Finite Temperature QCD from Lattice

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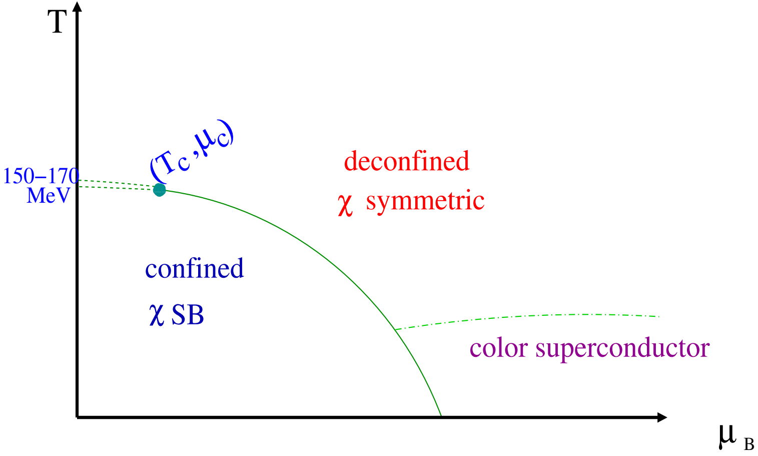
Outline

- QCD Phase diagram: lattice input
- Equation of state
- Nature of deconfined phase
- Transport coefficients

QCD at high temperatures and densities

At high temperatures and densities, QCD should be deconfining: effective degrees of freedom are quarks and gluons.

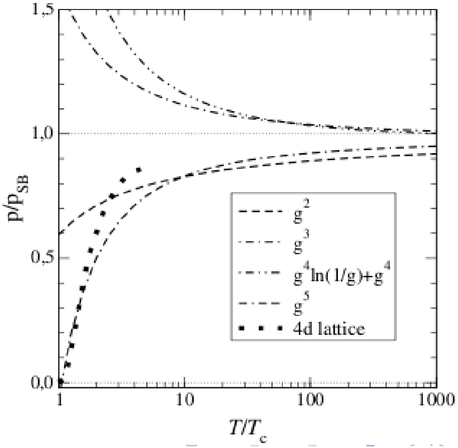


Study from QCD

Partition function $Z = \mathrm{Tr}\, e^{-H/T}$ can be written as

$$Z = \int_{U(0)=U(1/T)} dU \int_{\psi(0)=-\psi(1/T)} d(\bar{\psi},\, \psi)\; e^{-S[\bar{\psi},\psi,U]}$$

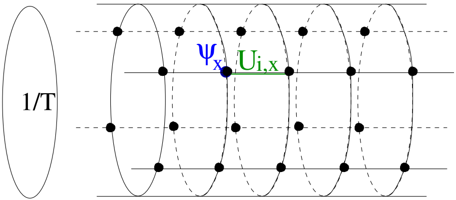
- Perturbation theory:
  Not good at the transition regime.
- Numerical calculation using lattice discretization:
  Allows calculation at large coupling constant.



Lattice discretization

$$Z \;=\; \int_{pbc} dUd(\bar{\psi},\;\psi)\; e^{-S[\bar{\psi},\psi,U]}$$

Discretize space-time, Monte Carlo integration with measure e^{-S} after doing the fermion integral.



- Continuum limit
- Finite volume: thermodynamic limit
- Fermions. Discretization problematic.
  "staggered" ⇛ flavors mixed up.
  "Wilson" ⇛ chiral symmetry broken.
  Other discretizations: issues with locality.

