An introduction to Regge Field Theory

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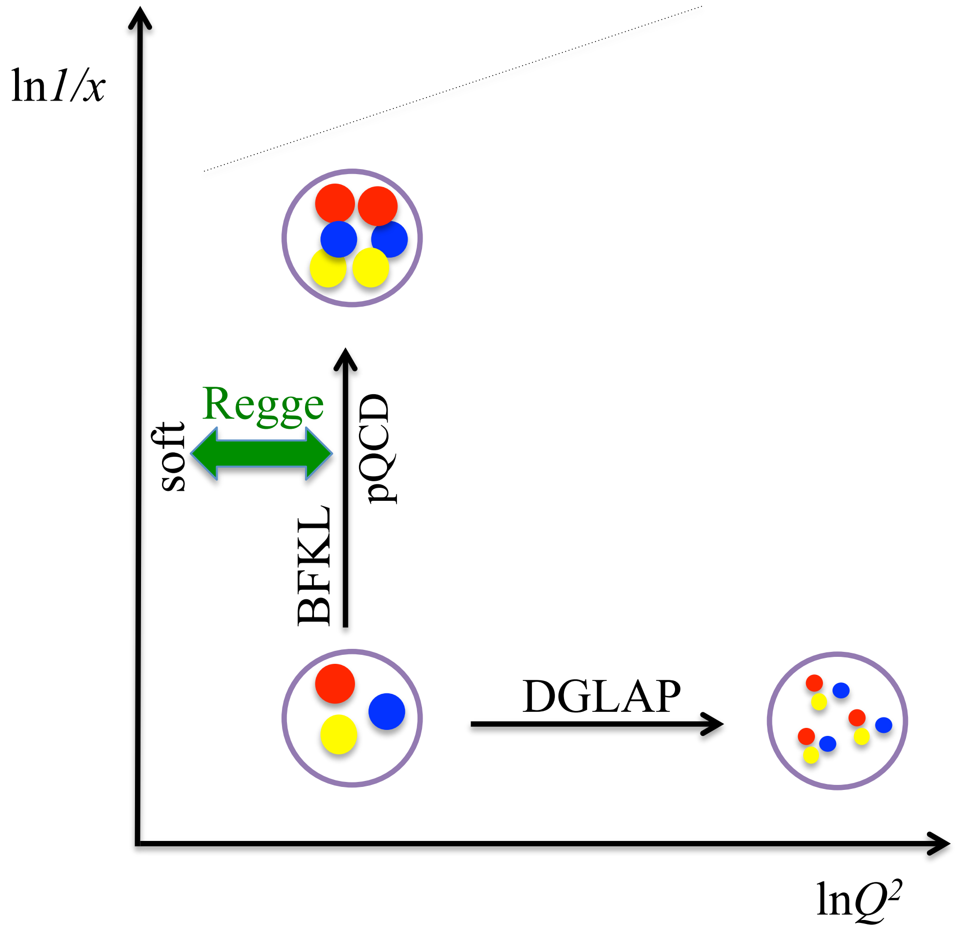
Map of High Energy Physics



The Scattering Matrix

The transition of a closed system of particles from an initial state $|k>$ to a final state $|f>$ is described in quantum theory by the S matrix:

$$|f> = S|k>$$

The matrix elements of the S matrix:

$$S_{fk} = <f|S|k>$$

$[(|f>)^{+} = <f|$ - Hermitian conjugate$]$

Can be represented in the form

$$S_{fk} = \delta_{fk} + i(2\pi)^4 \delta^{(4)}(P_i - P_k) T_{fk}$$

$\delta_{fk} = 1$ if the state does not change ($|f> = |k>$). No interaction

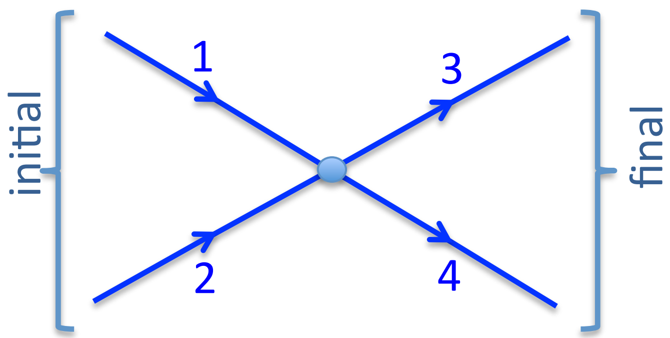
conservation of energy and momentum

T_{fk} is called the transition (scattering) amplitude from the state $|k>$ to the state $|f>$

T_{fk} (S_{fk}) is a function of 4-momentum and polarization of particles (and contains γ-matrixes in case of fermions).

The Scattering Amplitude

For spinless particles, T_{fk} is a function of the relativistically invariant variables formed from the 4-momentum of the particles.



$T_{fk}(P_1, P_2, P_3, P_4)$ $\quad P_i = \{E_i, \mathbf{p}_i\}$

4×4 = 16 variables

Not all 16 variables are independent

Energy-momentum conservation: $P_4 = P_1 + P_2 - P_3$

6 invariants can be formed with P_1, P_2 and P_3: $P_1^2, P_2^2, P_3^2, (P_1P_2), (P_1P_3), (P_2P_3)$

$P_i^2 = m_i^2 \ (i = 1, 2, 3, 4)$

$P_4^2 = (P_1 + P_2 - P_3)^2 = P_1^2 + P_2^2 + P_3^2 + 2(P_1P_2) - 2(P_1P_3) - 2(P_2P_3) = m_4^2$

T_{fk} is a function of 2 variables for binary reactions with spinless particles

Mandelstam variables

$s = (P_1 + P_2)^2 = (P_3 + P_4)^2$

$t = (P_1 - P_3)^2 = (P_2 - P_4)^2 \quad [1 + \bar{3} \rightarrow \bar{2} + 4]$

$u = (P_1 - P_4)^2 = (P_2 - P_3)^2 \quad [1 + \bar{4} \rightarrow \bar{2} + 3]$

$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$

$T_{fk} = T_{fk}(s, t)$

Crossing symmetry

In quantum field theory, absorption of a particle with 4-momentum $-\boldsymbol{p}$ and energy $E<-m$ corresponds to emission of an antiparticle with 4-momentum $\boldsymbol{p}$ and positive energy $E > m$.

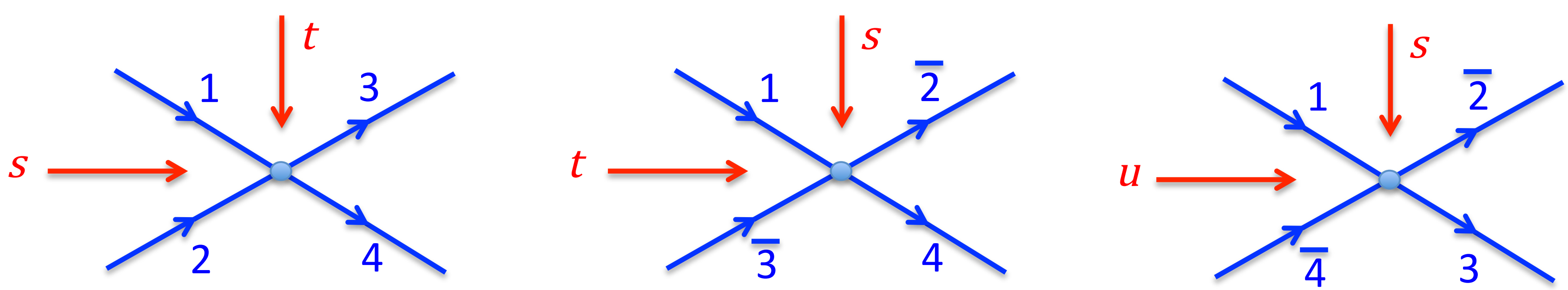
Since $T_{fk}(s, t)$ is a function of kinematical invariants (not on the sign of P_i), the same function describes the following reactions:

$1+2 \rightarrow 3+4$ for $P_1, P_2, P_3, P_4 > 0$ — s – channel ($s> 4m^2,\ \ t, u<0$)

$1+\bar{3} \rightarrow \bar{2}+4$ for $P_1, P_4 > 0$ *and* $P_2, P_3 < 0$ — t – channel ($t> 4m^2,\ \ s, u<0$)

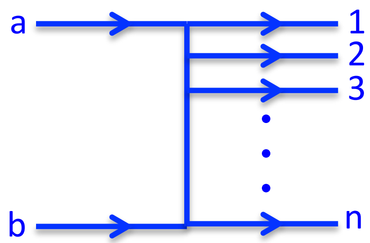
$1+\bar{4} \rightarrow \bar{2}+3$ for $P_1, P_3 > 0$ *and* $P_2, P_4 < 0$ — u – channel ($u> 4m^2,\ \ s, t<0$)

$1 \rightarrow \bar{2}+ 3+4$ for unstable particle ($P_1, P_3, P_4 > 0$ *and* $P_2 < 0$)



Unitarity

From the conservation of the probability norm in interaction processes:



$S^{+}S = 1$

The sum of probabilities of all processes which are possible at a given energy is equal to unity

$$i\left[T^{*}_{fk} - T_{fk}\right] = \sum_{n}\int\prod_{l=1}^{n}\frac{d^{3}p_{l}}{2E_{l}(2\pi)^{3}}T_{nk}T^{*}_{fn}\left(2\pi\right)^{4}\delta^{(4)}(P_{k} - \sum_{q=1}^{n}P_{q})$$

T_{nk} – the amplitude for a transition from the state $|k>$ to the state $|n>$ with n particles

$\sum_{n}\int$ – means integration over phase-space for each particle in the channel with n particles sum over all channels.

Optical theorem

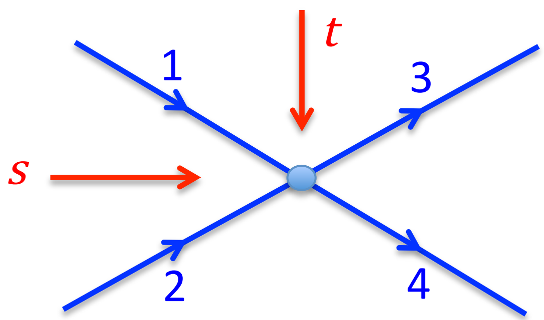
If $|f> = |k>$:

$$2\,\mathrm{Im}\,T(s,0) = \sum_{n}\int\prod_{l=1}^{n}\frac{d^{3}p_{l}}{2E_{l}(2\pi)^{3}}\left|T_{nk}\right|^{2}\left(2\pi\right)^{4}\delta^{(4)}(P_{k} - \sum_{q=1}^{n}P_{q}) = 4j\sigma_{tot}(s)$$

$$j = \sqrt{\left(p_{a}p_{b}\right)^{2} - m_{a}^{2}m_{b}^{2}}$$

$$\frac{d\sigma_{el}}{dt} = \frac{\left|T(s,t)\right|^{2}}{64\pi p_{a}^{*2}s} \qquad \sigma_{tot}(s) = \frac{\mathrm{Im}\,T(s,0)}{2p_{a}^{*}\sqrt{s}}$$

Kinematics of binary reactions



Let's assume $m_1 = m_2 = m_3 = m_4 = m$

In CM system: $\boldsymbol{p}_1 + \boldsymbol{p}_2 = \boldsymbol{p}_3 + \boldsymbol{p}_4 = \boldsymbol{0}$

$$E_i = \frac{1}{2}\sqrt{s} \qquad p_i^2 = \frac{s}{4} - m^2 \qquad i = 1,2,3,4$$

$$t = -(\boldsymbol{p}_1 - \boldsymbol{p}_3)^2 = -2\,\boldsymbol{p}_1^2(1 - \cos\theta_s)$$

$$-4p_1^2 \le t \le 0$$

$$\cos\theta_s = 1 + \frac{2t}{s - 4m^2} = -\left(1 + \frac{u}{s - 4m^2}\right)$$

$$T(s,t) = T(s, \cos\theta_s)$$

$$\frac{d\sigma_{el}}{d\Omega} = \left|\frac{T(s,t)}{8\pi\sqrt{s}}\right|^2$$

Partial wave expansion

$$f(s,\cos\theta) = \frac{1}{8\pi\sqrt{s}} T(s,t) = \frac{1}{p^*} \sum_{l=0}^{\infty} (2l+1) f_l(s) P_l(\cos\theta)$$

Represented in the form of a series in the partial-wave amplitudes $f_l(t)$, which characterize scattering in the state with relative orbital angular momentum l.

$$\int_{-1}^{1} dz P_l(z) P_{l'}(z) = \frac{2\delta_{ll'}}{2l+1}$$

+ unitarity

$$P_l(z) = \frac{1}{2^l l!} \frac{d^l}{(dz)^l} (z^2-1)^l$$ – Legendre polynomial

$$\operatorname{Im} f_l(s) = |f_l(s)|^2 + \sum_n \int |f_l^N(s,\tau_N)|^2 d\tau_N \Rightarrow \operatorname{Im} f_l(s) \geq |f_l(s)|^2 \Rightarrow \operatorname{Im} f_l(s) \leq 1$$

