Probing a Four Flavour vis-a-vis Three Flavour Neutrino mixing for Ultrahigh energy Neutrino Signals at a $1\mathrm{Km}^2$ Detector

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1 Introduction

- The existence of a fourth sterile neutrino has been hypothesised since a long time.
- Earlier neutrino oscillation data from experiments like LSND, miniBoone etc. as well as more recent data from several current experiments like MINOS, MINOS+, Daya Bay, Bugey, NOνA etc. could not be satisfactorily explained by 3 neutrino oscillation framework and their analyses give new bounds on **active-sterile mixing angles** (θ_{ij}) and Δm^2. Future long baseline experiments such as DUNE would be useful in case a third independent Δm^2 (Δm^2_{41}) exists, which signifies the existence of fourth mass (and flavour) eigenstate that mixes with the active species but arguably is sterile in nature, in order not to jeopardise the cosmological and particle physics bounds on the sum of neutrino masses.
- In this work we consider a 4-flavour neutrino (3 active + 1 sterile) scheme as well as the 3 active neutrino oscillation scenario. Our aim is to explore the possibility of an experimental signature that would or would not indicate the existence of a sterile neutrino.
- Keeping this in view we consider ultrahigh energy (UHE) neutrinos from distant high energy astrophysical sources such as Gamma Ray Bursts (GRBs) and we estimate the possible detection yield at a kilometer square detector like IceCube for a 4-flavour neutrino oscillation scheme as well as for 3 active neutrino oscillation framework.

2 Formalism

2.1 Four and Three Flavour Neutrino Oscillations

In general the probability for a neutrino $|\nu_\alpha\rangle$ of flavour α to oscillate to a neutrino $|\nu_\beta\rangle$ of flavour β is given by [1] (considering no CP violation in neutrino sector)

$$P_{\nu_\alpha\to\nu_\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}\sin^2\left(\frac{\pi L}{\lambda_{ij}}\right) \ . \quad (1)$$

For UHE neutrinos from distant GRBs or AGNs, the oscillatory part in the probability equation is averaged to half (L is large, $\Delta m^2 L/E \gg 1$) . The probability equation (Eq. (1)) is then reduced to

$$P_{\nu_\alpha\to\nu_\beta} = \sum_j |\ U_{\alpha j}\ |^2 |\ U_{\beta j}\ |^2 \ . \quad (2)$$

Considering the present 4-flavour scenario to be the minimal extension of 3-flavour case by a sterile neutrino, the mixing matrix $\tilde{U}$ can be written as

$$\tilde{U}_{(4\times4)} = \begin{pmatrix} c_{14}\mathcal{U}_{e1} & c_{14}\mathcal{U}_{e2} & c_{14}\mathcal{U}_{e3} & s_{14} \\ -s_{14}s_{24}\mathcal{U}_{e1}+c_{24}\mathcal{U}_{\mu1} & -s_{14}s_{24}\mathcal{U}_{e2}+c_{24}\mathcal{U}_{\mu2} & -s_{14}s_{24}\mathcal{U}_{e3}+c_{24}\mathcal{U}_{\mu3} & c_{14}s_{24} \\ \begin{matrix}-c_{24}s_{14}s_{34}\mathcal{U}_{e1}\\ -s24s34\mathcal{U}_{\mu1}\\ +c_{34}\mathcal{U}_{\tau1}\end{matrix} & \begin{matrix}-c_{24}s_{14}s_{34}\mathcal{U}_{e2}\\ -s24s34\mathcal{U}_{\mu2}\\ +c_{34}\mathcal{U}_{\tau2}\end{matrix} & \begin{matrix}-c_{24}s_{14}s_{34}\mathcal{U}_{e3}\\ -s24s34\mathcal{U}_{\mu3}\\ +c_{34}\mathcal{U}_{\tau3}\end{matrix} & c_{14}c_{24}s_{34} \\ \begin{matrix}-c_{24}c_{34}s_{14}\mathcal{U}_{e1}\\ -s_{24}c_{34}\mathcal{U}_{\mu1}\\ -s_{34}\mathcal{U}_{\tau1}\end{matrix} & \begin{matrix}-c_{24}c_{34}s_{14}\mathcal{U}_{e2}\\ -s_{24}c_{34}\mathcal{U}_{\mu2}\\ -s_{34}\mathcal{U}_{\tau2}\end{matrix} & \begin{matrix}-c_{24}c_{34}s_{14}\mathcal{U}_{e3}\\ -s_{24}c_{34}\mathcal{U}_{\mu3}\\ -s_{34}\mathcal{U}_{\tau3}\end{matrix} & c_{!4}c_{24}c_{34} \end{pmatrix} , \quad (3)$$

where $\mathcal{U}_{\alpha i}$ are the matrix elements of 3-flavour neutrino mixing matrix

$$\mathcal{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13} & c_{12}c_{23}-s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13} & -c_{12}s_{23}-s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} . \quad (4)$$