The "order parameters":

Properties of the transition:

- for $m_q \to 0$: chiral symmetry restored:

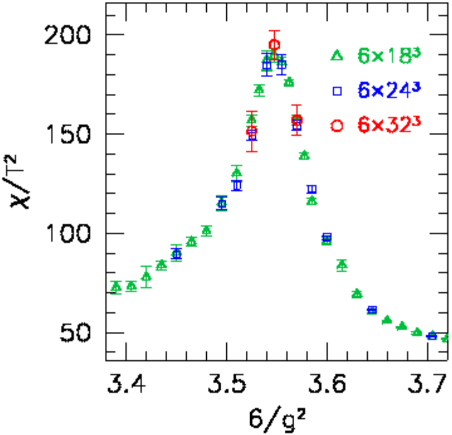
$$SU_V(N_f) \Rightarrow SU_L(N_f) \times SU_R(N_f)$$

  Order parameter: $\langle \bar{\psi}\psi \rangle$

- Deconfinement of quarks (and gluons)
  Polyakov loop $\langle L \rangle \sim exp(-F(\mathrm{heavy}\ Q))$
  $\langle L \rangle \sim 0$ in confined phase
- Release of degrees of freedom: energy density, pressure jump

The equilibrium transition at $\mu = 0$ is a smooth crossover

Y. Aoki et al., Nature 443, 675 (2006)



Crossover Temperature T_χ

Transition temperature T_c in the range 145-175 MeV, depending on observable.

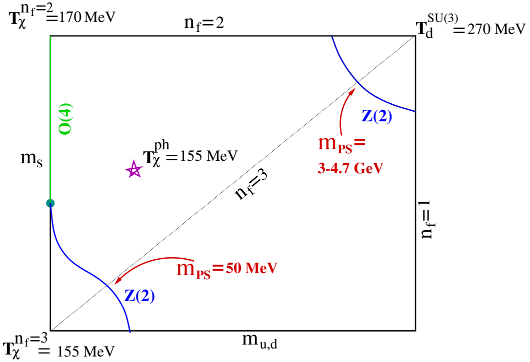
- "stout" fermions, $m_\pi = 135$ MeV, N_τ = 6-16
  Borsanyi et al., JHEP 1009 (2010) 073

$\frac{\chi_{\bar\psi\psi}}{T^4}$	$\langle\bar\psi\psi\rangle$	$\frac{\epsilon}{T^4}$	χ_2^s/T^2
147(2)(3)	155(3)(3)	157(4)(3)	165(5)(3)

- HISQ fermions, $m_\pi \geq 155$ MeV, N_τ = 6-12
  $T(\chi_{\bar\psi\psi}) = 154(9)$ MeV $\quad T(L) \sim 165$MeV
  HotQCD, arXiv:1111.1710
- Comparison of quark number susceptibilities with experiment
  $N_t = 6$ staggered fermions, $m_\pi = 230$ MeV
  $T_c = 175^{+1}_{-7}$ MeV
  S. Gupta, et al., Science 332 (2011) 1525

Similar results with Wilson and chiral fermions.

m_q dependence of transition



F. Cuteri et al., 1710.09304; HotQCD, Lattice 2015; Nakamura et al., PRD 92 (2015) 114511.

For $N_f\!=\!2$: $\langle\bar{\psi}\psi\rangle \sim m^{\frac{1}{\delta}}$ with O(4) δ

WHOTQCD(Umeda et al.), Lattice 2016

Diagram at nonzero μ_B

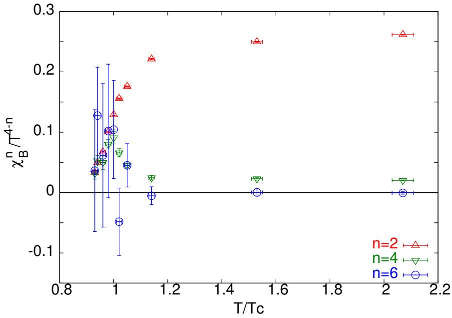
At finite μ_B, $S(\mu_B, T)$ not real, so cannot put e^{-S} in measure.
Direct numerical evaluation not possible.
But observables at small μ_B calculable by expanding in μ

