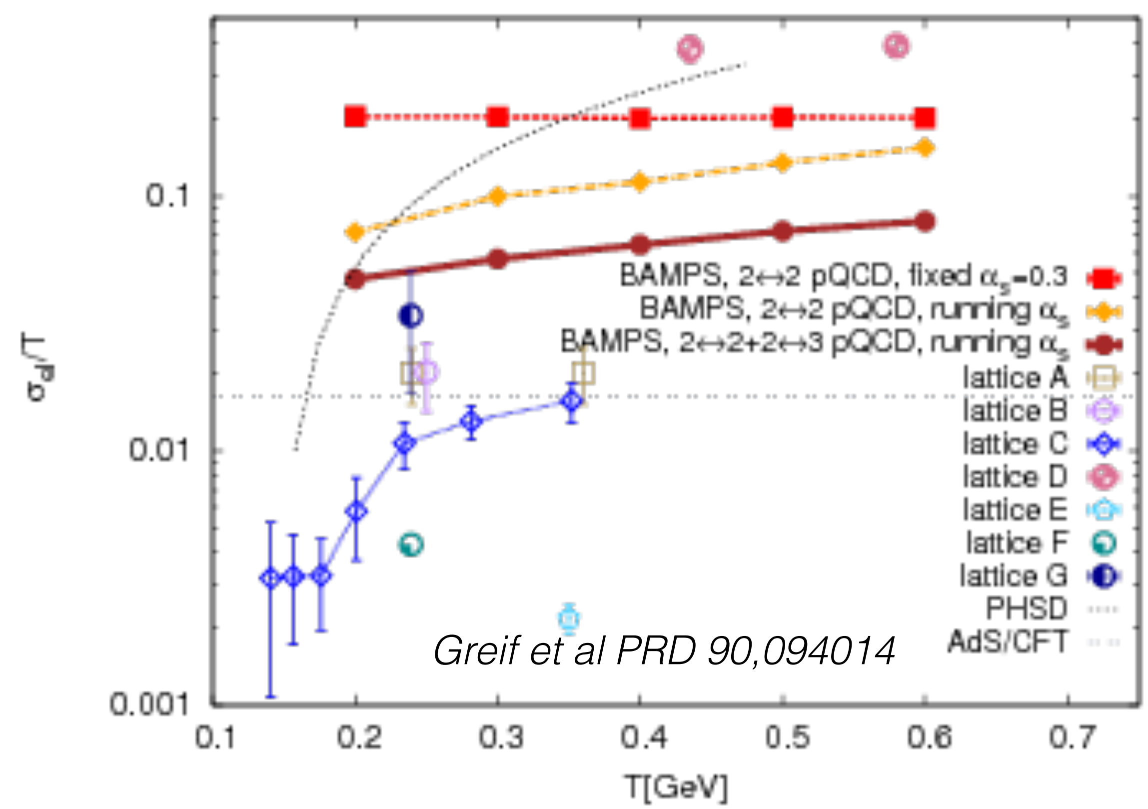
$$1 \ \text{MeV} \sim 1.69 \times 10^{11} \ \frac{\text{S}}{\text{m}} \ (\text{SI unit})$$

Copper at 20 c $5.96 \times 10^7 \ \frac{\text{S}}{\text{m}}$

QGP highly conductive

Electrical conductivity of QGP



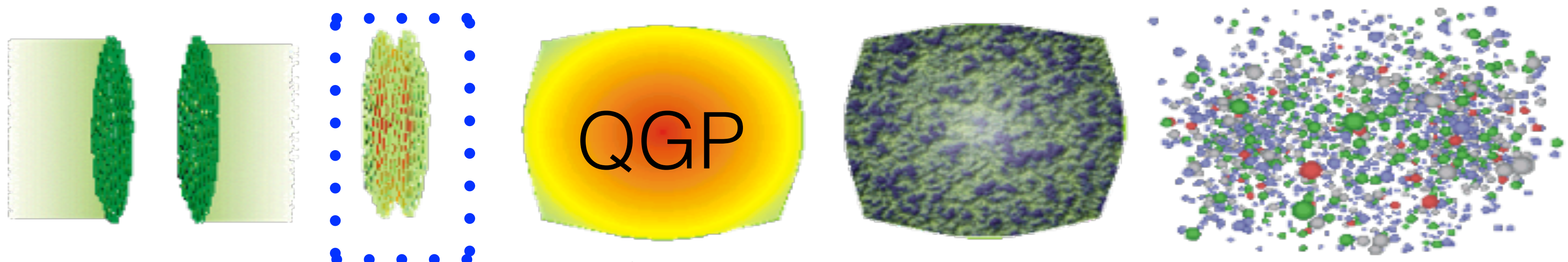


Evolution of mag field: $$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\sigma \mu}\left(\nabla^2 \mathbf{B} - \mu\epsilon \frac{\partial^2 \mathbf{B}}{\partial t^2}\right)$$

Magnetic Reynolds number: $R_m \equiv LU\sigma\mu,$

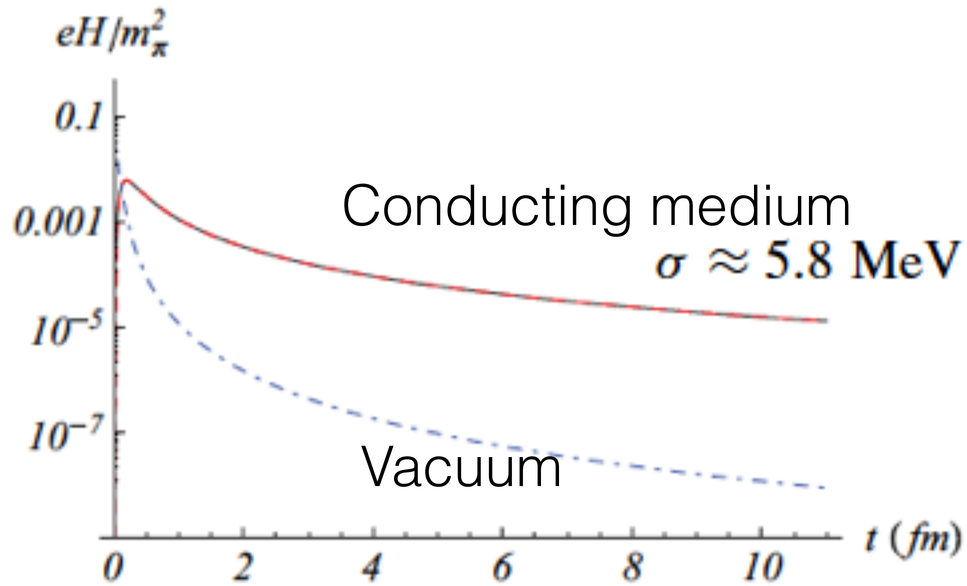
$R_m \gg 1$ Ideal MHD

Temporal evolution of fields



t<0 t=0 t>0

Field evaluation Hydrodynamic starts, t~0.6 fm



Tuchin, PRC 88,024911 (2013)

We consider
zero and
non-zero conductivity

Relativistic magnetohydrodynamics

fluid consists of charged particles moving under external electric/magnetic field experience force which changes fluid velocity

fluid constituents are electrically charged hence its motion generates electromagnetic field which changes the external field

In Magnetohydrodynamics we solve energy-momentum conservation and Maxwell equations self consistently

Formulation of Relativistic magnetohydrodynamics

conservation equations

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\nabla_\mu F^{\mu\nu} = j^\mu$$

$$\nabla_\mu N^\mu = 0$$

$$\nabla_\mu \mathcal{F}^{\mu\nu} = 0$$

$$N^\mu = n u^\mu \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad \mathcal{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

$$j^\mu = \rho u^\mu + \sigma^{\mu\beta} E_\beta$$

In case of vanishing net charge and isotropic system

$$\sigma^{\mu\beta} = \sigma g^{\mu\beta}$$
$$\rho \to 0$$

$$j^\mu = \sigma E^\mu$$

Formulation of Relativistic ideal MHD

$$E^\mu = F^{\mu\nu}u_\nu = \gamma\left[\left(\vec{v}\cdot\vec{E}\right), \vec{E}+\left(\vec{v}\times\vec{B}\right)\right]$$

In the fluid rest frame

$$\vec{j} = \sigma\left[\vec{E}+\left(\vec{v}\times\vec{B}\right)\right]$$

when $\sigma \to \infty$ but for finite $\vec{j}$

$$\vec{E} = -\vec{v}\times\vec{B}$$

Ideal MHD

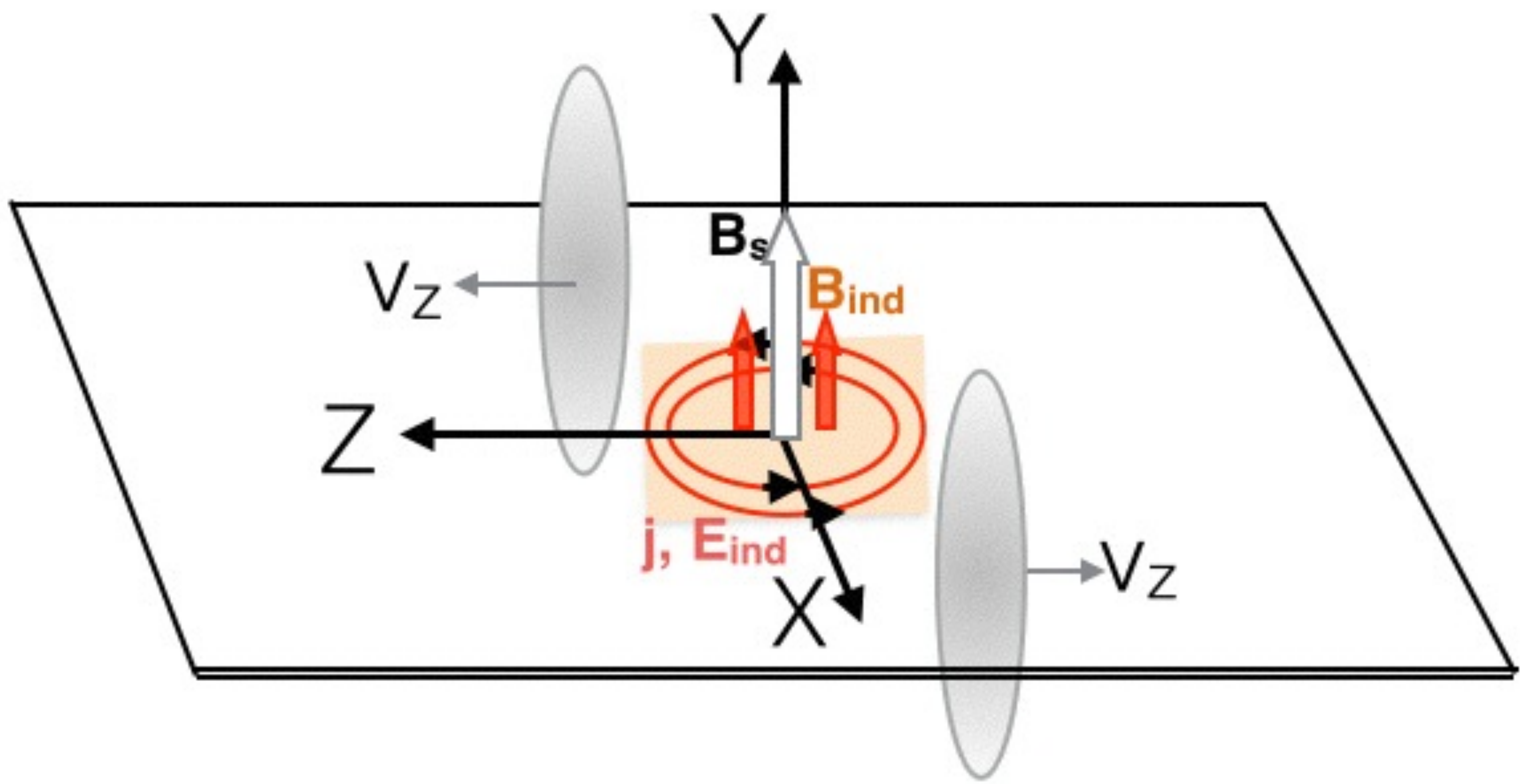
$$T^{\mu\nu}_{matter} = (\varepsilon + p)\,u^\mu u^\nu - pg^{\mu\nu}$$

$$T^{\mu\nu}_{field} = -F^{\mu\alpha}F^k_\alpha + \frac{1}{4}g^{\mu\nu}F_{\gamma\delta}F^{\gamma\delta} \Longrightarrow B^2u^\mu u^\nu - \frac{1}{2}B^2g^{\mu\nu} - B^2b^\mu b^\nu$$

Energy momentum tensor for ideal MHD

$$T^{\mu\nu} = \left(\varepsilon + p + B^2\right)u^\mu u^\nu - \left(p + \frac{1}{2}B^2\right)g^{\mu\nu} - B^2b^\mu b^\nu$$

