Introduction to Diffractive Processes in Hadron-Nucleus and Photon-Nucleus Reactions

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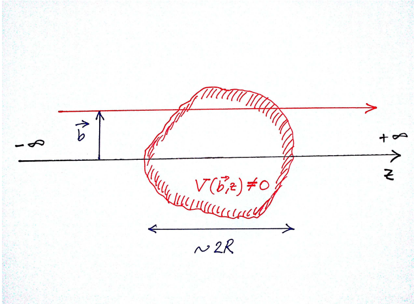
Outline

Eikonal approximation: scattering theory in the short wavelength limit $kR \gg 1$



- Eikonal phase:
  $$\delta(\boldsymbol{b}) = -\frac{1}{2v}\int_{-\infty}^{\infty} dz V(\boldsymbol{b}, z)\,, v = k/m$$
- scattered wave $\psi(\boldsymbol{b}, +\infty) = S(\boldsymbol{b})\psi(\boldsymbol{b}, -\infty)$, $S(\boldsymbol{b}) = \exp(2i\delta(\boldsymbol{b}))$
- profile function: $\Gamma(\boldsymbol{b}) \equiv 1 - S(\boldsymbol{b})\,.$
- scattering amplitude:
  $$f(\boldsymbol{q}) = \frac{ik}{2\pi}\int d^2\boldsymbol{b}\,\exp[-i\boldsymbol{q}\boldsymbol{b}]\Gamma(\boldsymbol{b})$$

- This is the counterpart of the *partial wave expansion*:
- $f(\theta) = \frac{i}{2k}\sum(2l+1)(1-S_l)P_l(cos\theta)$, where $l \sim kb$, $|\boldsymbol{q}| \sim k\theta$.
- $i(1-S_l) \leftrightarrow \Gamma(\boldsymbol{b})$.
- unitarity bound: $|\Gamma(\boldsymbol{b})| \leq 1$

From amplitudes to cross sections:

Elastic cross section:

$$\frac{d\sigma_{\rm el}}{d\Omega} = |f(\boldsymbol{q})|^2 = \frac{k^2}{4\pi^2} \int d^2\boldsymbol{b} d^2\boldsymbol{b}' \exp[-i\boldsymbol{q}(\boldsymbol{b}-\boldsymbol{b}')]\,\Gamma(\boldsymbol{b})\Gamma^*(\boldsymbol{b}')$$

Integrated cross sections:

$$\begin{aligned}
\sigma_{\rm tot} &= \frac{4\pi}{k}\Im m\, f(0) = 2\int d^2\boldsymbol{b}\,\Re e\,\Gamma(\boldsymbol{b}) \\
\sigma_{\rm el} &= \int d^2\boldsymbol{b}\,|\Gamma(\boldsymbol{b})|^2 \\
\sigma_{\rm inel} &= \int d^2\boldsymbol{b}\left(2\Re e\,\Gamma(\boldsymbol{b}) - |\Gamma(\boldsymbol{b})|^2\right)
\end{aligned}$$

The “black disc”:

Let’s imagine a situation where there is no scattering outside a disc of radius R, and strong aborption for $|\boldsymbol{b}| < R$:

$$\delta(\boldsymbol{b}) = 0 \,\mathrm{for}\, |\boldsymbol{b}| > R, \, , \Im m \delta(\boldsymbol{b}) \gg 1 \,\mathrm{for}\, |\boldsymbol{b}| < R\,.$$

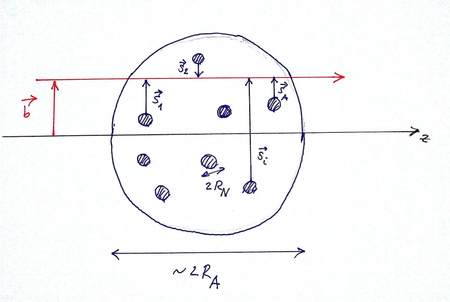
$$S(\boldsymbol{b}) = \theta(|\boldsymbol{b}| - R) \Rightarrow \Gamma(\boldsymbol{b}) = \theta(R - |\boldsymbol{b}|)\,.$$

Then we get:

Cross sections & elastic amplitude:

$$\begin{aligned}
\sigma_{\rm tot} &= 2 \int d^2\boldsymbol{b}\, \Re e\, \Gamma(\boldsymbol{b}) = 2\pi R^2 \\
\sigma_{\rm el} &= \int d^2\boldsymbol{b}\, |\Gamma(\boldsymbol{b})|^2 = \pi R^2 = \frac{1}{2}\sigma_{\rm tot} \\
\sigma_{\rm inel} &= \int d^2\boldsymbol{b} \left(2\Re e\, \Gamma(\boldsymbol{b}) - |\Gamma(\boldsymbol{b})|^2 \right) = \pi R^2 \\
f(\boldsymbol{q}) &= ikR^2 \, \frac{J_1(qR)}{qR}
\end{aligned}$$

Scattering on the composite target:



- Consider A scattering centers of size $R_N \ll R_A$, frozen at transverse coordinates $\boldsymbol{s}_1, \dots, \boldsymbol{s}_A$.
- Let's assume, that not only $kR_A \gg 1$, but also $kR_N \gg 1$. Then, following Glauber we can simply add the eikonal phase-shifts of the individual scattering centers:

$$\delta_A(\boldsymbol{b}; \boldsymbol{s}_1, \dots, \boldsymbol{s}_A) = \sum_{i=1}^{A} \delta_N(\boldsymbol{b} - \boldsymbol{s}_i)$$

Scattering off the composite target:

$$S_A(\boldsymbol{b}; \boldsymbol{s}_1, \dots \boldsymbol{s}_A) = \prod_{i=1}^{A} S_N(\boldsymbol{b} - \boldsymbol{s}_i) \Rightarrow \Gamma_A(\boldsymbol{b}; \boldsymbol{s}_1, \dots \boldsymbol{s}_A) = 1 - \prod_{i=1}^{A} [1 - \Gamma_N(\boldsymbol{b} - \boldsymbol{s}_i)]$$

Multiple scattering expansion:

Scattering off the composite target:

$$\begin{aligned}\Gamma_A(\boldsymbol{b};\boldsymbol{s}_1,\dots\boldsymbol{s}_A) &= 1-\prod_{i=1}^{A}[1-\Gamma_N(\boldsymbol{b}-\boldsymbol{s}_i)] \\ &= \sum_{i=1}^{A}\Gamma_N(\boldsymbol{b}-\boldsymbol{s}_i)-\sum_{i<j}\Gamma_N(\boldsymbol{b}-\boldsymbol{s}_i)\Gamma_N(\boldsymbol{b}-\boldsymbol{s}_j) \\ &\quad +\dots+(-1)^{A-1}\prod_{i=1}^{A}\Gamma_N(\boldsymbol{b}-\boldsymbol{s}_i)\end{aligned}$$

- Linear term gives the contribution of single scattering off each of the A scattering centers added coherently. (Impulse approximation).
- Higher orders are multiple scattering contributions. Notice that at a given order, each scattering center enters only once!
- the quadratic term interferes destructively with single scattering.