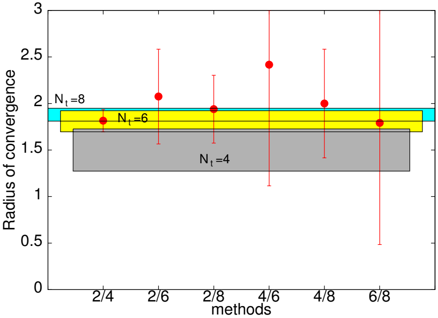
$$\frac{p}{T^4} = \sum_{ijk} \frac{1}{i!\, j!\, k!} \chi^{BSQ}_{ijk} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_q}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

Gavai & Gupta 2003; Allton et al. 2003

The χ^{BSQ}_{ijk} are exactly the fluctuation and correlation observables.

Estimation of μ_B^E



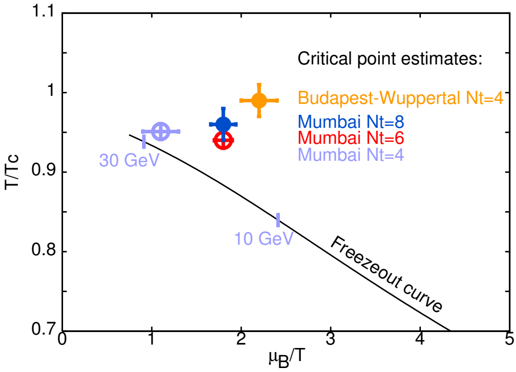


Estimate from 2-flavor, N_t=8 lattices with $m_\pi \sim 230$ MeV:

$$\frac{\mu_B^E}{T_E} = 1.85 \pm 0.04, \qquad \frac{T_E}{T_c} = 0.94 \pm 0.01$$

Datta, Gavai, Gupta, PRD 95(2017) 054512

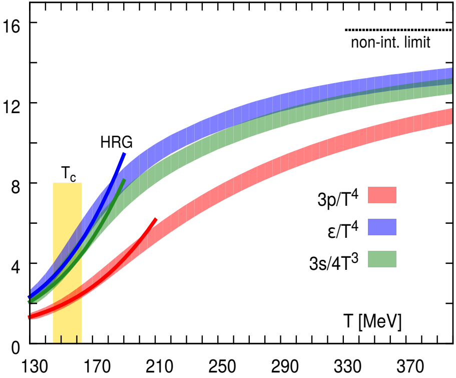
Critical point in (T, μ_B) plane

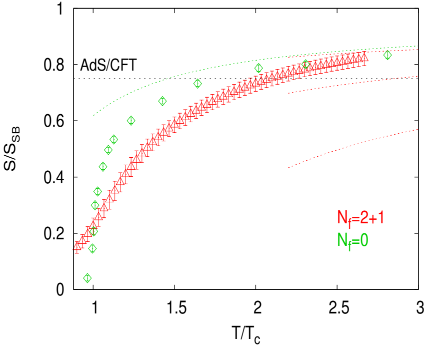


Other recent studies that put bounds:

HotQCD, PRD 95 (2017) 054504; D'Elia et al., 1611.08285

EoS at μ_B=0

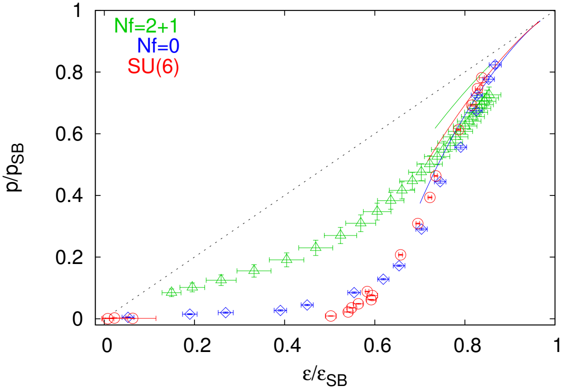




Continuum results with improved staggered. Other fermion formulations agree.