Initial magnetic field



We numerically solve energy-momentum tensor on a space-time lattice.
The space-time evolution of magnetic field (in lab frame) is taken as parametrised

$$B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$$

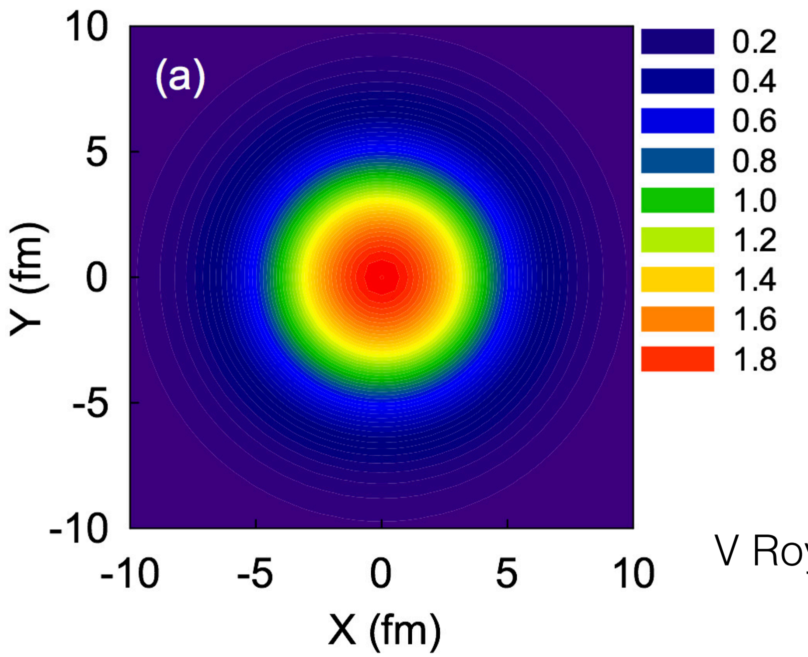
Fluid frame → B^μ ; Lab frame B → $F_{\alpha\beta}$

$$B^\mu B_\mu = -B^2$$

Lab frame: $$\frac{eB_y(\tau,x,y)}{m_\pi^2} = f(\tau)\,g(x,y)$$

Magnetic field in the transverse plane

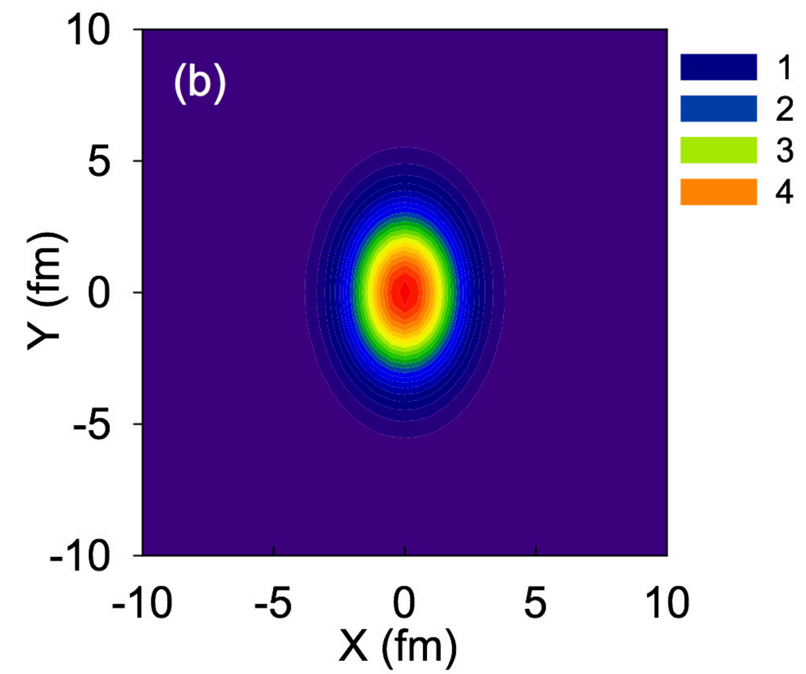
b=0 fm



$\sigma_x = \sigma_y = 3.5\text{fm}$

V Roy et al PRC, 92,064902 (2015)

b=10 fm

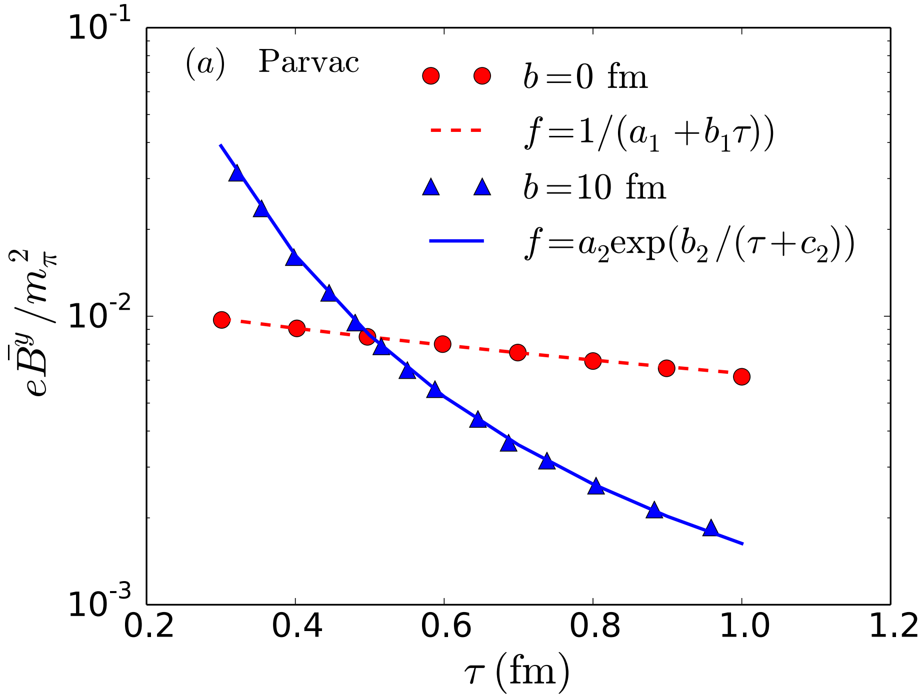


$\sigma_x = 1.5\text{fm} \;\; ; \;\; \sigma_y = 2.2\text{fm}$

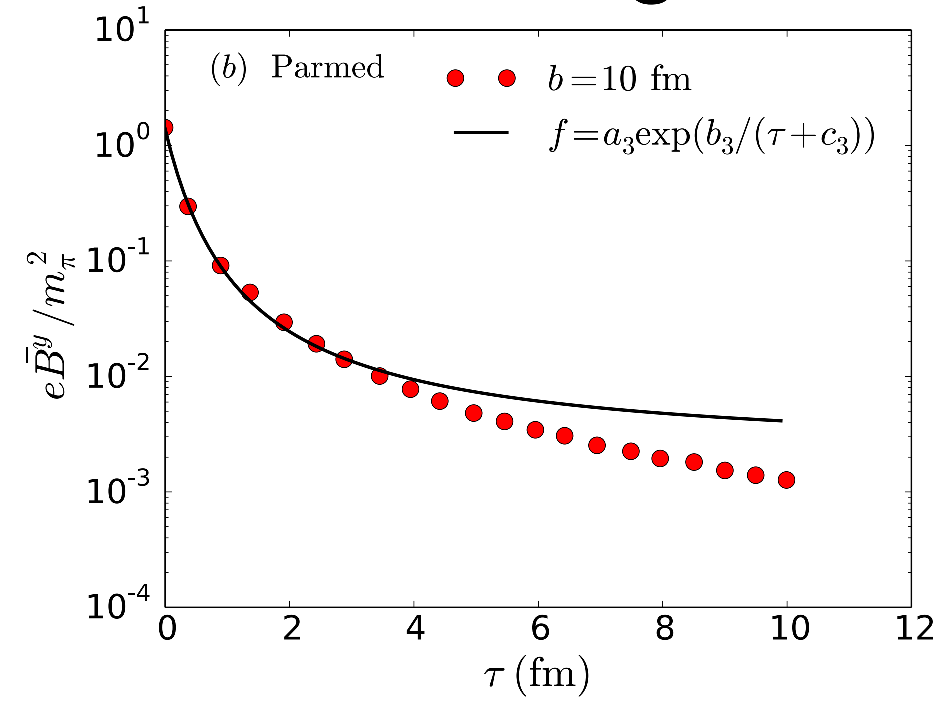
$$g\left(x,y\right) = exp\left(-\frac{\left(x-x_0\right)^2}{4\sigma_x^2} - \frac{\left(y-y_0\right)^2}{4\sigma_y^2}\right)$$

Time evolution of magnetic field

Vacuum

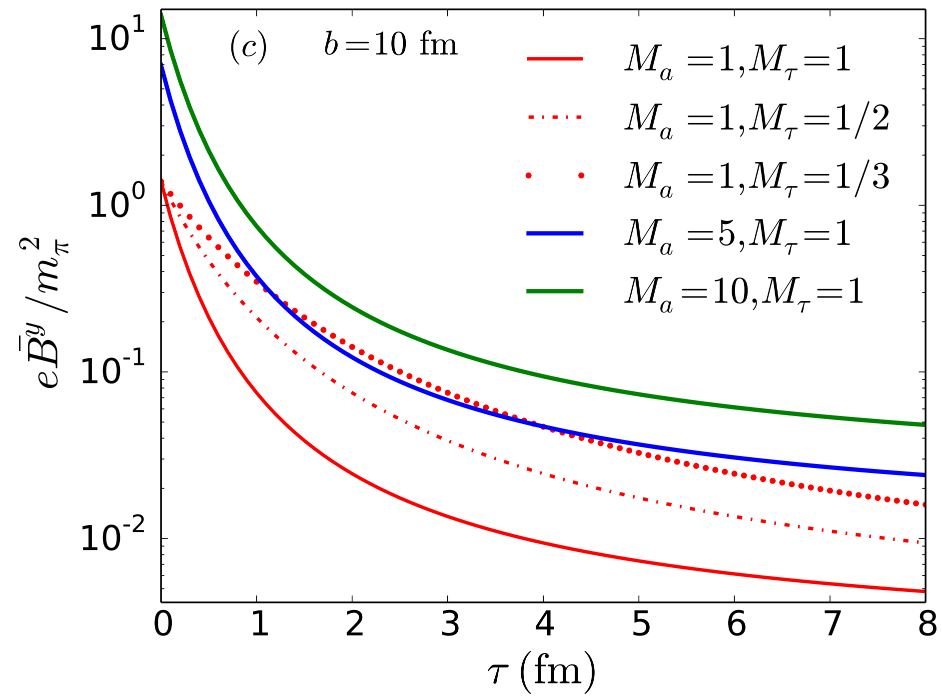


Conducting QGP



$$f(\tau) = M_a a_3 e^{(b_3/(M_\tau \tau + c_3))}$$

$M_a = 1 \Rightarrow \frac{eB}{m_\pi^2} = 1$ at $\tau = 0$



Other inputs to RMHD model

Initial energy density : $\varepsilon(x,y,b) = \varepsilon_0 \left[x_h N_{part}(x,y,b) + (1-x_h) N_{coll}(x,y,b)\right]$

Equation of State : Lattice+HRG (s95p-PCE165-v0)

Thermalisation time : 0.6 fm

Initial fluid velocity : $v_x(x,y) = v_y(x,y) = 0$

Freeze out temperature: 130 MeV

Equation of motion

$$\nabla_\mu T^{\mu\nu} = 0$$

$\nu = 0:$

$$\partial_\tau\left(\tilde{T}^{\tau\tau}\right) + \partial_x\left(\tilde{T}^{\tau\tau}\tilde{v_x}\right) + \partial_y\left(\tilde{T}^{\tau\tau}\tilde{v_y}\right) = -\left(p + \frac{B^2}{2}\right)$$

$\nu = 1:$

$$\partial_\tau\left(\tilde{T}^{\tau x}\right) + \partial_x\left[\left(\tilde{T}^{\tau x} + \tilde{B}^\tau\tilde{B}^x\right)v_x\right] + \partial_y\left[\left(\tilde{T}^{\tau x} + \tilde{B}^\tau\tilde{B}^x\right)v_y\right] = \partial_y\left[\tilde{B}^y\tilde{B}^x\right] - \partial_x\left[\tilde{p} + \frac{\tilde{B}^2}{2} - \tilde{B}^{x^2}\right]$$

$\nu = 2:$

$$\partial_\tau\left(\tilde{T}^{\tau y}\right) + \partial_x\left[\left(\tilde{T}^{\tau y} + \tilde{B}^\tau\tilde{B}^y\right)v_x\right] + \partial_y\left[\left(\tilde{T}^{\tau y} + \tilde{B}^\tau\tilde{B}^y\right)v_y\right] = \partial_x\left[\tilde{B}^y\tilde{B}^x\right] - \partial_y\left[\tilde{p} + \frac{\tilde{B}^2}{2} - \tilde{B}^{y^2}\right]$$

$$\tilde{T}^{\mu\nu} = \tau T^{\mu\nu}, \qquad \tilde{B}^\mu = \sqrt{\tau}B^\mu, \qquad \tilde{B}^2 = \tau\tilde{B}^2$$

Multidimensional Flux-Corrected algorithm SHASTA is used for the numerical solution of conservation equations

Conservative to Primitive variables

At each time step we need to find out velocity, energy density and pressure from the transported component of $T^{\mu\nu}$

$$
\begin{aligned}
E &:= T^{\tau\tau} = w\gamma^2 - p_B - B^2(b^\tau)^2 , \\
M^x &:= T^{\tau x} = w\gamma^2 v^x - B^2 b^\tau b^x , \\
M^y &:= T^{\tau y} = w\gamma^2 v^y - B^2 b^\tau b^y .
\end{aligned}
$$