Froissart bound

For $s \to \infty$ the contribution comes from terms with $l_{eff} \leq C\sqrt{s}\ln s$

$$\sigma_{tot}(s) = \frac{\operatorname{Im} T(s,0)}{2p_a^*\sqrt{s}} = \frac{4\pi}{p_a^{*2}} \sum_{l=0}^{l_{eff}} (2l+1) f_l(s) \leq \frac{4\pi}{p_a^{*2}} l_{eff}^2 \approx C'\ln^2 s$$

$$\sigma_{tot}(s) \leq C'\ln^2 s$$

(assumes unitarity, analyticity, short-range character of strong interactions)

Pomeranchuk theorem

$$\operatorname{Im} T_{ab}(s, t=0) = s\sigma_{tot}(ab)$$

$$\operatorname{Im} T_{ab}(u, t=0) = u\sigma_{tot}(a\bar{b})$$

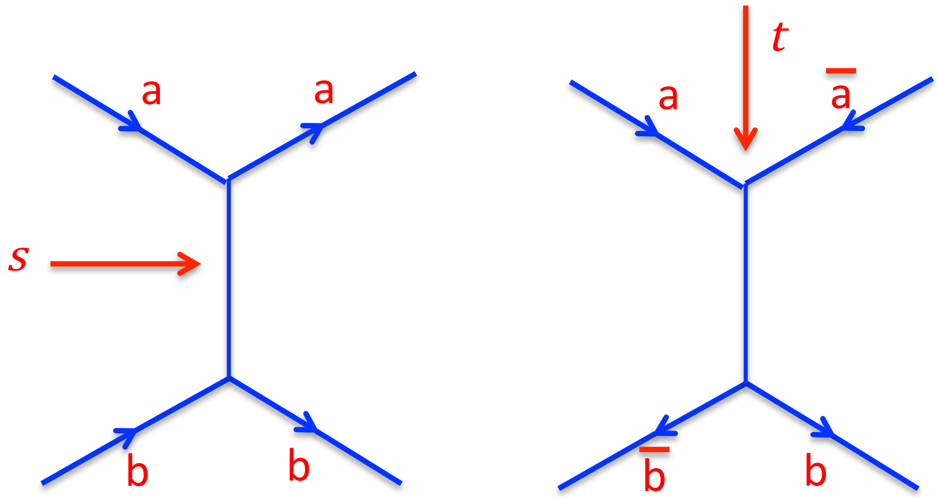
If T is not an oscillating function and $\frac{1}{\ln s}\frac{\operatorname{Re} T(s,0)}{\operatorname{Im} T(s,0)} \to 0$ at $s \to \infty$

⇓

$$\sigma_{tot}(ab) = \sigma_{tot}(a\bar{b}) \text{ at } s \to \infty$$

Pomeranchuk theorem may be violated. See O. Nachtmann's talk.

t-channel exchange picture



Suppose the $f_l(t)$ has a singularity of form

$$f_l(t) = \frac{r(t)}{l-\alpha(t)}$$

$$\frac{1}{2i}\left[f_l(t) - f_l^*(t)\right] \sim f_l(t) f_l^*(t)$$

$$f_l(t) - f_l^*(t) = \frac{r}{l-\alpha(t)} - \frac{r^*}{l-\alpha^*(t)} = \frac{r\left(l-\alpha^*(t)\right) - r^*\left(l-\alpha(t)\right)}{\left(l-\alpha(t)\right)\left(l-\alpha^*(t)\right)} \sim 2i\frac{rr^*}{\left(l-\alpha(t)\right)\left(l-\alpha^*(t)\right)}$$

$$r = r^*$$

$$\operatorname{Im}\alpha(t) \sim r \neq 0$$

No poles in the real axis in the l plane for $t > 4m^2$

Resonances

Assume for some $t = t_R \equiv M_R^2 > 4m^2,\ \mathrm{Re}\,\alpha(t_R) = l_R$ ⇒ $\alpha(t) \approx l_R + i\,\mathrm{Im}\,\alpha(t_R) + \alpha'(t_R)(t - t_R)$

Taylor expansion

$$f_{l_R}(t) \approx \frac{r(t_1)}{l_R - [l_R + i\,\mathrm{Im}\,\alpha(t_R) + \alpha'(t_R)(t - t_R)]} = -\frac{\mathrm{Im}\,\alpha(t_1)}{\alpha'(t_R)(t - t_R) + i\,\mathrm{Im}\,\alpha(t_R)} = \frac{\mathrm{Im}\,\alpha(t_R)/\alpha'(t_R)}{t_R - t - i\,\mathrm{Im}\,\alpha(t_R)/\alpha'(t_R)}$$

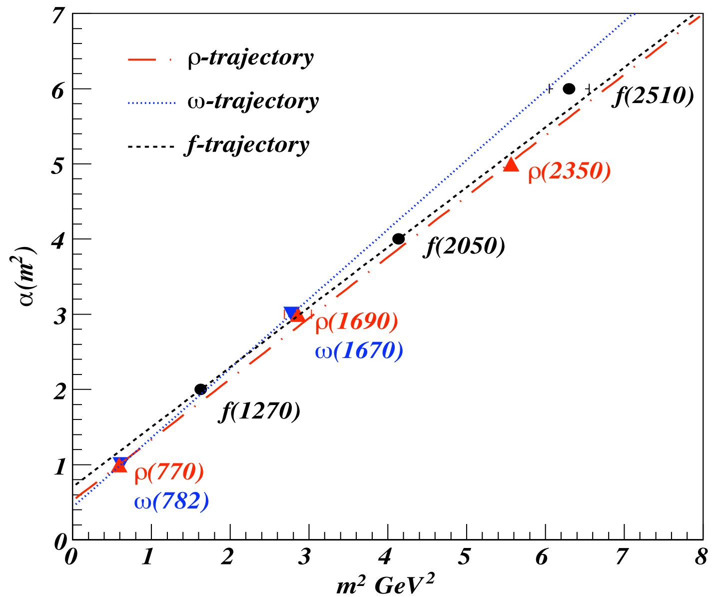
$$\mathrm{Im}\,\alpha(t_R)/\alpha'(t_R) \equiv M_R \Gamma$$

$$f_{l_R}(t) \sim \frac{1}{M_R^2 + t - iM_R\Gamma}$$

Breit-Wigner

Regge pole in the physical region of the t-channel ($t > 4m^2$) corresponds to a Briet-Wigner resonance with $\mathrm{Re}\,\alpha(t_1) = l_R$ (=spin of R)

Reggeon trajectory



Chew-Frautschi plot

Regge trajectories are almost straight lines and in standard Regge theory they are parameterized by $\alpha(t) = \alpha_0 + \alpha' \cdot t$

Regge pole gives a generalization of a particle exchange in the *t*-channel. It corresponds to an exchange in the *t*-channel by a state of noninteger spin $\alpha(t)$ (reggeon trajectory), which coincides with particles of spin J for $t = M_J^2$

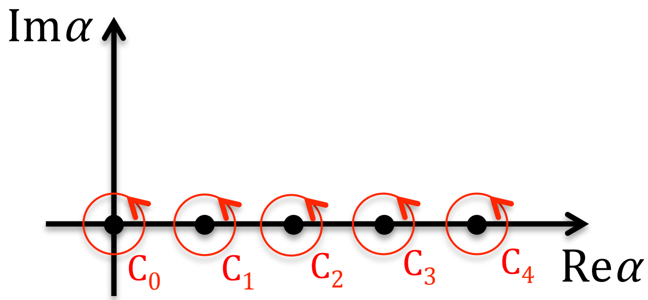
Partial wave expansion in t-channel and Sommerfeld-Watson transformation

$$T(t,s) = T(t,\cos\theta_t) = \sum_{l=0}^{\infty}(2l+1)f_l(t)P_l(\cos\theta_t)$$

$$\cos\theta_t = 1 + \frac{2s}{t-4m^2}$$

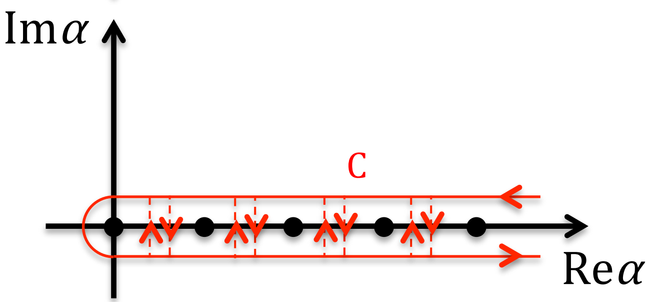
Cauchy's integral theorem

$$F(a) = \frac{1}{2\pi i}\int_C \frac{F(z)}{z-a}dz$$



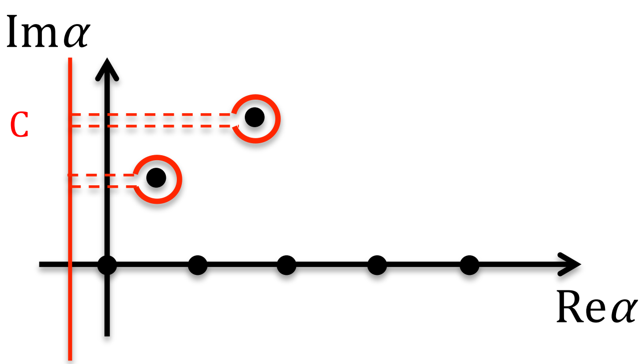
$$\sin(\pi\alpha) \approx \sin(\pi l) + \pi(\alpha - l)\cos(\pi l) = \pi(-1)^l(\alpha - l)$$

$$(2l+1)f_l(t)P_l(\cos\theta_t) = \frac{1}{2i}\int_{C_l}\frac{(-1)^{\alpha}(2\alpha+1)f(\alpha,t)P_{\alpha}(\cos\theta_t)}{\sin(\pi\alpha)}d\alpha$$



$$T(t,s) = \frac{1}{2i}\int_C \frac{e^{-i\pi\alpha}(2\alpha+1)f(\alpha,t)P_{\alpha}(\cos\theta_t)}{\sin(\pi\alpha)}d\alpha$$

$$T(t,s) = \frac{1}{2i}\int_C \sum_{\sigma=\pm 1}\frac{\left(1+\sigma e^{-i\pi\alpha}\right)(2\alpha+1)f^{\sigma}(\alpha,t)P_{\alpha}(\cos\theta_t)}{2\sin(\pi\alpha)}d\alpha$$



$$T(t,s) = \sum_{\sigma=\pm 1}\sum_{poles}\eta_{\sigma}\left(\alpha_i^{\sigma}(t)\right)r_i^{\sigma}(t)P_{\alpha_i^{\sigma}(t)}(\cos\theta_t)$$

$$\eta_{\sigma}(\alpha) = -\frac{\sigma + e^{-i\pi\alpha}}{\sin(\pi\alpha)}$$ signature factor

Regge pole exchange amplitude

"Physical" region in the t−channel corresponds to $t > 4m^2$, $s < 0$. Analytically continue the amplitude to $s > 4m^2$, $t < 0$ (s−channel).

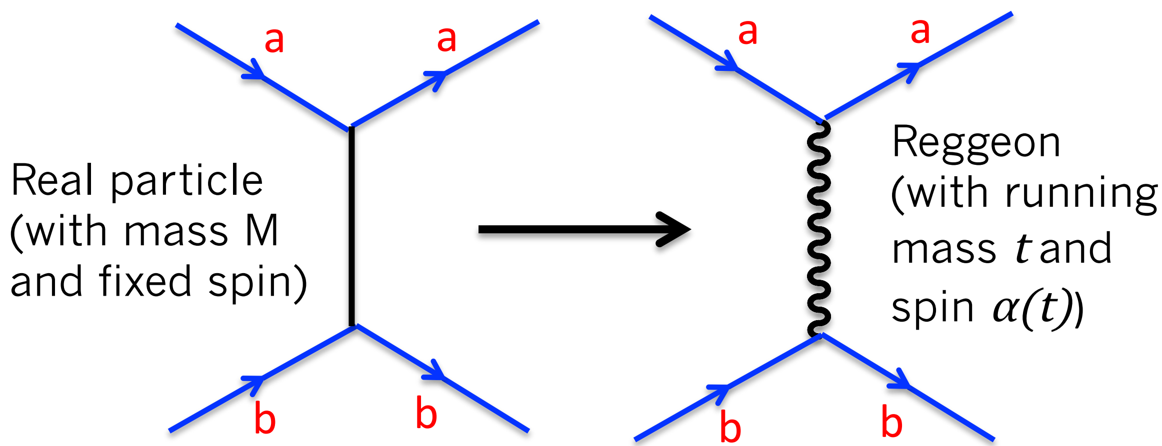
For $s >> 4m^2 > |t|$, $\cos\theta_t \sim \frac{s}{4m^2} >> 1$