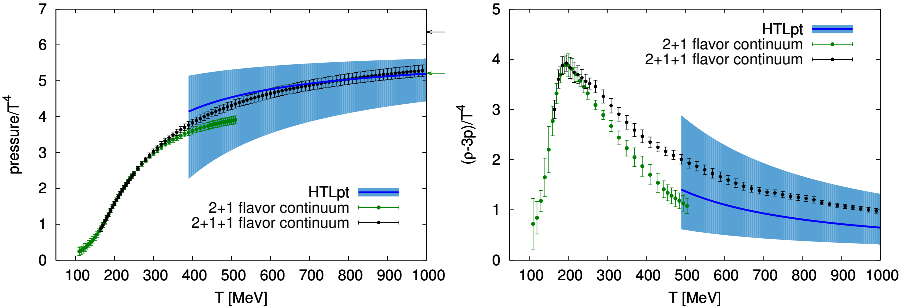
HotQCD (Bazavov ... Hegde et al.) PRD 90(2014)094503. See also Borsanyi et al., PL B 370(2014)99. PT: N.Haque et al, Laine et al

Strongly coupled conformal phase?



Datta and Gupta, PRD82(2010)114505; HotQCD(2014)

EoS at higher T



Charm contribution non-negligible at $T \gtrsim 400$ MeV.
Also bottom starts contributing for $T \gtrsim 1$ GeV.

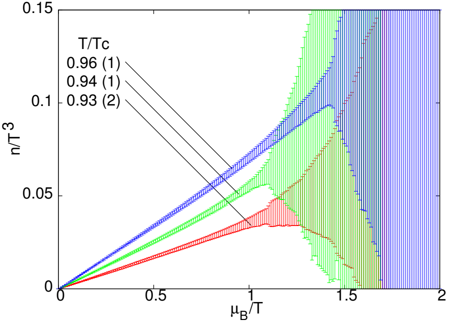
Borsanyi et al., 1606.07494

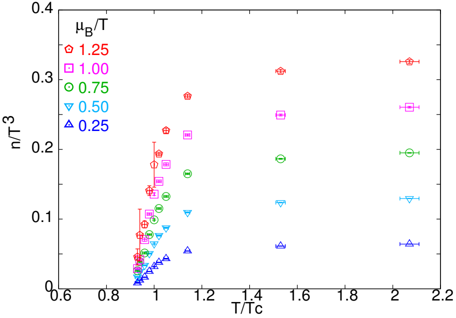
EoS at small μ_B

Can get EoS by summing the series in μ_B. But a finite sum will miss the critical point effect near μ_B^E:

$$\chi_2^B \sim \frac{1}{|{\mu_B}^2 - {\mu_B^E}^2|^{\psi}} \Rightarrow m_1 \;=\; \frac{\partial}{\partial \mu^2} \log \chi_2^B \;\sim\; \frac{\psi}{{\mu_B}^2 - {\mu_B^E}^2}$$

$\Rightarrow$ resum the series of m_1





Datta, Gavai, Gupta (PRD,2017)

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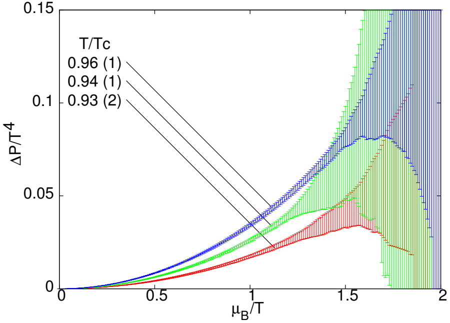
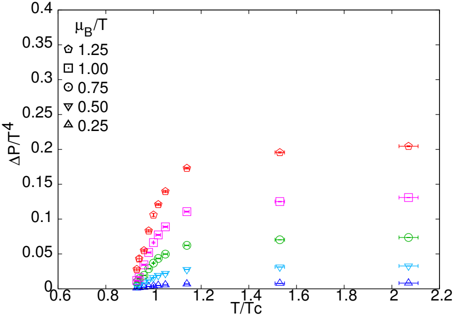