Redefine the variables

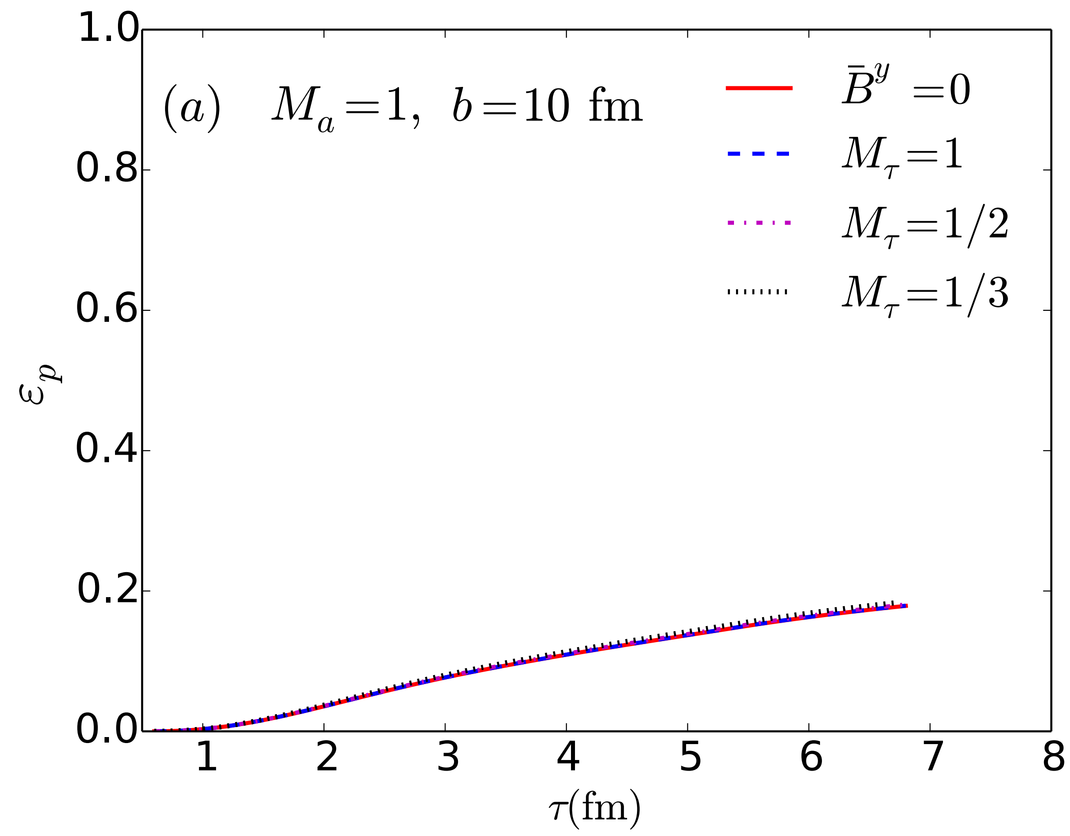
$$
\begin{aligned}
E^{'} &:= E + B^2(b^\tau)^2 = w\gamma^2 - p_B , \\
M^{x^{'}} &:= M^x + B^2 b^\tau b^x = w\gamma^2 v^x , \\
M^{y^{'}} &:= M_y + B^2 b^\tau b^y = w\gamma^2 v^y ,
\end{aligned}
$$

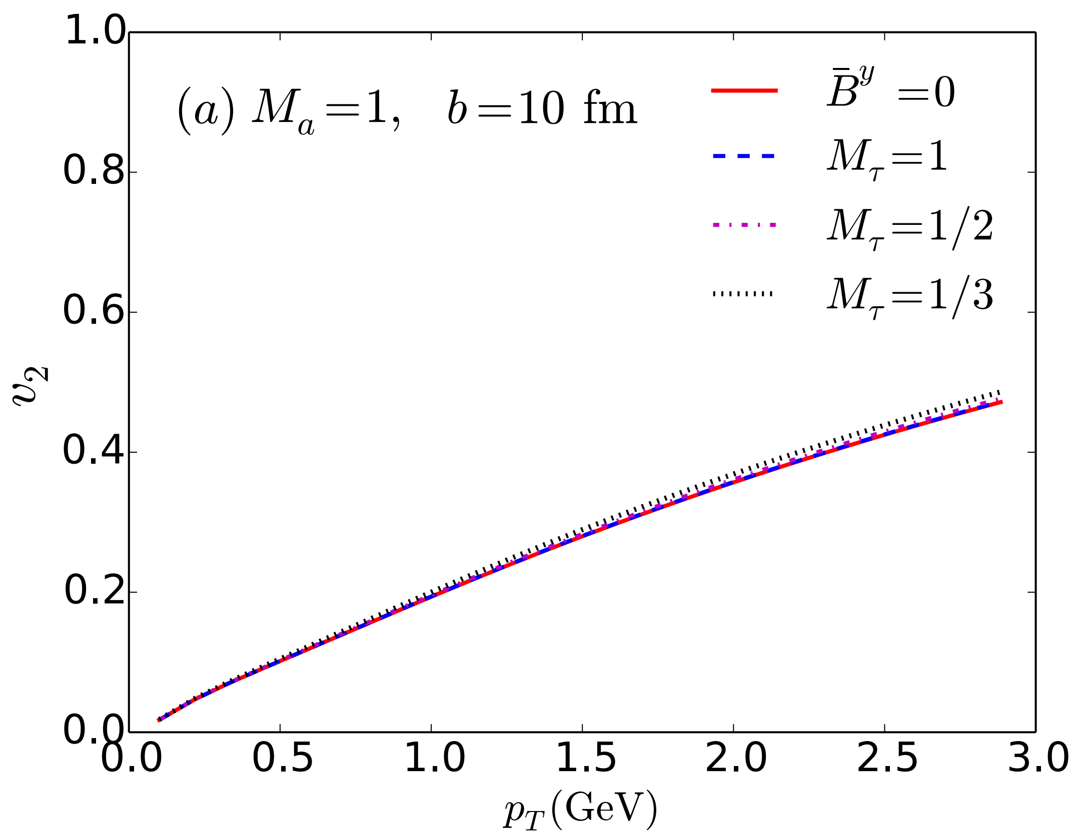
$$
\begin{aligned}
M^{'} &= \left(E^{'} + p_B\right) v , \\
\varepsilon &= E^{'} - M^{'} v - \frac{B^2}{2} ,
\end{aligned}
$$

$$
v = \left. \frac{M^{'}}{E^{'} + p(\varepsilon)} \right|_{\varepsilon = E^{'} - M^{'} v - B^2/2} .
$$

