Strong Electromagnetic Field

EFFECTS

in Ultra-Relativistic

Heavy-Ion Collisions

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Plan:

1. Introduction

2. Strong-field effects in the e^+e^- pair production

3. Strong-field effects in the $\mu^+\mu^-$ pair production

4. Large contribution of the virtual Delbrück scattering to nuclear bremsstrahlung

5. Production of bound-free e^+e^- pairs at LHC

Problems with strong-field (SF) effects were considered in many books and reviews, for example:

Heitler "The quantum theory of radiation," 1954;

Greiner, Müller, Rafelsky "QED of strong fields," 1985

Baur, Hencken, Trautmann. Phys. Rep. 453, 1 (2007)

Baltz et al. Phys. Rep. 458, 1 (2008)

This report is based mainly on papers (written at **Basel, Dresden, Jena, Heidelberg, Leipzig and Novosibirsk Universities**):

Ginzburg, Jenschura, Karshenboim, Krauss, Serbo, Soff. *Phys. Rev. C 58, 3565 (1998)*;
Ivanov, Schiller, Serbo. *Phys. Lett. B 454, 155 (1999)*;

Lee, Milstein, Serbo. *Phys. Rev. A* **65**, *022102 (2002)*;
Jentschura, Hencken, Serbo. *Eur. Phys. J. C* **58**, *281 (2008)*;
Jentschura, Serbo. *Eur. Phys. J.* **C 64**, 309 (2009);

Artemyev, Jentschura, Serbo, Surzhykov. *Eur. Phys. J.* **C 72**, 1935 (2012).
Artemyev, Serbo, Surzhykov. *in progress (2013)*.

1. Introduction

For the RHIC and LHC colliders, the charge numbers of nuclei $Z_1 = Z_2 \equiv Z$ and their Lorentz factors $\gamma_1 = \gamma_2 \equiv \gamma$ are given as follows:

Collider	Z	γ
RHIC, Au-Au	79	108
LHC, Pb-Pb	82	3000

Only a few EM processes are related to Fundamental Physics, but

some of EM processes are of great importance mainly for two reasons:

they are **dangerous** or they are **useful**.

Two examples:

1) The e^+e^- pair production. The number of the produced electrons is so huge that some of them can be captured by nuclei, that immediately leads to loss of these nuclei from the beam. Thus, this very process is determined mainly **the life time of the beam and a possible luminosity of a machine**.

2) Coherent bremsstrahlung, not ordinary bremsstrahlung

$$Z_1 \, Z_2 \to Z_1 \, Z_2 \, \gamma$$

but coherent one! The number of the produced photons at the RHIC is so huge in the region of the infrared light, that this process can be used for **monitoring beam collisions**:

R. Engel, A. Schiller, V.G. Serbo. A new possibility to monitor collisions of relativistic heavy ions at LHC and RHIC, Particle Accelerators **56**, *1 (1996)*

D. Trbojevic, D. Gasner, W. MacKay, G. McIntyre, S. Peggs, V. Serbo, G. Kotkin. Experimental set-up to measure coherent bremssrahlung and beam profiles in RHIC. 8th European Particle Accelerator Conference (EPAC 2002, 3–7 June, 2002, Paris) p. 1986

It means that various EM processes

have to be estimated

(their cross sections and distributions)

not to miss

something interesting or dangerous.

How **strong** is nuclear field?

The typical electric field of nucleus is of the order of

$$\mathcal{E} \sim \frac{Ze}{\rho^2}\gamma = \gamma\, Z\alpha\, \mathcal{E}_{\text{Schwinger}} \ \text{ at } \ \rho = \frac{\hbar}{m_e c},$$

$$\mathcal{E}_{\text{Schwinger}} = \frac{m_e^2 c^3}{e\hbar} = 1.3 \cdot 10^{16} \ \frac{\text{V}}{\text{cm}},$$

therefore,

$$\frac{\mathcal{E}}{\mathcal{E}_{\text{Schwinger}}} \ \sim \mathbf{60} \ \text{ for RHIC and } \ \sim \mathbf{1800} \ \text{ for LHC},$$

but interaction time is **very short**.
As a result, one can use **Perturbation Theory**, but the perturbation parameter $\mathbf{Z}\alpha \approx \mathbf{0.6}$ for Au-Au and Pb-Pb collisions.

2. Strong-field effects in the e^+e^- pair production

The cross section of **one pair production in the Born approximation** (described by Feynman diagram of Fig. 1)

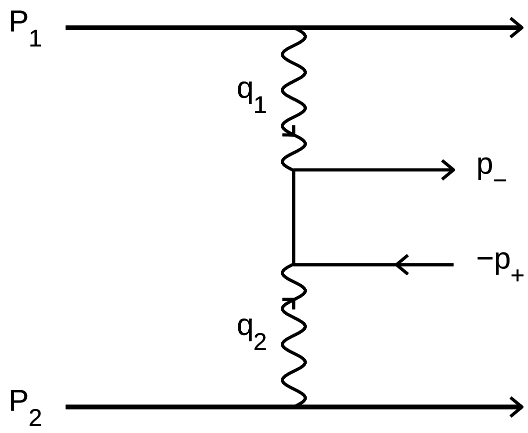

Fig. 1

with **two photon production** was obtained many years ago by

Landau, Lifshitz (1934) and *Racah (1937)*:

$$\sigma_{\mathrm{Born}} = \sigma_0 \left[L^3 - 2.198\, L^2 + 3.821\, L - 1.632 \right] ,$$

where

$$\sigma_0 = \frac{28}{27\pi} \frac{(Z_1 \alpha Z_2 \alpha)^2}{m_e^2} , \quad \alpha = \frac{1}{137} , \quad L = \ln(\gamma_1 \gamma_2) \gtrsim 10 ,$$

$\boldsymbol{m_e}$ is the electron mass and $c = 1,\ \hbar = 1$.

Since **the parameter $Z\alpha$ is not small**

the **whole** series in $Z\alpha$ **has to be summed**

to obtain the cross section with sufficient accuracy.

Fortunately, there is an **important small parameter**

$$\frac{1}{L} < 0.11\,, \qquad L = \ln(\gamma^2)\,,$$

and therefore, in some (**but not in all!**) cases it is sufficient to calculate the corrections
in **the leading logarithmic approximation (LLA) only**.

Note!

In the literature, there were **a lot of controversial and incorrect** statements in papers devoted to this subject.

For example, three groups had published papers with **the wrong statement that $Z\alpha$ corrections are absent in this process:**

B. Segev, J.C. Wells, Phys. Rev. A 57 (1998) 1849; physicsr9805013;
A.J. Baltz, L. McLerran, Phys. Rev. C 58 (1998) 1679;
U. Eichmann, J. Reinhardt, W. Greiner, nucl-thr9806031.

This mistake was criticize by

D.Yu. Ivanov, A. Schiller, V.G. Serbo. Phys. Lett. B 454 (1999) 155;
R.N. Lee, A.I. Milstein. Phys. Rev. A 61 (2000) 032103;
Phys. Rev. A 64 (2001) 032106 (2001).

Further **critical remarks and references** can be found in

Lee, Milstein, Serbo. Phys. Rev. A **65**, 022102 (2002);
Aste, Baur, Hencken, Trautmann, Scharf. Eur. Phys. J. C23 (2002) 545;
Jentschura, Hencken, Serbo. Eur. Phys. J. C58 (2008) 281;
Jentschura, Serbo. Eur. Phys. J. C 64 (2009) 309;
M. Klusek-Gawenda, A. Szczurek. Phys.Rev.C82 (2010) 014904.

The exact cross section for **one pair production σ_1** can be written in the form

$$\sigma_1 = \sigma_{\mathrm{Born}} + \sigma_{\mathrm{Coul}} + \sigma_{\mathrm{unit}}\,,$$

where **two different types of SF-corrections** have been distinguished.

