Unified description of VMP and DVCS.

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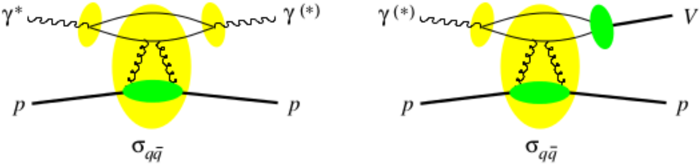
- Two "soft" and "hard" components Pomeron model;
- Balance between "soft" and "hard"

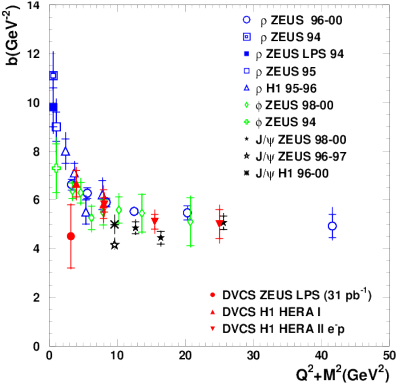
Objectives

- To construct the amplitude that will describe all VMP and (possibly) DVCS reactions;



- One of the incoming particle are virual (doesn't lay on the mass shell).
- The differactive process can be the instrument for investiggating the distributions of low energy partons inside the proton.

One component Pomeron (An idea)

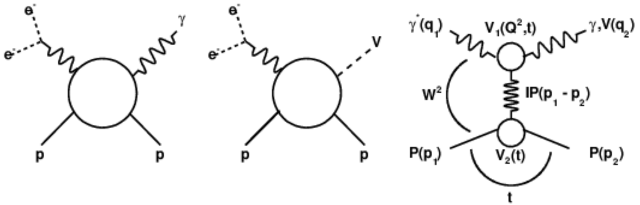


The scale: $\widetilde{Q^2} = Q^2 + M^2$

$$\beta(t, M^2, Q^2) = e^{2\left(\frac{a}{\widetilde{Q^2}} + \frac{b}{2m_N^2}\right)t},$$

$$A = \widetilde{H} e^{-\frac{i\pi\alpha(t)}{2}} \left(\frac{s}{s_0}\right)^{\alpha(t)} e^{2\left(\frac{a}{\widetilde{Q^2}} + \frac{b}{2m_N^2}\right)t},$$

$$\widetilde{H} = \frac{\widetilde{A_0}}{\left(1 + \frac{\widetilde{Q^2}}{Q_0^2}\right)^{n_s}}$$



(a) (b) (c)

By using the norm

$$\frac{d\sigma_{el}}{dt} = \frac{\pi}{s^2}|A(s,t,M,Q^2)|^2,$$

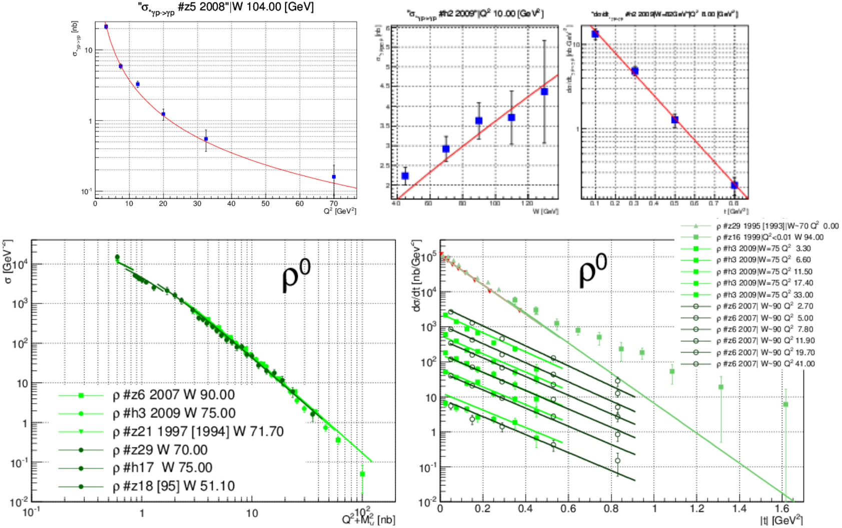
the differential and integrated elastic cross sections are