Scattering off a nucleus:

- To obtain the amplitudes for the nuclear target, we need to average quantum mechanically over the different frozen-nucleon configurations.

$$f_{fi}(\boldsymbol{q}) = \frac{ik}{2\pi} \int d^2\boldsymbol{b} \exp[-i\boldsymbol{q}\boldsymbol{b}] \, \langle A_f | \Gamma_A(\boldsymbol{b}; \boldsymbol{s}_1, \dots, \boldsymbol{s}_A) | A_i \rangle$$

- The necessary information is contained in the nuclear wavefunction:

$$\langle A_f | \Gamma_A(\boldsymbol{b}; \boldsymbol{s}_1, \dots, \boldsymbol{s}_A) | A_i \rangle = \int d^3\vec{r}_1 \dots d^3\vec{r}_A \, \psi_f^*(\vec{r}_1, \dots, \vec{r}_A) \psi_i(\vec{r}_1, \dots, \vec{r}_A) \, \Gamma_A(\boldsymbol{b}; \boldsymbol{s}_1, \dots, \boldsymbol{s}_A) \, .$$

- Of special interest is the elastic transition $A \to A$, which can be greatly simplified in the dilute gas approximation, where all nuclear correlations are neglected:

$$|\Psi_A(\vec{r}_1, \dots, \vec{r}_A)|^2 = \prod_{i=1}^{A} \frac{n_A(\boldsymbol{s}_i, z_i)}{A} \, , \; T_A(\boldsymbol{s}) \equiv \int_{-\infty}^{\infty} dz \, n_A(\boldsymbol{s}, z) \, .$$

- it leads to the nuclear average:

$$\begin{aligned} \langle A | \Gamma_N(\boldsymbol{b} - \boldsymbol{s}_i) | A \rangle &= \int d^3\vec{r}_1 \dots d^3\vec{r}_A \frac{n_A(\boldsymbol{s}_1, z_1)}{A} \dots \frac{n_A(\boldsymbol{s}_A, z_A)}{A} \Gamma_N(\boldsymbol{b} - \boldsymbol{s}_i) \\ &= \frac{1}{A} \int d^2\boldsymbol{s}_i \, T_A(\boldsymbol{s}_i) \Gamma(\boldsymbol{b} - \boldsymbol{s}_i) \approx \frac{1}{A} T_A(\boldsymbol{b}) \int d^2\boldsymbol{s} \Gamma(\boldsymbol{s}) = \frac{1}{2A} \sigma_{\rm tot}^{hN} T_A(\boldsymbol{b}) \, . \end{aligned}$$

Glauber theory

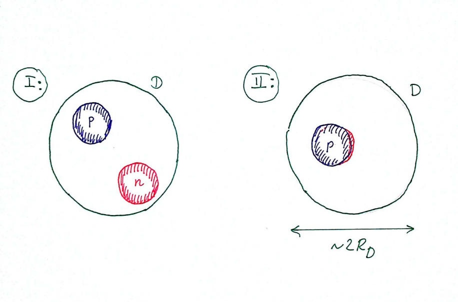
The nuclear amplitude:

$$\begin{aligned}
\langle A|\Gamma_A(\boldsymbol{b};\boldsymbol{s}_1,\dots\boldsymbol{s}_A)|A\rangle &= \langle A|1-\prod_{i=1}^{A}[1-\Gamma_N(\boldsymbol{b}-\boldsymbol{s}_i)]|A\rangle \\
&= 1-[1-\frac{1}{2A}\sigma^{hN}_{\rm tot}T_A(\boldsymbol{b})]^A \approx 1-\exp[-\frac{1}{2}\sigma^{hN}_{\rm tot}T_A(\boldsymbol{b})]
\end{aligned}$$

Glauber formulae for nuclear cross sections:

$$\begin{aligned}
\sigma^{hA}_{\rm tot} &= 2\int d^2\boldsymbol{b}\left(1-\exp[-\frac{1}{2}\sigma^{hN}_{\rm tot}T_A(\boldsymbol{b})]\right) \\
\sigma^{hA}_{\rm el} &= \int d^2\boldsymbol{b}\left(1-\exp[-\frac{1}{2}\sigma^{hN}_{\rm tot}T_A(\boldsymbol{b})]\right)^2 \approx \frac{1}{4}\int d^2\boldsymbol{b}\,T_A^2(\boldsymbol{b})\exp[-\sigma^{hN}_{\rm tot}T_A(\boldsymbol{b})](\sigma^{hN}_{\rm tot})^2 \\
\sigma_{\rm inel} &= \int d^2\boldsymbol{b}\left(1-\exp[-\sigma^{hN}_{\rm tot}T_A(\boldsymbol{b})]\right)
\end{aligned}$$

Shadowing on the deuteron



- For the deuteron target, the multiple scattering series truncates with double scattering, and one can obtain for the total hadron-deuteron cross section:

$$\sigma_{\rm tot}(hD) = \sigma_{\rm tot}(hp) + \sigma_{\rm tot}(hn) - \langle \frac{1}{4\pi r^2} \rangle \sigma_{\rm tot}(hp)\sigma_{\rm tot}(hn) \quad (1)$$

Glauber summary

- Glauber theory allows us to calculate *nuclear* observables from *free nucleon* input.
- In practice the most important requirement for Glauber to work is that scattering off individual nucleons is strongly forward peaked.
- We skipped over a lot of details: Inclusion of the real part, the account for nuclear correlations etc. All these things were already worked out by Glauber and can be found in his lecture notes.
- We used the notation borrowed from non-relativistic quantum mechanics, but there is obviously nothing intrinsically non-relativistic of the results presented. One could easily analyze high-energy potential scattering with a Klein-Gordon or Dirac equation along the same lines.
- Even the potential is not needed! This is why we insisted on the formulation involving the eikonal phase. Additivity of phases/factorization of the S-matrix translates even to Quantum Field Theory.
- Up to now, multiple scattering corrections include only elastic rescatterings. This works typically in a range of $0.8\,\mathrm{GeV} < p_{\mathrm{lab}} < 5\,\mathrm{GeV}$. Beyond these energies diffraction disscociation processes become sizable. Herein lies the reason for the eventual failure of Glauber theory at very high energies. Something new has to be introduced.