2.2 UHE Neutrino Fluxes from GRBs

From the GRBs, the neutrino (antineutrino) flavours are expected to be produced in the ratio $\nu_e : \nu_\mu : \nu_\tau = 1:2:0$ (3 flavour) and $\nu_e : \nu_\mu : \nu_\tau : \nu_s = 1:2:0:0$ (4 flavour).

The standard ratio of intrinsic neutrino flux is

$$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} : \phi_{\nu_s} = 1:2:0:0 \ . \quad (5)$$

Here we adopt GRB neutrino fluxes estimated from two different analyses and perform our calculations for each of them. These are

1. The diffused GRB flux estimated by Waxman and Bahcall [2,3] (Waxman - Bahcall flux).
2. The best fit power law flux from the analysis of high energy starting events (HESE) data at IceCube [4].

The Waxman - Bahcall flux for ν_μ and $\bar{\nu}_\mu$ estimated by summing over all the sources is given as (Gandhi *et al.*) [5]

$$\mathcal{F}(E_\nu) = \frac{dN_{\nu_\mu+\bar{\nu}_\mu}}{dE_\nu} = \mathcal{N}\left(\frac{E_\nu}{1\mathrm{GeV}}\right)^{-n} \mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}\mathrm{GeV}^{-1} \ . \quad (6)$$

In the above,

$$\mathcal{N} = 4.0\times10^{-13} \quad n=1 \text{ for } E_\nu < 10^5 \text{ GeV} ,$$
$$\mathcal{N} = 4.0\times10^{-8} \quad n=2 \text{ for } E_\nu > 10^5 \text{ GeV} .$$

Therefore the fluxes of the corresponding flavours (same for both neutrinos and antineutrinos since no CP violation is considered in the neutrino sector) can be expressed as

$$\frac{dN_{\nu_\mu}}{dE_\nu} = \phi_{\nu_\mu} = \frac{dN_{\bar{\nu}_\mu}}{dE_\nu} = \phi_{\bar{\nu}_\mu} = 0.5\mathcal{F}(E_\nu) ,$$
$$\frac{dN_{\nu_e}}{dE_\nu} = \phi_{\nu_e} = \frac{dN_{\bar{\nu}_e}}{dE_\nu} = \phi_{\bar{\nu}_e} = 0.25\mathcal{F}(E_\nu) \ . \quad (7)$$

The fluxes of neutrino flavours for four and three flavour cases, on reaching the earth will respectively be

$$F^4_{\nu_e} = P^4_{\nu_e\to\nu_e}\phi_{\nu_e} + P^4_{\nu_\mu\to\nu_e}\phi_{\nu_\mu} , F^4_{\nu_\mu} = P^4_{\nu_\mu\to\nu_\mu}\phi_{\nu_\mu} + P^4_{\nu_e\to\nu_\mu}\phi_{\nu_e} , F^4_{\nu_\tau} = P^4_{\nu_e\to\nu_\tau}\phi_{\nu_e} + P^4_{\nu_\mu\to\nu_\tau}\phi_{\nu_\mu} ,$$
$$F^4_{\nu_s} = P^4_{\nu_e\to\nu_s}\phi_{\nu_e} + P^4_{\nu_\mu\to\nu_s}\phi_{\nu_\mu} \quad (8)$$

and

$$F^3_{\nu_e} = P^3_{\nu_e\to\nu_e}\phi_{\nu_e} + P^3_{\nu_\mu\to\nu_e}\phi_{\nu_\mu} , F^3_{\nu_\mu} = P^3_{\nu_\mu\to\nu_\mu}\phi_{\nu_\mu} + P^3_{\nu_e\to\nu_\mu}\phi_{\nu_e} , F^3_{\nu_\tau} = P^3_{\nu_e\to\nu_\tau}\phi_{\nu_e} + P^3_{\nu_\mu\to\nu_\tau}\phi_{\nu_\mu} \ . \quad (9)$$