$$\frac{d\sigma_{el}}{dt} = \frac{A_0^2}{\left(1+\frac{\widetilde{Q^2}}{Q_0^2}\right)^{2n}} \left(\frac{s}{s_0}\right)^{2(\alpha(t)-1)} e^{4\left(\frac{a}{\widetilde{Q^2}}+\frac{b}{2m_N^2}\right)t},$$

where $A_0 = -\frac{\sqrt{\pi}}{s_0}\widetilde{A_0}$,

$$\sigma_{el} = \frac{A_0^2}{\left(1+\frac{\widetilde{Q^2}}{Q_0^2}\right)^{2n}} \frac{\left(\frac{s}{s_0}\right)^{2(\alpha_0-1)}}{4\left(\frac{a}{\widetilde{Q^2}}+\frac{b}{2m_N^2}\right)+2\alpha' \ln\left(\frac{s}{s_0}\right)}.$$

Typical fits



The single-term model has problem with describing both soft (photoproduction, low-Q^2) and hard (electroproduction, high-Q^2) regions for light ρ-meson.

VMP, DVCS HERA data.
The parameters with no uncertaintes specified were fixed.
The DVCS "mass parameter" was fixed to $M = 0$ GeV.
Each type of reaction was fitted separately.

$$A(t,s,\widetilde{Q^2}) = \frac{\widetilde{A_0}}{\left(1+\frac{\widetilde{Q^2}}{Q_0^2}\right)^{n_s}} e^{-\frac{i\pi\alpha(t)}{2}} \left(\frac{s}{s_0}\right)^{\alpha(t)} e^{2\left(\frac{a}{\widetilde{Q^2}}+\frac{b}{2m_N^2}\right)t}$$

	A_0 $\left[\frac{\sqrt{\text{nb}}}{\text{GeV}}\right]$	$\widetilde{Q_0^2}$ $\left[\text{GeV}^2\right]$	n	α_0	α' $\left[\frac{1}{\text{GeV}^2}\right]$	a	b	$\tilde{\chi}^2$
ρ^0	344±376	0.29±0.14	1.24±0.07	1.16±0.14	0.21±0.53	0.60±0.33	0.9±4.3	2.74
ϕ	58±112	0.89±1.40	1.30±0.28	1.14±0.19	0.17±0.78	0.0±19.8	1.3±5.1	1.22
J/ψ	30±31	2.3±2.2	1.45±0.32	1.21±0.09	0.07±0.07	1.72	1.16	0.27
Υ	37±100	0.93±1.75	1.45±0.53	1.29±0.25	0.006±0.6	1.90	1.03	0.4
γ	14.5±41.	0.28±0.98	0.90±0.18	1.23±0.14	0.04±0.71	1.6	1.9±2.5	1.05

DL: $\alpha_s(t) = 1.08 + 0.25t, \quad \alpha_h(t) = 1.40 + 0.1t.$

Two components Pomeron

$$A(s,t,Q^2,M_V^2) = \widetilde{H_s}\, e^{-i\frac{\pi}{2}\alpha_s(t)} \left(\frac{s}{s_{0s}}\right)^{\alpha_s(t)} e^{2\left(\frac{a_s}{\widetilde{Q^2}}+\frac{b_s}{2m_p^2}\right)t}$$

$$+\widetilde{H_h}\, e^{-i\frac{\pi}{2}\alpha_h(t)} \left(\frac{s}{s_{0h}}\right)^{\alpha_h(t)} e^{2\left(\frac{a_h}{\widetilde{Q^2}}+\frac{b_h}{2m_p^2}\right)t}$$

$$\alpha_s(t) = \alpha_{0s} + \alpha'_s t, \quad \alpha_h(t) = \alpha_{0h} + \alpha'_h t.$$

As an input we use the parameters suggested by DL:

$$\alpha_s(t) = 1.08 + 0.25t, \quad \alpha_h(t) = 1.40 + 0.1t.$$

The "softness" or "hardness" of the process depend on the relative, $\widetilde{Q^2}$-dependent weight:

$$\widetilde{H_s} = \frac{\widetilde{A_s}}{\left(1+\frac{\widetilde{Q^2}}{\widetilde{Q_s^2}}\right)^{n_s}}, \quad \widetilde{H_h} = \frac{\widetilde{A_h}\left(\frac{\widetilde{Q^2}}{\tilde{Q}_h^2}\right)}{\left(1+\frac{\widetilde{Q^2}}{\widetilde{Q_h^2}}\right)^{n_h+1}}.$$

Since the fitted parameters $a_{s,h}$ are close to 0, and since we have large ancertainties of the fitted parameters, we have an idea to simplify the model by $2\left(\frac{a_{s,h}}{\widetilde{Q^2}}+\frac{b_{s,h}}{2m_p^2}\right)\rightarrow b_{s,h}$.