Diffraction dissociation and inelastic shadowing

- Diffractive dissociation shares with elastic scattering the sharp forward peaking, and can be incorporated into a generalization of Glauber theory.
- The double scattering contribution can be related to the diffractive mass spectrum for the $hN \to XN$ reaction:

$$\delta\sigma = -4\pi \int_{(M_N+m_\pi)^2}^{(\sqrt{s}-m_N)^2} dM^2 \, \frac{d\sigma(hN \to XN; t=0)}{dt dM^2} \, G_A^2(q_L^2) \, .$$

- Here $q_L = \frac{M_X^2 - m_h^2}{2k}$ is the longitudinal momentum transfer. The formfactor $G_A^2(q_L^2)$ cuts off the inelastic contribution when either the diffractive mass becomes too large or the beam momentum too small. In the latter case we return to the elastic Glauber contribution only.
- The Gribov-formula gives a rigorous way to evaluate inelastic shadowing in a deuteron from the data on the diffractive mass spectrum.
- Application of a triple-Regge analysis of diffraction in pp scattering gives for pd scattering at $p_{\rm lab} = 10^3\,\mathrm{GeV}$:

$$\delta\sigma_{\rm Glauber} = 3.62\,\mathrm{mb} \;\; \delta\sigma_{\rm lowmass} = 0.52\,\mathrm{mb} \;\; \delta\sigma_{\rm highmass} = 0.55\,\mathrm{mb}$$

Diffraction scattering eigenstates

- For heavy nuclei, the inelastic shadowing corrections become a coupled channel problem, as we have to account for transitions $h \to h^* \to h^{**} \to \cdots \to h$. There is no way to fix the off-diagonal transitions from experiments on the free nucleon target.
- A different approach starts from the Good-Walker diffractive scattering eigenstates (DSE), which diagonalize the diffractive amplitude $\hat{f} = i\hat{\sigma}$ and which are orthogonal:

$$\hat{f}|\alpha\rangle = f_\alpha|\alpha\rangle = i\sigma_\alpha|\alpha\rangle \, , \, \langle\beta||\alpha\rangle = \delta_{\alpha\beta} \, .$$

- Expand the incoming hadron into DSE's:

$$|h\rangle = \sum_\alpha \psi_\alpha \, |\alpha\rangle \, .$$

Diffraction scattering eigenstates

Averages of the cross section operator:

$$\begin{aligned}
\sigma_{\rm tot}^{hN} &= \langle h|\hat{\sigma}|h\rangle = \sum_\alpha |\psi_\alpha|^2 \sigma_\alpha = \langle \hat{\sigma}\rangle \\
\frac{d\sigma_{\rm el}(t=0)}{dt} &= \frac{|\langle h|\hat{\sigma}|h\rangle|^2}{16\pi} = \frac{\langle\hat{\sigma}\rangle^2}{16\pi} \\
\frac{d\sigma_{\rm DD}(t=0)}{dt} &= \frac{|\sum_{X\neq h}\langle X|\hat{\sigma}|h\rangle|^2}{16\pi} = \frac{\langle\hat{\sigma}^2\rangle - \langle\hat{\sigma}\rangle^2}{16\pi} = \frac{\langle\Delta\hat{\sigma}^2\rangle}{16\pi}
\end{aligned}$$

Diffractive scattering eigenstates: nuclear target

- to extend the DSE method to the nucleus we do not need to rederive the multiple scattering theory: Glauber theory now applies for each DSE, and we subsequently average over them. The Glauber S-matrix simply becomes an operator:

$$\hat{S}_A(\boldsymbol{b}) = \exp[-\frac{1}{2}\hat{\sigma}\, T_A(\boldsymbol{b})] = \exp[-\frac{1}{2}\langle\hat{\sigma}\rangle\, T_A(\boldsymbol{b})]\, \exp[-\frac{1}{2}\Delta\hat{\sigma}\, T_A(\boldsymbol{b})]$$

- deviations from the single channel case (the importance of inelastic transitions) are quantified by the fluctuation of the cross section $\Delta\hat{\sigma} = \hat{\sigma} - \langle\hat{\sigma}\rangle$:

Nuclear target:

$$\begin{aligned}
\sigma^{hA}_{\rm tot} &= 2\int d^2\boldsymbol{b}\langle h|1-\exp[-\frac{1}{2}\hat{\sigma}\, T_A(\boldsymbol{b})]|h\rangle = 2\int d^2\boldsymbol{b}\sum_\alpha |\psi_\alpha|^2(1-\exp[-\frac{1}{2}\hat{\sigma}_\alpha T_A(\boldsymbol{b})]) \\
&= 2\int d^2\boldsymbol{b}(1-\exp[-\frac{1}{2}\langle\sigma\rangle\, T_A(\boldsymbol{b})]) + \delta\sigma_{\rm inel} \\
\delta\sigma_{\rm inel} &= 2\int d^2\boldsymbol{b}\exp[-\frac{1}{2}\langle\sigma\rangle\, T_A(\boldsymbol{b})](1-\langle\exp[-\frac{1}{2}\Delta\hat{\sigma}\, T_A(\boldsymbol{b})]\rangle) \\
&\approx -\frac{\langle\Delta\hat{\sigma}^2\rangle}{4}\int d^2\boldsymbol{b}\, T_A^2(\boldsymbol{b})\exp[-\frac{1}{2}\sigma_{hN} T_A(\boldsymbol{b})] + \dots
\end{aligned}$$