Now with Eqs. (2, 5), Eq. (8) can be rewritten as

$$\begin{pmatrix} F^4_{\nu_e} \\ F^4_{\nu_\mu} \\ F^4_{\nu_\tau} \\ F^4_{\nu_s} \end{pmatrix} = \begin{pmatrix} |\tilde{U}_{e1}|^2 & |\tilde{U}_{e2}|^2 & |\tilde{U}_{e3}|^2 & |\tilde{U}_{e4}|^2 \\ |\tilde{U}_{\mu1}|^2 & |\tilde{U}_{\mu2}|^2 & |\tilde{U}_{\mu3}|^2 & |\tilde{U}_{\mu4}|^2 \\ |\tilde{U}_{\tau1}|^2 & |\tilde{U}_{\tau2}|^2 & |\tilde{U}_{\tau3}|^2 & |\tilde{U}_{\tau4}|^2 \\ |\tilde{U}_{s1}|^2 & |\tilde{U}_{s2}|^2 & |\tilde{U}_{s3}|^2 & |\tilde{U}_{s4}|^2 \end{pmatrix} \begin{pmatrix} |\tilde{U}_{e1}|^2 & |\tilde{U}_{\mu1}|^2 & |\tilde{U}_{\tau1}|^2 & |\tilde{U}_{s1}|^2 \\ |\tilde{U}_{e2}|^2 & |\tilde{U}_{\mu2}|^2 & |\tilde{U}_{\tau2}|^2 & |\tilde{U}_{s2}|^2 \\ |\tilde{U}_{e3}|^2 & |\tilde{U}_{\mu3}|^2 & |\tilde{U}_{\tau3}|^2 & |\tilde{U}_{s3}|^2 \\ |\tilde{U}_{e4}|^2 & |\tilde{U}_{\mu4}|^2 & |\tilde{U}_{\tau4}|^2 & |\tilde{U}_{s4}|^2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix} \phi_{\nu_e} \ . \quad (10)$$

Similarly for 3-flavour scenario, we can write Eq. (9) as

$$\begin{pmatrix} F^3_{\nu_e} \\ F^3_{\nu_\mu} \\ F^3_{\nu_\tau} \end{pmatrix} = \begin{pmatrix} |\mathcal{U}_{e1}|^2 & |\mathcal{U}_{e2}|^2 & |\mathcal{U}_{e3}|^2 \\ |\mathcal{U}_{\mu1}|^2 & |\mathcal{U}_{\mu2}|^2 & |\mathcal{U}_{\mu3}|^2 \\ |\mathcal{U}_{\tau1}|^2 & |\mathcal{U}_{\tau2}|^2 & |\mathcal{U}_{\tau3}|^2 \end{pmatrix} \begin{pmatrix} |\mathcal{U}_{e1}|^2 & |\mathcal{U}_{\mu1}|^2 & |\mathcal{U}_{\tau1}|^2 \\ |\mathcal{U}_{e2}|^2 & |\mathcal{U}_{\mu2}|^2 & |\mathcal{U}_{\tau2}|^2 \\ |\mathcal{U}_{e3}|^2 & |\mathcal{U}_{\mu3}|^2 & |\mathcal{U}_{\tau3}|^2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \phi_{\nu_e} \ . \quad (11)$$

3 Calculations and Results

3.1 Diffused Neutrino Flux

For the purpose of our analysis, we have considered a ratio R between the muon and the shower events, which is defined as

$$R = \frac{T_\mu}{T_{\mathrm{sh}}} , \quad (12)$$

where

$$T_\mu = S(\text{for } \nu_\mu) + S(\text{for } \nu_\tau)$$
$$T_{\mathrm{sh}} = S_{\mathrm{sh}}(\text{for } \nu_e \text{ CC interaction}) + S_{\mathrm{sh}}(\text{for } \nu_e \text{ NC interaction}) + S_{\mathrm{sh}}(\text{for } \nu_\mu \text{ NC interaction}) + S_{\mathrm{sh}}(\text{for } \nu_\tau \text{ NC interaction}) \quad (13)$$