$P_l(z) \sim z^l$ for $z >> 1$

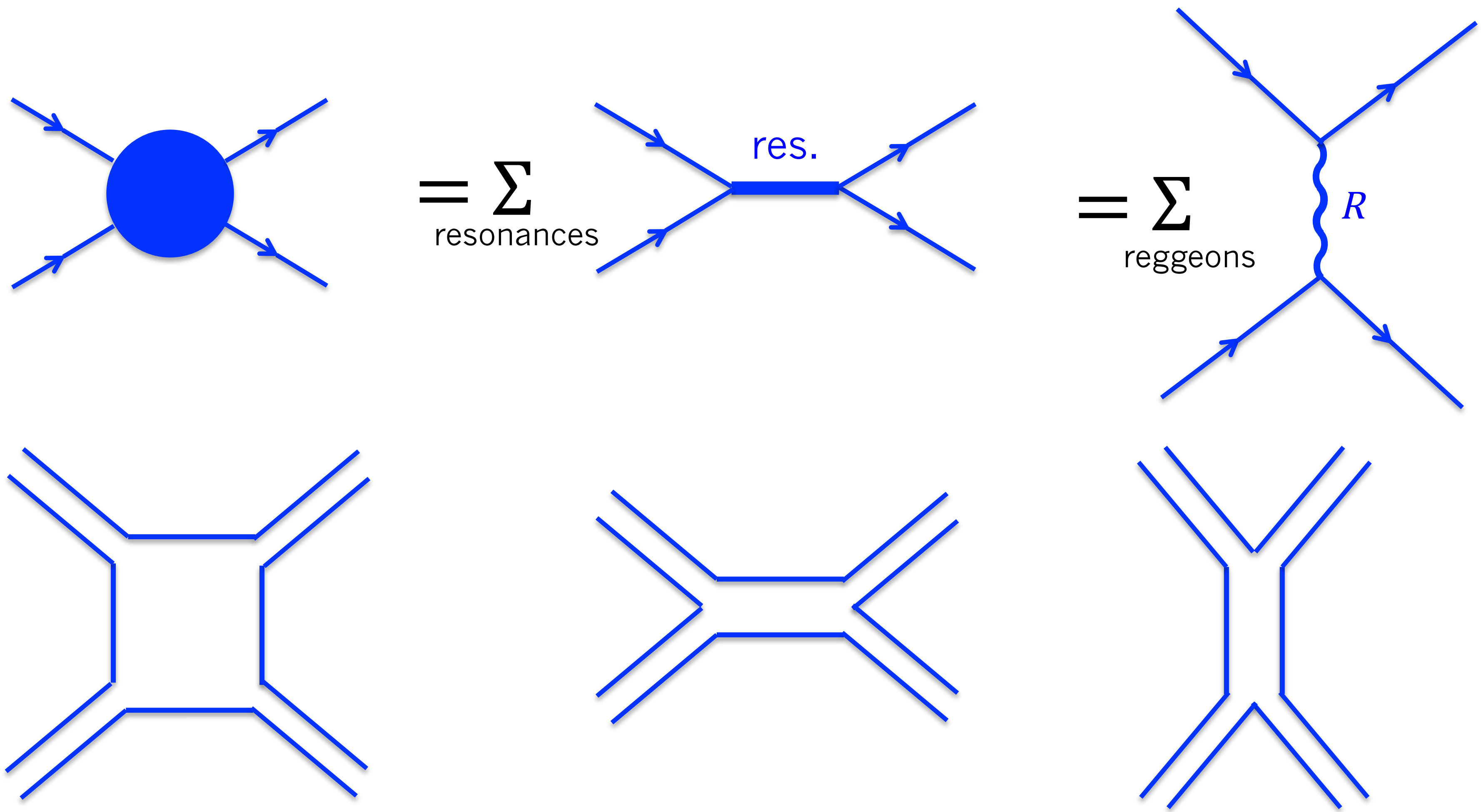
$$T(t,s) = \sum_{\sigma=\pm1} \sum_{poles} \eta_\sigma \left(\alpha_i^\sigma(t)\right) \gamma_i^\sigma(t) \left(\frac{s}{s_0}\right)^{\alpha_i^\sigma(t)}$$

$$\sigma_{tot}(s) = \frac{1}{s} \mathrm{Im}\, T(s,0) \sim s^{\alpha(0)-1}$$

s_0 is a constant scale factor, usually chosen to be $s_0 = 1\ \mathrm{GeV}^2$.



Duality

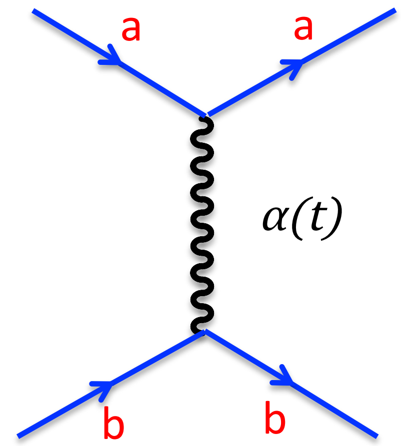


Factorization

What is the meaning of $\gamma(t)$?

In fact, all information about incoming and outgoing particles (baryon number, strangeness, etc.) are absorbed in $\gamma(t)$ and it does not depend on s.

$\gamma(t)$ should be related to Reggeon-hadron interaction vertex!



One can assume the initial state does not know anything about the final state: in the cross-channel the initial particles first transform into an intermediate state, which then gets converted into the final particles, with the amplitude independent of the properties of the initial state.

$$\gamma(t) = g_{aa}(t) g_{bb}(t)$$

It is not possible to predict the explicit form of $g_{aa}(t)$ from the analytical properties of the matrix element (model dependent).

Regge pole approximation

At fixed t, with $s >> t$

- Amplitude for a process governed by the exchange of a trajectory α(t) is

$$T(s,t) \propto \left(s / s_0\right)^{\alpha(t)}$$

- No prediction for *t* dependence

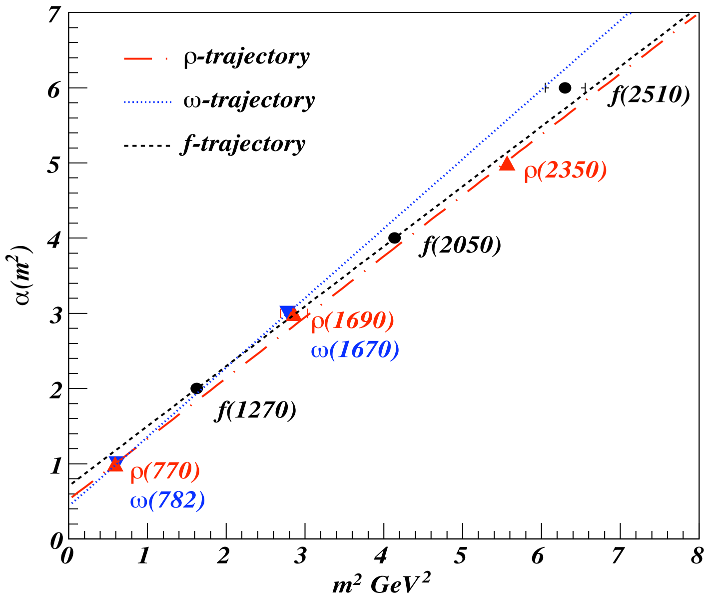
- Elastic cross section

$$\frac{d\sigma_{el}}{dt} \approx \frac{1}{s^2}\left|T(s,t)\right|^2 \propto s^{2(\alpha(t)-1)}$$

- Total cross section considering the optical theorem

$$\sigma_{tot} \approx \frac{1}{s}\operatorname{Im}T(s,0) \propto s^{\alpha(0)-1}$$

Reggeons



$$\alpha_i(t) = \alpha_i(0) + \alpha_i' \cdot t, \quad i = f, \rho, \omega.$$

$$\alpha_f(0) = 0.703 \pm 0.023 \quad \alpha_f' = 0.797 \pm 0.014 GeV^{-2}$$

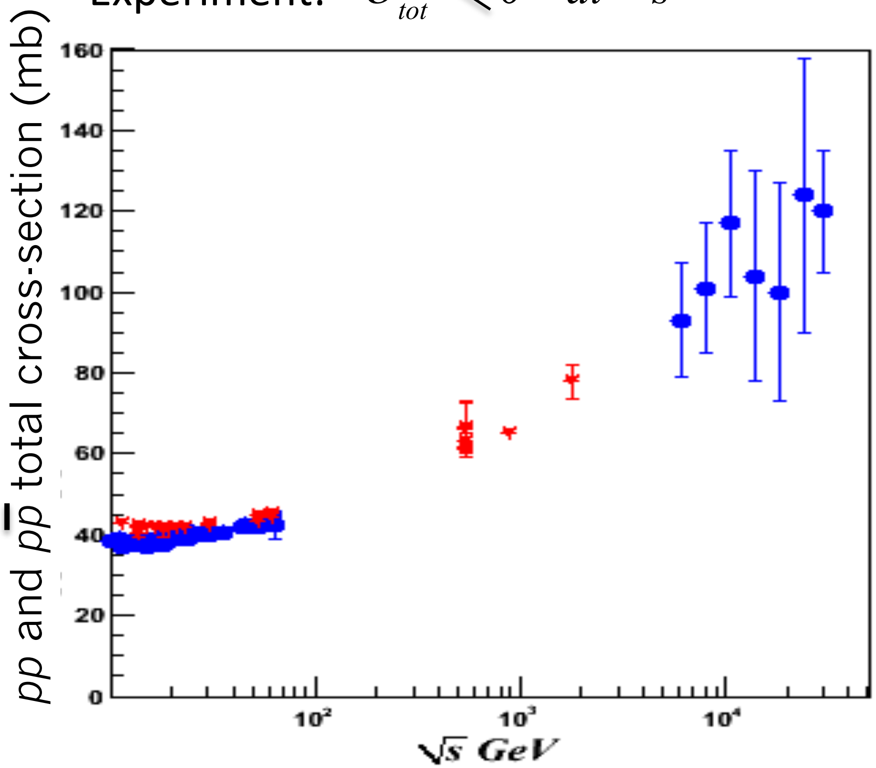
$$\alpha_\rho(0) = 0.522 \pm 0.009 \quad \alpha_\rho' = 0.809 \pm 0.015 GeV^{-2}$$

$$\alpha_\omega(0) = 0.435 \pm 0.033 \quad \alpha_\omega' = 0.923 \pm 0.054 GeV^{-2}$$

$$\sigma_{tot} \propto \left(\frac{s}{s_0}\right)^{\alpha_i(0)-1} \Longrightarrow \sigma_{tot} \to 0 \quad at \quad s \to \infty$$

Unexpected Reggeon?

Experiment: $\sigma_{tot} \not\to 0 \quad at \quad s \to \infty$

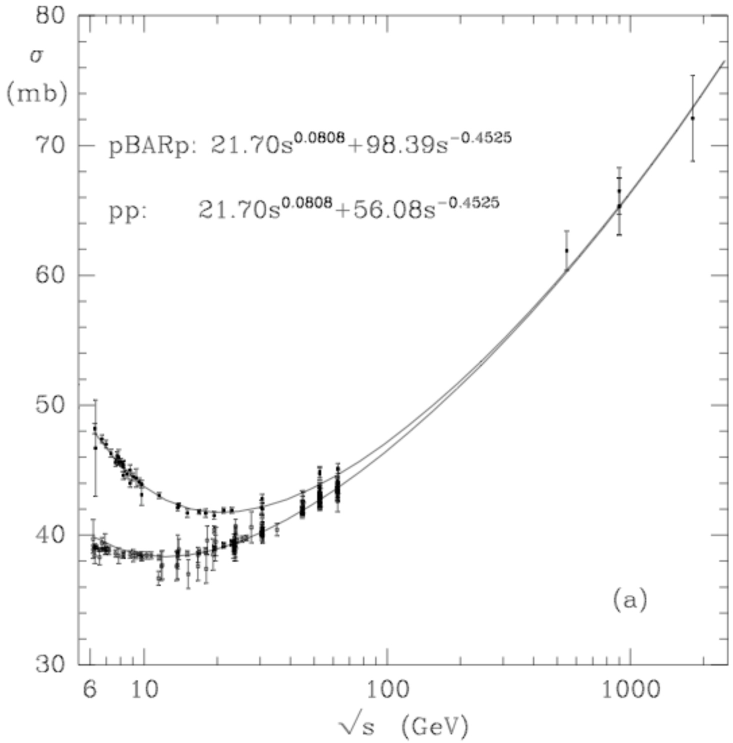


$$\sigma_{tot} \sim \left(\frac{s}{s_0}\right)^{\alpha_i(0)-1}$$

$\Downarrow$

An object with $\alpha(0) = 1 + \Delta > 1$ is needed

Pomeron



Donnachie and Landshoff (1992)

$$\sigma_{tot} = As^{0.0808}$$

grows as a power function of s

Unitarity requires that the total cross section at very high energies should not grow faster than $\ln^2 s$ (Froissart bound).

For describing DIS data

$$F_2(x,Q^2) = f(Q^2)x^{\Delta(Q^2)}$$ (CKMT 1992)

$$F_2(x,Q^2) = f_1(Q^2)x^{-0.08} + f_2(Q^2)x^{-0.42}$$ (DL 1998)

soft Pomeron hard Pomeron

Pomeron

It is usually assumed that the Pomeron in QCD is related to gluonic exchanges in the t–channel.

Δ_{eff} determined from fits to data are in general different from Δ.