2.1. Results for the SF-corrections

The **Coulomb** corrections σ_{Coul} correspond to **multi-photon exchanges** of the produced $e^{\pm}$ with the nuclei:

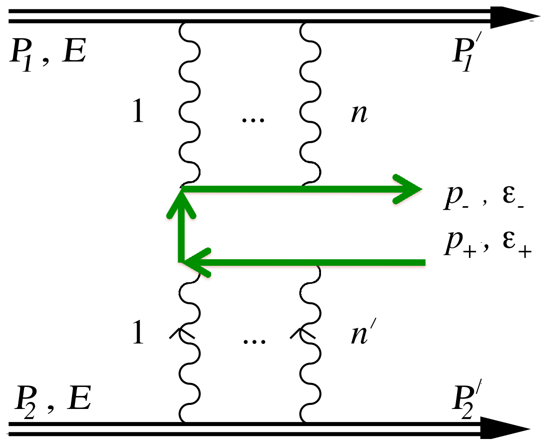

Fig. 2

$$\sigma_{\text{Coul}} = -A(Z\alpha)\,[L^2 - B(Z\alpha)\,L]\,\sigma_0\,,$$

where the leading coefficient

$$A(Z\alpha) = 6f(Z\alpha) = 6\,(Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + (Z\alpha)^2)} \approx 1.9$$

was calculated about ten years ago by
D.Yu. Ivanov, A. Schiller, V.G. Serbo. Phys. Lett. B 454 (1999) 155
and next-to-leading coefficient $B(Z\alpha) \approx 5.5$ was calculated by
R.N. Lee, A.I. Milstein. ЖЭТФ 136 (2009) 1121.

It was also shown by ISS that **the Coulomb corrections disappear** for **large transverse momenta of the produced leptons**,
at $p_{\pm\perp} \gg m_e$.

The **unitarity** corrections σ_{unit} correspond to the exchange of the virtual light-by-light blocks between the nuclei (Fig. 3)

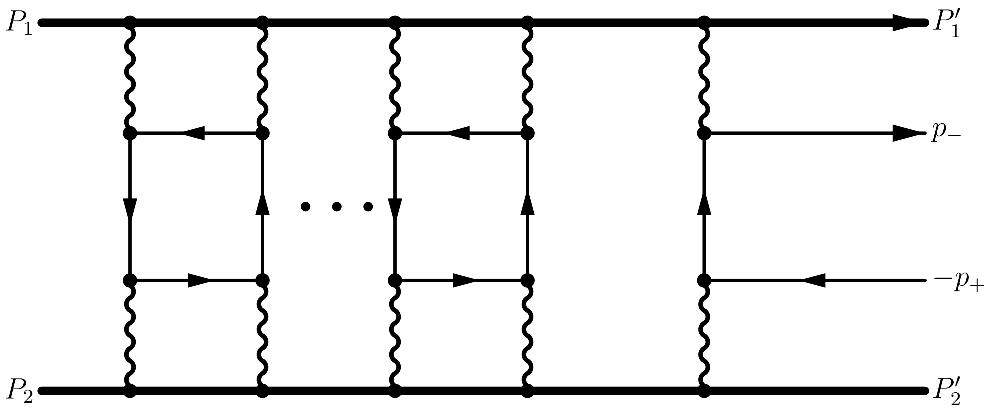

Fig. 3

They were calculated by *R.N. Lee, A.I. Milstein, V.G. Serbo. Phys. Rev. A 65 (2002) 022102*
and updated by
U.D. Jentschura, K. Hencken, V.G. Serbo. EPJ C58 (2008) 281.

It was found that **the Coulomb corrections are about 10 %** while **the unitarity corrections are about two times smaller**:

Coulomb and unitarity corrections to the e^+e^- pair production

Collider	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{unit}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$[Baltz]
RHIC, Au-Au	-10%	-5.0%	-17%
LHC, Pb-Pb	-9.4%	-4.0%	-11%

In the last column is shown the result of A. Baltz. Phys.Rev. C71 (2005) 024901; Erratum-ibid. C71 (2005) 039901 obtained by numerical calculations using formula for the cross section resulting from "exact solution of the semiclassical Dirac equations". In fact, this formula allows to calculate the Coulomb correction in **the LLA only**, which is insufficient in this case.

Multiple production of e^+e^- pairs

$$Z + Z \to Z + Z + n(e^+e^-)$$

If $Z\alpha$ is small, the corresponding cross section grows as L^n:

$$\sigma_n = C_n \frac{(Z\alpha)^{4n}}{m_e^2} L^n , \;\; n \geq 2 ,$$

$$C_2 = 2.21 , \; C_3 = 0.443 , \; C_4 = 0.119 .$$

R.N. Lee, A.I. Milstein, V.G. Serbo. Phys. Rev. A 65 (2002) 022102
U.D. Jentschura, K. Hencken, V.G. Serbo (EPJ C58 (2008) 281)

For large values of $Z\alpha$ there are only numerical calculations of σ_n for a particular values of γ

A. Alscher, K. Henken, D. Trautman, G. Baur. Phys. Rev. C 59 (1999) 811

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2.2. Unitarity corrections and σ_n

For heavy ultra-relativistic nuclei, it is possible to treat the nuclei as **sources of the external field** and calculate the probability of n-pair production $P_n(\rho)$ in collision of two nuclei at a given impact parameter ρ.

The cross section is then found as:

$$\sigma_n = \int P_n(\rho)\, d^2\rho\,.$$

What we know about

$$P_n(\rho)\,?$$

It was realized many years ago that in the Born approximation

$$P_1(\rho) \sim (Z\alpha)^4 L \quad \text{at } \rho \sim 1/m_e$$

and, therefore, this probability **can be greater than 1**
Baur. Phys. Rev. A 42 (1990) 5736.

It means:

1) that one **should take into account the unitarity corrections**, which come from **the unitarity requirement for the S-matrix**;

2) that the cross section for multiple pair production should be large enough.

It was argued in papers

Baur. Phys. Rev. **D 41**, *3535 (1990); Roades-Brown, Wenes. Phys. Rev.* **A 44**, *330 (1991); Best, Greiner, Soff. Phys. Rev* **A 46**, *261 (1992); Henken, Trautman, Baur. Phys. Rev.* **A 51**, *998 (1995)*

that the factorization of the multiple pair production probability is valid with a good accuracy given by **the Poisson distribution**:

$$P_n(\rho) = \frac{[\bar{n}(\rho)]^n}{n!} \mathrm{e}^{-\bar{n}(\rho)} ,$$

where $\bar{n}(\rho)$ is **the average number of pairs**.

It was proved in paper

Bartoš, Gevorkyan, Kuraev, Nikolaev. Phys. Lett. B 538 (2002) 45

by a direct summation of the Feynman diagrams in LLA.

The unitarity requirement is fulfilled by the Poisson distribution, whose sum over n gives one.

The probability for producing **one** pair, given in perturbation theory by $\bar{n}_e(\rho)$, should be modified to read $\bar{n}_e(\rho) \cdot \exp[-\bar{n}_e(\rho)]$.

For the one-pair production it corresponds to replacement:

$$\sigma_{e^+e^-} = \int \bar{n}_e(\rho)\, \mathrm{d}^2\rho \quad \to \quad \sigma_{e^+e^-} + \sigma^{\mathrm{unit}}_{e^+e^-} = \int \bar{n}_e(\rho)\, \mathrm{e}^{-\bar{n}_e(\rho)}\, \mathrm{d}^2\rho\,,$$

where

$$\sigma^{\mathrm{unit}}_{e^+e^-} = -\int \bar{n}_e(\rho) \left[1 - \mathrm{e}^{-\bar{n}_e(\rho)}\right] \mathrm{d}^2\rho$$

is the **unitarity** correction.

The main contribution to $\sigma_{e^+e^-}$ comes from $\rho \gg 1/m_e$,
But, the main contribution to $\sigma^{\mathrm{unit}}_{e^+e^-}$ comes from $\rho \sim 1/m_e$.

The function $\bar{\boldsymbol{n}}_e(\rho)$ is a very important quantity for the evaluation of unitarity corrections.

It was found for $\gamma \gg 1$ in closed form
(taken into account $(Z\alpha)^n$ terms exactly) by
Baltz, McLerran. Phys. Rev. C 58 (1998) 1679;
Segev, Wells. Phys. Rev. A 57 (1998) 1849;
Baltz, Gelis, McLerran, Peshier. Nucl. Phys. A 695 (2001) 395 .

The problem of its **proper regularization** was solved by
Lee, Milstein. Phys. Rev. A 64 (2001) 032106.

But! The obtained close form for $\bar{\boldsymbol{n}}_e(\rho)$ is, in fact, a **nine-fold integral** and its calculation is **very laborious**.