The event rate of muons (S) and the same for the shower (S_{sh}) are expressed as

$$S = \int_{E_{\mathrm{thr}}}^{E_{\nu\max}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\mathrm{shadow}}(E_\nu) P_\mu(E_\nu, E_\mu^{\min}) . \quad (14)$$

$$S_{\mathrm{sh}} = V\int_{E_{\mathrm{thr}}}^{E_{\nu\max}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\mathrm{shadow}}(E_\nu) \int dy \frac{1}{\sigma^i}\frac{d\sigma^i}{dy} P_{\mathrm{int}}(E_\nu, y) \ . \quad (15)$$

For the isotropic fluxes the shadow factor is given by

$$P_{\mathrm{shadow}}(E_\nu) = \frac{1}{2\pi}\int_{-1}^{0} d\cos\theta_z \int d\phi \ exp[-z(\theta_z)/L_{\mathrm{int}}(E_\nu)] , \quad (16)$$

θ_{14}	θ_{24}	θ_{34}	R_4 (in 4fl)	R_3 (in 3fl)
3°	5°	20°	9.48	1.80
4°	6°	15°	9.68	1.80

Table 1: Comparison of the muon to shower ratio for a diffused GRB neutrino flux for the 4 flavour (3+1) case compared with the same for 3-flavour case for two sets of active sterile neutrino mixing angles.

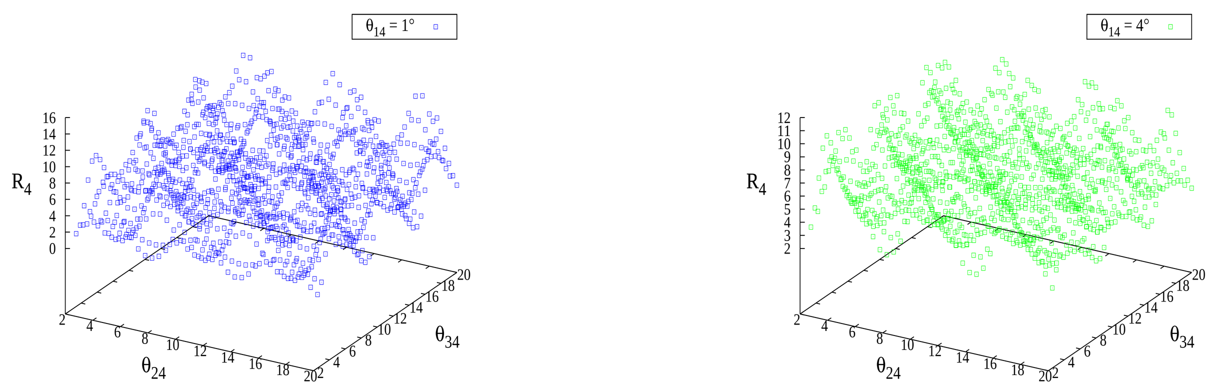

Figure 1: . Variation of R_4 with θ_{24} and θ_{34} for (a) $\theta_{14}=1°$ and (b) $\theta_{14}=4°$.

θ_{14}	θ_{24}	θ_{34}	R_4 (in 4fl)	R_3 (in 3fl)
3°	5°	20°	2.01	0.55
4°	6°	15°	2.01	0.55

Table 2: Same as Table 1, whereas we consider diffused flux of UHE neutrinos obtained from the recent analysis of the IceCube (HESE) data. HESE data furnished a best fit power law for neutrino flux as $E^2\phi(E) = 2.46\pm0.8\times10^{-8}\left(\frac{E}{100\mathrm{TeV}}\right)^{-0.92}$ GeV cm^{-2} s^{-1} sr^{-1}, where for one component fit the neutrino flux $\phi(E)\sim E^{-\gamma}$, with the index $\gamma=2.92$ ($\gamma = 2.92^{+0.33}_{-0.29}$). The energy range above 60 TeV is to be considered for such calculations.