Momentum anisotropy and v_2

$$eB = m_\pi^2$$



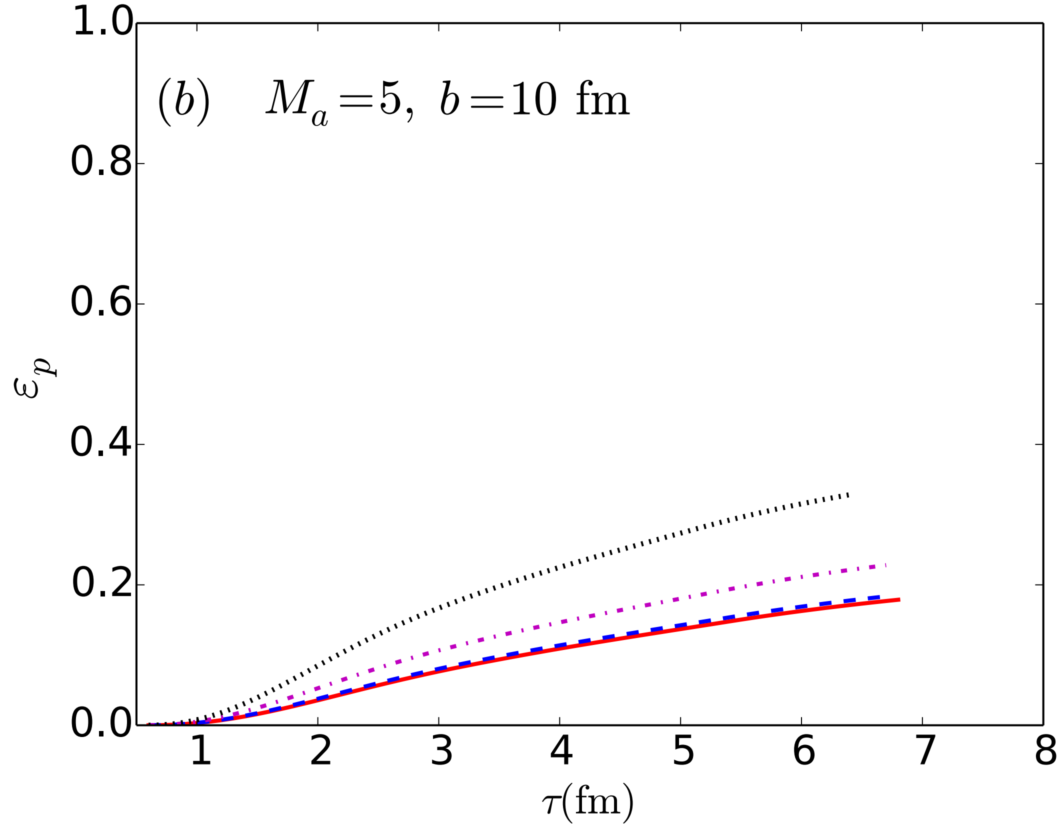


$$\varepsilon_p = \frac{< T^{xx} - T^{yy} >}{< T^{xx} + T^{yy} >}$$

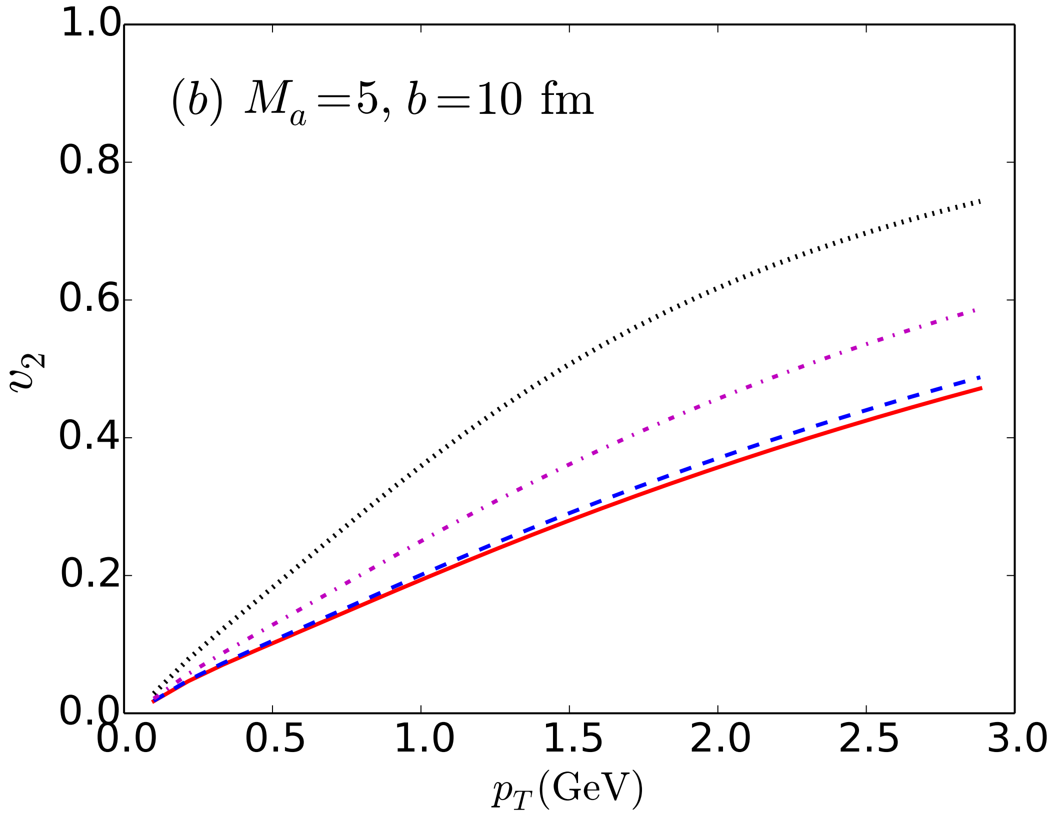
$$v_2 =< cos(2\phi) >$$

Momentum anisotropy and v_2

$$eB = 5m_\pi^2$$



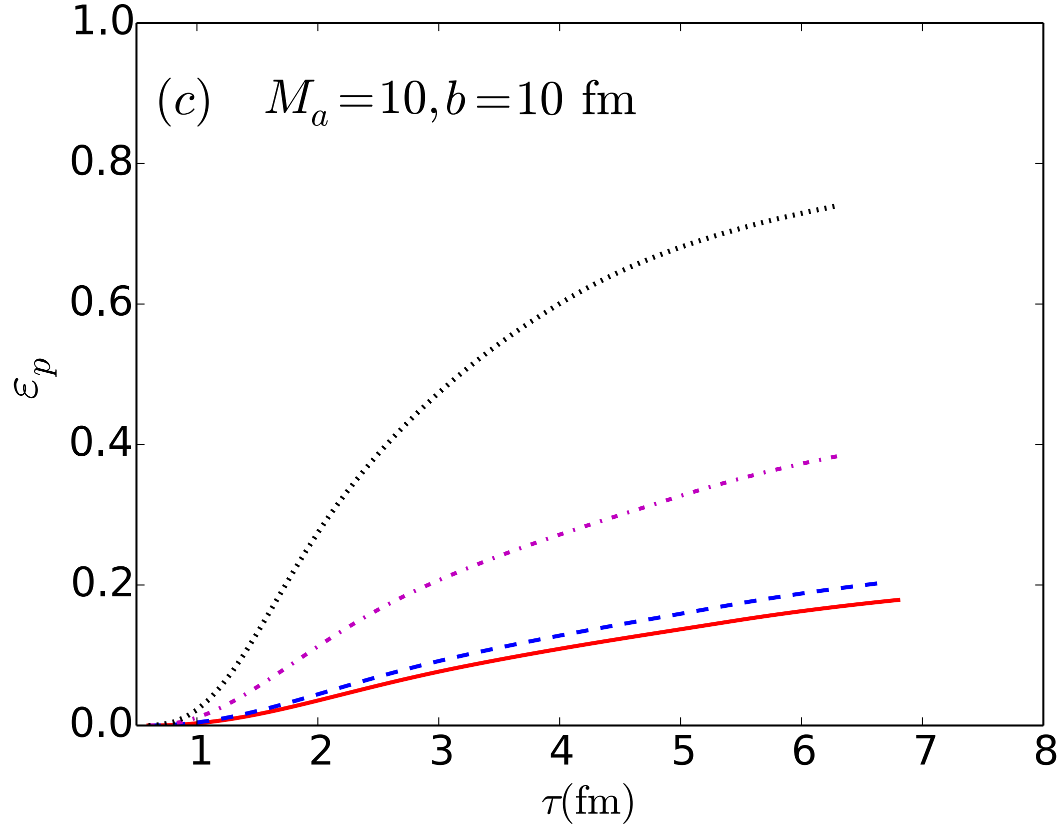
Momentum anisotropy



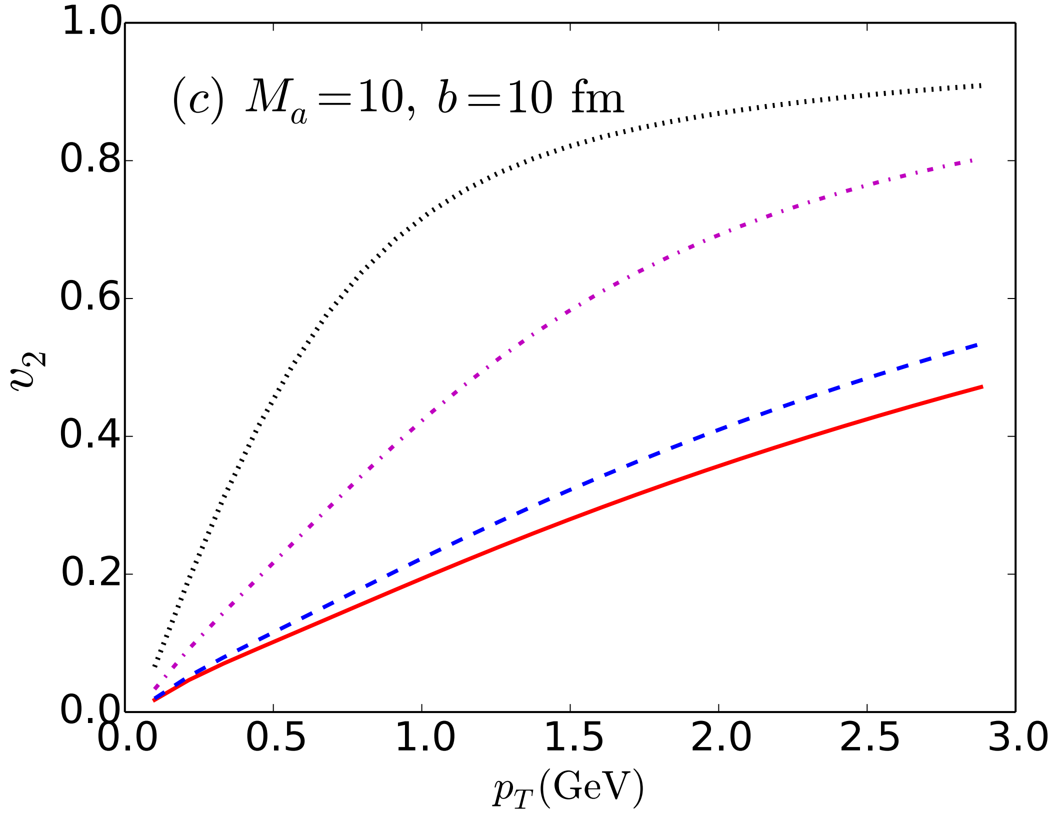
Elliptic flow

Momentum anisotropy and v_2

$$eB = 10m_\pi^2$$



Momentum anisotropy



Elliptic flow

$$T^{\tau x} \Rightarrow \partial_y \left[\tilde{B}^y \tilde{B}^x\right] - \partial_x \left[\tilde{p} + \frac{\tilde{B}^2}{2} - \tilde{B}^{x2}\right]$$

$$T^{\tau y} \Rightarrow \partial_x \left[\tilde{B}^y \tilde{B}^x\right] - \partial_y \left[\tilde{p} + \frac{\tilde{B}^2}{2} - \tilde{B}^{y2}\right]$$

Summary & outlook

- For realistic values of magnetic field we observe enhanced elliptic flow within reduced MHD formalism.
- In order to extract transport coefficient of QGP more precisely it is important to consider the effect of magnetic field on flow harmonics.
- A self consistent Relativistic non-ideal magnetohydrodynamics code is needed in order to more accurately study the effect of magnetic field.

Extra slides

Finding velocity in each time step

$$
\begin{aligned}
E &:= T^{\tau\tau} = w\gamma^2 - p_B - B^2(b^\tau)^2 \ , \\
M^x &:= T^{\tau x} = w\gamma^2 v^x - B^2 b^\tau b^x \ , \\
M^y &:= T^{\tau y} = w\gamma^2 v^y - B^2 b^\tau b^y \ .
\end{aligned}
$$

$$
\begin{aligned}
E^{'} &:= \ E + B^2(b^\tau)^2 = w\gamma^2 - p_B \ , \\
M^{x^{'}} &:= \ M^x + B^2 b^\tau b^x = w\gamma^2 v^x \ , \\
M^{y^{'}} &:= \ M_y + B^2 b^\tau b^y = w\gamma^2 v^y \ ,
\end{aligned}
$$

$$
\begin{aligned}
M^{'} &= \left(E^{'} + p_B\right) v \ , \\
\varepsilon &= E^{'} - M^{'} v - \frac{B^2}{2} \ ,
\end{aligned}
$$

$$
v = \left. \frac{M^{'}}{E^{'} + p(\varepsilon)} \right|_{\varepsilon = E^{'} - M^{'} v - B^2/2} \ .
$$

Energy momentum conservation

$$\partial_\tau \left(\tilde{T}^{\tau\tau}\right) + \partial_x \left(\tilde{T}^{\tau\tau}\tilde{v_x}\right) + \partial_y \left(\tilde{T}^{\tau\tau}\tilde{v_y}\right) = -\left(p + \frac{B^2}{2}\right)$$

$$\partial_\tau \left(\tilde{T}^{\tau x}\right) + \partial_x \left[\left(\tilde{T}^{\tau x} + \tilde{B}^\tau \tilde{B}^x\right) v_x\right] + \partial_y \left[\left(\tilde{T}^{\tau x} + \tilde{B}^\tau \tilde{B}^x\right) v_y\right] = \partial_y \left[\tilde{B}^y \tilde{B}^x\right] - \partial_x \left[\tilde{p} + \frac{\tilde{B}^2}{2} - \tilde{B}^{x2}\right]$$

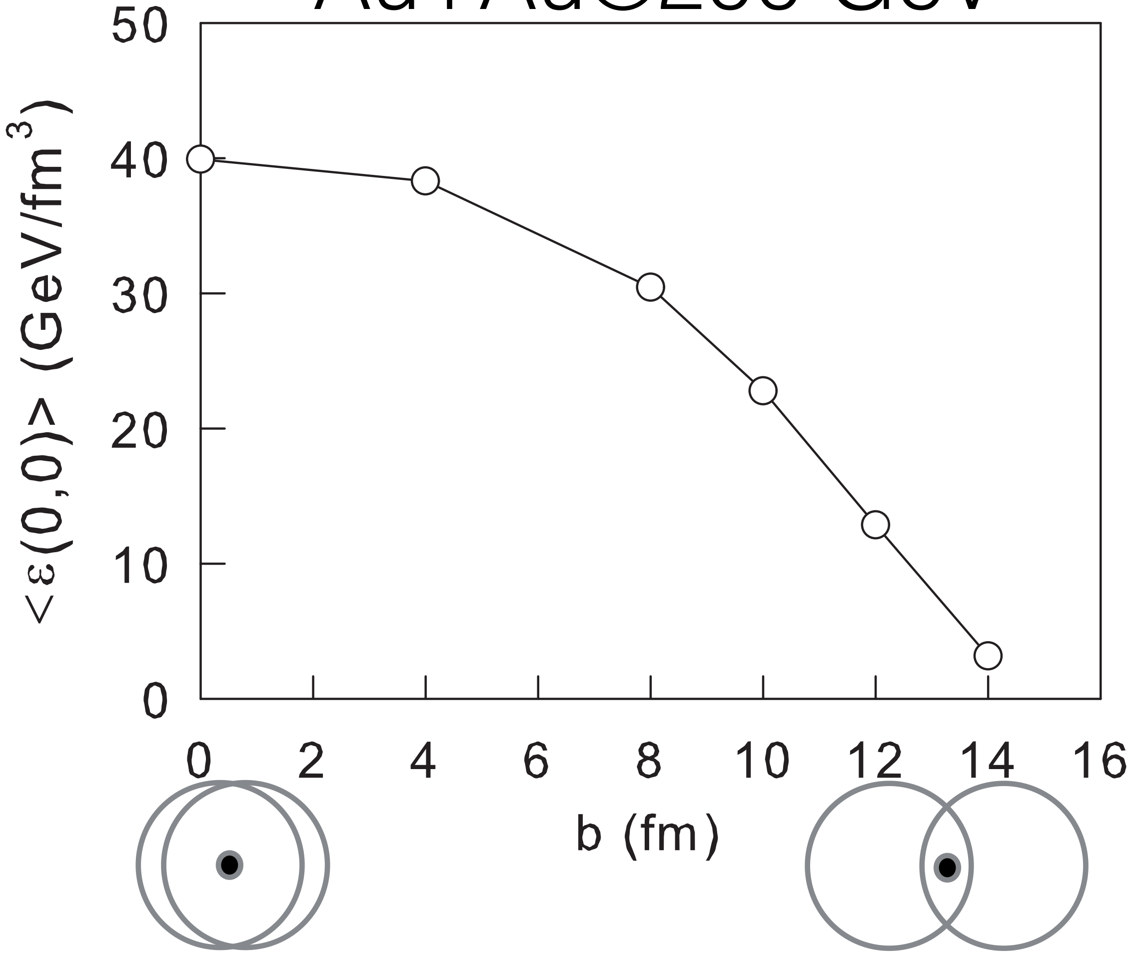
$$\partial_\tau \left(\tilde{T}^{\tau y}\right) + \partial_x \left[\left(\tilde{T}^{\tau y} + \tilde{B}^\tau \tilde{B}^y\right) v_x\right] + \partial_y \left[\left(\tilde{T}^{\tau y} + \tilde{B}^\tau \tilde{B}^y\right) v_y\right] = \partial_x \left[\tilde{B}^y \tilde{B}^x\right] - \partial_y \left[\tilde{p} + \frac{\tilde{B}^2}{2} - \tilde{B}^{y2}\right]$$

- $T^{\tau\tau} = w\gamma^2 - p_B - B^\tau B^\tau$
- $T^{x\tau} = w\gamma^2 v^x - B^\tau B^x$
- $T^{y\tau} = w\gamma^2 v^y - B^\tau B^y$

$w = \left(\varepsilon + p + \frac{B^2}{2}\right)$ and $p_B = \left(p + \frac{B^2}{2}\right)$.