A simpler approximate expression for $\bar{\boldsymbol{n}}_e(\rho)$ is very desirable.
The functional form of this function in the region of interest reads

$$\bar{n}_e(\rho, \gamma, Z) = (Z\alpha)^4 F(x, Z) \, [L - G(x, Z)] \, , \quad L = \ln(\gamma^2) \, , \; x = m_e \, \rho \, .$$

The simple analytical expressions for functions $F(x,Z)$ and $G(x,Z)$ is obtained by Lee, Milstein, Serbo (2002) only at large values of the impact parameters, $\boldsymbol{\rho \gg 1/m_e}$.

On the other hand, for the calculation of the unitarity corrections we need $F(x,Z)$ and $G(x,Z)$ in the range $\boldsymbol{\rho \sim 1/m_e}$.

In the paper by *Lee, Milstein. J.Exp.Theor.Phys.104 (2007) 423*: detailed consideration of the function $F(x,Z)$ including **tables and compact integral form — (“only” a five-fold integral)**.

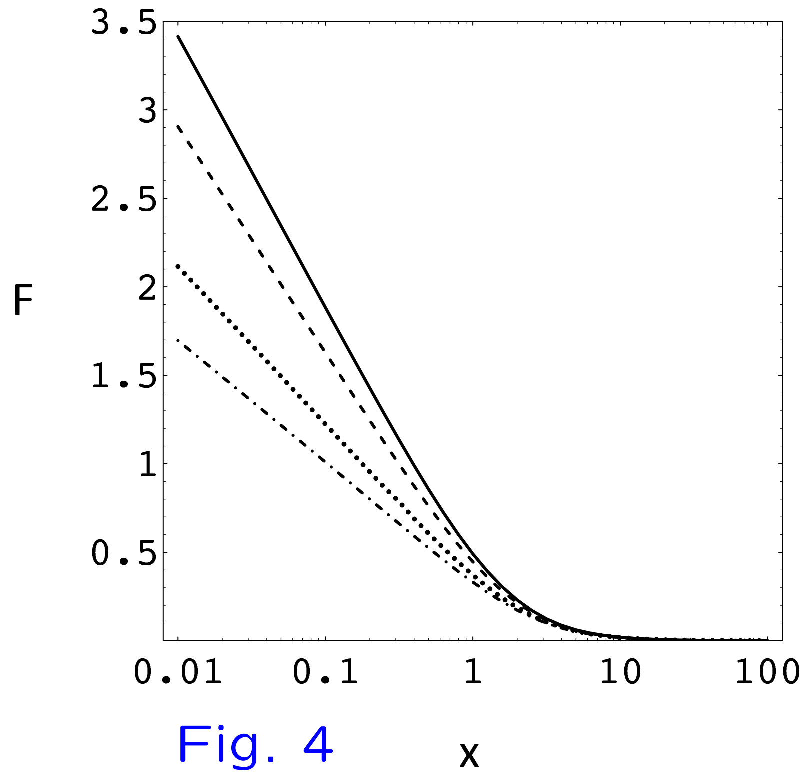


Fig. 4

As an example, in Fig. 4 it is shown the function $F(x = m_e\rho, Z)$ from *Lee, Milstein* paper, for $Z = 92$ (dash-dotted line) , $Z = 79$ (dotted line), $Z = 47$ (dashed line), and the Born approximation (solid line).

Using some numerical calculations for the function $\bar{\boldsymbol{n}}_e(\boldsymbol{\rho}, \boldsymbol{\gamma}, \boldsymbol{Z})$, **we find a simple approximation**

Jentschura, Hencken, Serbo. *EPJ C58 (2008) 281*

$$G(x, Z) \approx 1.5 \ln(x + 1.4) + 1.9 \,.$$

As a result, the approximate expression

$$\bar{n}_e(\rho, \gamma, Z) = (Z\alpha)^4 \, F(x, Z) \; [L - 1.5 \ln(x + 1.4) - 1.9] \; ,$$

$$L = \ln\left(\gamma^2\right), \quad x = m_e \, \rho$$

with the function $F(x, Z)$ from the paper of Lee, Milstein (2006) can be used for calculation of unitarity corrections with an accuracy on the order of few percents.

3. Strong-field effects in the $\mu^+\mu^-$ pair production

Motivation: **muon pair production may be easier for an experimental observation**.

This process was considered in detail by
Hencken, Kuraev, Serbo. Phys. Rev. C 75 (2007) 034903;
Jetschura, Hencken, Serbo. EPJ C58 (2008) 281;
Jentschura, Serbo. Eur. Phys. J. **C 64**, *309 (2009);*
M. Klusek-Gawenda, A. Szczurek Phys.Rev.C82 (2010) 014904.

It was found out that:

1. The Coulomb corrections are **small**. This result **justifies using the Born approximation for numerical simulations** of the discussed process at RHIC and LHC.

2. Unitarity corrections are **large**. **The exclusive** cross section **differs** considerable from its Born value, but an experimental observation is difficult;

3. The inclusive cross section **coincides** with the Born cross section.

Born cross section for one $\mu^+\mu^-$ pair production

Let us consider the production of **one $\mu^+\mu^-$ pair**

$$Z_1 + Z_2 \to Z_1 + Z_2 + \mu^+\mu^- \,,$$

using EPA, but **taking into account** nuclear electromagnetic **form factors** (Fig. 5):

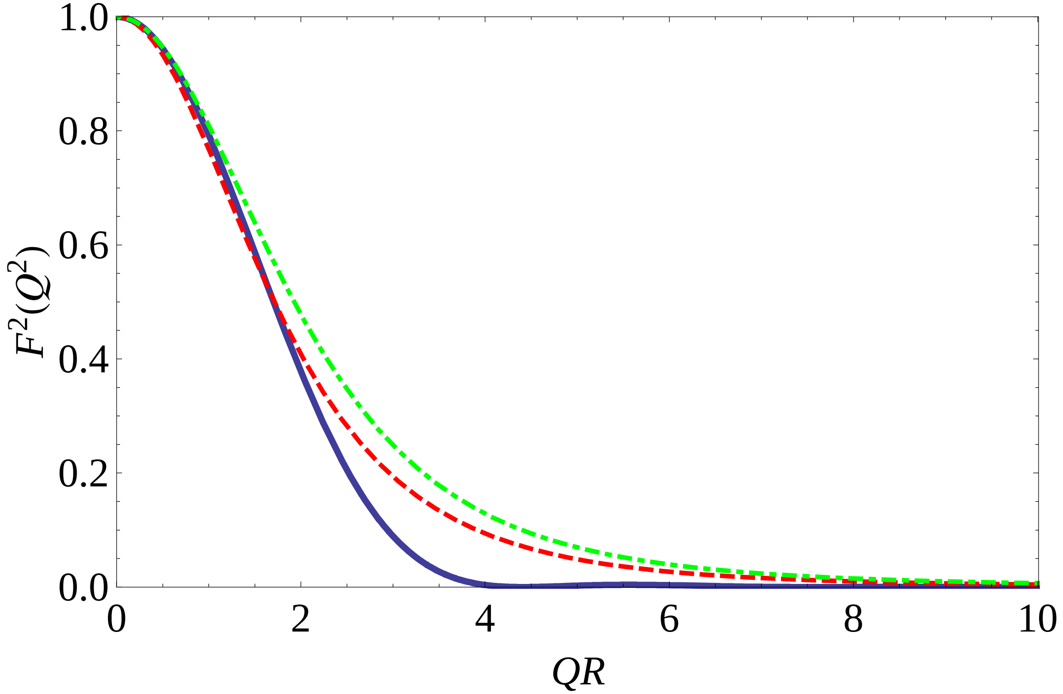


Fig. 5. Realistic (solid line) and simplified (dashed and dot-dashed lines) form factors vs. QR for Au; here R is the radius of nucleus

The Born differential cross section $d\sigma_{\mathrm{B}}$ for the considered process is related to the cross section $\sigma_{\gamma\gamma}$ for **the real $\gamma\gamma \to \mu^+\mu^-$ process** by the equation

$$d\sigma_{\mathrm{B}} = dn_1 dn_2\, d\sigma_{\gamma\gamma}\,,$$

where dn_i is the number of equivalent photons.

As a result, the cross section for the case of **the realistic nuclear form factor** reads:

$$\sigma_{\mathrm{B}} = \mathbf{0.21}\ \text{barn for RHIC and}\ \mathbf{2.5}\ \text{barn for LHC}\,.$$

The accuracy of this calculation is of the order of few percents.

The Coulomb correction corresponds to the Feynman diagram of Fig. 6 with **a multi-photon exchange**.