See talks by
L. Jenkovszky
A. Martin
O. Nachtmann
W. Schäfer

DIFFRACTION:
In HEP any process involving Pomeron exchange

A simple parameterization of Regge residues

$$g_{aa}(t) = g_{aa}\exp\left\{R_{aa}^2 t\right\}$$ R_{aa} – Regge radius of hadron *a*

$$\alpha(t) = \alpha_0 + \alpha' t$$

$$\eta_\sigma(\alpha) = -\frac{\sigma + e^{-i\pi\alpha}}{\sin(\pi\alpha)} = e^{-i\pi\alpha/2}\begin{cases}\dfrac{-1}{\sin(\pi\alpha/2)} & \text{for } \sigma = +1 \\ \dfrac{i}{\cos(\pi\alpha/2)} & \text{for } \sigma = -1\end{cases}$$

$$T(s,t) = g_{aa}g_{bb}\eta(\alpha(0))\exp\{\lambda t\}\left(\frac{s}{s_0}\right)^{\alpha_0}$$

$$\lambda \equiv R_{aa}^2 + R_{bb}^2 + \alpha'\left(\ln\left(s/s_0\right) - i\pi/2\right)$$

Impact parameter representation

$$f_{ab}(s,\boldsymbol{b}) \sim \int d^2q_\perp \exp\left\{-i\mathbf{b}\mathbf{q}_\perp\right\}T(s,q_\perp^2) \sim \frac{\left(s/s_0\right)^{\alpha_0-1}}{\lambda}\exp\left\{-\frac{b^2}{4\lambda}\right\}$$

$$\sqrt{\overline{b}^2} = 2|\lambda| \approx 2\sqrt{R_a^2 + R_b^2 + \alpha'\ln\left(s/s_0\right)}$$

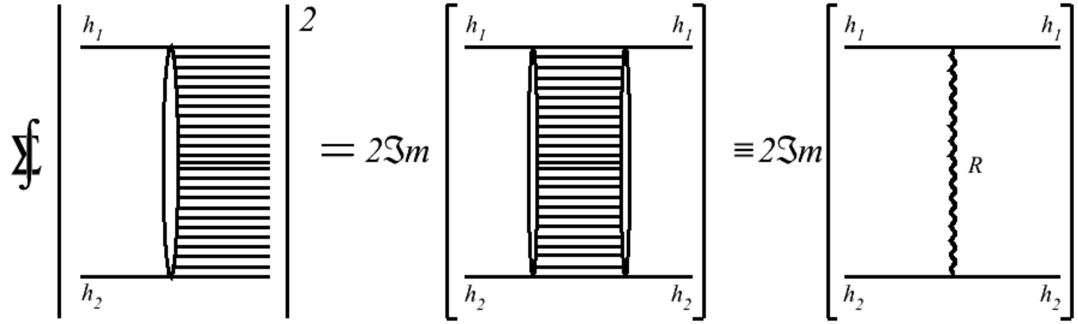
Radius of interaction increases with increasing *s*

$$\frac{d\sigma}{dt} \sim \left(\frac{s}{s_0}\right)^{2\alpha-2} \times \exp\left\{-2\left(R_{aa}^2 + R_{bb}^2 + \alpha'\ln\left(s/s_0\right)\right)|t|\right\}$$

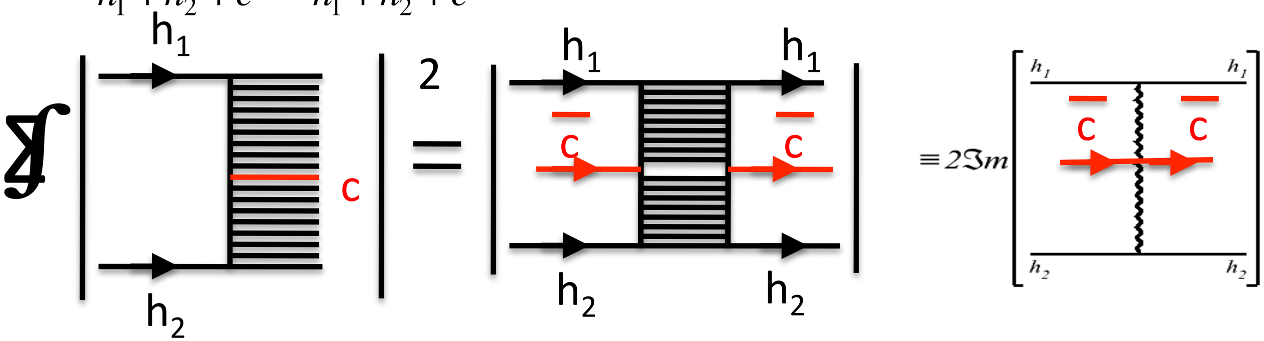
Increases with increasing *s*. Diffraction peak shrinkage.

Unitarity and two-body & three-body reactions

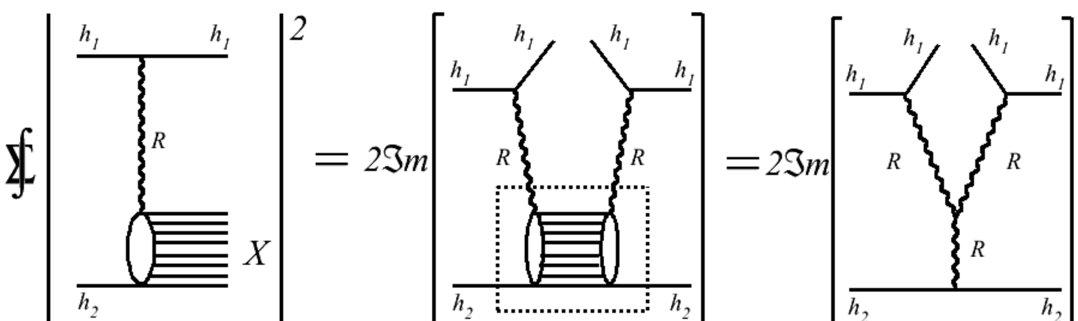
$$\sum_c T_{ac} T^*_{ac} = 2\Im m T_{aa}$$



Analogous to the optical theorem, Muller's theorem relates the inclusive cross-section for the reaction $h_1 + h_2 \to c + X$ to the forward scattering amplitude of the three-body hadronic process $h_1 + h_2 + \bar{c} \to h_1 + h_2 + \bar{c}$.

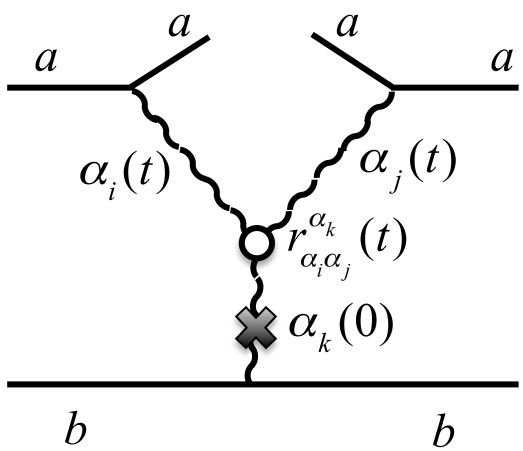


double Regge limit



triple Regge limit

Triple-Regge diagram



$$\frac{d\sigma_{SD}}{dM^2dt} = \left(\frac{s_0}{s}\right)^2 \sum_{i,j,k} G_{ijk}(t)\left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)}\left(\frac{M^2}{s_0}\right)^{\alpha_k(0)}$$

$$G_{ijk}(t) = 4\pi g^{\alpha_i}_{aa}(t)\, g^{\alpha_j}_{aa}(t)\, g^{\alpha_k}_{bb}(0)\, r^{\alpha_k}_{\alpha_i\alpha_j}(t)\,\eta(\alpha_i(t))\,\eta^*(\alpha_j(t))$$

$$(PP)P \quad \propto \frac{s^{2\Delta}}{(M^2)^{1+\Delta}}$$

$$(PP)R \quad \propto \frac{s^{2\Delta}}{(M^2)^{1.5+2\Delta}}$$

$$(RR)P \quad \propto \frac{(M^2)^{\Delta}}{s}$$

$$(RR)R \quad \propto \frac{1}{s(M^2)^{0.5}}$$

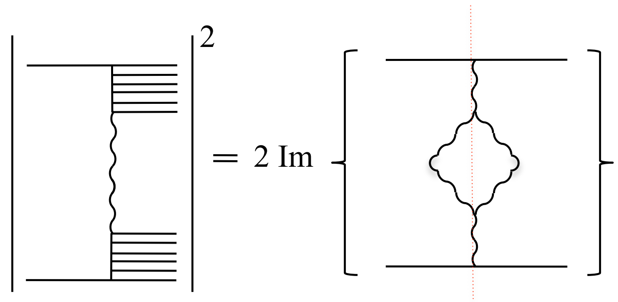
$$(\pi\pi)P \quad \propto \frac{(M^2)^{1+\Delta}}{s^2}$$

$$(\pi\pi)R \quad \propto \frac{(M^2)^{0.5}}{s^2}$$

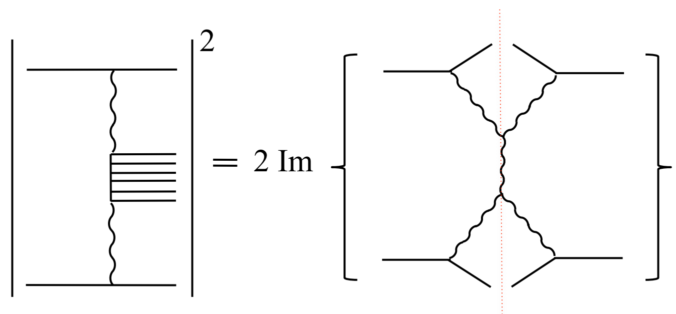
$$(PR)P \quad \propto \frac{s^{\Delta-0.5}}{(M^2)^{0.5}}$$

$$(PR)R \quad \propto \frac{s^{\Delta-0.5}}{(M^2)^{1+\Delta}}$$

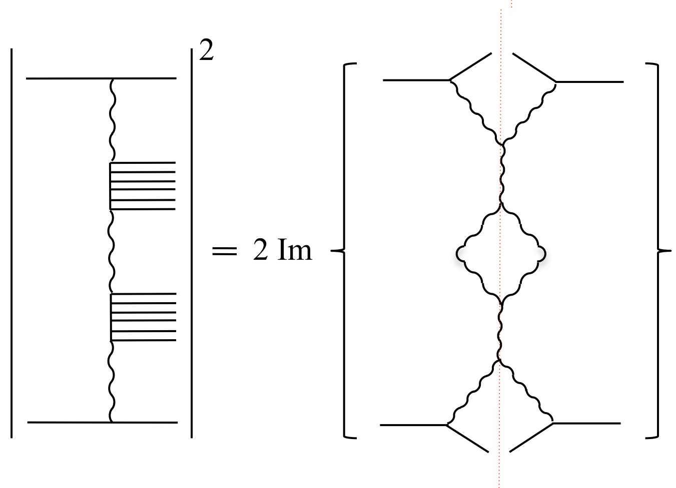
See talks by
A. Martin
L. Jenkovszky



Double diffraction



Central production



Double gap topology

s-channel picture of Reggeons

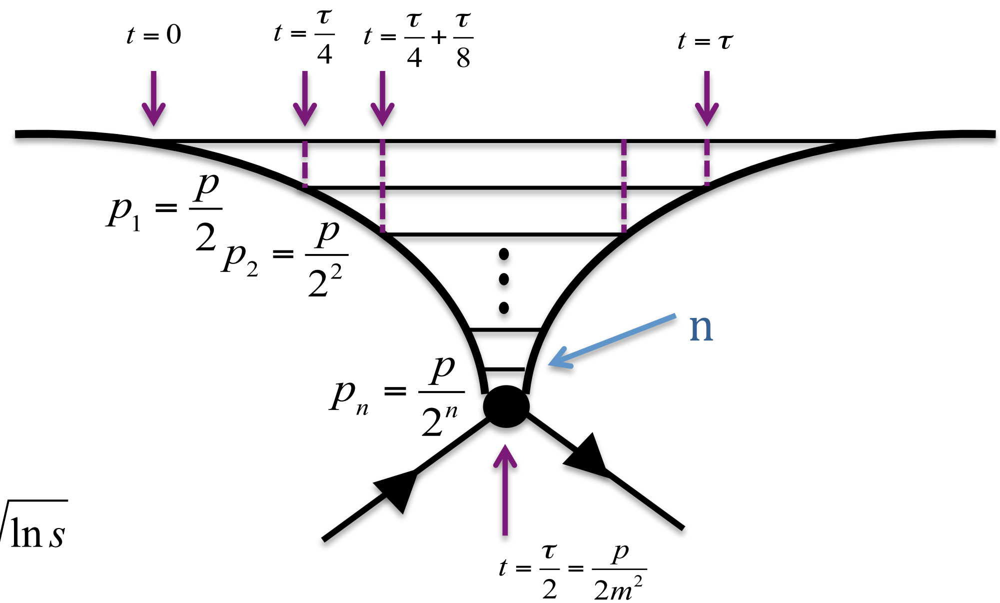
Multiperipheral fluctuation development time:

$$\tau = \frac{p}{m^2}$$

Slow partons interact: $p_n \approx m$

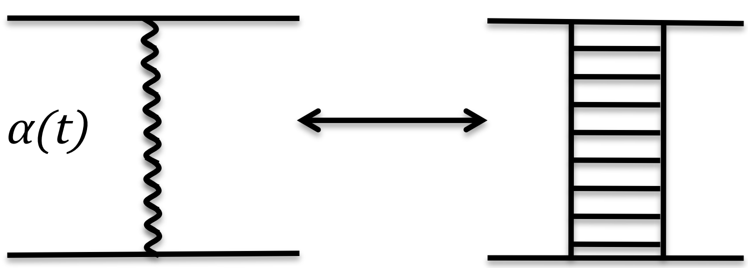
$$n \sim \ln p \sim \ln s$$

Random walk in b space: $\sqrt{\overline{b}^2} \sim n \sim \sqrt{\ln s}$



High-energy hadronic interactions are essentially non-local.