Energy density of fluid

Fluid energy density
Au+Au@200 GeV



$$\varepsilon_{\text{fluid}} = k \sum_{nucleon} e^{-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma_g^2}}$$

for initial time ~0.5 fm

$$k = 6$$
$$\sigma_g = 0.5 fm$$

$$\sigma = \frac{\varepsilon_{\text{magfield}}}{\varepsilon_{\text{fluid}}}$$

$$\langle\sigma\rangle = \langle\frac{\varepsilon_{magfield}}{\varepsilon_{fluid}}\rangle \neq \langle\sigma\rangle = \frac{\langle\varepsilon_{magfield}\rangle}{\langle\varepsilon_{fluid}\rangle}$$

Ratio of field to fluid energy density

$\frac{\text{energy of field}}{\text{energy of fluid}}$ $\sigma >\sim 1$ —> **B** Important

$\sigma \ll 1$ —> **B** may be neglected

Consensus : since mag field decay quickly

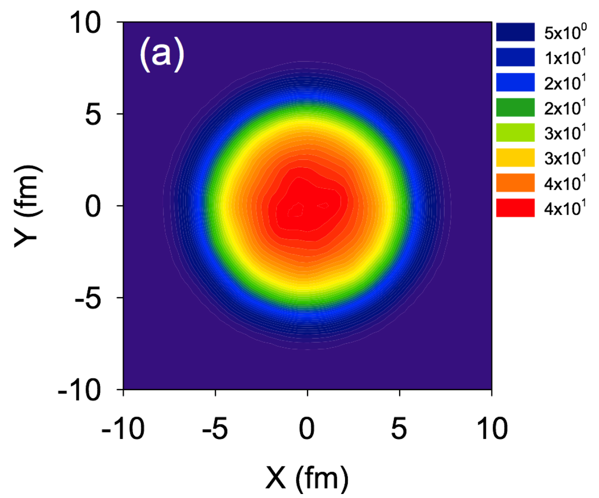
$$\langle\sigma\rangle = \frac{\langle\varepsilon_{magfield}\rangle}{\langle\varepsilon_{fluid}\rangle} \quad <<1$$

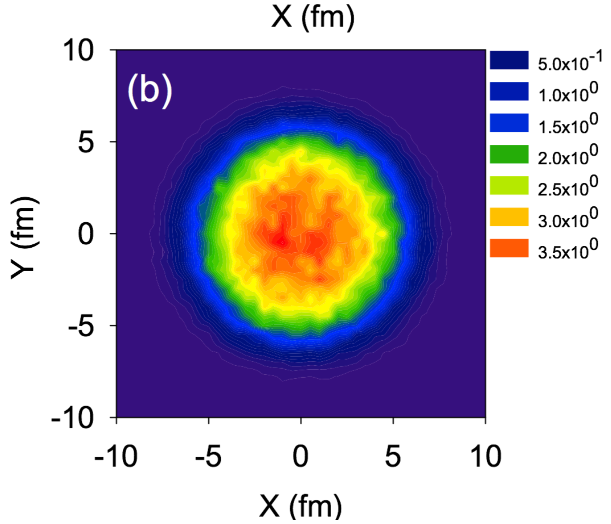
Reality: not always true because of the fluctuating IC

$$\langle\sigma\rangle = \langle\frac{\varepsilon_{magfield}}{\varepsilon_{fluid}}\rangle \quad \sim 1$$

Event average energy

b=0 fm

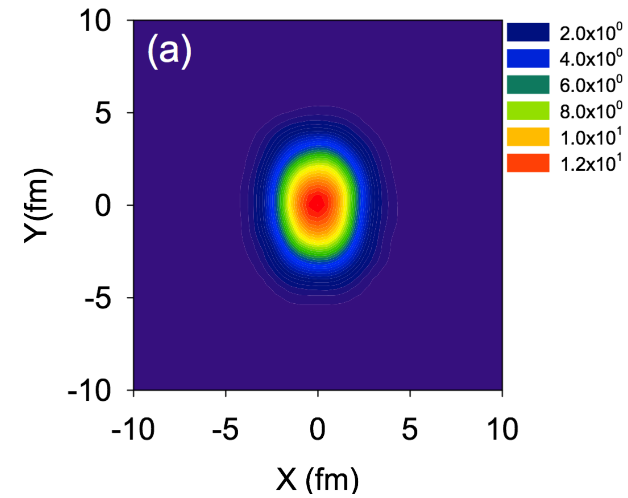


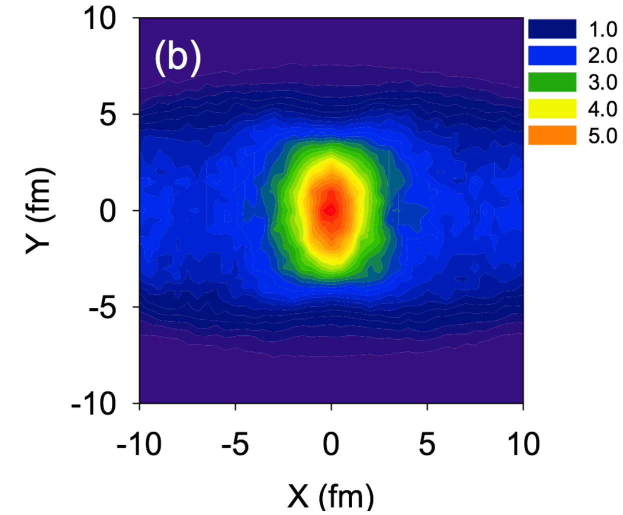


Fluid energy

$$\frac{(eB_y)^2}{2}$$

b=12 fm

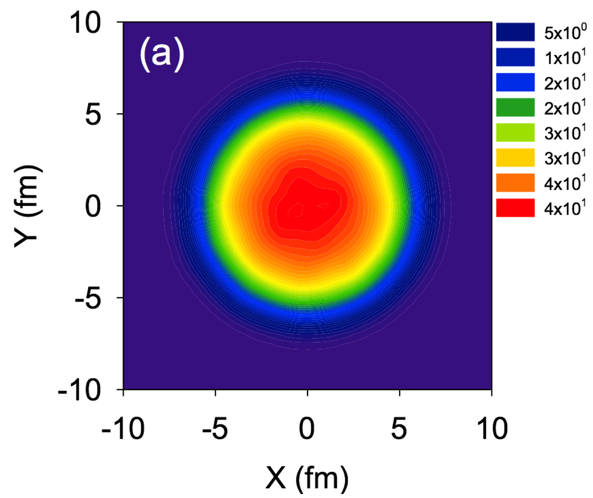


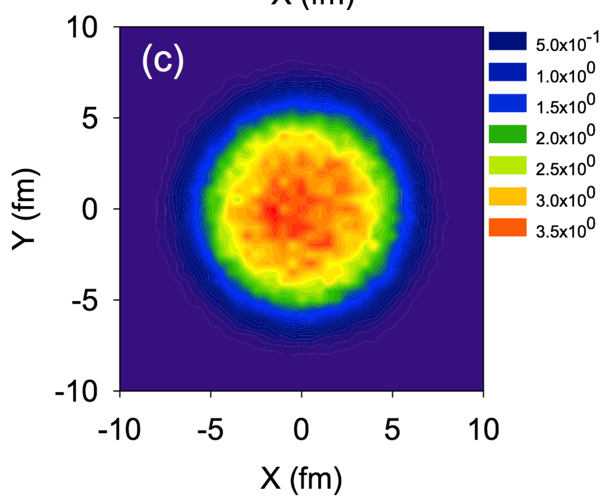


V Roy et al PRC, 92,064902 (2015)

Event average energy

b=0 fm

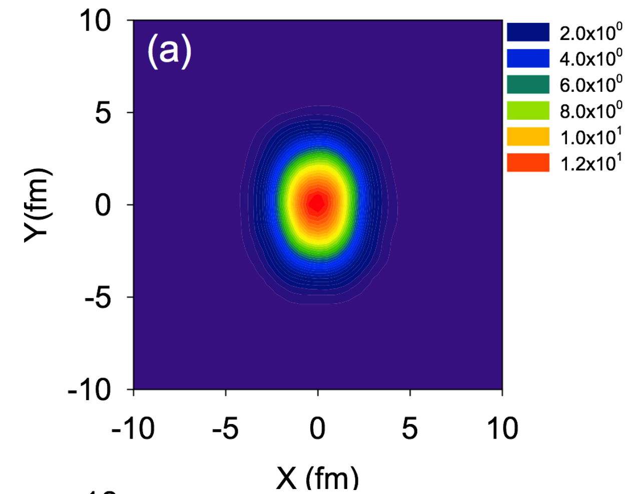


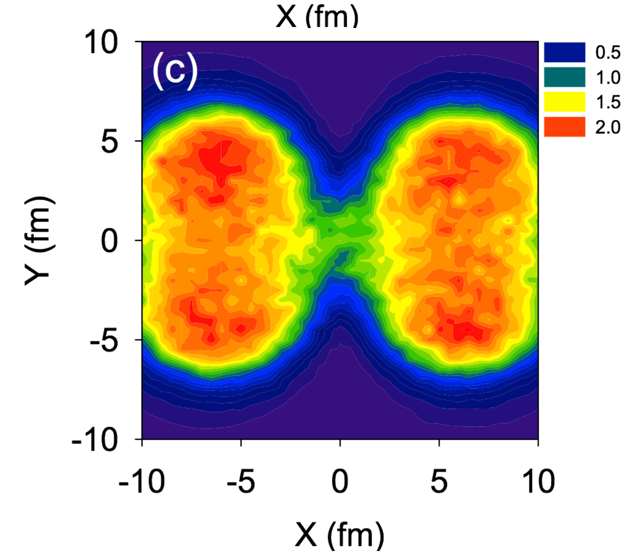


Fluid energy

$$\frac{(eB_x)^2}{2}$$

b=12 fm

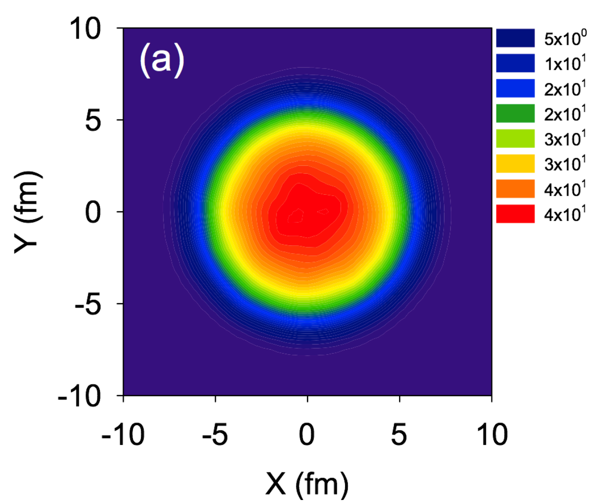




V Roy et al PRC, 92,064902 (2015)