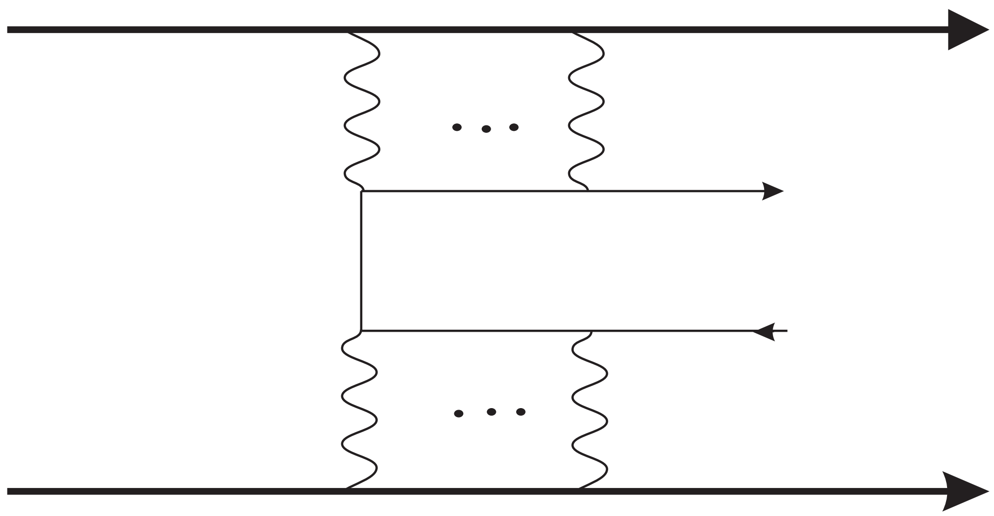


Fig. 6

Estimation: Due to the **restriction of transverse momenta** of additional exchange photons on the level of $1/R$ (**nuclear form factor!**), the effective parameter of the perturbation series is not $(Z\alpha)^2$, **the real suppression parameter** is of the order of

$$\eta_2 = \frac{(Z\alpha)^2}{(R\mu)^2 L}, \quad L = \ln\left(\gamma^2\right), \quad \frac{1}{R} \approx 30 \;\; \text{MeV},$$

which corresponds to **a Coulomb correction of the order of a percent**.

Our recent calculation shows that **Coulomb corrections** to the $\mu^+\mu^-$ pair production is **small**:

Coulomb corrections to the $\mu^+\mu^-$ pair production

Collider	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$ [Baltz]
RHIC, Au-Au	-3.7%	-22%
LHC, Pb-Pb	-1.3%	-14%

In the last column is shown the recent result of A. Baltz. Phys. Rev. C80 (2009) 034901. In fact, this calculations **do not** take into account the nuclear form factors **properly** and, therefore, may be **incorrect**. Their trend contradicts the physical requirement that Coulomb corrections should vanish for an infinite mass of the produced lepton pair, not grow with the lepton mass.

The unitarity correction σ_{unit} to **one muon pair production** is described by the exchange of blocks, corresponding to light-by-light scattering via a **virtual electron loop**, between the nuclei (Fig. 7).

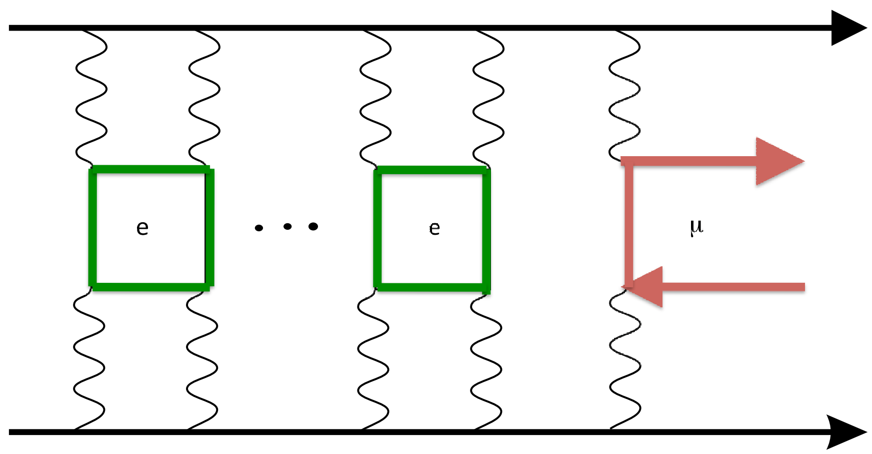


Fig. 7

As usual,

$$\sigma_{\mathrm{B}} = \int_{2R}^{\infty} P_{\mathrm{B}}(\rho)\ d^2\rho \to \sigma_{\mathrm{B}} + \sigma_{\mathrm{unit}} = \int_{2R}^{\infty} P_{\mathrm{B}}(\rho)\, \mathrm{e}^{-\bar{n}_e(\rho)}\, d^2\rho$$

and

$$\sigma_{\mathrm{unit}} = -\int_{2R}^{\infty} \left[1 - \mathrm{e}^{-\bar{n}_e(\rho)}\right]\ P_{\mathrm{B}}(\rho)\, d^2\rho$$

is **the unitarity correction for the exclusive production of one muon pair**. **In LLA we find**

$$\delta_{\mathrm{unit}} = \frac{\sigma_{\mathrm{unit}}}{\sigma_{\mathrm{B}}} = -49\ \%\ \text{for the Pb-Pb collisions at LHC.}$$

The correction is large because there is a logarithmic enhancement from the region of small impact parameters $2R < \rho < 1/m_e$.

It is seen that unitarity corrections are **large**, in other words, **the exclusive production of one muon pair differs considerable from its Born value**.

However, the experimental study of the **exclusive** muon pair production seems to be **a very difficult task**.

Indeed, this process requires that the muon pair should be registered **without any electron–positron pair production**, including $e^{\pm}$ emitted at **very small angles**.

Otherwise, the corresponding inclusive cross section will be **close to the Born cross section** (for detail see

Hencken, Kuraev, Serbo. Phys. Rev. C 75 (2007) 034903).

4. Large contribution of the virtual Delbrück scattering into nuclear bremsstrahlung

4.1. Introduction

Ordinary nuclear bremsstrahlung

The ordinary nuclear bremsstrahlung without excitation of the final nuclei is given by Feynman diagrams of Fig. 8

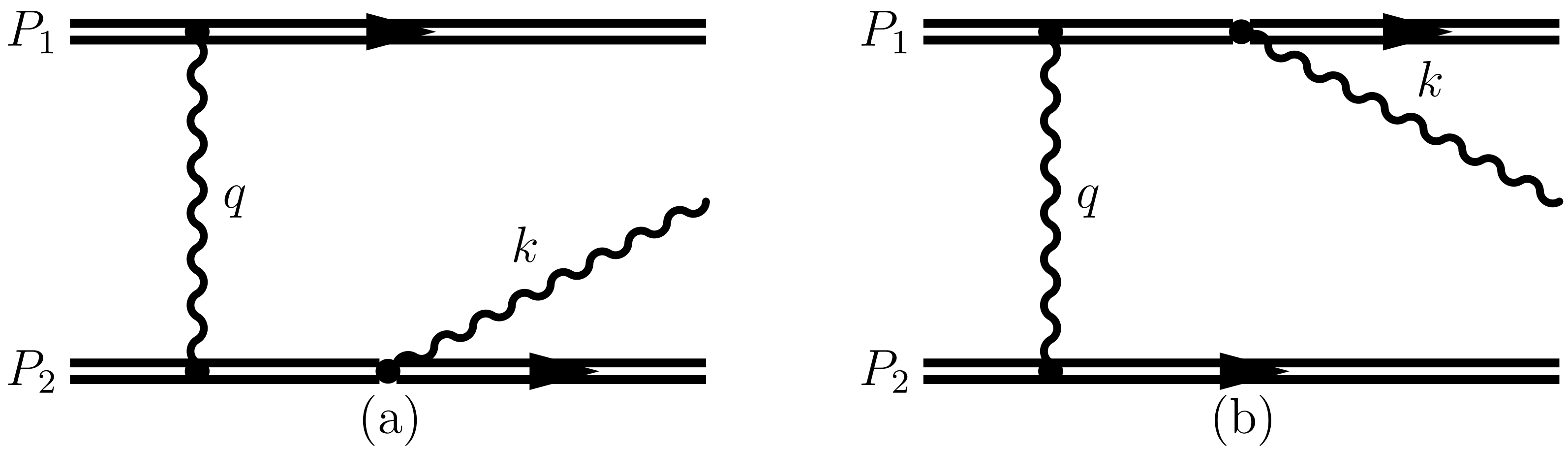


Fig. 8

and was known in detail many years ago

Bertulany, Baur Phys. Rep. **163**, *299 (1988)*

It can be described as **the Compton scattering** of the equivalent photon off opposite nucleus:

$$d\sigma_{\text{br}} = d\sigma^a_{\text{br}} + d\sigma^b_{\text{br}}\,,$$

and

$$d\sigma^a_{\text{br}} = dn_1\; d\sigma_{\text{C}}(\omega, E_\gamma, E_2, Z_2)\,.$$

Here, dn_1 is the number of equivalent photons emitted by nucleus 1 and $d\sigma_{\text{C}}(\omega, E_\gamma, E_2, Z_2)$ is the differential cross section for the Compton scattering off nucleus Z_2.