Summation of mutiperipheral diagrams leads to regge behavior



Ladder diagram

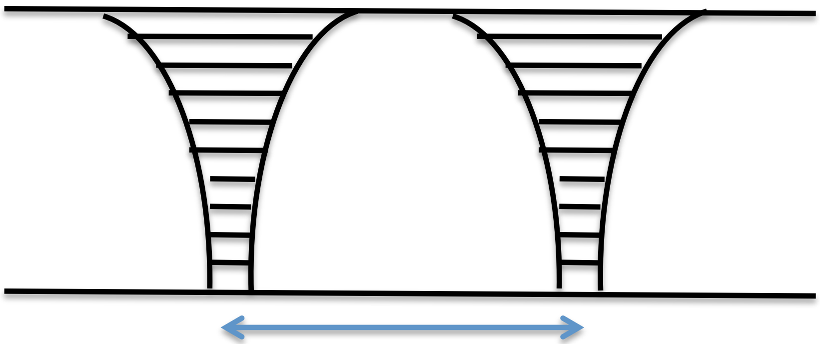
See A. Martin's talk

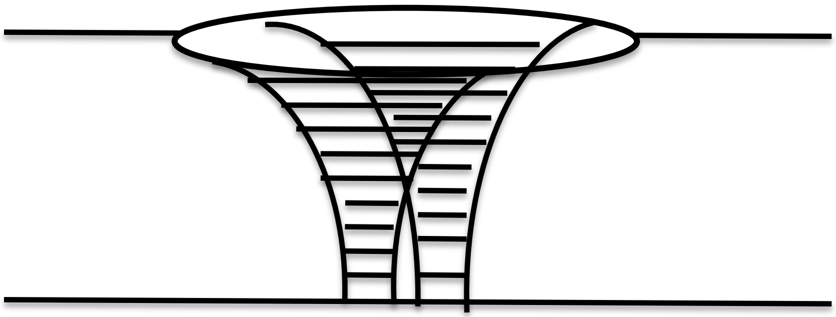
Reggeon is a non-local object!

Space-time picture of high-energy hh interactions

AFS (successive)

Mandelstam (simultaneous)



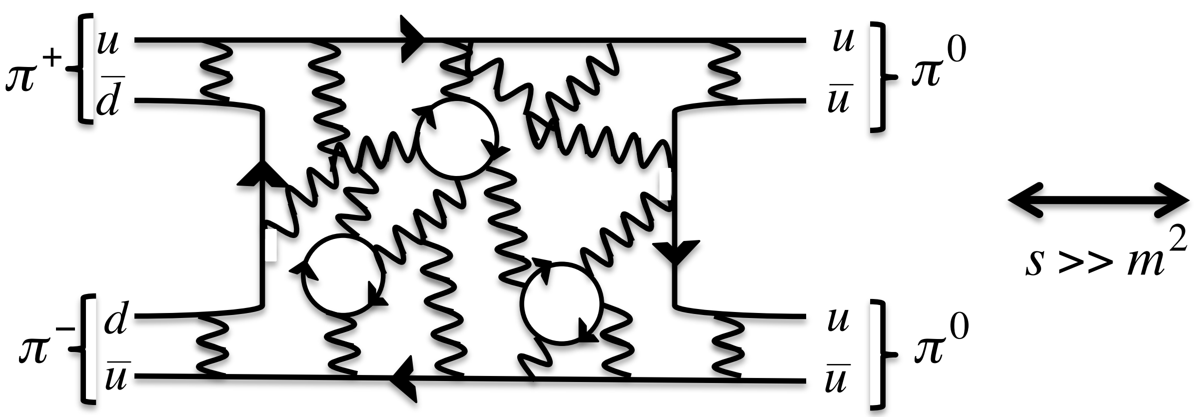


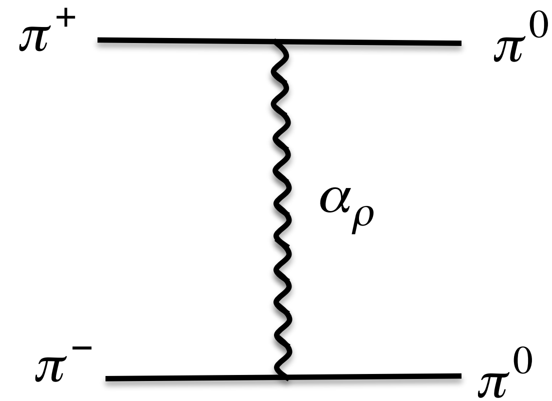
$\tau \sim E$

$$\sigma \to 0 \quad at \quad s \to \infty$$

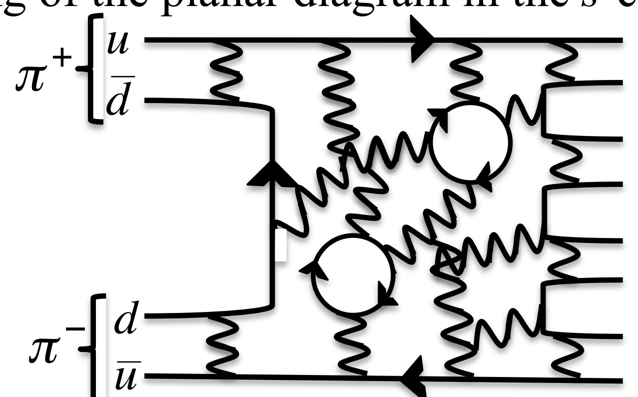
Regge poles in QCD

1/N-expansion is a useful non-perturbative method to study soft interaction dynamics.
$N_c >> 1$ (t'Hooft)
$N_c \approx N_f >> 1$ (Veneziano)
All diagrams are classified according to their topology. Amplitudes are expanded in *1/N* (*1/N*^{2}). The first term corresponds to planar diagrams.





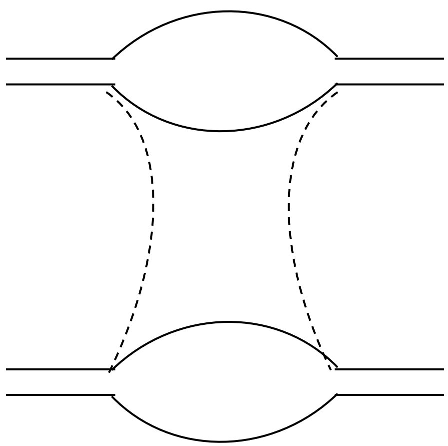
Cutting of the planar diagram in the *s*-channel

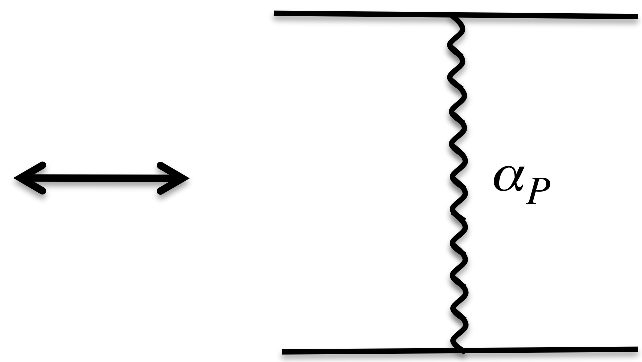


Configuration of the final state particles.

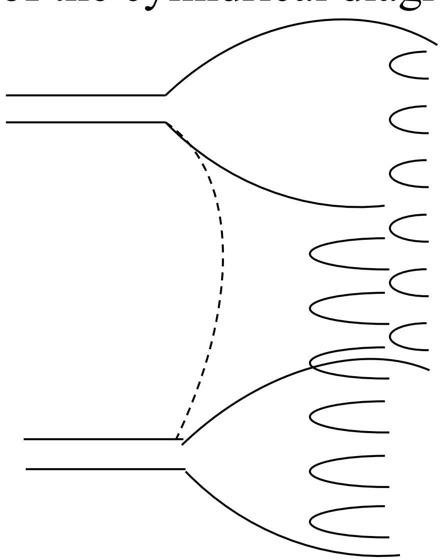
Pomeron in QCD

Pomeron is usually related to gluonic exchange in the *t*-channel.
From the point of view of 1/N-expansion Pomeron corresponds to cylinder-type diagrams.

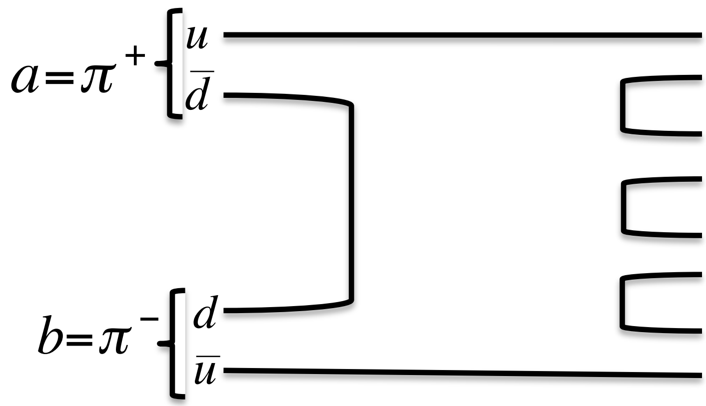




Cutting of the cylindrical diagram in the *s*-channel



Configuration of the final state particles.



$$\sigma_{ab}^{an}(y_a - y_b) = w(y_{q_a} - y_{\bar{q}_a})w(y_{q_b} - y_{\bar{q}_b}) \cdot \sigma_{\bar{q}_a q_b} \cdot P_{q_a \bar{q}_b \to X}$$

$$w(y_{q_a} - y_{\bar{q}_a})w(y_{q_b} - y_{\bar{q}_b}) = Aw(y_a - y_b)$$

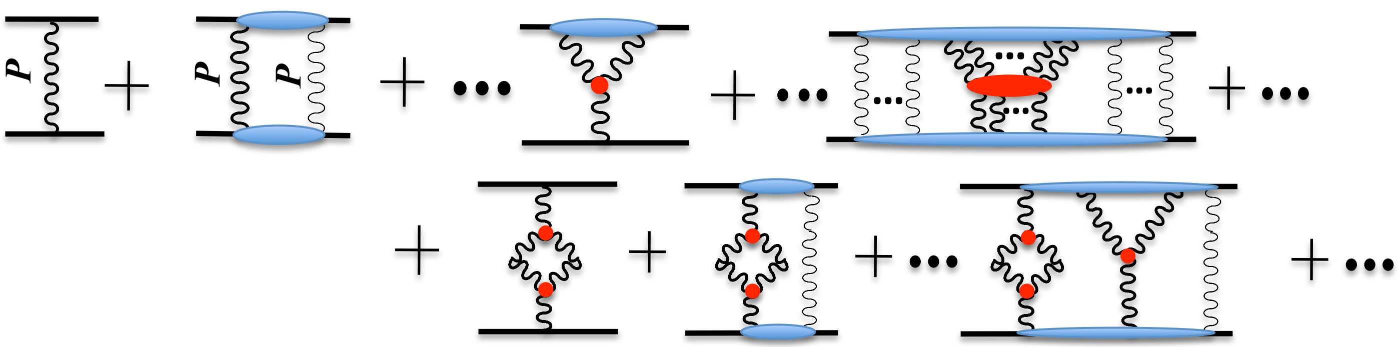
$$w(y_{q_i} - y_{\bar{q}_j}) = A\exp\left\{-\beta\left(y_{q_i} - y_{\bar{q}_j}\right)\right\}$$

$$\int d^2 b_q w(y_a - y_{\bar{q}}, b_a - b_{\bar{q}}) w(y_b - y_q, b_q - b_b) = Aw(y_a - y_b, b_a - b_b)$$

$$w(y_i - y_k, b_i - b_k) = \frac{A}{4\pi\gamma(y_i - y_k)}\exp\left\{-\beta(y_i - y_k) - \frac{(b_i - b_k)^2}{4\gamma(y_i - y_k)}\right\}$$

$$\beta = 1 - \alpha_R(0), \quad \gamma = \alpha_R'$$

Reggeon Field Theory



Gribov-1986

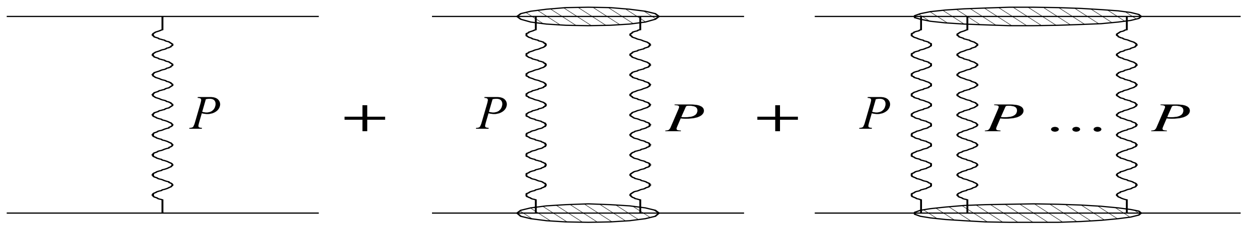
At high energies (parton densities) the interaction between Pomerons starts to play an important role. The Regge theory becomes unsafe. Interaction vertices (multi-Pomeron and Pomeron-hadron) are not known theoretically.

models based on RFT:
Kaidalov-Ponomarev-Ter-Martirosyan, Khoze-Martin-Ryskin, Gotsman-Levin-Maor, Ostapchenko, L. Jenkovszky et al., Kaidalov-Poghosyan, ...