Diffractive scattering eigenstates: hadronic diffraction on nuclei

Cross section for $hA \to XA$:

$$\begin{aligned}
\sigma^{hA}_{\rm DD} &= \int d^2\boldsymbol{b} \Big(\langle \exp[-\hat\sigma T_A(\boldsymbol{b})] \rangle - \langle \exp[-\frac{1}{2}\hat\sigma T_A(\boldsymbol{b}) \rangle^2 \Big) \\
&= \int d^2\boldsymbol{b} \exp[-\langle\hat\sigma\rangle T_A(b)] \Big(\langle \exp[-\Delta\hat\sigma T_A(\boldsymbol{b})] \rangle - \langle \exp[-\frac{1}{2}\Delta\hat\sigma T_A(\boldsymbol{b}) \rangle \Big) \\
&= \frac{\langle \Delta\sigma^2 \rangle}{4} \int d^2\boldsymbol{b}\, T_A^2(\boldsymbol{b}) \exp[-\sigma_{hN} T_A(\boldsymbol{b})] + \dots \\
&= 4\pi \frac{d\sigma^{hN}_{\rm DD}(t=0)}{dt} \int d^2\boldsymbol{b}\, T_A^2(\boldsymbol{b}) \exp[-\sigma_{hN} T_A(\boldsymbol{b})] + \dots
\end{aligned}$$

- In hadronic interactions the typical dispersion of the diffraction operator is not large:

$$\frac{\langle\hat\sigma^2\rangle - \langle\hat\sigma\rangle^2}{\langle\hat\sigma\rangle^2} \approx 0.3$$

- To lowest order the diffractive cross section is proportional to diffraction on a nucleon times a "gap survival factor". Absorption in the nucleus is controlled by σ_{hN} – it is large. The fraction of gap events is negligibly small.

A historical remark

Great attention was paid to diffraction production of particles in $p-p$ and p-nucleus collisions at high energies (see e.g. [68]-[70]). The main idea was that the minimal momentum transfer from proton to nucleus in, say, pion production in pA collisions is equal to $q = m\mu/E$, where m and μ are the proton and pion masses, and E is the proton energy. If $1/q \gg R$ – the nuclear radius, – then the pion production process proceeds outside the nucleus and the characteristics of such a process can be calculated phenomenologically without the use of perturbation theory.

By this method a number of processes were calculated: elastic diffraction scattering in pp and pA collisions, production of photons, mesons and meson pairs in pA collisions, diffraction phenomena in deuteron-nucleus scattering, photon production in collisions of mesons with nuclei etc. When Pomeranchuk reported the results of these calculation at a seminar of the Lebedev Institute, Academician Skobelzyn asked: "How can it be that the production process proceeds outside the nucleus?" Pomeranchuk explained that the wave function of the incoming particles overlaps with the shadow of the nucleus, which results in a distortion of the wave function and gives rise to the production processes. Then he continued his talk. After some time Skobelzyn repeated his question. Pomeranchuk gave the same explanation, but in more detail. After another while Skobelzyn repeated his question for a third time. Pomeranchuk's reply was: "If you like, you can consider this effect as immaculate conception".

- quoted from B. L. Ioffe, "The first dozen years of the history of ITEP Theoretical Physics Laboratory," Eur. Phys. J. H **38** (2013) 83 [arXiv:1208.1386 [physics.hist-ph]].

Color dipoles as diffraction scattering eigenstates

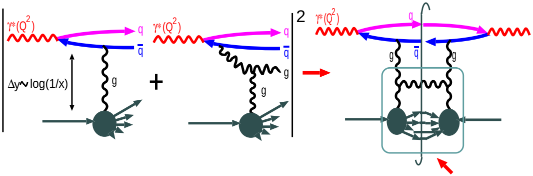
- The familiar dipole representation of the total photoabsorption cross section is a specific example of an expansion over DSE's!

$$\sigma(\gamma^* p) = \int dz d^2\mathbf{r} |\psi_{q\bar{q}}(z,\mathbf{r})|^2 \sigma(x,\mathbf{r}) = \langle \hat{\sigma} \rangle$$

-

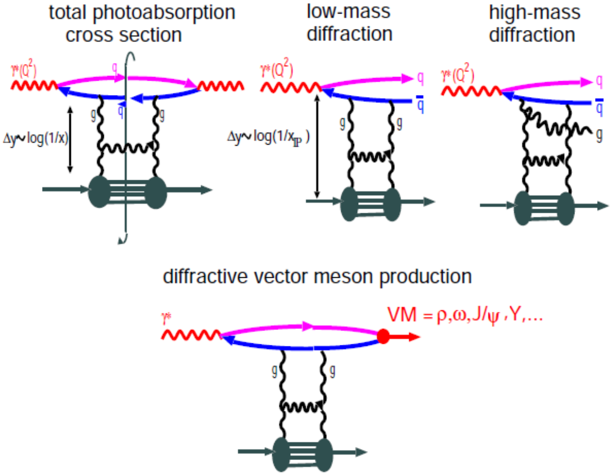
total photoabsorption cross section

sum over final states → imaginary part of forward amplitude



gluon density

Color dipoles as diffraction scattering eigenstates



- On one footing, one can access total cross section, inclusive diffraction (see L. Goerlich's talk), as well as exclusive diffraction (see J. Figiel's talk).