Now a little more about **method of equivalent photons** or **Weizsäcker-Williams** method....

Carl Friedrich von Weizsaecker (1912—2007)

Let us consider emission of photons not via the virtual Compton subprocess, but via another one –

the virtual Delbrück scattering subprocess (Fig. 9)

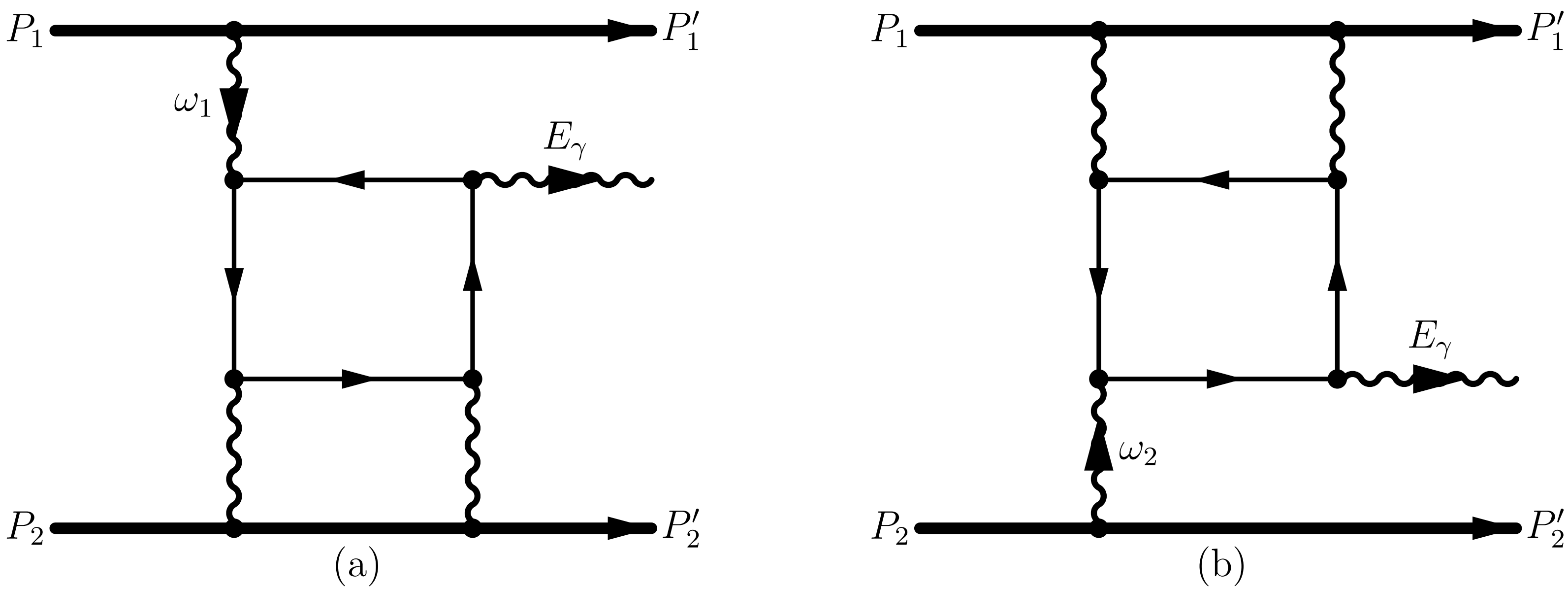


Fig. 9

First note: *Baur, Bertulany Z. f. Phys.* **A 330**, *77 (1988)*

At first sight, this is a process of **a very small cross section** since

$$\sigma \propto \alpha^7.$$

But at second sight, we should **add a very large factor**

$$Z^6 \sim 10^{11}$$

and take into account that the cross section **scale** is

$$1/m_e^2\,.$$

And the last, but not the least, we found that this cross section has **an additional logarithmic enhancement** of the order of

$$L^2 \gtrsim 100\,, \quad L = \ln\left(\gamma^2\right).$$

Thus, the estimate is

$$\sigma \sim \frac{(Z\alpha)^6\,\alpha}{m_e^2}\,L^2\,.$$

Our analytical result

Ginzburg, Jentschura, Serbo, *Phys. Lett.* B **658**, 125 (2008);
Ginzburg, Jentschura, Serbo, *Eur. Phys. J.* C **54**, 267 (2008)

$$\sigma = C \frac{(Z\alpha)^6 \, \alpha}{m_e^2} L^2$$

with

$$C \approx 0.4 \, .$$

This cross sections is **considerably larger** than that for ordinary nuclear bremsstrahlung in the photon energy range:

$$m_e \ll E_\gamma \ll m_e \, \gamma \, .$$

Thus, the discussed cross section for Au-Au collisions at the RHIC collider is

$$\sigma = \mathbf{14 \ \ barn}$$

and for Pb-Pb collisions at the LHC collider is

$$\sigma = \mathbf{50 \ \ barn}\,.$$

That is quite a serious number!

Note for comparison, that the last cross section is **6 times larger than for the total hadronic/nuclear cross section in Pb–Pb collisions**, which is roughly **8 barn**.

Few words about calculations

First of all about Delbrück scattering (DS)

The DS is an elastic scattering of a photon in the Coulomb field of a nucleus via a **virtual electron-positron loop** (Fig. 10)

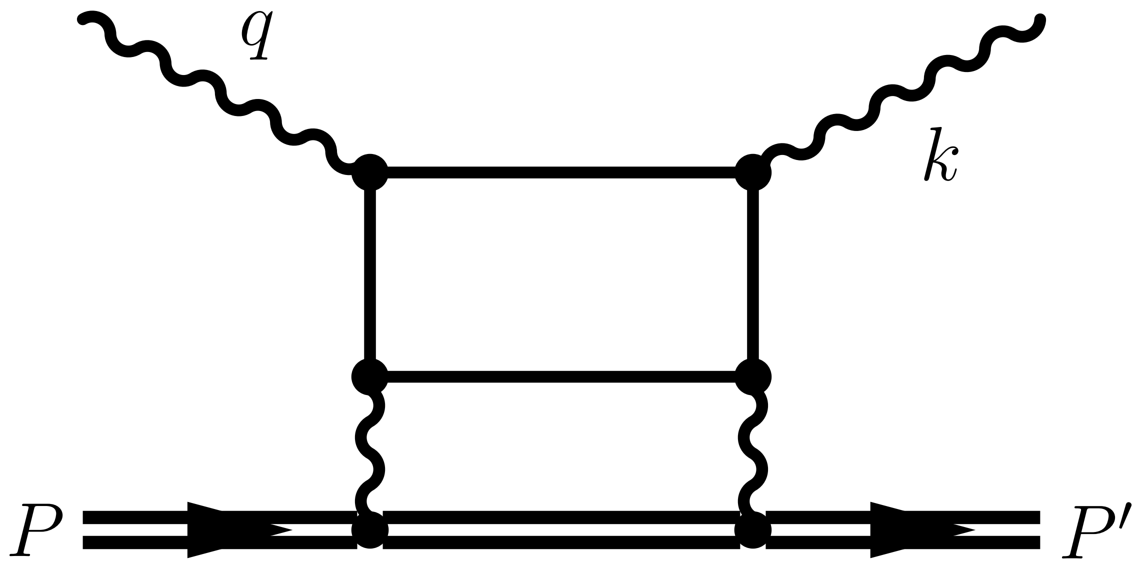


Fig. 10

Its properties are well known

see review *Milstein, Schumacher*, *Phys. Rep.* **243**, *183 (1994)*

The total cross section of this process **vanishes** at small energies

$$\sigma_{\mathrm{D}}(\omega_L, Z) \sim (Z\alpha)^4 \frac{\alpha^2}{m^2} \left(\frac{\omega_L}{m}\right)^4 \text{ at } \omega_L = qP/M \ll m\,, \quad m \equiv m_e\,,$$

but **tends to constant** at $\omega_L \gg m$.
In the lowest order of the perturbative theory this constant is

$$\sigma_{\text{D}}^{(0)}(Z) = 1.07\,(Z\alpha)^4\,\frac{\alpha^2}{m^2} \quad \text{at} \quad \omega_L \gg m\,.$$