Main difference in implementing the GW mechanism, in used sets of diagrams, and in parameterizing interaction vertices (+AGK).

Gribov's reggeon calculus

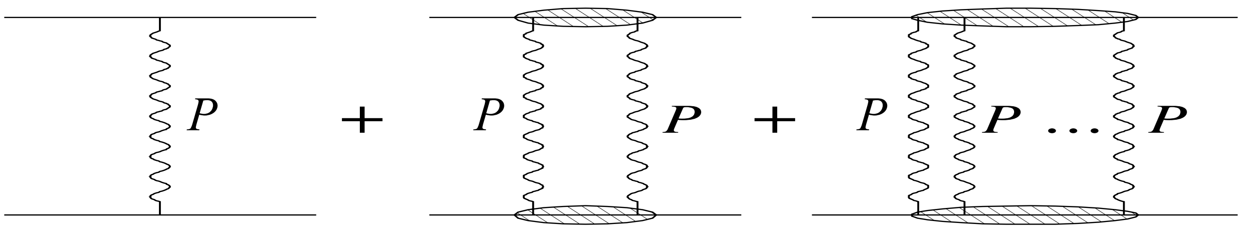
Regge-poles are not the only singularities of the amplitude. There are also branch points which correspond to the exchange of several Reggeons. A Regge pole can be interpreted as corresponding to a single scattering. Regge cuts – multiple scatterings of hardons' constituents.



$$iM^{(n)}(s,t) = \frac{1}{n!}\int \prod_{i=1}^{n}\left[iM^{(1)}(s,\mathbf{q}_{i\perp}^2)\frac{d^2\mathbf{q}_{i\perp}}{\pi}\right] C^{(n)}(\{\mathbf{q}_{i\perp}\})\delta\left(\mathbf{q}_\perp - \sum_{i=1}^{n}\mathbf{q}_{i\perp}\right)$$

$$M^{(1)}(s,t) = \frac{T(s,t)}{8\pi s} = \gamma\eta\left(\alpha(0)\right)e^{\lambda t}\left(\frac{s}{s_0}\right)^{\Delta} \qquad \Delta \equiv \alpha_P - 1$$

Multi-Pomeron exchange



$$M_P^{(n)}(s,t) = -i\lambda \left(\frac{\gamma}{\lambda}\right)^n \frac{\exp\{-\lambda t/n\}}{n \cdot n!} \left(-\frac{s}{s_0}\right)^{n\Delta}$$

$\Delta_{nP} = n\Delta_P$ → for $\Delta_P > 0$ all nP exchanges should be taken into account

$$M(s,t) = \sum_{n=1}^{\infty} M_P^{(n)}(s,t)$$

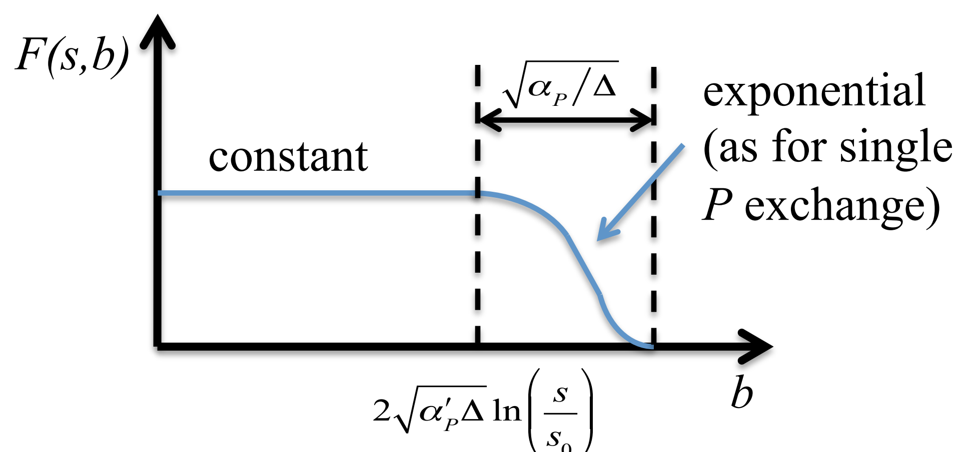
$$\sigma_{tot} \sim \ln^2 s$$

Impact parameter representation

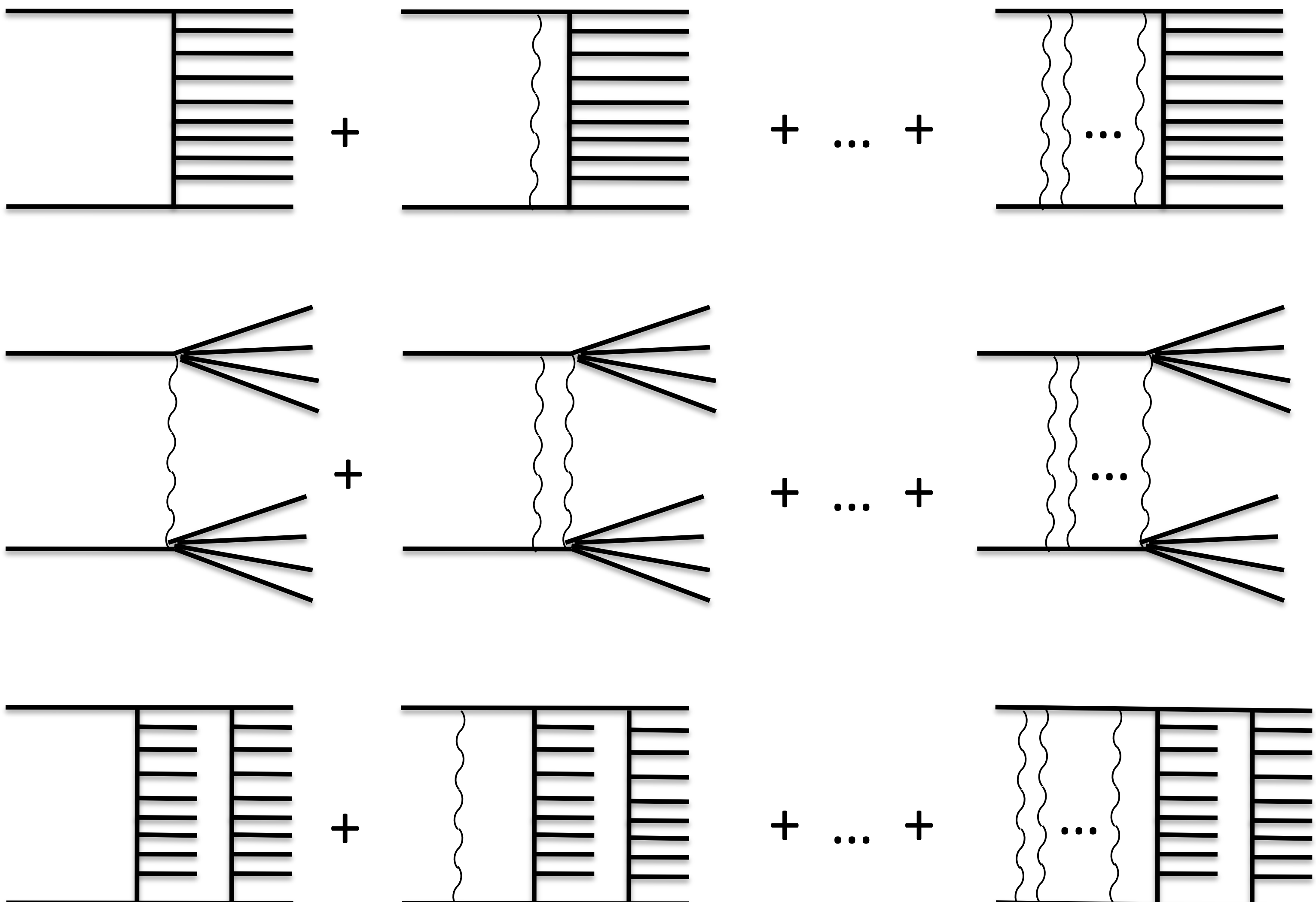
$$F(s,b) = 1 - \exp\left[\chi_P(s,b)\right]$$

$$\chi_P(s,b) = -\frac{\gamma}{\lambda}\exp\left\{-\Delta \ln\left(s/s_0\right) - b^2/4\lambda\right\}$$

– eikonal



How to calculate the cross-section of a given process?



Abramovsky-Gribov-Kancheli cutting rules

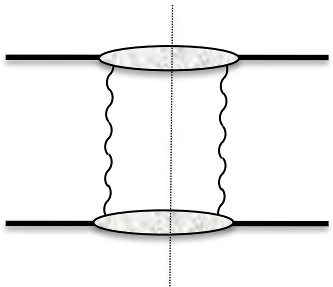
AGK cutting rules allow:

- to relate to each other the different s-channel discontinuities of a given graph
- to calculate the contribution of each graph in the total cross-section.

- If the Pomeron is not cut entirely, its contribution is suppressed exponentially.
- No particle production from interaction vertices
- All the vertices for various cuts are the same and real.

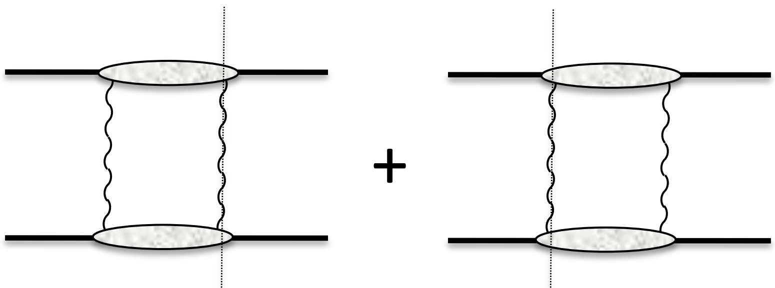
- There is one cut-plane which separates the initial and final states
- Each cut-pomeron obtains an extra factor of (-2) due to the discontinuity of the pomeron amplitude (for a cut Pomeron replace the factor $iM^{(1)}(s,t)$ by $2\mathscr{Im}M^{(1)}(s,t)$)
- Each un-cut pomeron obtains an extra factor of 2 since it can be placed on both sides of the cut-plane (the factors $iM^{(1)}(s,t)$ for the Pomerons to the right of the cut are placed by the complex-conjugate values)

AGK for PP exchange



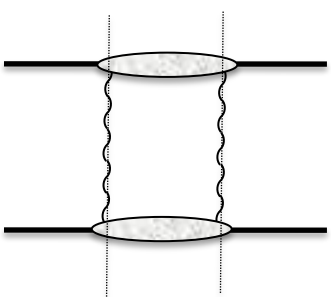
$$2\Delta M_0^{(2)} = 2\left[ReM^{(1)}(s,t_1)ReM^{(1)}(s,t_2) + ImM^{(1)}(s,t_1)ImM^{(1)}(s,t_2)\right]$$

Diffractive cutting (between Pomerons)



$$2\Delta M_1^{(2)} = -8\left[ImM^{(1)}(s,t_1)ImM^{(1)}(s,t_2)\right]$$

Cutting through one of Pomerons



$$2\Delta M_2^{(2)} = 4\left[ImM^{(1)}(s,t_1)ImM^{(1)}(s,t_2)\right]$$

Cutting both Pomerons

$$2\Delta M_0^{(2)} + 2\Delta M_1^{(2)} + 2\Delta M_2^{(2)} = 2\left[ReM^{(1)}(s,t_1)ReM^{(1)}(s,t_2) - ImM^{(1)}(s,t_1)ImM^{(1)}(s,t_2)\right] = 2\Delta M^{(2)}$$

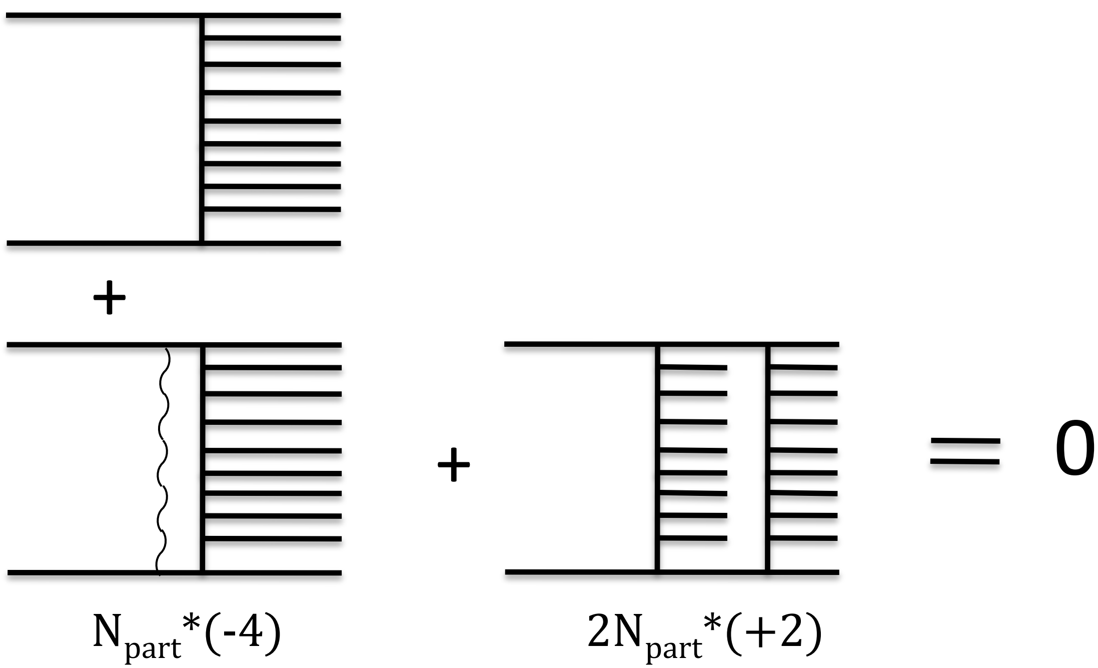
$$\Delta M_1^{(2)} + 2 \bullet \Delta M_2^{(2)} = 0$$