The "Reggeometric" combination $2\left(\frac{a_{s,h}}{\widetilde{Q^2}}+\frac{b_{s,h}}{2m_p^2}\right)$ was important for the description of $B(Q^2)$ within the single-term pomeron model (see previous Section), but in the case of two terms the Q^2-dependence of the slope B can be reproduced without this extra combination.
The proper variation of the slope B with $\tilde{Q}^2$ will be provided by the factors $\widetilde{H_s}(\widetilde{Q^2})$ and $\widetilde{H_h}(\widetilde{Q^2})$.

Two components pomeron Amplitude

Thus the scattering amplitude assumes the form:

$$A_{s+h} = \widetilde{H_s}\, e^{-i\frac{\pi}{2}\alpha_s(t)} \left(\frac{s}{s_{0s}}\right)^{\alpha_s(t)} e^{b_s t} + \widetilde{H_h}\, e^{-i\frac{\pi}{2}\alpha_h(t)} \left(\frac{s}{s_{0h}}\right)^{\alpha_h(t)} e^{b_h t},$$

Using

$$\frac{d\sigma_{el}}{dt} = \frac{\pi}{s^2}|A(s,t,M,Q^2)|^2,$$

we can calculate differential and integrated cross sections.

$$\frac{d\sigma_{el}}{dt} = H_s^2 e^{2\{L(\alpha_s(t)-1)+b_s t\}} + H_h^2 e^{2\{L(\alpha_h(t)-1)+b_h t\}}$$

$$+2H_s H_h e^{\{L(\alpha_s(t)-1)+L(\alpha_h(t)-1)+(b_s+b_h)t\}} \cos\left(\frac{\pi}{2}(\alpha_s(t)-\alpha_h(t))\right),$$

$$\sigma_{el} = \frac{H_s^2 e^{2L(\alpha_{0s}-1)}}{2(\alpha_s' L + b_s)} + \frac{H_h^2 e^{2L(\alpha_{0h}-1)}}{2(\alpha_h' L + b_h)} + 2H_s H_h e^{L(\alpha_{0s}-1)+L(\alpha_{0h}-1)} \frac{\mathfrak{B}\cos\phi_0 + \mathfrak{L}\sin\phi_0}{\mathfrak{B}^2 + \mathfrak{L}^2},$$

where

$$L = \ln(s/s_0), \qquad \mathfrak{B} = L\alpha_s' + L\alpha_h' + (b_s + b_h),$$
$$\phi_0 = \frac{\pi}{2}(\alpha_{0s} - \alpha_{0h}), \qquad \mathfrak{L} = \frac{\pi}{2}(\alpha_s' - \alpha_h'),$$

$$H_s(\widetilde{Q^2}) = \frac{A_s}{\left(1+\frac{\widetilde{Q^2}}{\widetilde{Q_s^2}}\right)^{n_s}}, \quad H_h(\widetilde{Q^2}) = \frac{A_h\left(\frac{\widetilde{Q^2}}{\tilde{Q}_h^2}\right)}{\left(1+\frac{\widetilde{Q^2}}{\widetilde{Q_h^2}}\right)^{n_h+1}},$$

here $A_{s,h} = -\frac{\sqrt{\pi}}{s_{0\,s,h}}\widetilde{A}_{s,h}$.