Color dipoles as diffraction scattering eigenstates

- Besides the dipole cross section also the cross section for the $q\bar{q}g$ state is important:

$$\sigma_{q\bar{q}g}(\boldsymbol{r},\boldsymbol{\rho}) = \frac{N_c^2}{N_c^2-1}[\sigma(\boldsymbol{\rho})+\sigma(\boldsymbol{\rho}+\boldsymbol{r})] - \frac{1}{N_c^2-1}\sigma(\boldsymbol{r}) \, .$$

Cross sections in the color dipole approach:

$$\sigma(\gamma^* p)_{\rm tot} = \int_0^1 dz \int d^2\boldsymbol{r} |\psi_{q\bar{q}}(z,\boldsymbol{r})|^2 \sigma(x,\boldsymbol{r})$$

$$\frac{d\sigma(\gamma^* p \to q\bar{q}p; t=0}{dt} = \frac{1}{16\pi}\int_0^1 dz d^2\boldsymbol{r} |\psi_{q\bar{q}}(z,\boldsymbol{r})|^2 \sigma^2(x,\boldsymbol{r})$$

$$\frac{d\sigma(\gamma^* p \to q\bar{q}gp; t=0)}{dt} = \frac{1}{16\pi}\int_0^1 dz \frac{dz_z}{z_g} d^2\boldsymbol{r} d^2\boldsymbol{\rho} z_g |\psi_{q\bar{q}g}(z,z_g,\boldsymbol{r},\boldsymbol{\rho})|^2 [\sigma^2_{q\bar{q}g}(\boldsymbol{r},\boldsymbol{\rho}) - \sigma^2(x,\boldsymbol{r})]$$

$$\frac{d\sigma(\gamma^* p \to Vp; t=0)}{dt} = \frac{1}{16\pi}\left| \int_0^1 dz d^2\boldsymbol{r} \psi^*_V(z,\boldsymbol{r})\psi_{q\bar{q}}(z,\boldsymbol{r})\sigma(x,\boldsymbol{r})\right|^2$$

Color dipoles: the nuclear target

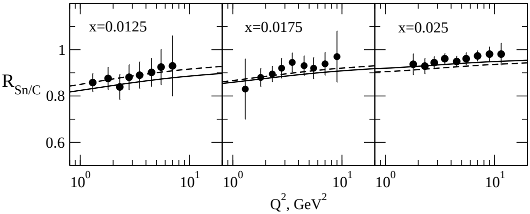
- Realizing that color dipoles are the diffraction scattering eigenstates, we can easily calculate the dipole-*nucleus* cross sections for $q\bar{q}$ as well as $q\bar{q}g$ states: dipoles scatter only elastically, like in Glauber theory!

Nuclear cross section in the color dipole approach:

$$
\begin{aligned}
\sigma_A(\boldsymbol{r}) &= 2\int d^2\boldsymbol{b}\Gamma_A(\boldsymbol{b},\boldsymbol{r}) = 2\int d^2\boldsymbol{b}(1-\exp[-\frac{1}{2}\sigma(\boldsymbol{r})T_A(\boldsymbol{b})]) \\
\sigma_{A,q\bar{q}g}(\boldsymbol{r},\boldsymbol{\rho}) &= 2\int d^2\boldsymbol{b}(1-\exp[-\frac{1}{2}\sigma_{q\bar{q}g}(\boldsymbol{r},\boldsymbol{\rho})T_A(\boldsymbol{b})]) \\
&\approx 2\int d^2\boldsymbol{b}(1-\exp[-\frac{1}{2}\sigma(\boldsymbol{\rho})T_A(\boldsymbol{b})]\exp[-\frac{1}{2}\sigma(\boldsymbol{\rho}+\boldsymbol{r})T_A(\boldsymbol{b})])
\end{aligned}
$$

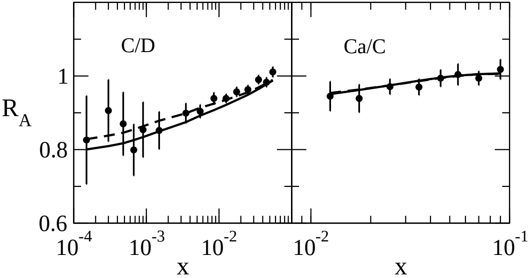
- together with the $q\bar{q}g$ wavefunction that is all the input we need to evaluate observables for DIS off nuclear targets.

Shadowing of nuclear structure functions



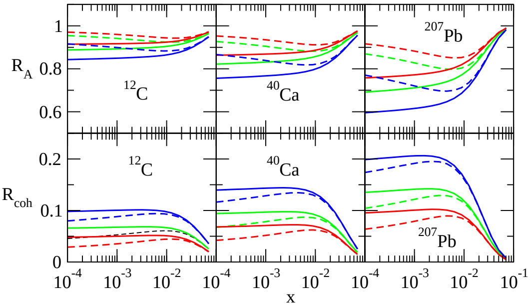
- $R_A = \frac{\sigma(\gamma^* A)}{A\sigma(\gamma^* p)}$, $R_{A_1/A_2} = \frac{R_{A_1}}{R_{A_2}}$
- data from NMC Collab. ('95)
- dashed = $q\bar{q}$, solid = $q\bar{q} + q\bar{q}g$ contributions
- calculation from Nikolaev, WS, Zoller & Zakharov '07

Shado



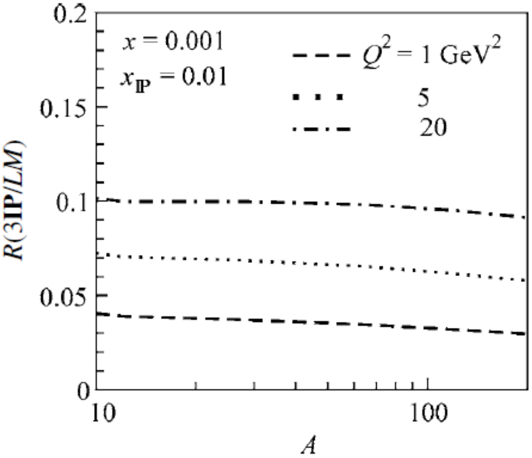
- $R_A = \frac{\sigma(\gamma^* A)}{A\sigma(\gamma^* p)}$, $R_{A_1/A_2} = \frac{R_{A_1}}{R_{A_2}}$
- data from NMC Collab. ('95)
- x and Q^2 are correlated
- calculation from Nikolaev, WS, Zoller & Zakharov '07
- dashed $= q\bar{q}$, solid $= q\bar{q} + q\bar{q}g$ contributions

Predictio



- Predictions for a future EIC: $Q^2 = 1, 5, 20\,\mathrm{GeV}^2$
- $R_A = \frac{\sigma(\gamma^* A)}{A\sigma(\gamma^* p)}$, $R_{coh} = \frac{\text{coherent diffraction}}{\text{total}}$
- calculation from Nikolaev, WS, Zoller & Zakharov '07
- dashed $= q\bar{q}$, solid $= q\bar{q} + q\bar{q}g$ contributions