Datta, Gavai, Gupta (PRD,2017)

Screening of charges

- Probe: screening of a charge
- e.g., Debye screening of static charge in QED:

T=0 — V(r) ~ 1/r

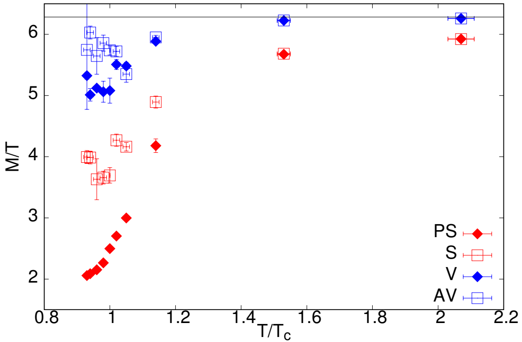
T > 0 — V(r) ~ 1/r $e^{-m_D r}$

$$\lim_{x\to\infty}\langle E_i(\vec{x})E_j(\vec{0})\rangle \sim e^{-m_D x}$$

$$m_D = \frac{eT}{\sqrt{3}} + O(e^2)$$

- Screening of gluonic sources: Debye screening, "glueball" screening lengths:
  effective symmetry of finite temperature system
- Screening of mesonic sources.

Screening of mesonic charges



$$\bar{\psi}\,\gamma_5\,\tau^a\,\psi \quad \overset{SU(2)_A}{\Longleftrightarrow} \quad \bar{\psi}\,\psi$$

$$\bar{\psi}\,\gamma_5\,\tau^a\,\psi \quad \overset{U(1)_A}{\Longleftrightarrow} \quad \bar{\psi}\,\tau^a\,\psi$$

$$\bar{\psi}\,\gamma_i\,\tau^a\,\psi \quad \overset{SU(2)_A}{\Longleftrightarrow} \quad \bar{\psi}\,\gamma_i\,\gamma_5\,\psi$$

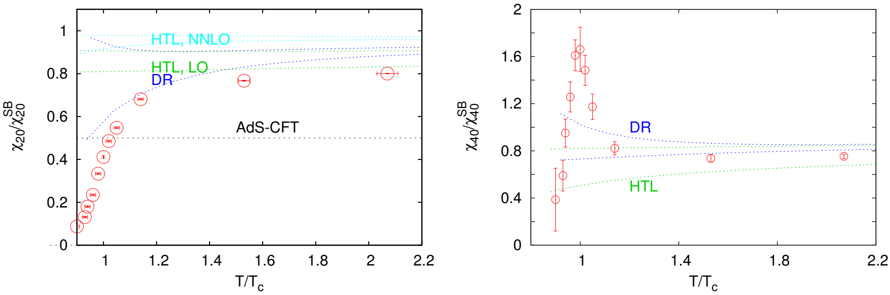
Datta, Gupta, Karthik, in prep.

While $SU(2)_L \times SU(2)_R$ restored at $T_c \sim 150$ MeV, $U(1)_A$ gets restored only at temperatures $\gtrsim 1.2\,T_c$

Similar conclusion from earlier studies, and also from susceptibilities $\chi_\pi,\ \chi_\delta,\ \chi_\sigma$.

HotQCD, PRD 89 (2014) 054514

Nonlinear Susceptibilities



Datta, Gupta, Gavai; Vuorinen et al.; Anderson, Haque, Mustafa, Strickland, Su

Lattice results: $N_t = 8$ staggered

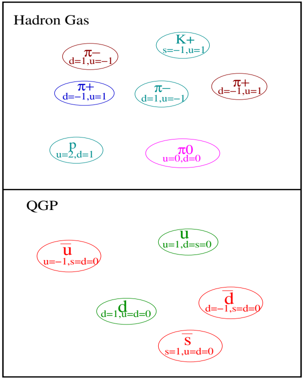
HotQCD: detailed study of susceptibilities in 2+1 flavor, and correlations between conserved quantum numbers.