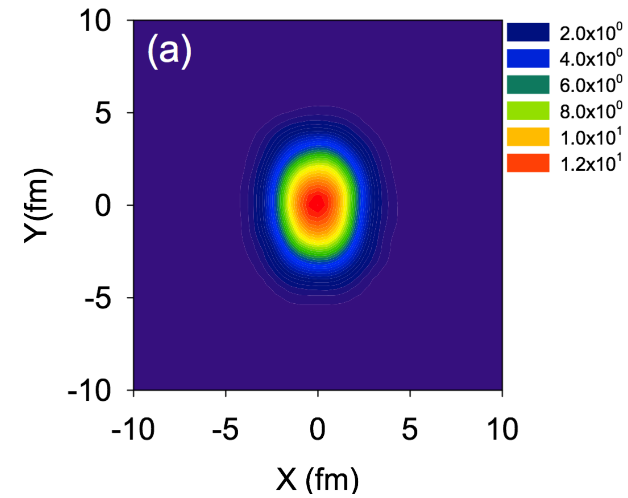
Event average sigma

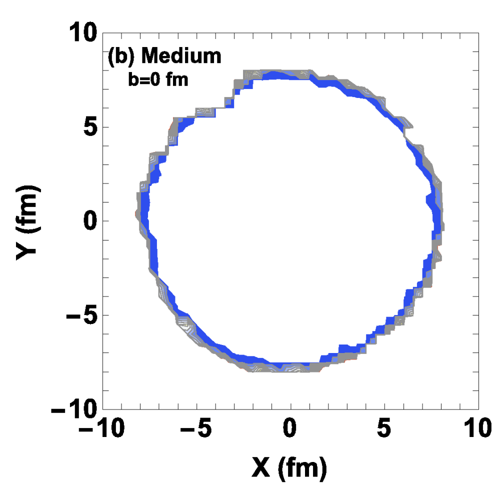
b=0 fm



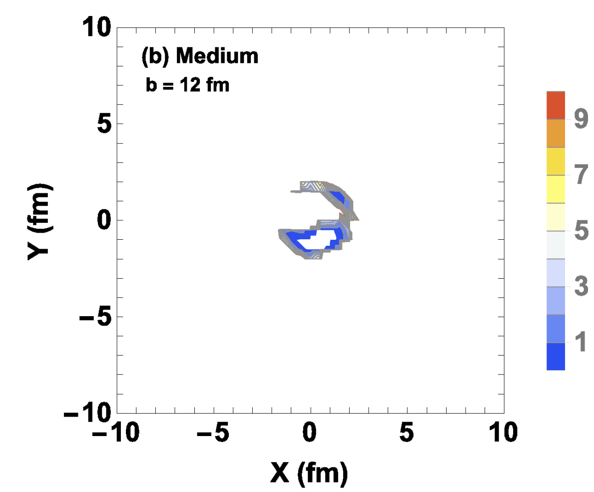
Fluid energy

b=12 fm





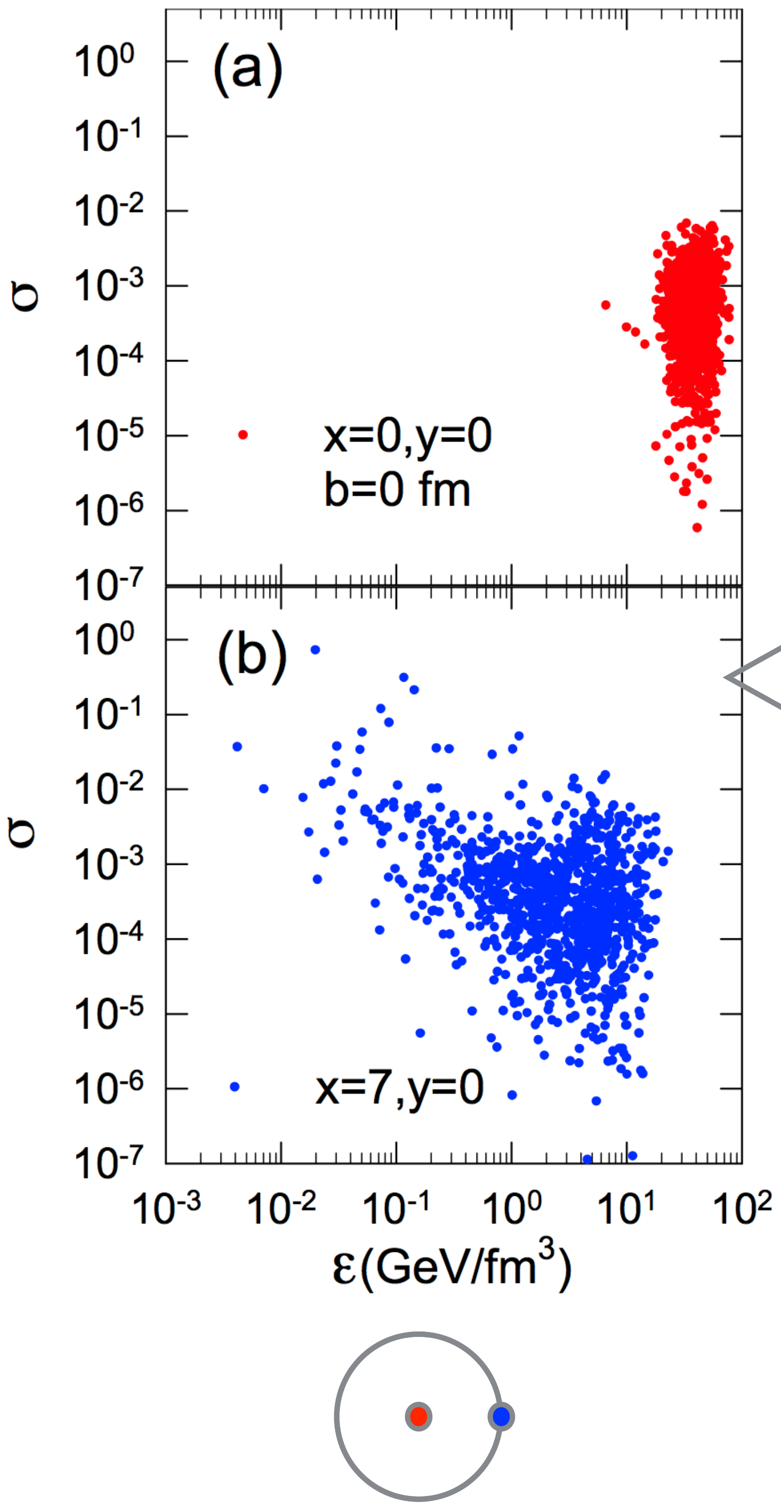
$$0.01 < \langle \sigma(x,y) \rangle < 10$$



V Roy et al PRC, 92,064902 (2015)

Event-by-event distribution of sigma

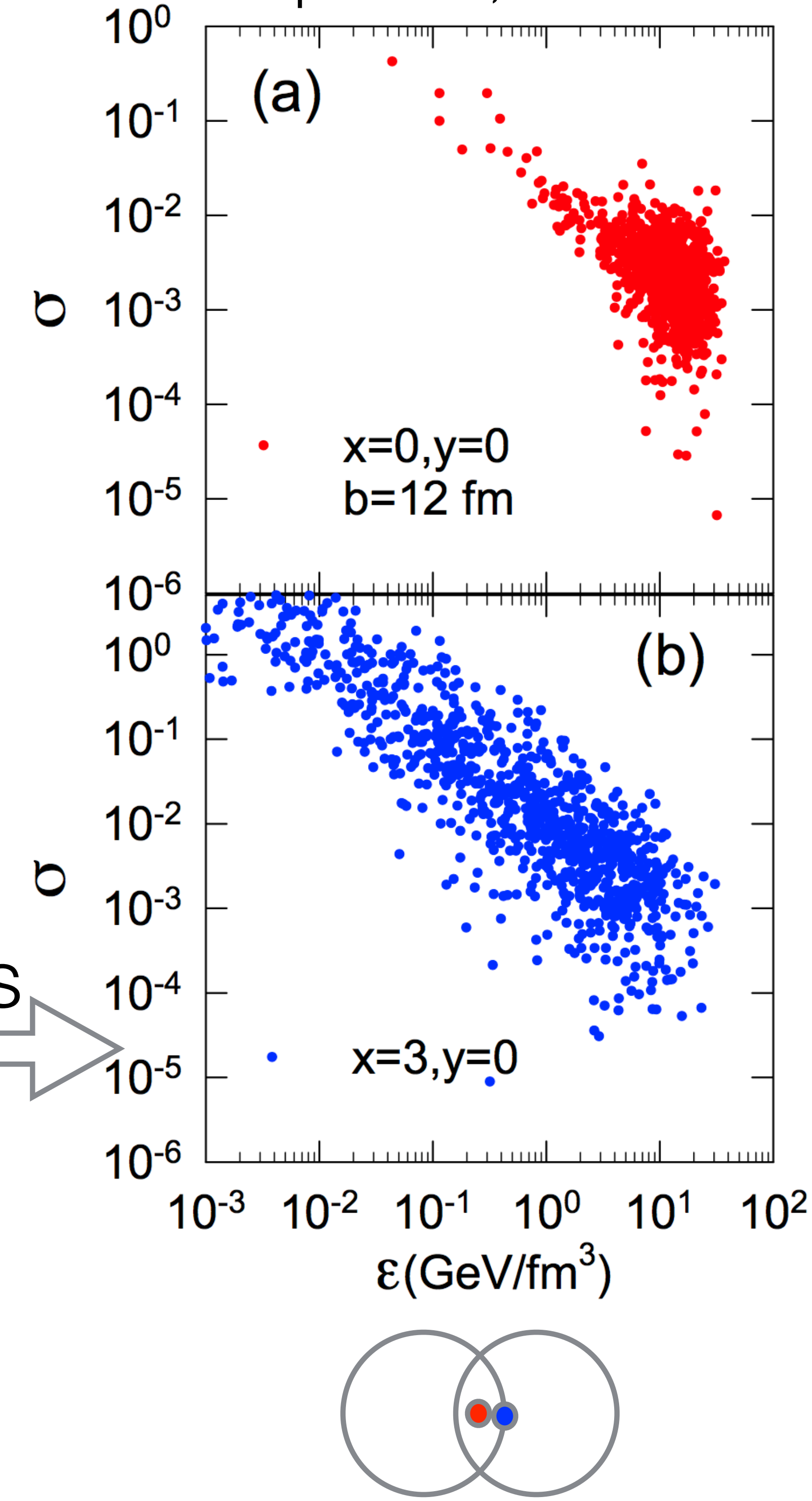
Central, b=0 fm



Central collisions
$\sigma \ll 1$

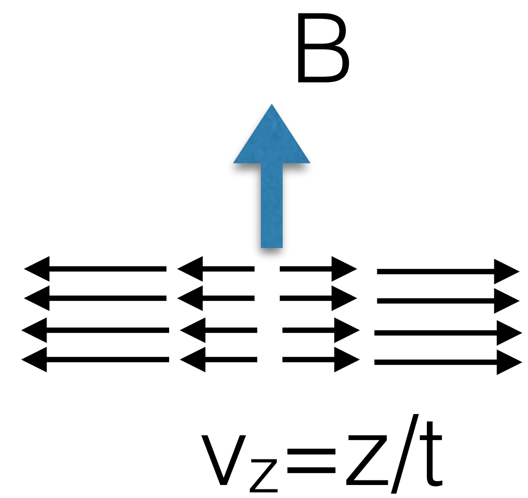
Peripheral collisions
$\sigma \sim 1$

Peripheral, b=12 fm



Magneto-hydrodynamics in one dimension

Most simple possible model



- Bjorken flow,
- ideal fluid (zero shear, bulk viscosity and zero electrical resistivity),
- Transverse magnetic field,
- Constant speed of sound,
- temporal evolution of magnetic field:

(a) “frozen in flux theorem”

$$B\left(\tau\right) = B_0 \frac{s}{s_0} = B_0 \frac{\tau_0}{\tau}$$

or

(b) “Power law decay”.

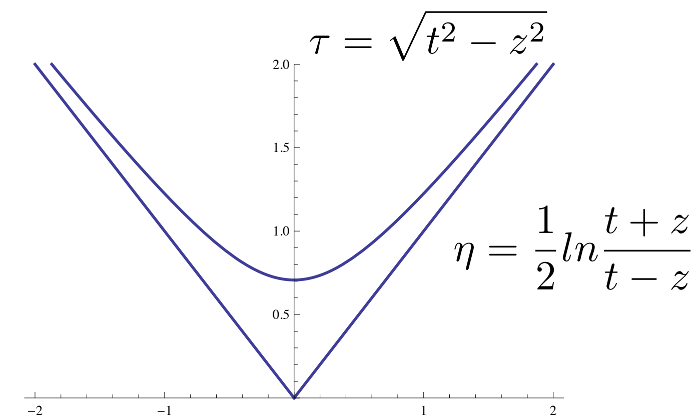
$$B(\tau) = B_0 \Big(\frac{\tau_0}{\tau}\Big)^a$$

Bjorken hydro in a nutshell

Energy-momentum tensor

$$T^{\mu\nu} = (e+p)\, u^{\mu} u^{\nu} - p g^{\mu\nu}$$

$$\partial_{\nu} T^{\mu\nu} = 0$$



$$u^{\mu} = (cosh\eta, 0, 0, sinh\eta)$$

$\parallel$ to four velocity

$$u_{\mu} \partial_{\nu} T^{\mu\nu} = 0$$

$$\partial_{\tau} e + \frac{e+p}{\tau} = 0$$

Evolution of energy density

$\perp$ to four velocity

$$(\eta_{\mu\nu} - u_{\mu} u_{\nu}) \partial_{\alpha} T^{\alpha\nu} = 0$$

$$\partial_{\eta} p = 0$$

Euler equation

Bjorken magneto-hydro in a nutshell

Energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{fluid} + T^{\mu\nu}_{field}$$

$$T^{\mu\nu} = \left(e + p + B^2\right) u^\mu u^\nu - \left(p + \frac{B^2}{2}\right) g^{\mu\nu} - B^\mu B^\nu$$

$\parallel$ to four velocity

$$u_\mu \partial_\nu T^{\mu\nu} = 0$$

$$\partial_\tau \left(e + \frac{B^2}{2}\right) + \frac{e + p + B^2}{\tau} = 0$$

Evolution of energy density

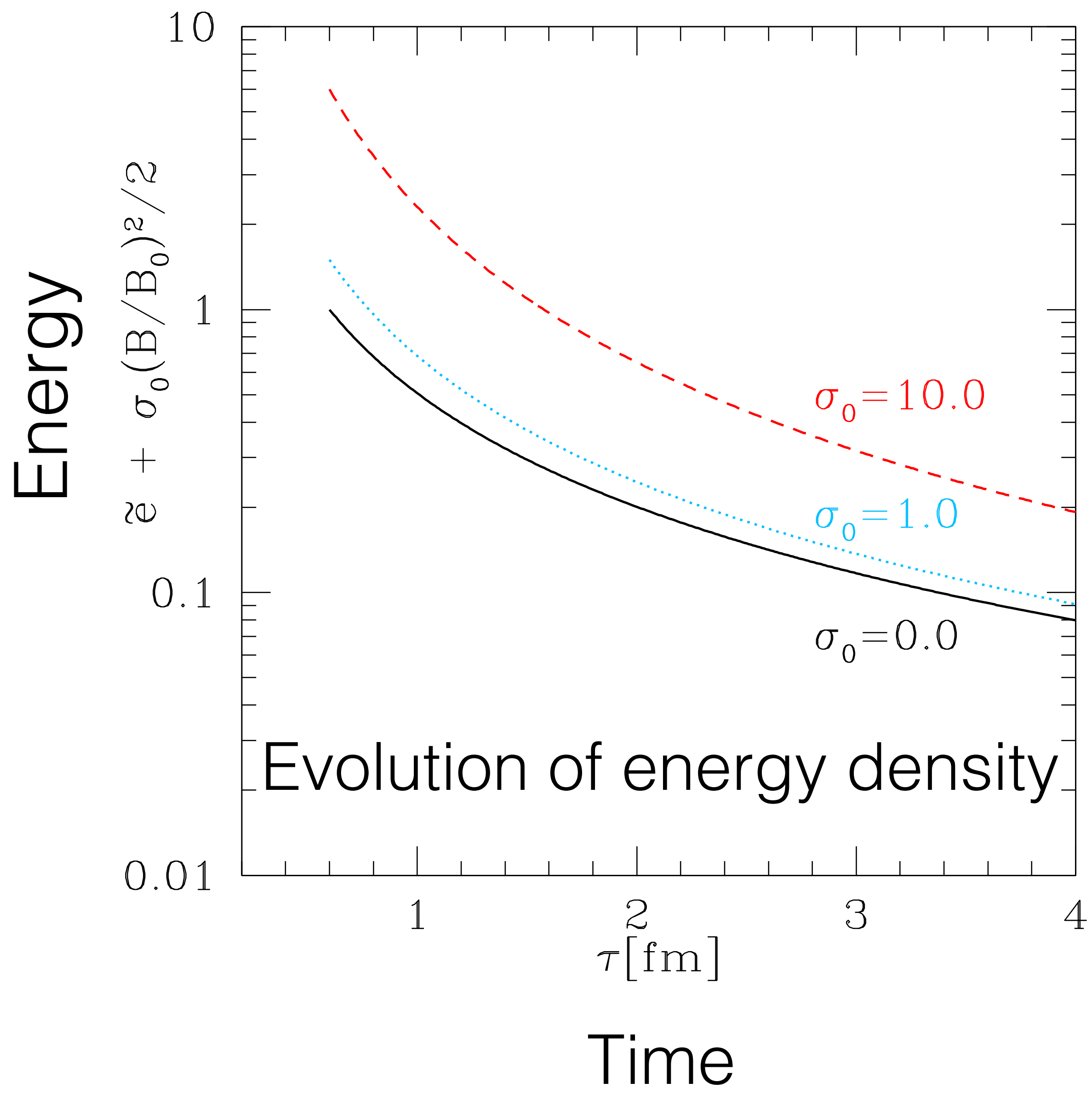
$\perp$ to four velocity

$$(\eta_{\mu\nu} - u_\mu u_\nu)\partial_\alpha T^{\alpha\nu} = 0$$

$$\frac{\partial}{\partial \eta}\left(p + \frac{1}{2}B^2\right) = 0$$

Euler equation

Magneto-hydrodynamics in one dimension: Frozen in flux



V Roy et al PLB, 750 , 45-52 (2015)

$$B(\tau) = B_0 \frac{s}{s_0} = B_0 \frac{\tau_0}{\tau}$$

$$\partial_\tau \left[\tilde{e} + \frac{\sigma_0}{2} \tilde{e}^{2/(1+c_s^2)} \right] + \frac{\tilde{e} + \tilde{p} + \sigma_0 \tilde{e}^{2/(1+c_s^2)}}{\tau} = 0$$