Normalization of VM cross sections

We would like to construct the model, describing the whole set of VM with the same scattering amplitude. For this reason we need to normalise all VM cross section to the same level, i.e. we neeed to find (or calculate)
some f_i factors, such that all normalized cross section would lay on the same surface $f(W, \widetilde{Q^2})$:

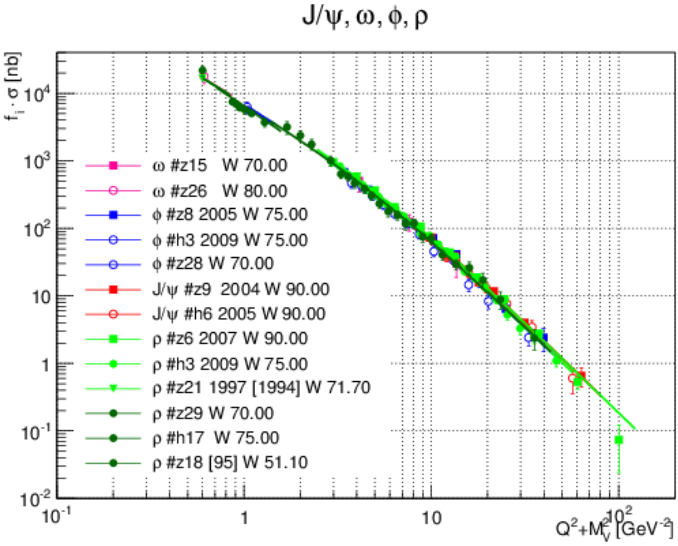
$$f(W, \widetilde{Q^2}) = f_{\rho^0}\sigma_{\rho^0} = f_\omega\sigma_\omega = f_\phi\sigma_\phi = f_{J/\psi}\sigma_{J/\psi},$$

J/ψ was taken as a reference point, so the normalisation parameters are just ratioes: $f_i = \frac{\sigma_{J/\psi}}{\sigma_i} = const.$
In our calculations we used:

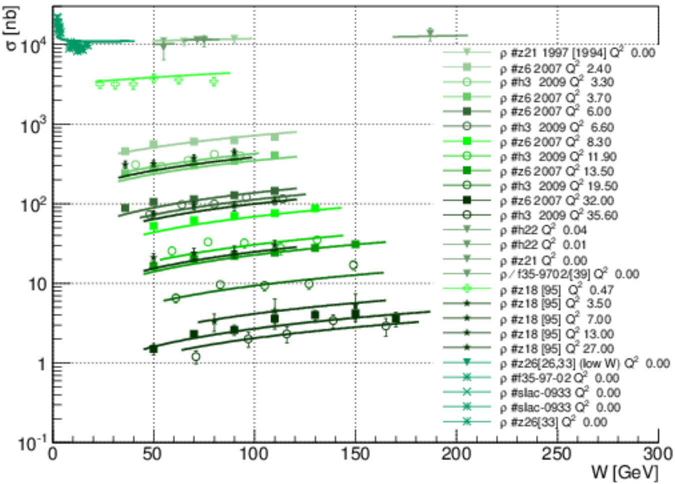
$$f_{\rho^0} : f_\omega : f_\phi : f_{J/\psi} : f_\Upsilon = 0.68 : 0.068 : 0.155 : 1 : 0.75.$$

in the next article there are three sets of normalization parameters
I. Ivanov, N. Nikolaev and A. Savin, Phys. Part. Nucl. **37**, *1 (2006) [hep-ph/0501034].*

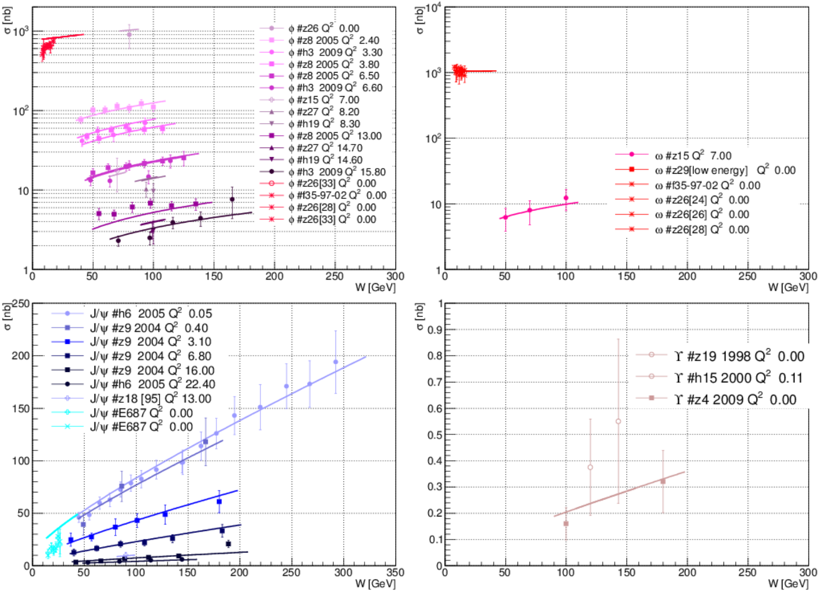
Fit to the data on normalized cross sections $f_i \cdot \sigma(Q^2)$