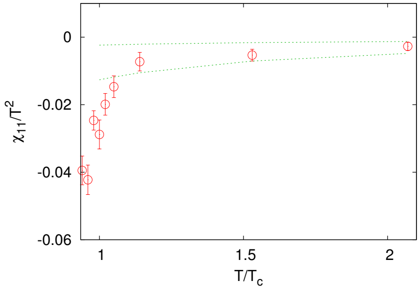
Bazavov ... Hegde, Sharma, et al., PRD 95 (2017) 054504.

Off-diagonal susceptibilities

Quantities like χ_{ud} probe correlations between quantum numbers: very different between bound state phase and free quarks.

Koch, Majumdar & Randrup, PRL 95(05)





Dynaical properties: Transport coefficients

Linear response and fluctuation-dissipation theorem allows estimation of transport coefficients from retarded correlators: Under a small perturbation $\int d^3x J_{\rm ext}(x,t)\Phi(x,t)$ added at $t=0$

$$\delta\langle\Phi(x,t)\rangle = \int dt' d^3x' G^R(x,t;x',t') J_{\rm ext}(x',t')$$

$$\begin{aligned} G^R(\Phi(x)\Phi^+(0)) &= \theta(x^0)\langle[\Phi(x),\Phi^+(0)]\rangle, \\ G^R(q_0,\vec{q}) &= G^E(q_E \to -i(q_0+i\epsilon),\vec{q}) \end{aligned}$$

Spectral function

Spectral function $\rho(q) = \mathcal{F}.\mathcal{T}.\langle \frac{1}{2}[\phi(x), \phi^{+}(0)] \rangle$

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh(\omega(\beta/2 - t))}{\sinh(\omega\beta/2)}$$

Transport coefficients associated with low-ω behavior of ρ: e.g.,

$$\eta \;=\; \lim_{\omega \to 0} \; \frac{\rho^{T_{12},T_{12}}(\omega)}{\omega}$$

Various Bayesian techniques have been employed to extract some information for transport coefficients.

- The first attempt to calculate transport coefficient on the lattice was for shear viscosity η.
  Karsch & Wyld, PR D 35 ('87) 2518
- Hydrodynamic modelling of RHIC data require a very small shear viscosity-to-entropy ratio, $\frac{\eta}{s} \sim \frac{1-3}{4\pi}$.
  Luzum & Romatschke, PR C 78 ('08) 034915
- LOPT: $\eta \sim \frac{T^3}{\alpha_s^2 \ln \alpha_s}$ large in weak coupling limit.
  $\eta/s \sim 1$ for $\alpha_s \sim 0.25$.
  Arnold, Moore, Yaffe, JHEP 05 ('03) 051
- $\eta/s = \frac{1}{4\pi}$ for N=4 SYM theory in the strong coupling limit.
  Kovtun, Son, Starinets, PRL 94 ('05) 111601
- Lattice results for N_f=0:
  Meyer, PRD 76('07) 101701

$$\frac{\eta}{s} = 0.134(33) \qquad \text{at } 1.65\,T_c$$
$$= 0.102(56) \qquad \text{at } 1.24\,T_c$$

Electric conductivity

- $$\sigma(T) = \frac{\sum_f q_f^2}{6} \lim_{\omega \to 0} \frac{\rho_{J_i\ J_i}(\omega, T)}{\omega}, \qquad j_i(x) = \sum_f q_f \bar{f} \gamma_i f$$

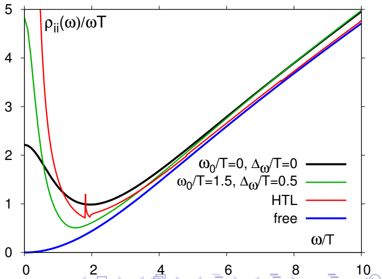
S. Gupta, PL B 597 ('04) 57. Aarts et al, PRL 99 (2007) 022002