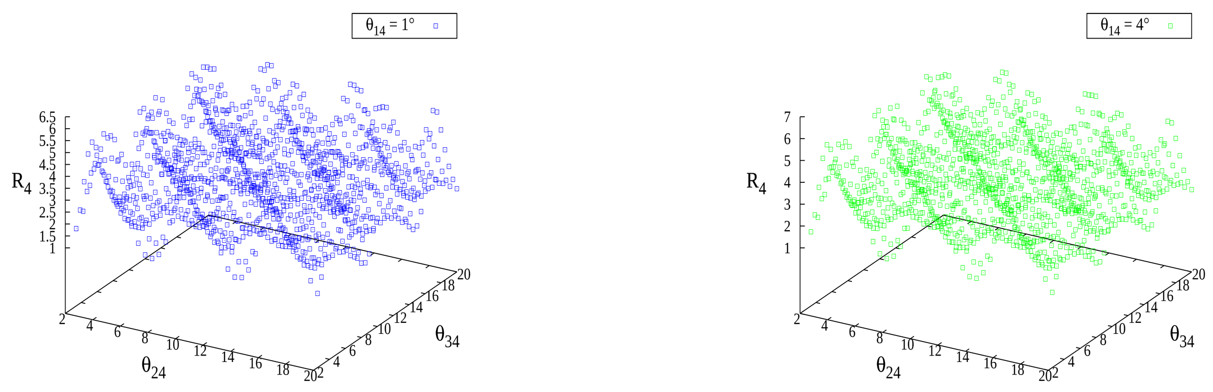

Figure 2: . Variation of R_4 with θ_{24} and θ_{34} for (a) $\theta_{14}=1°$ and (b) $\theta_{14}=4°$.

3.2 Single GRB

For a particular GRB the zenith angle θ_z is fixed and thus P_{shadow} is now modified as

$$P_{\mathrm{shadow}} = exp[-z(\theta_z)/l_{int}(E_\nu)] \ . \quad (17)$$

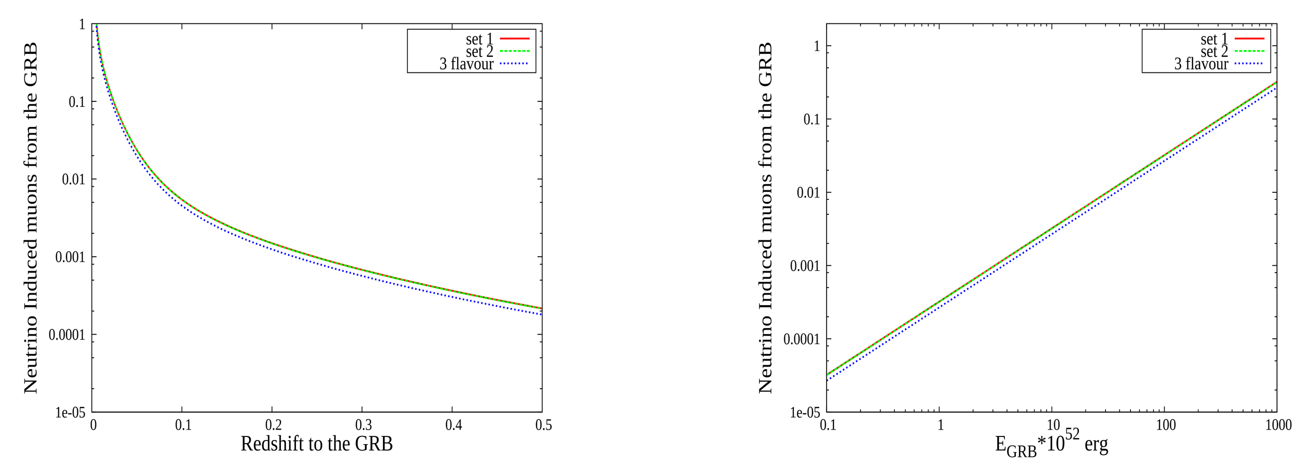

Figure 3: . Variations of the neutrino induced muons from the GRB with (a) different redshifts, (b) different GRB energies at a fixed zenith angle ($\theta_z = 10°$) (set 1: $\theta_{14}=3°, \theta_{24}=5°, \theta_{34}=20°$; set 2: $\theta_{14}=4°, \theta_{24}=6°, \theta_{34}=15°$).

4 Summary

The maximum value of the muon to shower ratio can be six to eight times larger for 3+1 scenario (R_4) when compared to that for normal three active neutrino (R_3) formalism if the Waxman-Bahcall flux is considered and R_4 can be three to four times of R_3 for the flux given by IceCube data (HESE data) analysis.

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