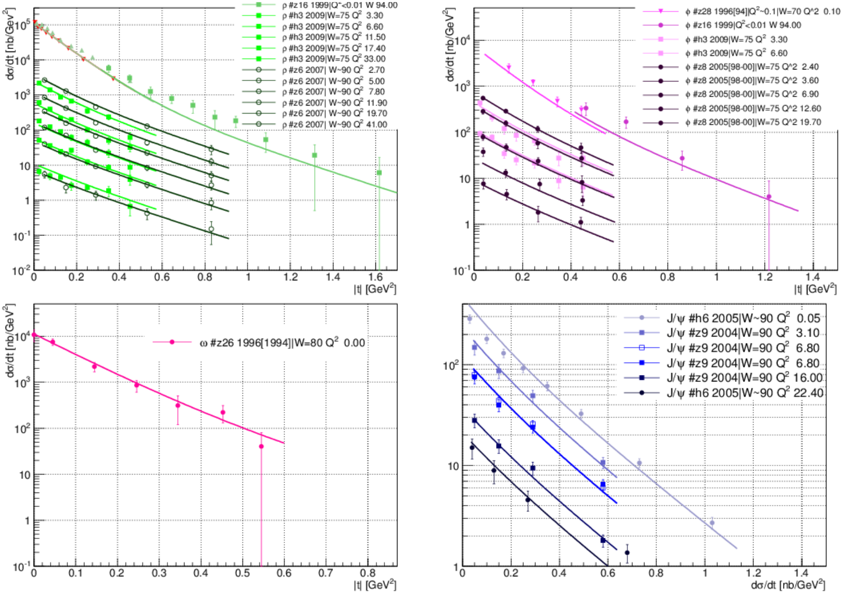
Integrated cross sections $\sigma(W)$ for ρ^0 for different Q^2



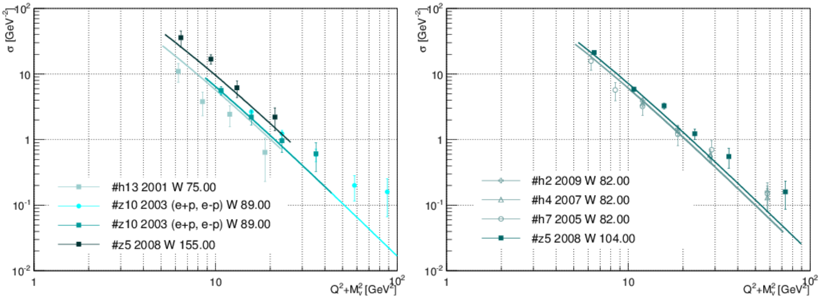
Integrated cross sections $\sigma(W)$ for $\phi^0, \omega, J/\psi, \Upsilon(1S)$



Differential cross sections $\frac{d\sigma(W)}{dt}$ for ρ^0, $\phi^0, \omega, J/\psi$