The Coulomb corrections $\sim (Z\alpha)^{2n}$ **decrease** it significantly

$$\sigma_{\text{D}}(\omega_L, Z)_{\omega_L \gg m} \to \sigma_{\text{D}}(Z) = \frac{\sigma_{\text{D}}^{(0)}(Z)}{r_Z}\,.$$

For example, for DS off the Au ($Z = 79$) and Pb ($Z = 82$) nuclei

$$\sigma_{\text{D}}(Z = 79) = 5.5 \text{ mb}\,, \quad \sigma_{\text{D}}(Z = 82) = 6.2 \text{ mb}\,,$$

this corresponds to $r_{79} =$ **1.7** and $r_{82} =$ **1.8**.

Comparison:

Cross section for **the nuclear Thomson scattering** is

$$\sigma_{\mathsf{T}}(Z) = \frac{8\pi}{3}\frac{Z^4\alpha^2}{M^2}\,,$$

where $M \approx Am_p$.

The ratio

$$\frac{\sigma_{\mathsf{T}}(Z)}{\sigma_{\mathsf{D}}(Z)} = 7.83\, r_Z \left(\frac{m}{\alpha^2 A m_p}\right)^2 \approx \frac{1}{30} \quad \text{for} \quad {}^{208}\mathrm{Pb}$$

is small for heavy nuclei.

The cross section for the nuclear bremsstrahlung is given by (Fig. 11)

$$d\sigma = d\sigma_a + d\sigma_b$$

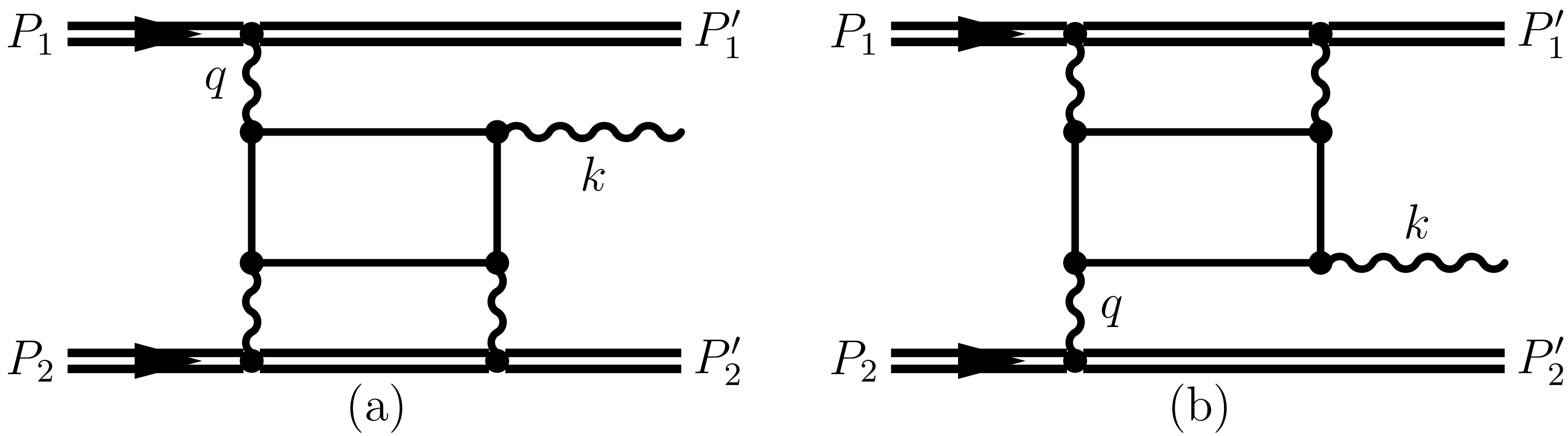


Fig. 11

The interference term is small and can be safely neglected.

In **the equivalent photon approximation**

$$d\sigma_a = dn_1(\omega)\, \sigma_{\mathrm{D}}(\omega_L, Z)\,,$$

where the number of equivalent photons is

$$dn_1(\omega) = 2\,\frac{Z^2\alpha}{\pi}\,\frac{d\omega}{\omega}\,\ln\frac{m\gamma}{\omega}\,.$$

Then integrating this cross section over ω in the region

$$\frac{m}{\gamma} \lesssim \omega \lesssim m\gamma\,,$$

we obtain **the total cross section** in the leading log approximation

$$\sigma = \sigma_a + \sigma_b = 2\frac{Z^2\alpha}{\pi}\,\sigma_{\mathrm{D}}(Z)\,L^2\,, \quad L = \ln\frac{P_1P_2}{2M_1M_2} = \ln(\gamma^2)\,. \qquad (1)$$

We also calculated the energy and angular distribution of photons

Concluding Remarks:

Coulomb and unitarity corrections, and loop effects (virtual Delbrück scattering) are essential for an accurate quantitative understanding of photon and lepton production in ultrarelativistic heavy-ion collisions.

The extremely strong fields encountered in these processes lead to a physical situation not encountered anywhere else in nature, and thus, surprising effects (like loop-dominance over the tree-level graphs for photon production) represent testimonies of the extreme state of matter.

5. Production of bound-free e^+e^- pair at LHC

This part of the report based on the paper:

A. N. Artemyev, U. D. Jentschura, V. G. Serbo, A. Surzhykov

"Bound-free pair production in ultra-relativistic ion collisions at the LHC collider: Analytic approach to the total and differential cross sections"
European Phisical Journal C 72 (2012) 1935

In collaboration with Anton Artemyev,


Andrey Surzhykov

and Ulrich Jentschura

5.1. Introduction

January 2010 — TRENTO (Italy) and ALICE (CERN)
Reiner Schicker

Summer 2011 — Physikalisches Institut der Universiät Heidelberg
Group of **Andrey Surzhykov**,
Questions from **Reiner Schicker**

The Landau-Lifshitz process
difficult for observation
has no clear trigger

A process with **an electron capture** (on the K-shell, for definiteness)

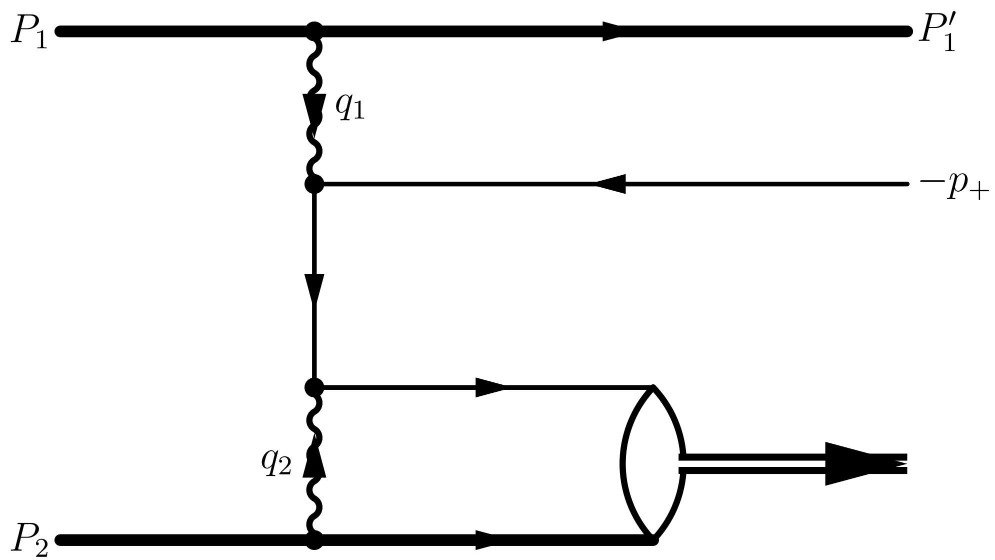
$$Z_1 + Z_2 \to Z_1 + e^+ + (Z_2 + e^-)_{1s} \tag{2}$$

has considerable smaller cross section $\sim$ 100 barn, but it is very important — see reviews and discussions in:

G. Baur et al., Phys. Rep. 364, 359 (2002);

J. M. Jowett, R. Bruce, S. Gilardoni, Proc. of the Particle Accelerator Conf. 2005, Knoxville p. 1306 (2005);

R. Bruce, D. Bocian, S. Gilardoni, J. M. Jowett, Phys. Rev. ST Accel. Beams 12, 071002 (2009).


P_1
P_1'
q_1
$-p_+$
q_2
P_2

WHY?