Predictions for a future EIC



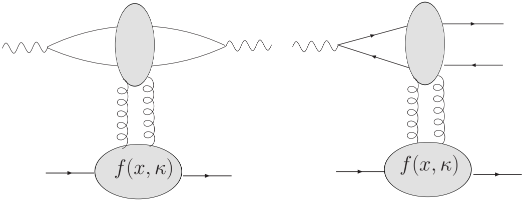
- the ratio of high mass to low mass ($M^2 \sim Q^2$) diffraction as a funtion of the nuclear mass number.
- rise with Q^2: QCD evolution of the diffractive structure function.

color dipole cross section $\leftrightarrow$ unintegrated glue

- at high energies, when $\Lambda_{QCD} \ll p_\perp \ll \sqrt{s}$, we should take parton transverse momenta explicitly into account $\to$ unintegrated parton distributions.
- equivalence of color dipole-cross section and unintegrated gluon distribution (Nikolaev & Zakharov '94):

$$\sigma(x, \boldsymbol{r}) = \int d^2\boldsymbol{\kappa} f(x, \boldsymbol{\kappa}) \Big[1 - \exp(i\boldsymbol{\kappa}\boldsymbol{r}) \Big] , f(x, \boldsymbol{\kappa}) = \frac{4\pi\alpha_S}{N_c} \frac{1}{\boldsymbol{\kappa}^4} \frac{\partial G(x, \boldsymbol{\kappa}^2)}{\partial \log \boldsymbol{\kappa}^2};$$

- virtual photoabsorption: $\sigma(\gamma^* p) = \int dz d^2\boldsymbol{r} |\psi_{q\bar{q}}(z, \boldsymbol{r})|^2 \sigma(x, \boldsymbol{r})$
- diffractive amplitude $\propto \displaystyle\int d^2\boldsymbol{r} \exp(-i\boldsymbol{p}\boldsymbol{r})\, \sigma(x, \boldsymbol{r})\, \psi_{q\bar{q}}(z, \boldsymbol{r})$,



Nuclear unintegrated glue at $x \sim x_A$

- at not too small $x \sim x_A = (R_A m_p)^{-1} \sim 0.01$ only the $\bar{q}q$ state is coherent over the nucleus, and $\Gamma(\boldsymbol{b}, x, \boldsymbol{r})$ can be constructed from Glauber-Gribov theory:

$$\Gamma(\boldsymbol{b}, x_A, \boldsymbol{r}) = 1 - \exp[-\sigma(x_A, \boldsymbol{r}) T_A(\boldsymbol{b})/2] = \int d^2\boldsymbol{\kappa}[1 - e^{i\boldsymbol{\kappa}\boldsymbol{r}}]\phi(\boldsymbol{b}, x_A, \boldsymbol{\kappa}).$$

- nuclear coherent glue per unit area in impact parameter space:

$$\phi(\boldsymbol{b}, x_A, \boldsymbol{\kappa}) = \sum w_j(\boldsymbol{b}, x_A) f^{(j)}(x_A, \boldsymbol{\kappa}) \,,\; f^{(1)}(x, \boldsymbol{\kappa}) = \frac{4\pi\alpha_S}{N_c} \frac{1}{\boldsymbol{\kappa}^4} \frac{\partial G(x, \boldsymbol{\kappa}^2)}{\partial \log(\boldsymbol{\kappa}^2)}$$

- collective glue of j overlapping nucleons :

$$f^{(j)}(x_A, \boldsymbol{\kappa}) = \int \Big[\prod^{j} d^2\boldsymbol{\kappa}_i f^{(1)}(x_A, \boldsymbol{\kappa}_i)\Big] \delta^{(2)}(\boldsymbol{\kappa} - \sum \boldsymbol{\kappa}_i)$$

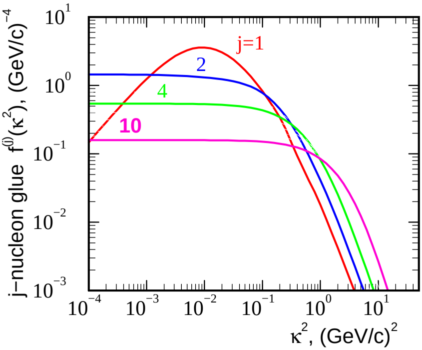
- probab. to find j overlapping nucleons

$$w_j(\boldsymbol{b}, x_A) = \frac{\nu_A^j(\boldsymbol{b}, x_A)}{j!} \exp[-\nu_A(\boldsymbol{b}, x_A)] \,,\; \nu_A(\boldsymbol{b}, x_A) = \frac{1}{2}\alpha_S(q^2)\, \sigma_0(x_A) T_A(\boldsymbol{b}) \,,$$

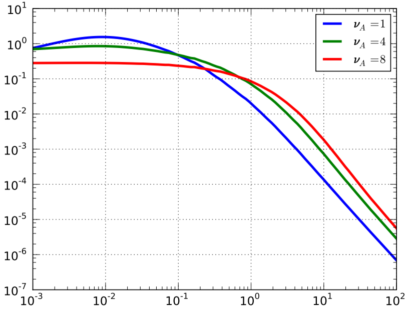
- impact parameter $\boldsymbol{b} \to$ effective opacity ν_A, $q^2 =$ the relevant hard scale.

Salient features of the nuclear unintegrated glue

collective glue $f^{(j)}(x_A, \boldsymbol{\kappa})$



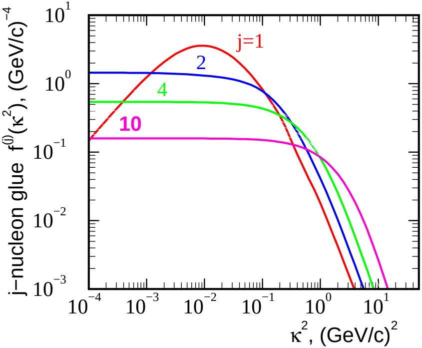
nuclear glue $\phi(\nu_A, x_A, \boldsymbol{\kappa})$