- Recent study: with Wilson quarks and a model for $\rho_{J_i\ J_i}(\omega, T)$

$\frac{\sigma}{C_{em} T} = \frac{1}{3} - 1$ at 1.5 T_c

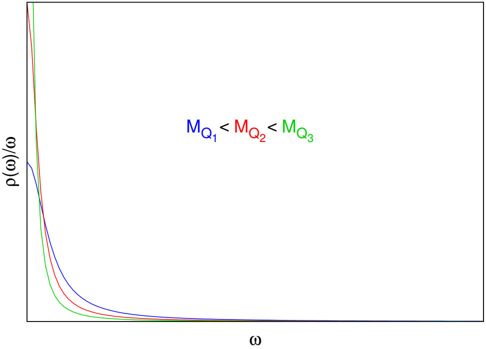
Ding et al., PR D 83 ('11) 034504

Consistent with Aarts et al.:
$\frac{\sigma}{C_{em} T} \sim 0.4 \pm 0.1$
at 1.5 T_c and 2.25 T_c



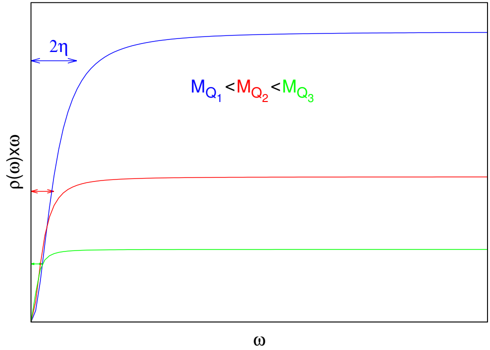
$\bar{Q}\gamma_i Q$ current and heavy quark diffusion

$$\rho_V(\omega) \underset{low\ \omega}{\sim} \chi_{00} D \frac{\omega\eta^2}{\eta^2+\omega^2}, \qquad \eta \sim \frac{1}{M}$$



Extracting the diffusion part

$\frac{1}{\chi_{00}} \quad \frac{\rho(\omega)}{\omega} \qquad x\omega^2$



$\Rightarrow$ spectral function for $M\frac{dJ_i}{dt}$

In NRQCD, $M\dot{\vec{J}}_i \;=\; \phi^\dagger g\vec{E}\phi - \theta^\dagger g\vec{E}\theta + \mathcal{O}(\frac{1}{M})$

Force-force correlator and Langevin description of heavy quark in plasma

In the static limit,

$$G_E(\tau) = \frac{\mathrm{Re\ Tr}\ U(\beta,\tau_1)\vec{E}(\tau_1)U(\tau_1,\tau_2)\vec{E}(\tau_2)U(\tau_2,0)}{\mathrm{Re\ Tr}\ U(\beta,0)}$$

Caron-Huot, Laine, Moore, JHEP(2009)

For thermal heavy quark, $M \gg T, \quad p \sim \sqrt{MT}$
Langevin description:

$$\frac{dp_i}{dt} = \xi_i(t) - \eta p_i, \qquad \langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')$$

$$\langle p^2\rangle = 3MT \rightarrow \eta = \frac{\kappa}{2MT} \qquad \langle x_i(t)x_j(t)\rangle = 2Dt\delta_{ij} \rightarrow D = \frac{2T^2}{\kappa}$$

Svetitsky '88; Moore & Teaney '05; Rapp & van Hees '05; Mustafa '05

Analysis

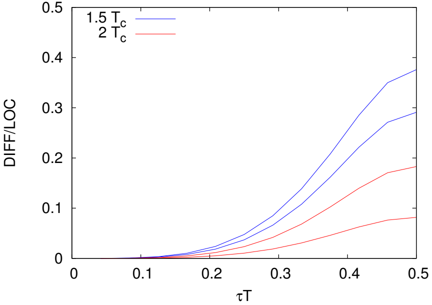
- $$\kappa = \lim_{\omega\to 0} \frac{2T}{\omega}\ \rho(\omega)$$
- correlator fitted to form $\rho_0 = (c_1\omega, c_2\omega^3)$ with IR and UV parts matched smoothly
  also test a correction,
  $\rho_0 \left(1 + c' \sin \pi \frac{x}{1+x}\right)$, $\qquad x = \log(1 + \frac{\omega}{\pi T})$.
- Systematics of IR form estimated by taking instead the form $2\kappa\ \tanh \frac{w}{2T}$
- Early results: $\frac{\kappa}{T^3} \sim 3$, rather flat, for $T \lesssim 1.5\,T_c$