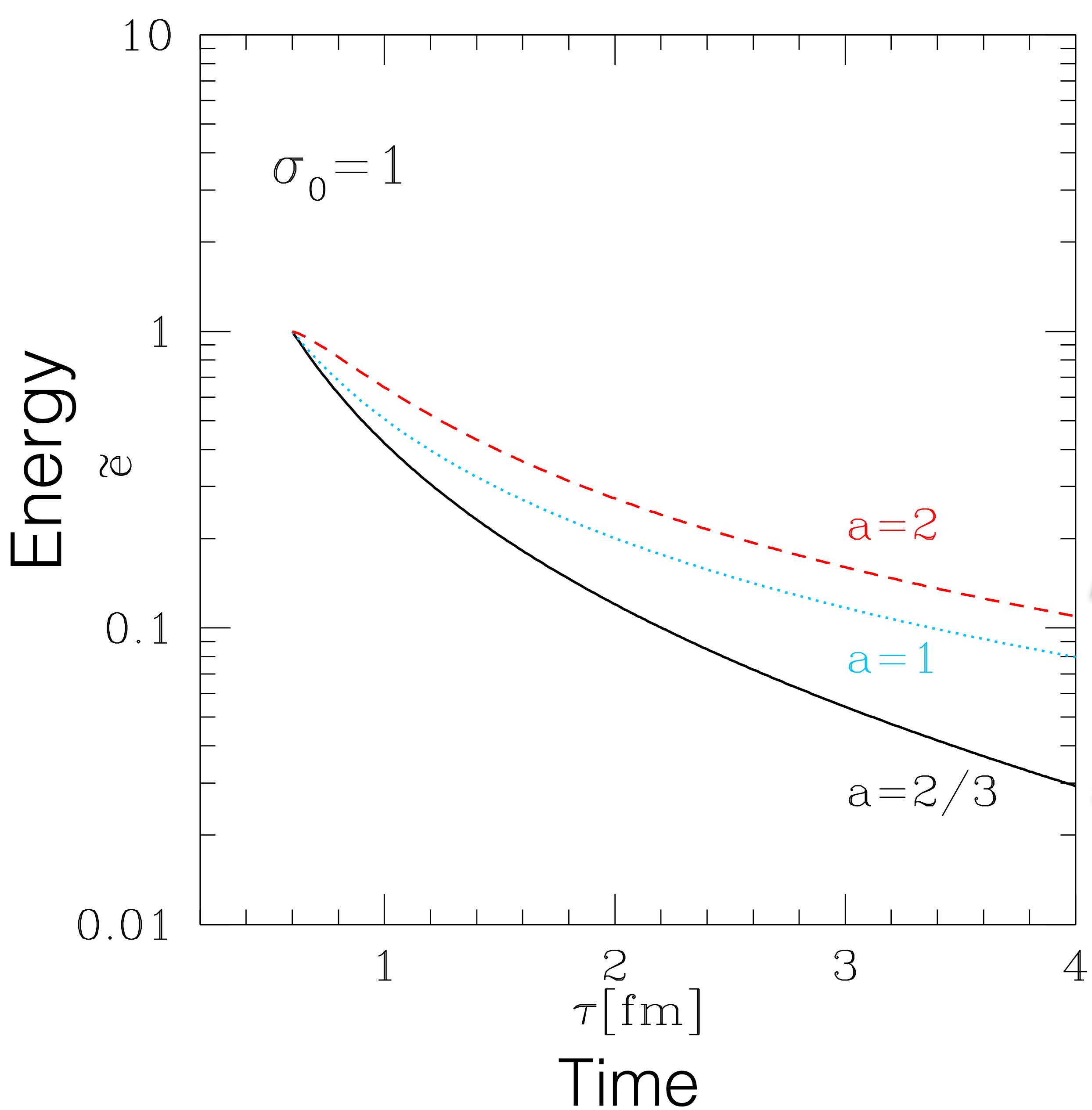
$$\tilde{e} = \frac{e}{e_0} \qquad \sigma_0 = \frac{B_0^2}{e_0}$$

$$\partial_\tau \tilde{e} = -(1 + c_s^2) \frac{\tilde{e}}{\tau}$$

Same as Bjorken expansion without mag field !!

Solution : $\tilde{e} = \left(\frac{\tau_0}{\tau} \right)^{1+c_s^2}$

Power law decay of magnetic fields



V Roy et al PLB, 750 , 45-52 (2015)

$$B(\tau) = B_0 \left(\frac{\tau_0}{\tau} \right)^a$$

$a > 1$ Quickly decaying B

Fluid gain energy

$a = 1$ Frozen flux, 0 resistivity

Same as without B

$a < 1$ Slowly decaying B

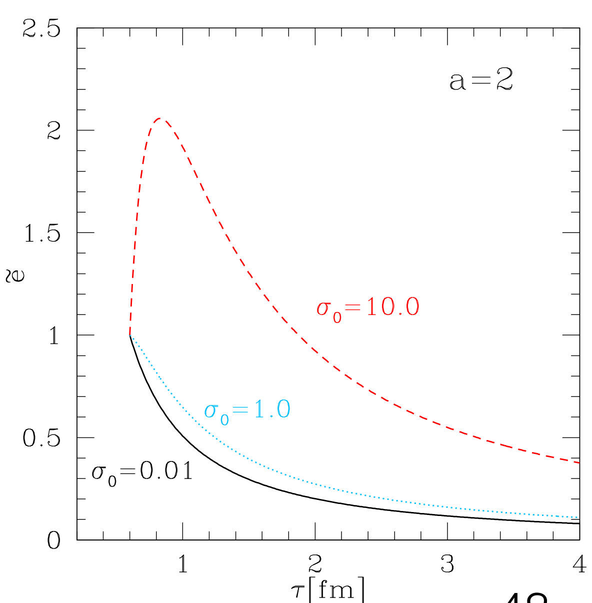
Fluid losses energy

Solution for power law decay of magnetic field

$$\partial_\tau \left[\tilde{e} + \frac{\sigma_0}{2} \left(\frac{\tau_0}{\tau}\right)^{2a} \right] + (1 + c_s^2)\, \frac{\tilde{e}}{\tau} + \frac{\sigma_0}{\tau} \left(\frac{\tau_0}{\tau}\right)^{2a} = 0$$

The solution is

$$\tilde{e}(\tau) = \left(\frac{\tau_0}{\tau}\right)^{1+c_s^2} + \sigma_0 \frac{1-a}{1+c_s^2-2a} \left[\left(\frac{\tau_0}{\tau}\right)^{1+c_s^2} - \left(\frac{\tau_0}{\tau}\right)^{2a} \right]$$



Equations of magneto-hydro

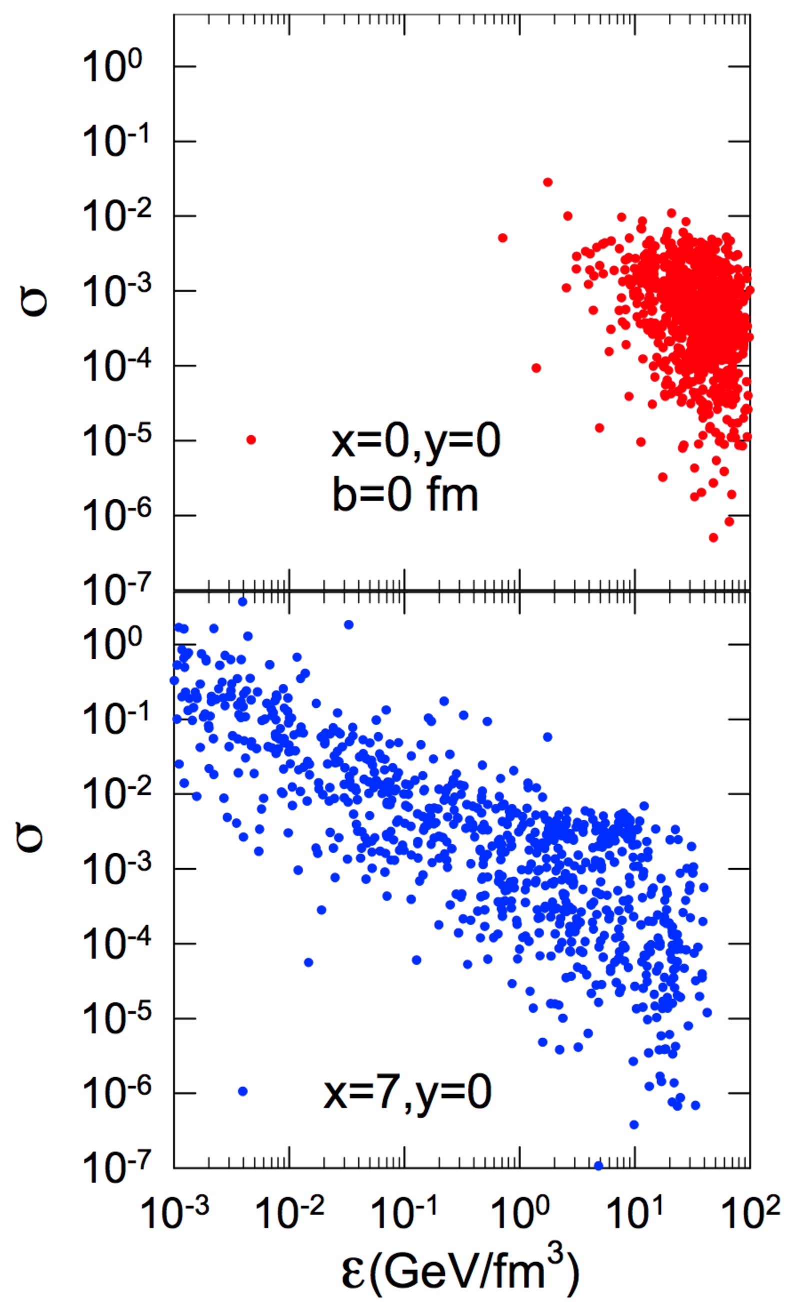
Current density : $\mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right),$

Magnetic field : $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\sigma\mu}\left(\nabla^2 \mathbf{B} - \mu\epsilon\frac{\partial^2 \mathbf{B}}{\partial t^2}\right)$