1. The hydrogen-like ion Pb^{81+} is bent out from the beam.->

limitation of the luminosity $L_{\mathrm{Pb-Pb}} \sim L_{\mathrm{pp}}/10^7$.

2. The secondary beam of down-charged ions hit beam-pipe and deposit a considerable portion of energy at a small spot, which may in turn lead to $->$

the quenching of superconducting magnets

SPS experiments ($\gamma_L = 168$) ultra–relativistic collisions of highly–charged Pb ions with solid–state and gas targets (there was a qualitative agreement with theory):

H. F. Krause et al., Phys. Rev. Lett. 80, 1190 (1998);

H. F. Krause et al., Phys. Rev. A 63, 032711 (2001).

Recently — **the first observation** of the beam losses at RHIC with nuclei of Cu^{29+} (energy 100 GeV/nucleon):

R. Bruce, et al. Phys. Rev. lett. 99 (2007) 144801.

But all these experiments are related to **the total cross section, i.e. to** $p_{+\perp} \lesssim m_e$**.**

This region was studied in the theoretical papers:

R. H. Pratt, Phys. Rev. 117 (1960) 1017;

A. I. Milstein and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. 103 (1993) 1584;

C.K. Agger, A.H. Sørensen. Phys. Rev. A 55 (1997) 402;

H. Meier, Z. Halabuka, K. Hencken, D. Trautmann, G. Baur, Eur. Phys. Jour. C 5 (1998) 287; Phys. Rev. A 63 (2001) 032713;

A. Aste. EPL 81 (2007) 61001;

G. Baur et al. Phys. Rep. 364 (2002) 359

In the LHC collider the bound-free pair production could be measured, in principle, in the following set-up: **a positron is registered in the center detector** with $p_{+\perp} \gg m_e$ **in coincidence with the bent hydrogen-like ion** Pb^{81+} in the very forward detector.

It demands new calculations!

The exact calculations in this region is very difficult.

We present here **the approximate calculations for the ALICE group.**

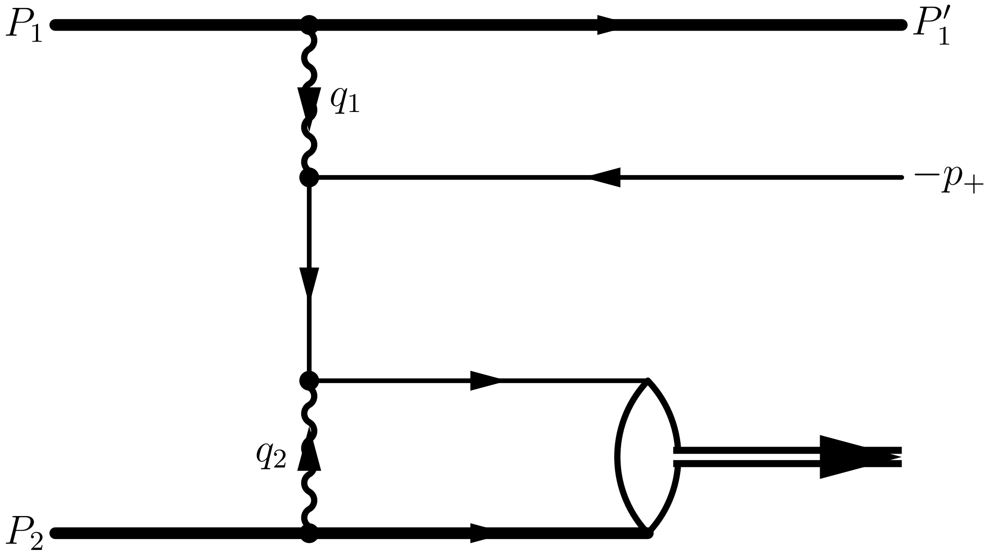
Besides, we present the simple approximate analytical formulae **for the total cross section also.**

5.2. Kinematics

Colliding nuclei of lead:

charges $Z_1 = Z_2 = Z$=82, masses M_1=M_2=M,
4–momenta $P_{1,2} = (E_{1,2}, \mathbf{P}_{1,2})$
Lorentz-factors $\gamma_1 = \gamma_2 = \gamma$=1500

For the virtual photoprocess it is convenient to use the rest frame of the second nucleus in which the first nucleus has Lorentz-factor $\gamma_L = 2\gamma^2 - 1 = 4.5 \cdot 10^6$.



Positrons are observed in **the central detector** with limitations on **a transverse momentum and rapidity** ($m \equiv m_e$):

$$p_{+\perp} \geq p_{\text{min}} \gg m \tag{3}$$

$$y_+ = \frac{1}{2} \ln \frac{\varepsilon_+ + p_{+z}}{\varepsilon_+ - p_{+z}} \approx -\ln \left[\tan \left(\frac{1}{2} \theta_+ \right) \right], \quad -y_{\text{min}} \leq y_+ \leq y_{\text{min}} \tag{4}$$

As a result, **the invariant mass of the lepton pair** is

$$W \approx \sqrt{2p_+p_-} \geq \sqrt{2\, p_{\mathrm{min}}\, m\gamma\, \tan\left(\frac{1}{2}\,\theta_{\mathrm{min}}\right)}\,, \tag{5}$$

the energy of the first virtual photon equals

$$\omega_1 = \frac{1}{2} p_{+\perp}\, \tan\left(\frac{1}{2}\,\theta_+\right) \tag{6}$$

and

$$\min\{\omega_1\} = \frac{1}{2}\, p_{\mathrm{min}}\, \tan\left(\frac{1}{2}\,\theta_{\mathrm{min}}/2\right)\,. \tag{7}$$

The first scenario:

$$p_{\mathsf{min}} = 1 \text{ GeV}\,, \qquad y_{\mathsf{min}} = 1 \tag{8}$$

then

$$\theta_{\mathsf{min}} = 40^\circ\,, W \geq \; 0.75 \text{ GeV}\,, \tag{9}$$

The second scenario:

$$p_{\mathsf{min}} = 0.05 \text{ GeV}\,, \qquad y_{\mathsf{min}} = 1.5\,, \tag{10}$$

then

$$\theta_{\mathsf{min}} = 25^\circ\,, \; W \geq \; 0.13 \text{ GeV}\,. \tag{11}$$

5.3. Method of calculation

EQUIVALENT PHOTON APPROXIMATION (EPA)

The cross section of the discussed process can be presented in the form

$$\mathrm{d}\sigma_{ZZ} = \mathrm{d}n_T\, \mathrm{d}\sigma^T_{\gamma * Z} + \mathrm{d}n_S\, \mathrm{d}\sigma^S_{\gamma * Z}\,, \tag{12}$$

where $\mathrm{d}n_T$ and $\mathrm{d}n_S$ — the number of T and S equivalent photons, and $\sigma^{T,S}_{\gamma * Z}$ — cross sections of the virtual photoprocess:

$$\gamma^* + Z_2 \to e^+ + (Z_2 + e^-)_{1s}\,, \tag{13}$$

these $\sigma^{T,S}_{\gamma * Z}$ depend on the energy of the virtual photon (in the rest frame of the second nucleus) $\omega_L = \frac{q_1 P_2}{M} \approx 2\gamma\omega_1$.

In a good approximation we can simplified the above expression

$$\mathrm{d}\sigma_{ZZ}^{\mathrm{EPA}} = \mathrm{d}n_\gamma(\omega_L)\,\mathrm{d}\sigma_{\gamma Z}(\omega_L, p_{+\perp})\,, \tag{14}$$

where

[Jentschura, Serbo. *Eur. Phys. J.* **C 64**, 309 (2009)]

$$\mathrm{d}n_\gamma(\omega_L) = \frac{Z_1^2\alpha}{\pi}\frac{\mathrm{d}\omega_L}{\omega_L}\left[2\ln\frac{\gamma_L}{\omega_L\,R} - 0.163\right] \tag{15}$$

and $R = 1/(28\ \mathrm{MeV})$ is the radius of nucleus.