For any *n ≥ k > 0*

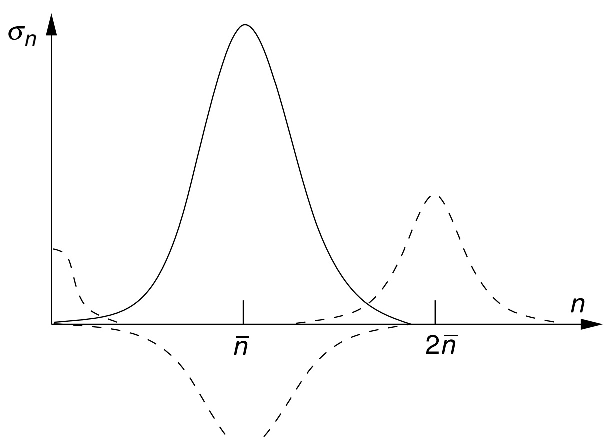
$$2\Delta M_k^{(n)}(s,t) = (-1)^{n-k} 2^n C_n^k \left[\prod_{i=1}^{n} ImM^{(1)}(s,t_i) \right]$$

$$2\sum_{k=1}^{n} k\Delta M_k^{(n)}(s,t) = \left[(-2)^n \prod_{i=1}^{n} ImM^{(1)}(s,t_i) \right] \sum_{k=1}^{n} (-1)^k k C_n^k = 0 \qquad n \geq 2$$

AGK and multiparticle production



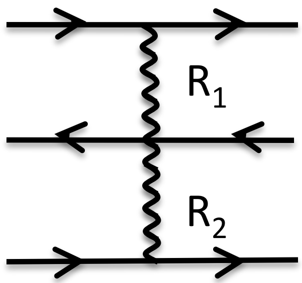
(AGK cancellation)



$\sigma_1^{(2)}$ is negative, it is a correction to the pole diagram.
Reducing it opens a room for new production processes

Inclusive cross-section

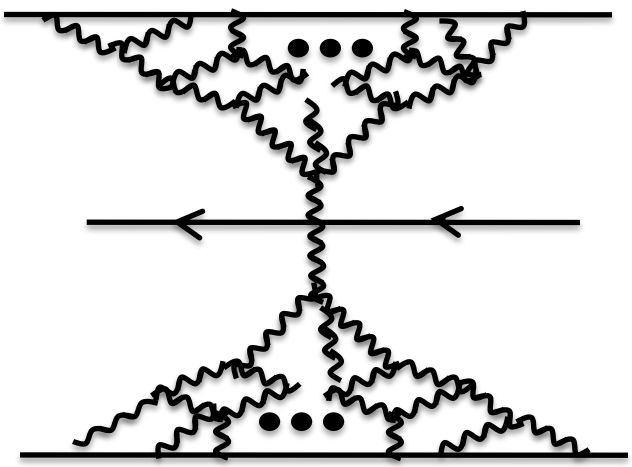
The central part of the inclusive spectrum is determined by Mueller-Kancheli diagram:

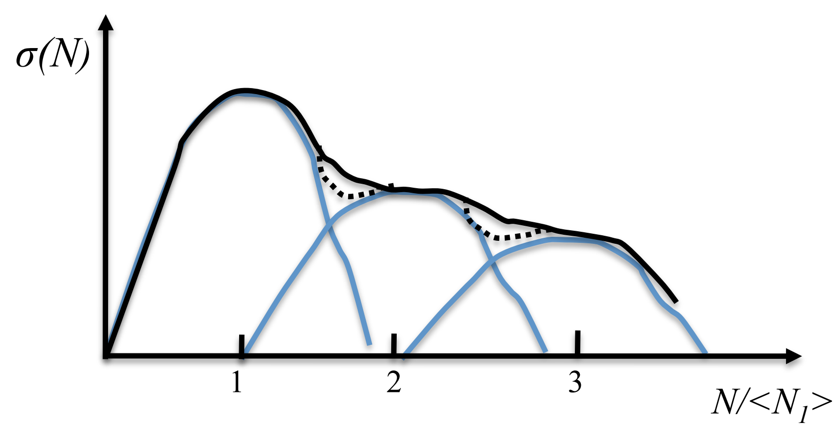


$$\left(\frac{d\sigma}{dy}\right)_{y=0} \sim s^{(\alpha_{R_1}+\alpha_{R_2}-2)/2}$$

if $R_1 = R_2 \equiv P$: $\left(\frac{d\sigma}{dy}\right)_{y=0} \sim s^{\Delta}$

With account of enhanced diagrams only Mueller-Kancheli type diagrams survive



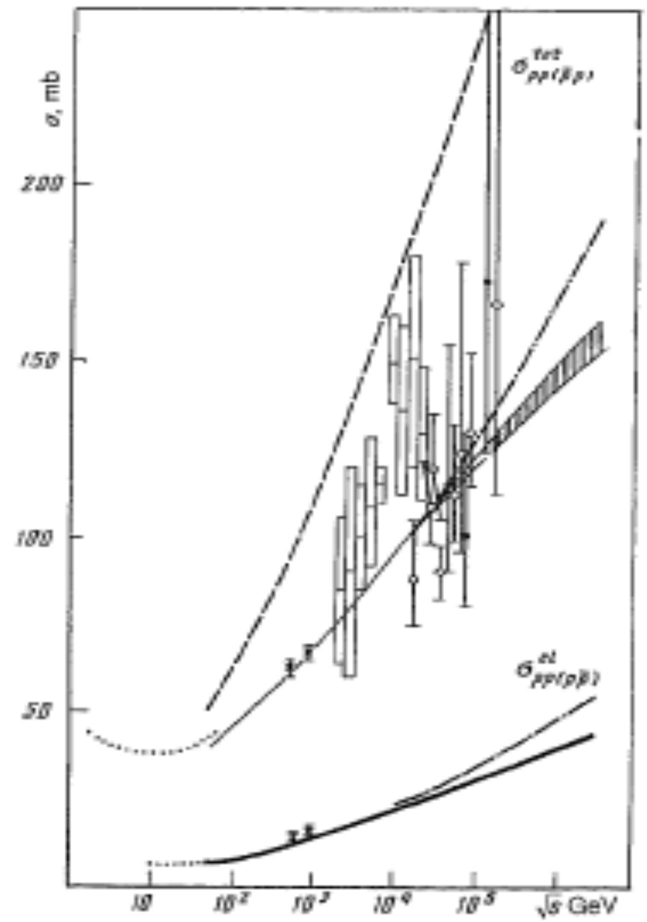


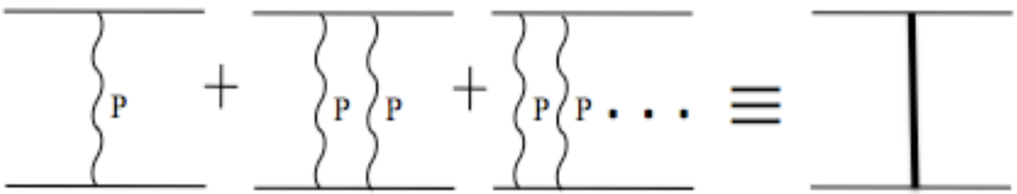
Schwimmer model: $\left(\frac{d\sigma}{dy}\right)_{y=0} \propto \frac{s^{\Delta}}{1+\frac{g_p r_{3P}}{\Delta}\left(s^{\Delta}-1\right)}$

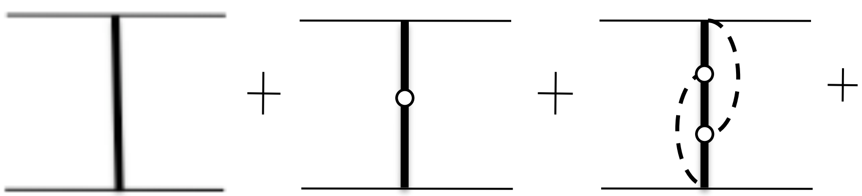
First estimate of the influence of enhanced graphs on physical observables

Dubovikov et al., Nucl. Phys. B123
Kopelovich and Lapidus, Sov. Phys. JETP 44
Dubovikov and Ter-Martirosyan, Nucl. Phys. B124
Kaidalov et al., Sov. J. N.P. 44

Kaidalov et al., Sov. J. N.P. **44**







$$\Delta_{eff} = \Delta - 4\pi G_{PPP}$$

$\Delta \approx 0.2$ $\Delta_{eff} \approx 0.12$

Fir… luence of enhanced graphs … l observables

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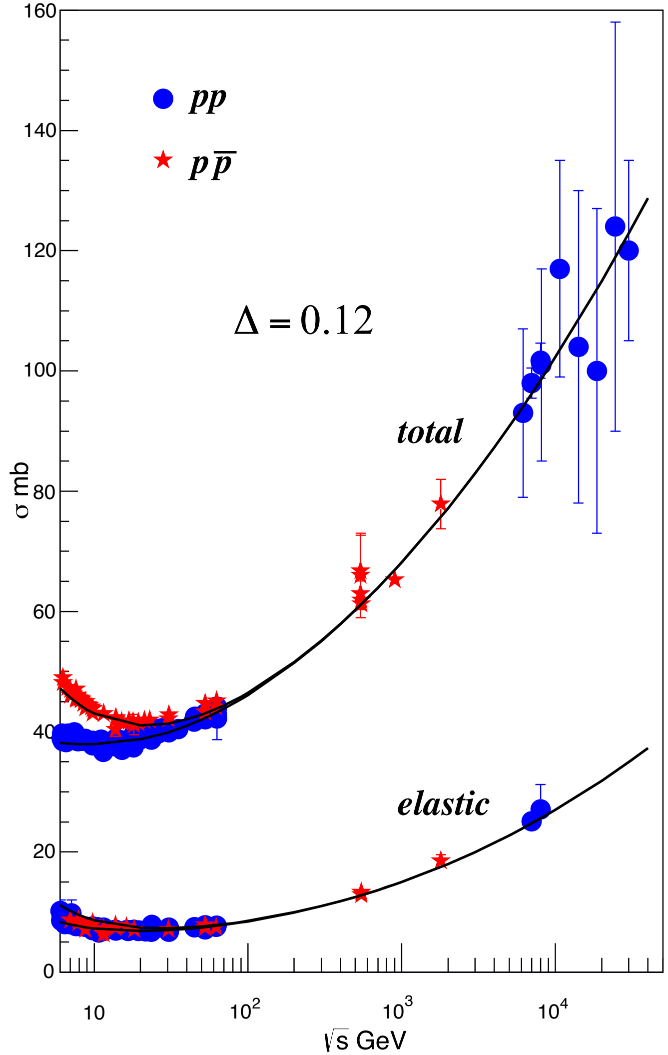
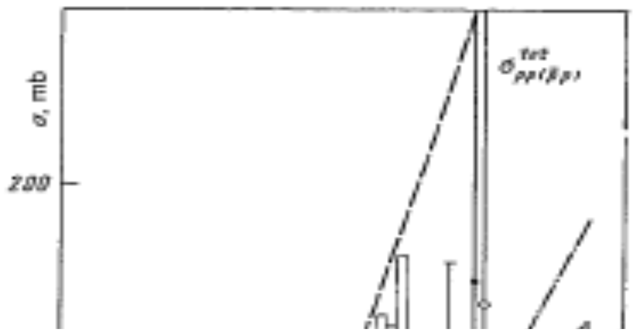
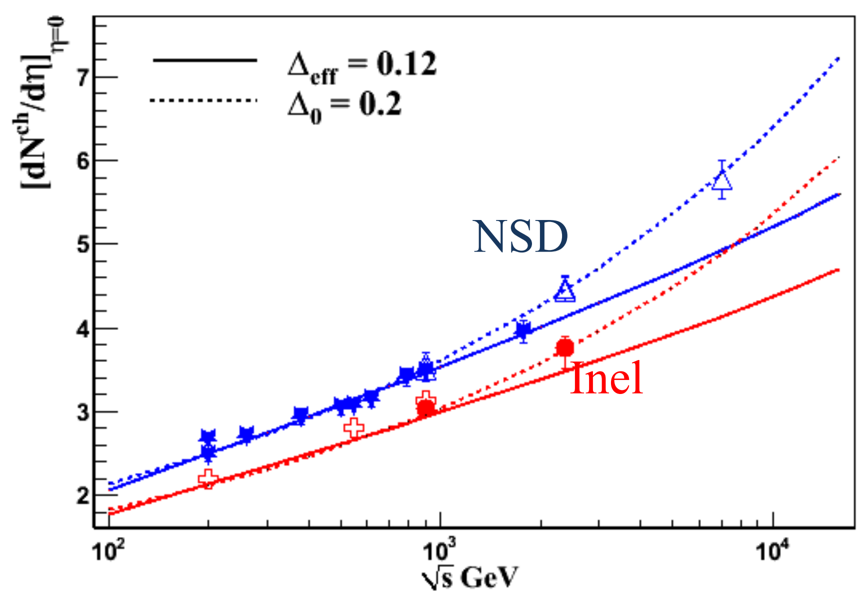