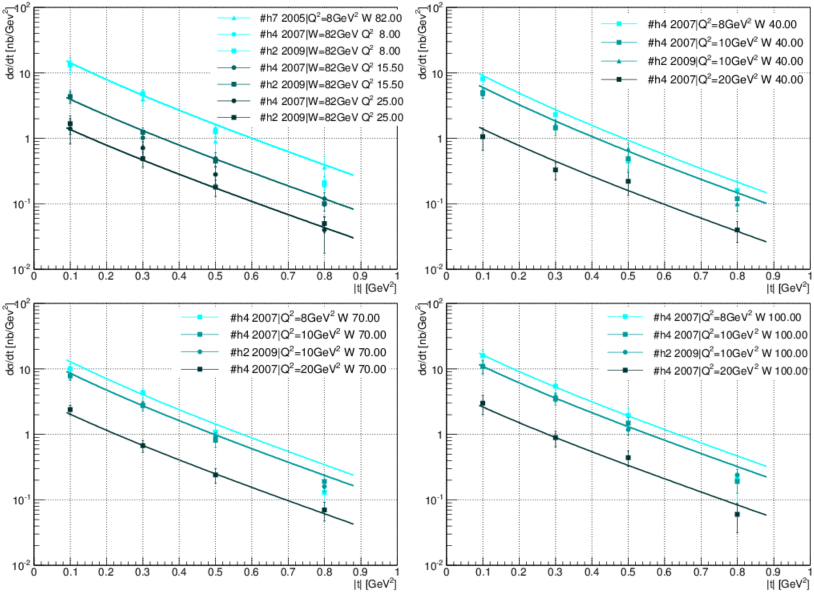


DVCS and VMP have similar behaveour, however there also differences between the two because of the vanishing rest mass of the produced real photon. The unified desctiption of these two types of related reactions does not work by simply setting $M_\gamma = 0$. We need to introduce some effective mass $M_{DVCS}^{eff} = 1.8$ GeV and set a normalization factor to $f_{DVCS} = 0.091$.

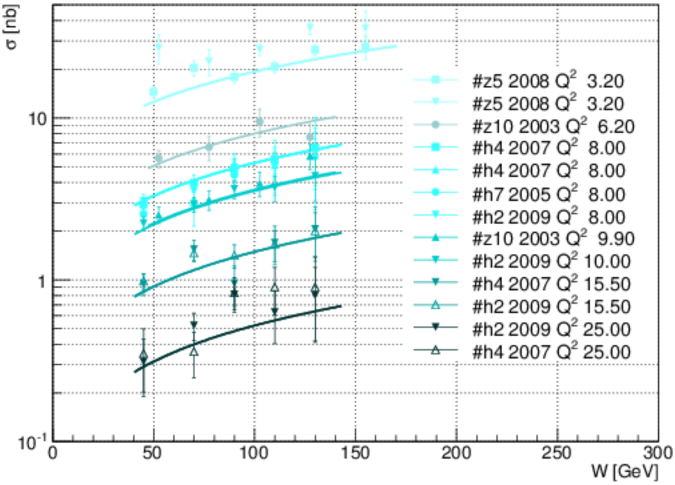


(see also p.9)

The differential DVCS cross sections



The integrated DVCS cross sections



Results

$\tilde{\chi}^2/Ndf = 0.986$

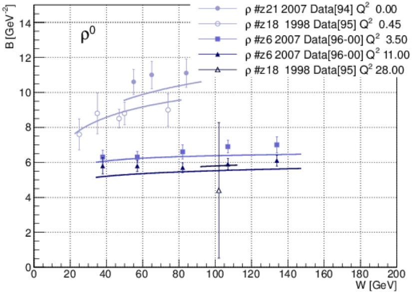
$A_{0s,h}$	$\widetilde{Q^2_{s,h}}$	$n_{s,h}$	$\alpha_{0\,s,h}$	$\alpha'_{s,h}$	$b_{s,h}$
2104±1749	0.29±0.20	1.63±0.40	1.005±0.090	0.32±0.57	2.93±5.06
44± 22	1.15±0.52	1.34±0.16	1.225±0.055	0.0±17	2.22±3.09

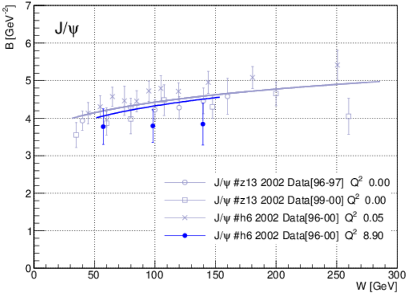
$$[A_{0s,h}] = \frac{\sqrt{\text{nb}}}{\text{GeV}};\ \left[\widetilde{Q^2_{s,h}}\right] = \text{GeV}^2;\ [n_{s,h}] = [\alpha_{0\,s,h}] = 1;\ \left[\alpha'_{s,h}\right] = \frac{1}{\text{GeV}^2};\ [b_{s,h}] = \frac{1}{\text{GeV}^2}$$