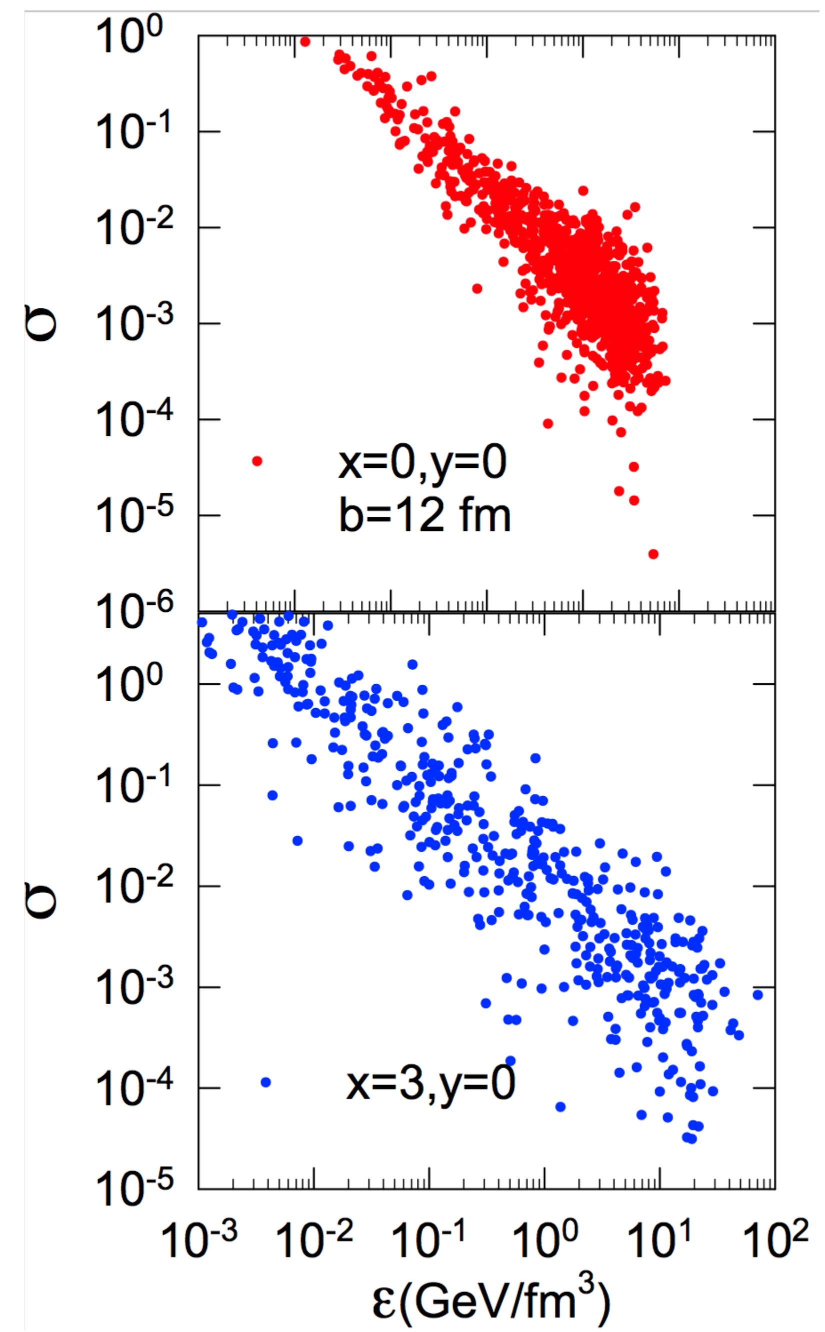
Electric Field : $\frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{v}}{\partial t} \times \mathbf{B} = \mathbf{v} \times (\nabla \times \mathbf{E}) + \frac{1}{\sigma\mu}\left(\nabla^2 \mathbf{E} - \mu\epsilon\frac{\partial^2 \mathbf{E}}{\partial t^2}\right)$

Magnetic Reynolds Number $R_m \equiv LU\sigma\mu,$

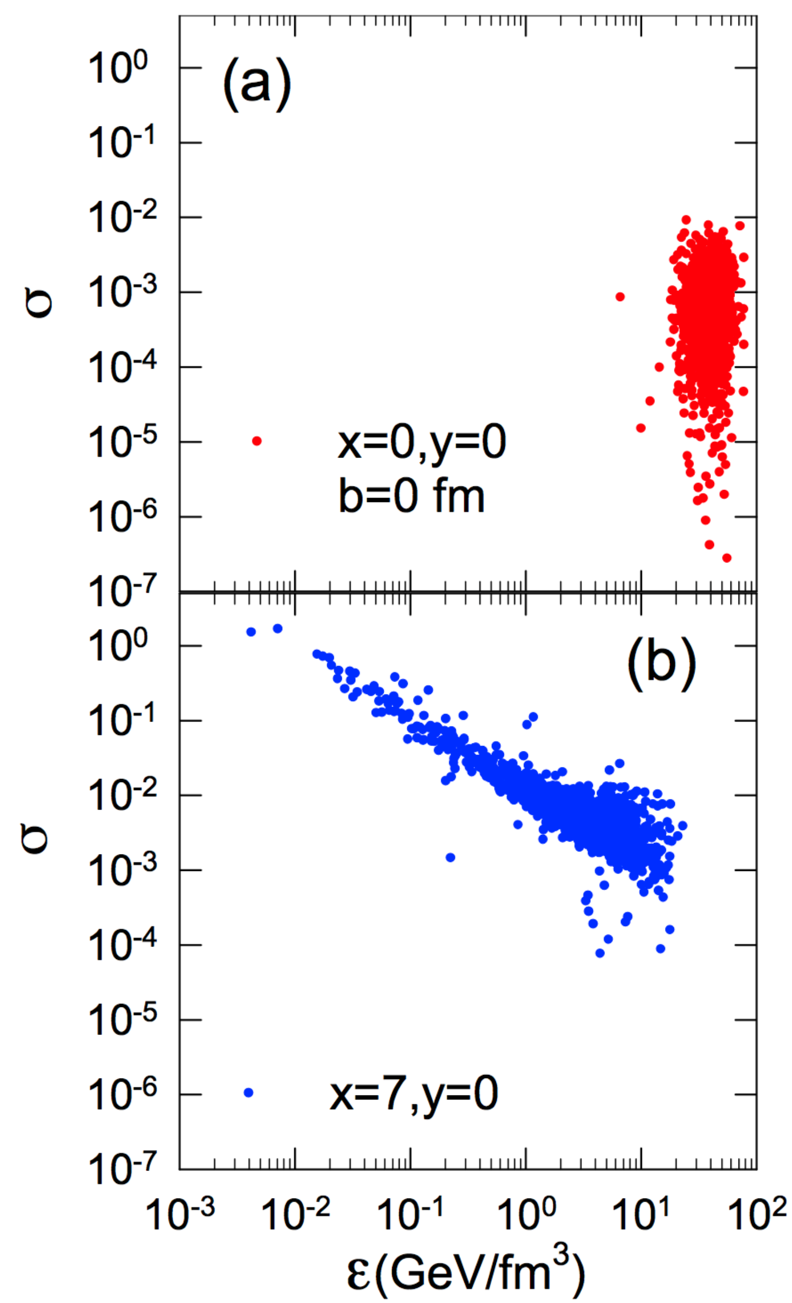
E-by-e sigma for different gaussian smearing

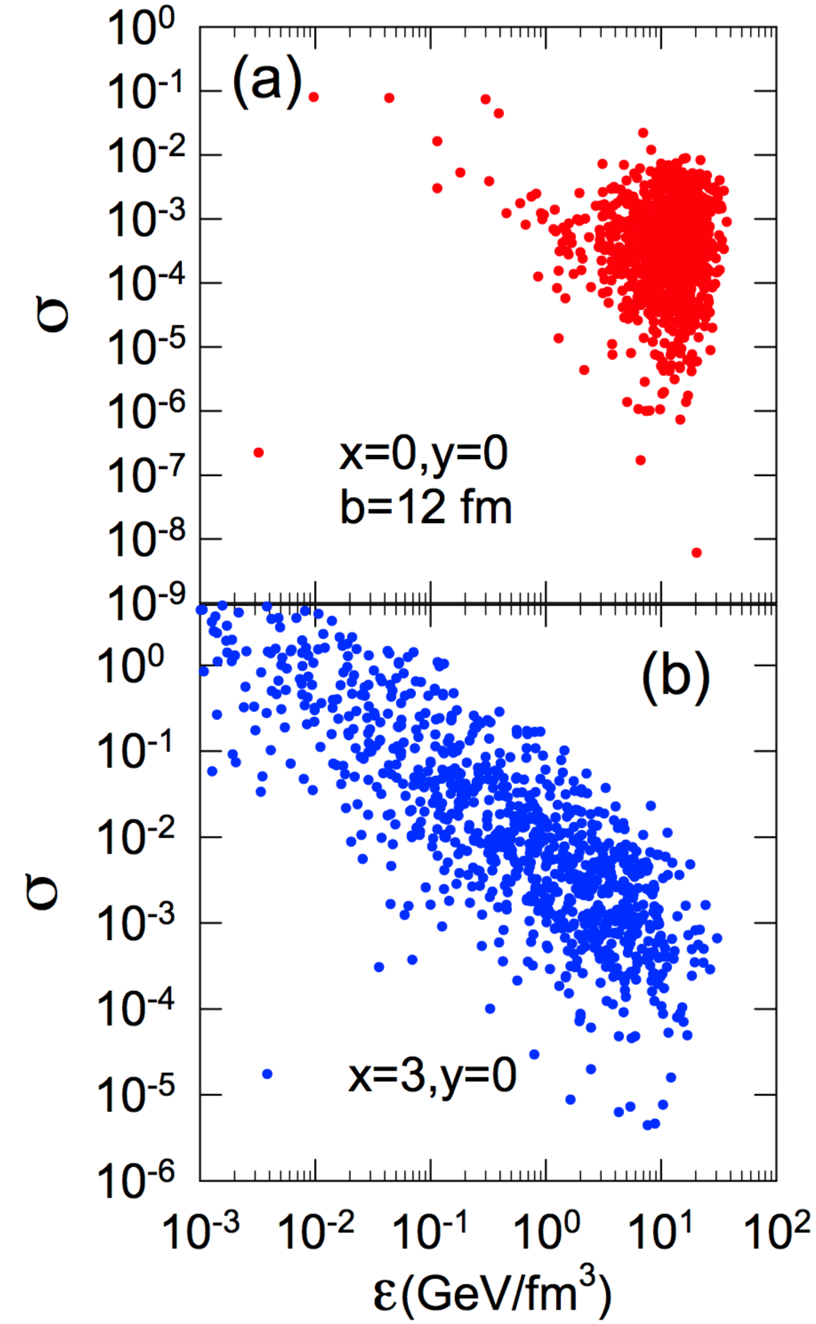


sigma_g=0.25



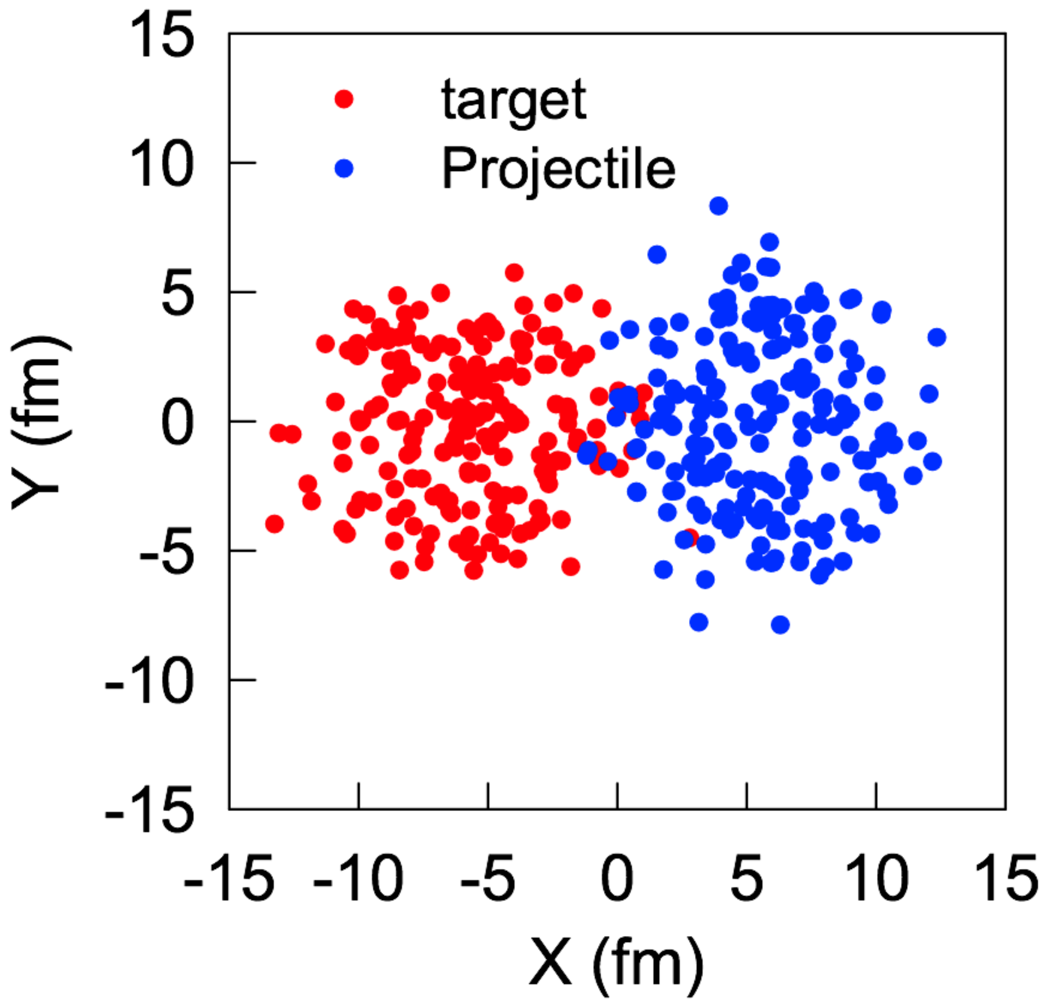
Event-by-event sigma for electric field





E-by-e calculation of fields

Monte-Carlo Glauber
Au+Au



In reality the nucleon's position fluctuates

Total field = $\sum_{proton} \vec{B}_{proton}$

Consequence:

- $\vec{B} \neq 0$ in central collision
- non uniform E/B

Motivation

Linear response theory: $\vec{j} = \sigma \vec{E}$ static medium

Moving conducting fluid : $\vec{j} = \sigma \left(\vec{E} + \vec{v} \times \vec{B} \right)$

- There can be charged current when charged fluid is in motion either in electric or magnetic field

⇩

Non zero Lorentz force $\vec{j} \times \vec{B}$

- QGP evolve under intense EM field
- Electrical conductivity of QGP is non-negligible