44
/s. B124

Kaidalov et al., Sov. J. N.P. **44**





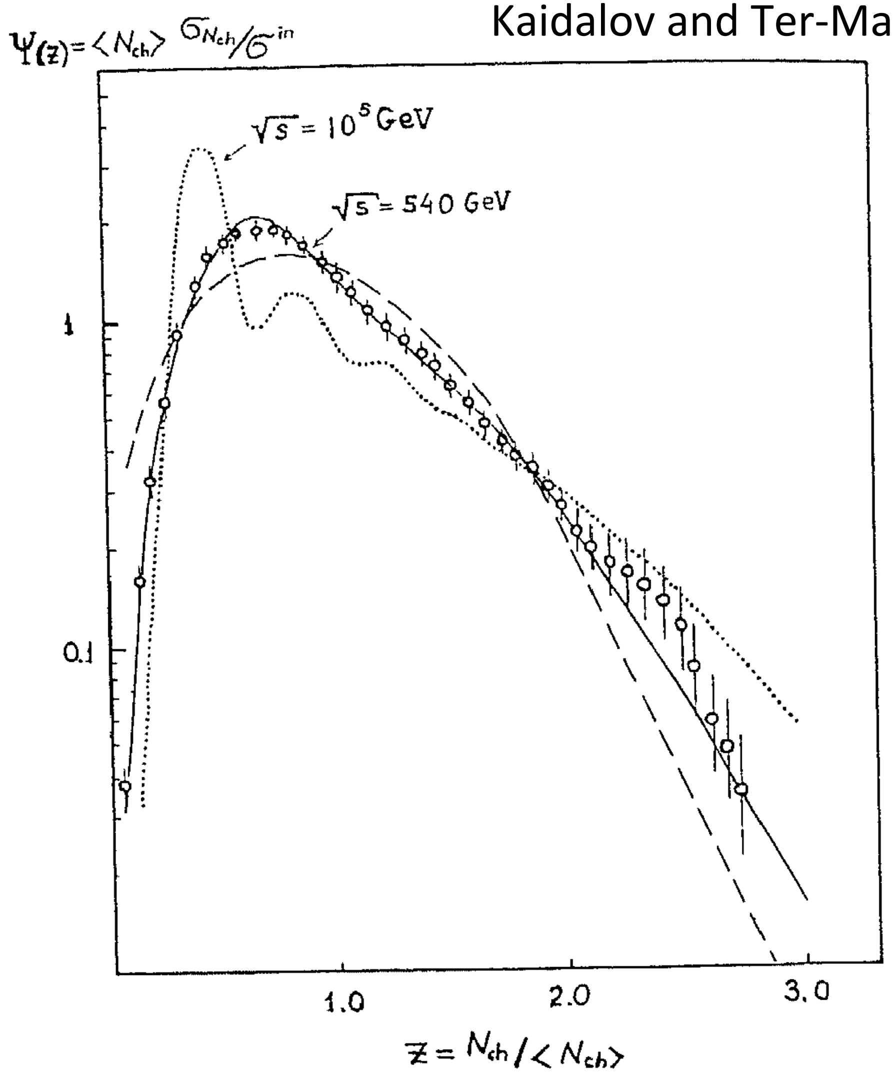
'PP

$\Delta \approx 0.2$ $\Delta_{eff} \approx 0.12$

KNO-scaling violation was predicted

Number of chains increases with energy ➔ no KNO scaling at high energies

Kaidalov and Ter-Martirosyan 1982



The role of multiple rescattering in hard processes

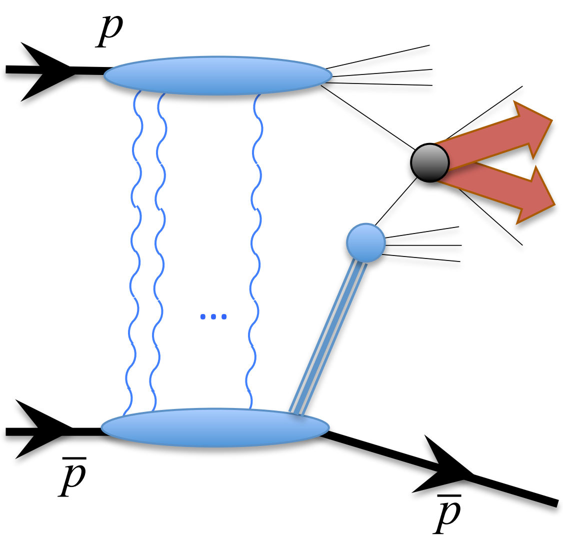
Survival probability (Bjorken 1992) : no other interactions occur except the hard coll. of interest

$$S^2 = \frac{\int |M(s,b)|^2 P(s,b) d^2 b}{\int |M(s,b)|^2 d^2 b}$$

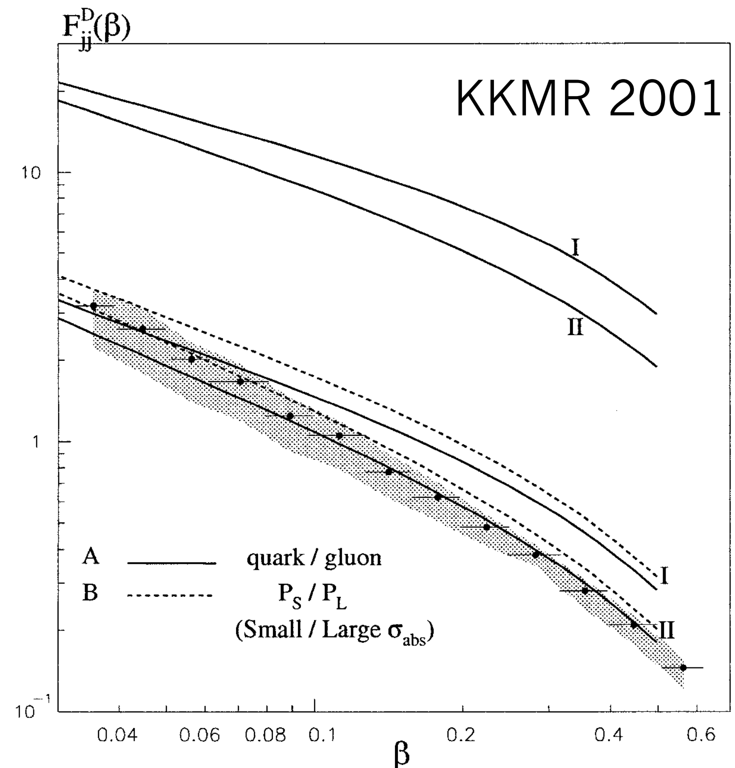
M(s,b) - amplitude (in *b*-space) of the **particular** process

P(s,b) - probability that no inelastic interaction occurs between scattered hadrons

Interplay of "soft" and "hard" dynamics in QCD.



Strong suppression of inelastic diffraction in the region of small b ($P \rightarrow 0$). Inelastic diff. occurs at the periphery of interaction region, where nonperturbative effects are essential.



fraction of *P* momentum carried by parton

The role of enhanced diagrams in AA

At $E < E_c$ ($E_c \sim mR$) an elastic *hA* –scattering amplitude can be considered as successive rescatterings of an initial hadron on nucleons of a nucleus. (Glauber)

At high energies hadronic (nuclear) fluctuations are “prepared” long before the interaction.

For $E > E_c$ there is a coherent interaction of constituents of a hadron with nucleons of a nucleus. *hA* elastic amplitude can be calculated as in the Glauber model, but with account of diffractive intermediate states.
(Gribov)

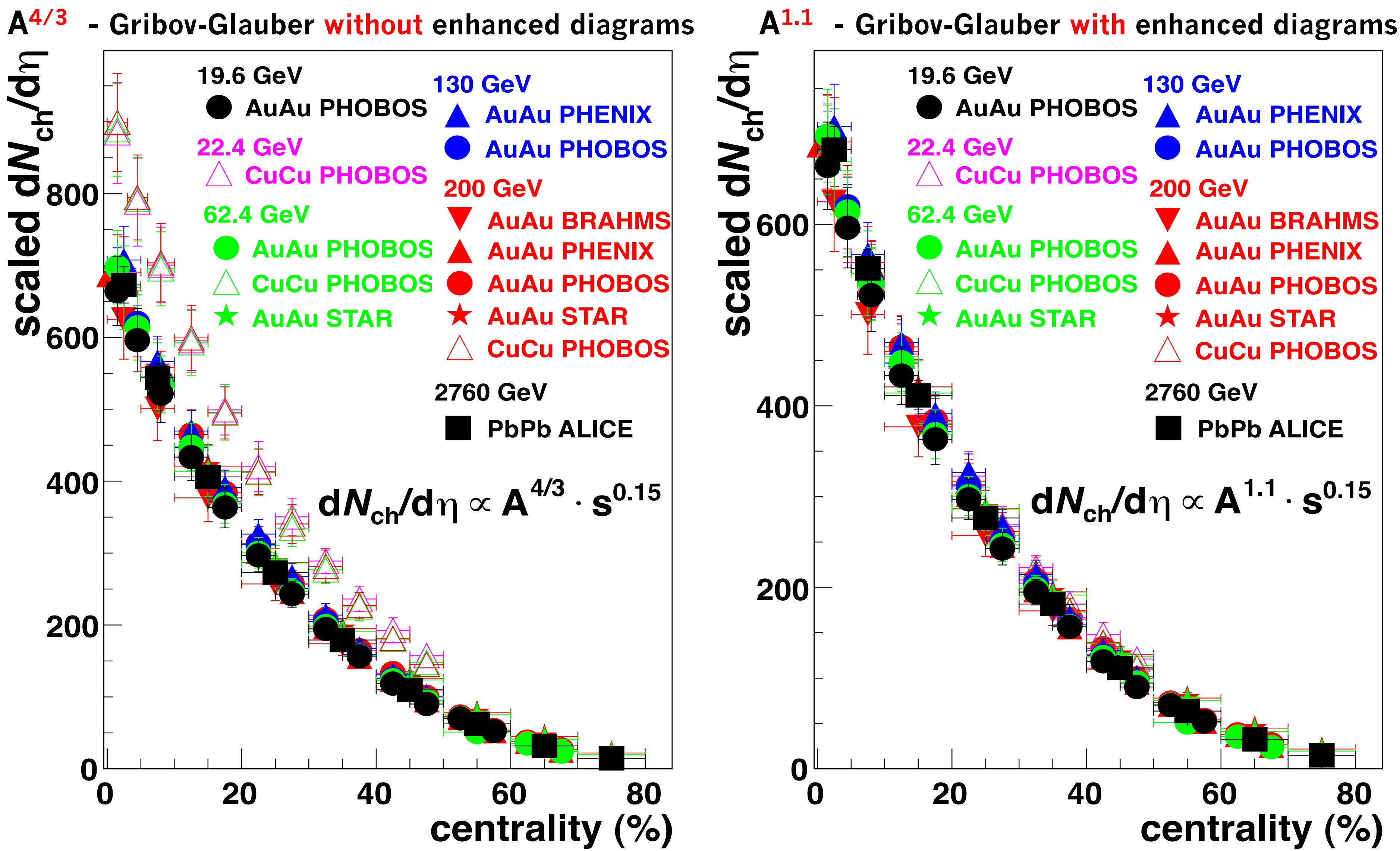
GRIBOV THEORY OF NUCLEAR INTERACTIONS AND PARTICLE DENSITIES AT FUTURE HEAVY-ION COLLIDERS .

A. Capella[a)], A. Kaidalov[a),b)] and J. Tran Thanh Van[a)]

Heavy Ion Phys. 9 (1999, published before the RHIC era!)

For collisions of identical nuclei (SS, PbPb) the $A^{4/3}$–dependence of particle densities of eq. (10) typical for the Glauber model changes to the behaviour A^{δ}. The value of delta is a weak function of energy and it is equal to $\delta \approx 1.1$ at LHC energies.

$dN/d\eta$ depends on $A^{4/3}$ or $A^{1.1}$?



Books and review papers

- P.D.B. Collins, *An introduction to Regge theory and high energy physics*, 1977.
- V.N. Gribov, *The Theory of Complex Angular Momenta*, 2003
- V.N. Gribov, *Strong Interactions of Hadrons at High Energies*, 2009

- M. Baker and K.A. Ter-Martirosyan, *Gribov's Reggeon Calculus: Its physical basis and implications*, Phys. Rep. C**28** (1976) 1.
- K. G. Boreskov, A. B. Kaidalov, and O. V. Kancheli, *Strong Interactions at High Energies in the Reggeon Approach*, Phys. Atomic Nuclei, 69 (2006) 1765.
- L.L. Jenkovszky, *High-energy Elastic Hadron Scattering*, Riv. Nuovo Cim. 10 (1987) 1.
- A.B.Kaidalov. *Regge poles in QCD*. arXiv:hep-ph/0103011.
- A.B.Kaidalov. *Pomeranchuk singularity and high- energy hadronic interactions*, Usp. Fiz. Nauk, 46 (2003) 1153 .
- E.M. Levin, *Everything about Reggeons*, arXiv:hep-ph/9710546.