DL: $\alpha_s(t) = 1.08 + 0.25t, \quad \alpha_h(t) = 1.40 + 0.1t.$

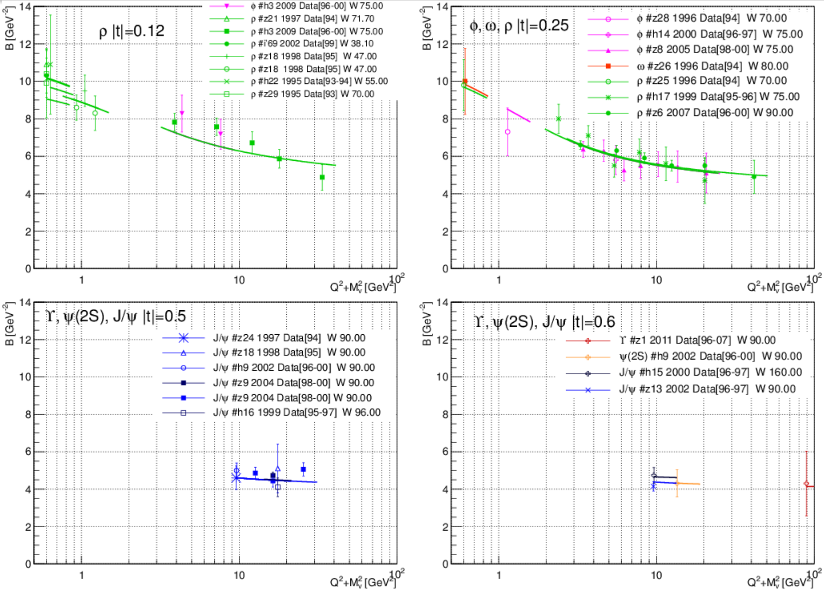
Meson production	$\sigma(W)$		$\sigma(Q^2)$		$\frac{d\sigma}{dt}$		
	χ/N	N	χ/N	N	χ/N	N	χ_i
Υ	0.47	4	0.00	1	0.00	1	0.469
$J\psi$	0.47	43	0.47	16	2.37	92	1.105
ω	0.10	3	0.09	4	0.33	7	0.174
ϕ	1.19	46	1.42	22	1.10	85	1.238
ρ	1.49	112	0.97	64	3.85	94	2.104
DVCS	1.83	89	2.20	38	1.41	84	1.815

The slope parameters for ρ^0 and J/ψ as functions of W

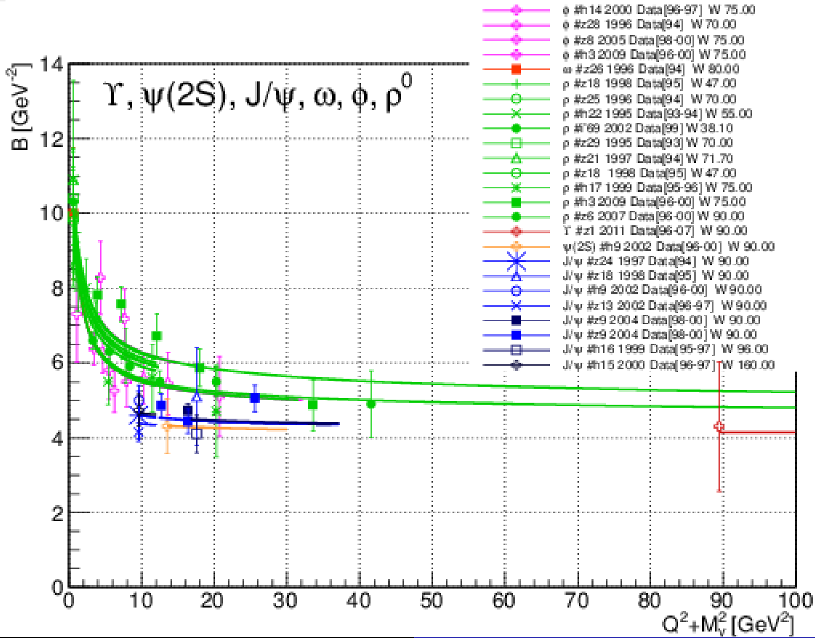




The slope parameters for $\rho^0, \phi, J/\psi$ and Υ at $|t| = 0.12,\ 0.25,\ 0.5,\ 0.6\ \mathrm{GeV}^{-2}$