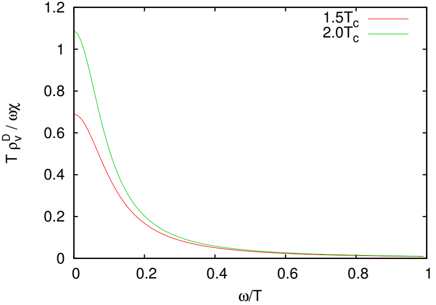
  Banerjee, Datta, Gavai, Majumdar (2012)

  In agreement with

  Francis, Kaczmarek, Laine, Neuhaus & Ohno (2015)
- Ongoing study: finite volume and a effect, and approach to higher temperature.

Diffusion contribution





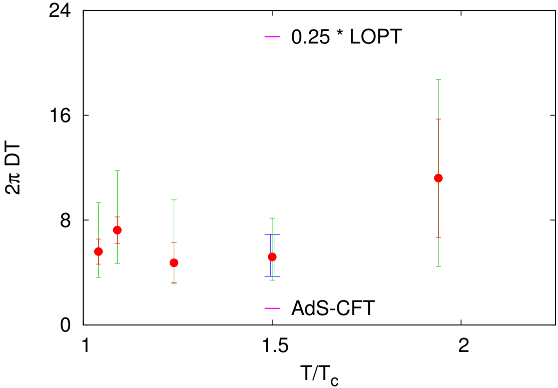
System NR: can use Einstein relations

$$D = \frac{2T^2}{\kappa} \qquad \eta = \frac{\kappa}{2MT}$$

to get the diffusive peak in vector current correlator:

$$\rho_{VV} = \chi_{00} \, D \frac{\omega \eta^2}{\omega^2 + \eta^2}$$

Diffusion coefficient for QGP



Banerjee, Datta, Gavai, Majumdar; Francis et al.

D much smaller than leading order perturbation theory

Closer to the strong coupling limit for $\mathcal{N}$=4 SYM

Near-flat in the temperature range 1.06-1.5 T_c

In the right ballpark for RHIC experimental results

Summary

- **QCD phase diagram**
  - Crossover at $\mu_B = 0, \;\; T \sim 145 - 175$ MeV
  - Evidence for a critical point at

$$\frac{\mu_B^E}{T_E} \;=\; 1.85 \pm 0.04, \qquad \frac{T_E}{T_c} \;=\; 0.94 \pm 0.01$$

- **Equation of state**
  - Rapid growth of energy density, entropy at T_c, reaching $\sim 80\%$ of free value by $2T_c$.
  - Large conformality breaking. No evidence for nontrivial strong coupling dynamics.
  - Contribution of c important for $T \gtrsim 400$ MeV.
  - To get EoS at finite μ_B using an expansion in μ_B, a resummation is necessary.

Summary

- **Nature of QGP**
  - $SU(2)_L \times SU(2)_R$ symmetry restored at T_c, but $U(1)_A$ breaking effects seen till 1.2 T_c
  - Diagonal and off-diagonal susceptibilities qualitatively consistent with weakly interacting deconfined medium by $1.5\,T_c$
- **Transport coefficients**
  - Calculation of transport coefficients at a primitive stage.
  - Estimates of shear viscosity, electric conductivity, heavy quark diffusion coefficient.
  - In general, transport coefficients at $T \lesssim 2\,T_c$ qualitatively different from weak coupling estimates.