PHOTO-PRODUCTION OF BOUND-FREE e^+e^- PIAR

Photo-process

$$\gamma + Z \to e^+ + (Z + e^-)_{1s}\,. \tag{16}$$

has been considered in a number of papers

Meier, Halabuka, Hencken, Trautmann, Baur, Eur. Phys. J. C 5, 287 (1998)

Agger, Sörensen, Phys. Rev. A 55, 402 (1997)

but in the region of small transverse momenta of positrons only. Problems...

Our approach. We start with the Sauter approximation ($Z\alpha \ll 1$):

$$\frac{\mathrm{d}\sigma^{\mathrm{SA}}_{\gamma Z}}{d\Omega_+} = \frac{Z^5\alpha^6}{m^2}\frac{v_+ \sin^2\vartheta_+}{(\gamma_L+1)^4\left(1-v_+\cos\vartheta_+\right)^4} \times\left[v_+^2(\gamma_L+2)\left(1-v_+\cos\vartheta_+\right)-2\frac{\gamma_L-1}{\gamma_L^3}\right]. \qquad (17)$$

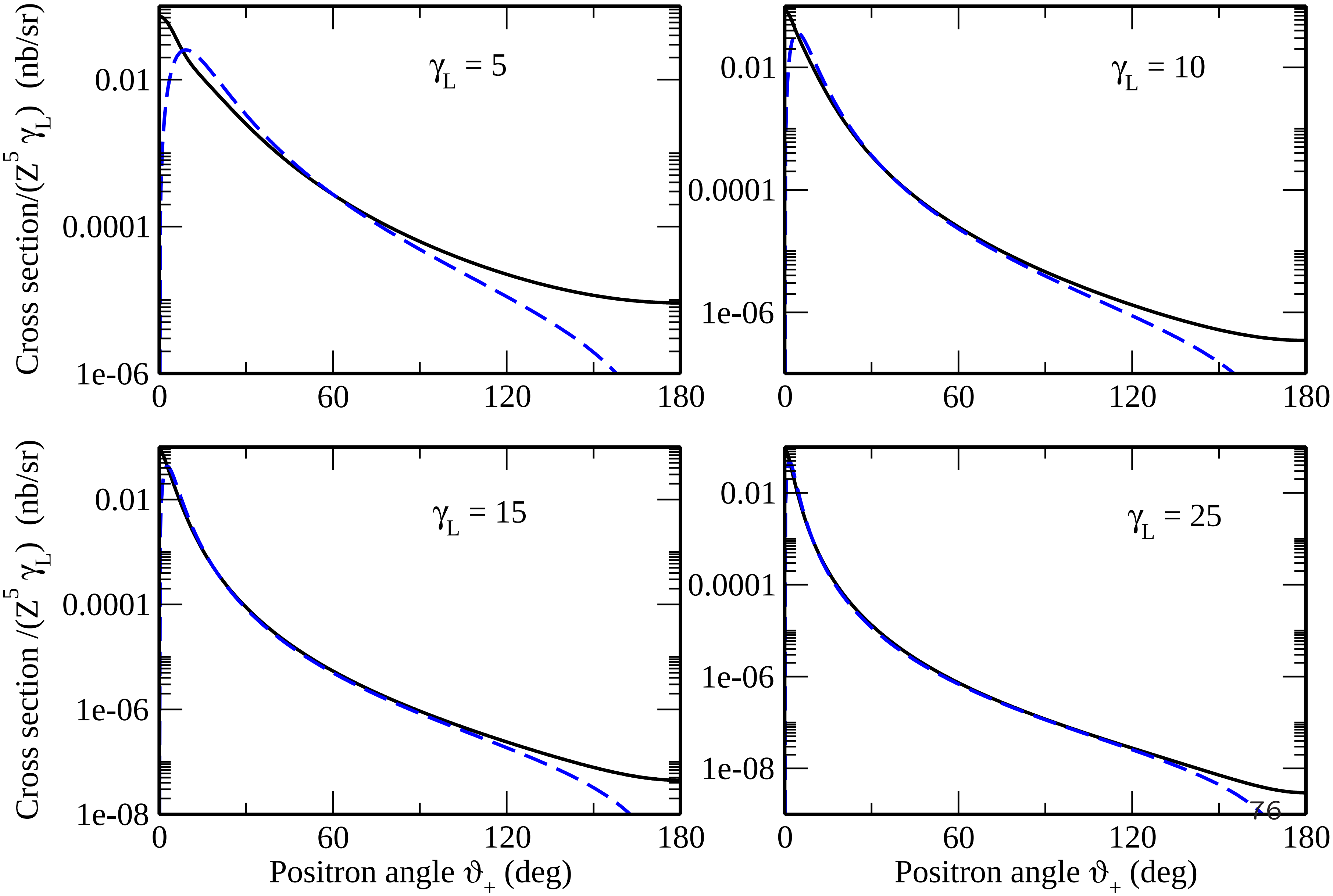
At high-energy and large transverse momentum $\gamma_L m \gg p_{+\perp} = m\gamma_L\vartheta_+ \gg m$ it gives

$$\mathrm{d}\sigma^{\mathrm{SA}}_{\gamma Z}(\omega_L, p_{+\perp}) = 16\pi\,\frac{Z^5\alpha^6}{m^2}\,\frac{m}{\omega_L}\,\frac{m^2\,\mathrm{d}p_{+\perp}}{p_{+\perp}^3}. \qquad (18)$$

Our conjecture: taking into account $Z\alpha$ corrections leads to additional factor $f(Z) \approx 0.22$:

$$d\sigma_{\gamma Z}^{\text{exact}} = f(Z)\, d\sigma_{\gamma Z}^{\text{SA}}\,, \qquad \omega_L \to \infty\,, \tag{19}$$

We check this idea by comparison with the exact numerical calculations for positron energy up to $\varepsilon_+ = 25m$:



Cross section/($Z^5\gamma_L$) (nb/sr)
Cross section /($Z^5\gamma_L$) (nb/sr)
γ_L = 5
γ_L = 10
γ_L = 15
γ_L = 25
0.01
0.0001
1e-06
1e-08
0
60
120
180
Positron angle ϑ_+ (deg)

Besides the direct numerical "proof", yet another confirmation of our conjecture was recently received

Di Piazzo, Milstein, Phys. Rev. A85 (2012) 042107

$$f^{\mathrm{asymp}}(Z) = \frac{2\,(2\eta)^{2\tilde{\gamma}-2}}{\Gamma(2\tilde{\gamma}+1)}\,|\Gamma(\tilde{\gamma}-\mathrm{i}\eta)|^3\,\mathrm{e}^{-2\eta\,\arccos\eta} \approx 0.29\,,$$

where $\eta = Z\alpha$ and $\tilde{\gamma} = \sqrt{1-\eta^2}$. Our assumption is close to this value (it is by about 25 % smaller).

5.4. Results

To estimate the number of events for the possible LHC experiment we integrate the differential cross section taking into account the experimental limitations. It gives:

$$\Delta\sigma_{ZZ} \approx \frac{32}{3} f(Z)\, \frac{(Z\alpha)^7}{m^2}\, \frac{\mathrm{e}^{y_{\mathrm{min}}}}{\gamma} \left(\frac{m}{p_{\mathrm{min}}}\right)^3 L\,, \tag{20}$$

where

$$L = \left[2 \ln\left(\frac{\gamma}{R\, p_{\mathrm{min}}}\right) + 2\, y_{\mathrm{min}} - 1.44\right] \left(1 - \mathrm{e}^{-2y_{\mathrm{min}}}\right) + 4y_{\mathrm{min}}\, \mathrm{e}^{-2y_{\mathrm{min}}}\,.$$

Assuming luminosity $L = 10^{27}$ cm^{-2} s^{-1} we have

For the first scenario: one event per 67 days

For the second scenario: by about 16 events per hour

WHAT NEXT?

We were asked to make estimates for two following processes at LHC:

1. Production of two bound-free e^+e^- pairs;

2. Production of bound-free e^+e^- pair and free $\mu^+\mu^-$ pair.

Preliminary estimates by

A. N. Artemyev, V. G. Serbo, A. Surzhykov, 2013

1. $\sim 40\,000$ **events per hour**

2. $\sim 8\,000$ **events per hour**

THANK YOU FOR YOUR ATTENTION!