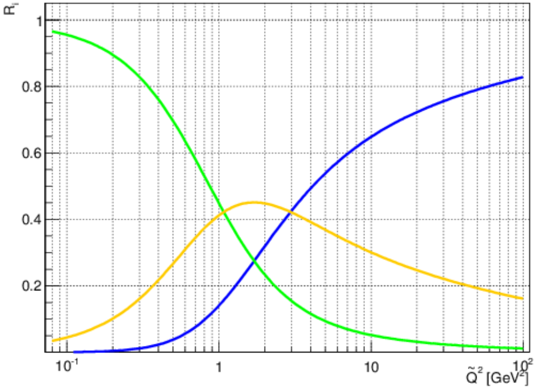
The slope parameters for $\rho^0, \phi, J/\psi$ and Υ (compilation)



Balancing between the "soft" and "hard" dynamics

$$A(t, s, \widetilde{Q^2}) = A_s(t, s, \widetilde{Q^2}) + A_h(t, s, \widetilde{Q^2}),$$

$$\sigma_{el} = \sigma_{s,el} + \sigma_{h,el} + \sigma_{interf,el}, \quad \frac{d\sigma_{el}}{dt} = \frac{d\sigma_{s,el}}{dt} + \frac{d\sigma_{h,el}}{dt} + \frac{d\sigma_{interf,el}}{dt}$$

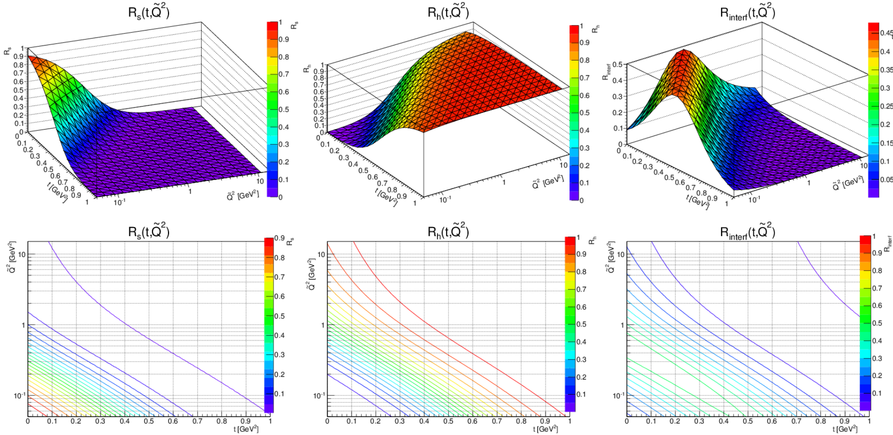


$$R_i(s, \widetilde{Q^2}) = \frac{\sigma_{i,el}}{\sigma_{el}}$$

($W = 70$ GeV) Soft (green line), hard (blue) and interference (yellow) components of cross section

Balancing between the "soft" and "hard" dynamics

$R_i(t, \widetilde{Q^2}, s) = \frac{d\sigma_{i,el}}{dt} / \frac{d\sigma_{el}}{dt}$, at $\sqrt{s} = 70$ GeV.
Soft (left), interference (middle) and hard (right) components.



Conclusions

1. We have developed a universal model for the pomeron, applicable *both to "soft" and "hard"* processes.
2. A global fit to all VMP (ρ^0, ω, ϕ, J/ψ, Υ) and DVCS measurables at HERA is performed with a small number of free parameters universal for all reactions.
   (8 free par. + 4 pomeron trajectories + 5 normalization par.),
3. The model reasonably well describes the scatter of the slope parameters B as a function of $\widetilde{Q^2}$, by taking into account the W- and t-dependence of B. (see p.18)
4. We have neglected sub-leading Regge contributions. They must be included at low energies (below 30 GeV).
5. The "soft" coomponent of the pomeron dominates in the region of small t and small $\widetilde{Q^2}$. A parameter, responsible for "softness" or "hardness", can be constructed as a combination of t and Q^2. A simple solution was suggested in *M. Capua, Phys. Lett.* **B645** *(2007) 161, [hep-ph/0605319]*, where a variable $z = t - Q^2$ was introduced.