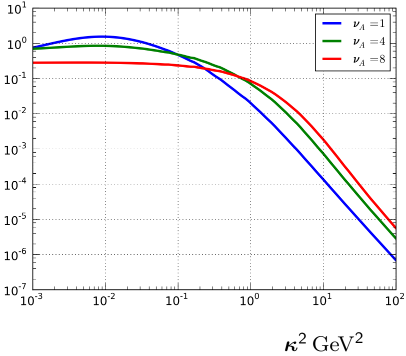
- a plateau at small $\boldsymbol{\kappa}^2$, which displays shadowing: $\phi(\nu_A, x_A, \boldsymbol{\kappa}) \propto 1/\nu_A$
- transition from plateau to tail is controlled by the saturation scale $Q_A^2(\nu_A, x)$

Salient features of the nuclear unintegrated glue

collective glue $f^{(j)}(x_A, \boldsymbol{\kappa})$



nuclear glue $\phi(\nu_A, x_A, \boldsymbol{\kappa})$

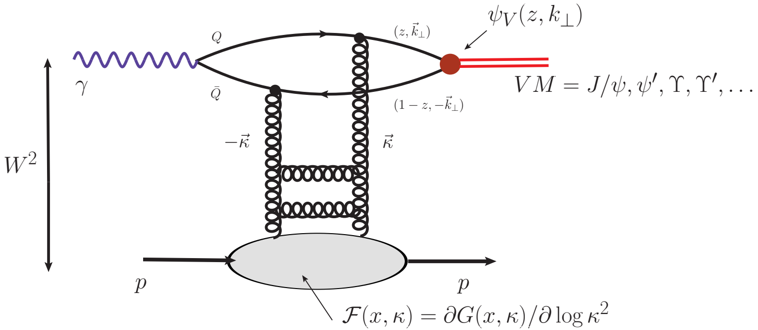


- using that $f(x_A, \boldsymbol{\kappa}^2) \sim \boldsymbol{\kappa}^{-2\gamma}$, $\gamma \approx 2$ manifestly positive higher twist at large $\boldsymbol{\kappa}^2$:

$$\phi(\nu_A, x_A, \boldsymbol{\kappa}) = \nu_A f(x_A, \boldsymbol{\kappa}) \cdot \left(1 + \nu_A \frac{2\pi^2\gamma^2\alpha_S G(x_A, \boldsymbol{\kappa}^2)}{N_c \boldsymbol{\kappa}^2} + \dots\right)$$

Nikolaev, WS & Schwiete (2000)

Diffractive Photoproduction $\gamma p \to Vp$



- $J/\psi = c\bar{c}$, $\Upsilon = b\bar{b}$: (almost) nonrelativistic bound states of heavy quarks. Wavefunctions constrained by their leptonic decay widths.
- Large quark mass $\to$ hard scale necessary for (perturbative) QCD.
- $\mathcal{F}(x,\kappa) \equiv$ unintegrated gluon density, $x \sim M_{VM}^2/W^2$, constrained by HERA inclusive data.
- for an extensive phenomenology, see Ivanov, Nikolaev, Savin (2006)
- topical subject: glue at small-x: nonlinear evolution, gluon fusion, saturation...

When do small dipoles dominate ?

- the photon shrinks with Q^2 - photon wavefunction at large r:

$$\psi_{\gamma^*}(z, \mathbf{r}, Q^2) \propto \exp[-\varepsilon r] \, , \, \varepsilon = \sqrt{m_f^2 + z(1-z)Q^2}$$

- the integrand receives its main contribution from

$$r \sim r_S \approx \frac{6}{\sqrt{Q^2 + M_V^2}}$$

Kopeliovich, Nikolaev, Zakharov '93

- a large quark mass (bottom, charm) can be a hard scale even at $Q^2 \to 0$.
- for small dipoles we can approximate

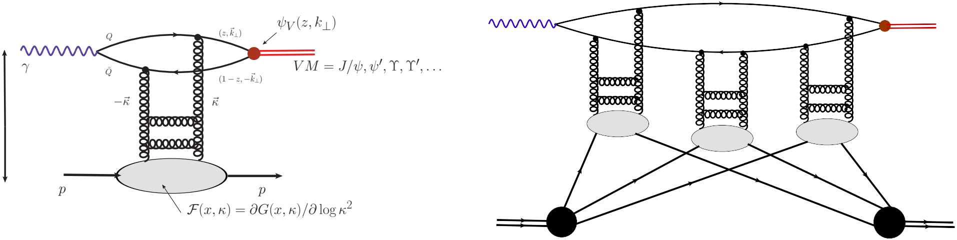
$$\sigma(x, r) = \frac{\pi^2}{3} r^2 \alpha_S(q^2) xg(x, q^2) \, , \, q^2 \approx \frac{10}{r^2}$$

- for large $Q^2 + M_V^2$ we then obtain the asymptotics

$$A(\gamma^* p \to Vp) \propto r_S^2 \sigma(x, r_S) \propto \frac{1}{Q^2 + M_V^2} \times \frac{1}{Q^2 + M_V^2} xg(x, Q^2 + M_V^2)$$

- probes the gluon distribution, which drives the energy dependence.
- From DGLAP fits: $xg(x, \mu^2) = (1/x)^{\lambda(\mu^2)}$ with $\lambda(\mu^2) \sim 0.1 \div 0.4$ for $\mu^2 = 1 \div 10^2 \mathrm{GeV}^2$.

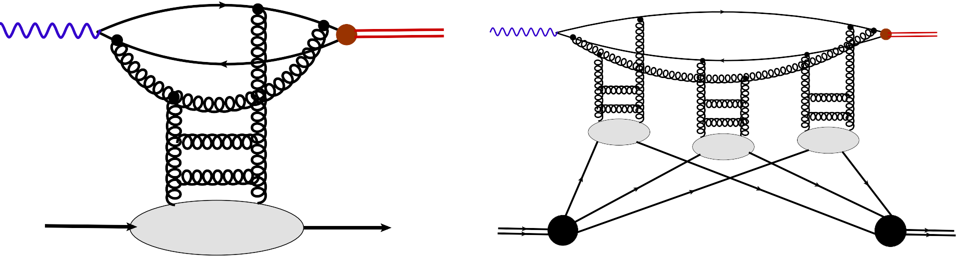
VM photoproduction from nucleon to nucleus:



- for heavy nuclei rescattering/absorption effects are enhanced by the large nuclear size
- $q\bar{q}$ rescattering is easily dealt with in impact parameter space
- the final state might as well be a (virtual) photon (total photoabsorption cross section) or a $q\bar{q}$-pair (inclusive low-mass diffraction).
- Color-dipole amplitude

$$\Gamma(\boldsymbol{b}, x, \boldsymbol{r}) = 1 - \frac{\langle A | Tr[S_q(\boldsymbol{b}) S_q^\dagger(\boldsymbol{b}+\boldsymbol{r})] | A\rangle}{\langle A | Tr[\mathbb{1}] | A\rangle}$$

Small-x evolution: adding $q\bar{q}(ng)$ Fock-states



- the effect of higher $q\bar{q}g$-Fock-states is absorbed into the x–dependent dipole-nucleus interaction Nikolaev, Zakharov, Zoller / Mueller '94
- evolution of unintegrated glue $\mathrm{Balitsky - Kovchegov\ '96 -' 98}$:

$$\frac{\partial \phi(\boldsymbol{b}, x, \boldsymbol{p})}{\partial \log(1/x)} = \mathcal{K}_{BFKL} \otimes \phi(\boldsymbol{b}, x, \boldsymbol{p}) + \mathcal{Q}[\phi](\boldsymbol{b}, x, \boldsymbol{p})$$

- corresponds to taking the contribution to shadowing from high–mass diffraction into account $\leftrightarrow$ Gribov's unitarity relation between nuclear shadowing and diffraction on the nucleon.

properties of the nonlinear term:

- first piece of the nonlinear term looks like a diffractive cut of a triple-Pomeron vertex Nikolaev & WS '05:

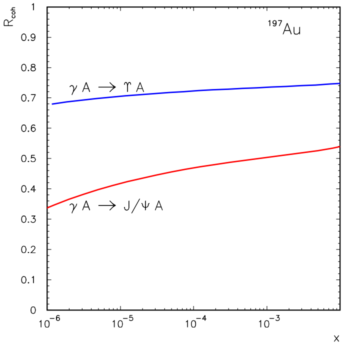
$$\int d^2\boldsymbol{q} d^2\boldsymbol{\kappa} \phi(\boldsymbol{b},x,\boldsymbol{q}) \Big[K(\boldsymbol{p}+\boldsymbol{\kappa},\boldsymbol{p}+\boldsymbol{q}) - K(\boldsymbol{p},\boldsymbol{\kappa}+\boldsymbol{p}) - K(\boldsymbol{p},\boldsymbol{q}+\boldsymbol{p})\Big] \phi(\boldsymbol{b},x,\boldsymbol{\kappa})$$
$$= -2K_0 \Big| \int d^2\boldsymbol{\kappa}\, \phi(\boldsymbol{b},x,\boldsymbol{\kappa}) \Big[\frac{\boldsymbol{p}}{\boldsymbol{p}^2+\mu_G^2} - \frac{\boldsymbol{p}+\boldsymbol{\kappa}}{(\boldsymbol{p}+\boldsymbol{\kappa})^2+\mu_G^2} \Big] \Big|^2$$

- at large $\boldsymbol{p}^2$ the nonlinear term is a pure higher twist, it is dominated by the 'anticollinear' region $\boldsymbol{\kappa}^2 > \boldsymbol{p}^2$. (see also Bartels & Kutak (2007)) It cannot be written as a square of the integrated gluon distribution.

$$\begin{aligned} \mathcal{Q}[\phi](\boldsymbol{b},x,\boldsymbol{p}) \quad &\approx \quad -\frac{2K_0}{\boldsymbol{p}^2} \Big| \int_{\boldsymbol{p}^2} \frac{d^2\boldsymbol{\kappa}}{\boldsymbol{\kappa}^2} \phi(\boldsymbol{b},x,\boldsymbol{\kappa}^2) \Big|^2 \\ & \quad -2K_0 \phi(\boldsymbol{b},x,\boldsymbol{p}^2) \int_{\boldsymbol{p}^2} \frac{d^2\boldsymbol{\kappa}}{\boldsymbol{\kappa}^2} \int_{\boldsymbol{\kappa}^2} d^2\boldsymbol{q} \phi(\boldsymbol{b},x,\boldsymbol{q}^2) \end{aligned}$$

- in that regard it differs from the earlier Mueller-Qiu and Gribov-Levin-Ryskin gluon fusion corrections.

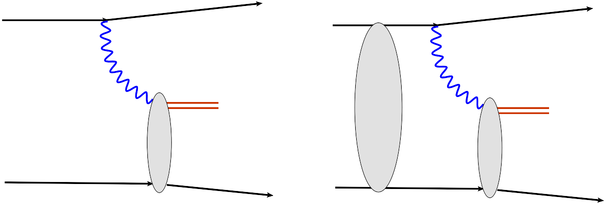
Coherent diffractive production of $J/\Psi, \Upsilon$ on ^{208}Pb



- A. Cisek, WS, A. Szczurek Phys. Rev **C86** (2012) 014905..
- Ratio of coherent production cross section to impulse approximation
- putative "gluon shadowing": $R_{\rm coh} \sim [g_A(x,\bar{Q}^2)/(A \cdot g_N(x,\bar{Q}^2))]^2$.

$$R_{\rm coh}(W) = \frac{\sigma(\gamma A \to VA; W)}{\sigma_{IA}(\gamma A \to VA; W)}\,, \, \sigma_{IA} = 4\pi \int d^2\boldsymbol{b} T_A^2(\boldsymbol{b}) \frac{d\sigma(\gamma N \to VN)}{dt}_{|t=0}$$

Absorption corrected flux of photons



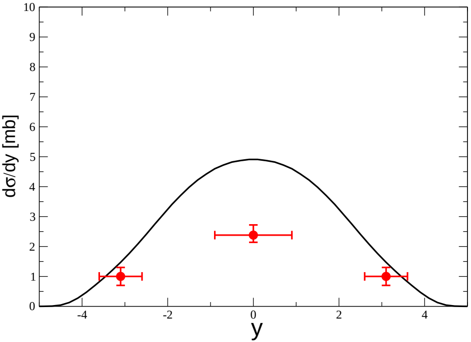
$$\sigma(A_1 A_2 \to A_1 A_2 f; s) = \int d\omega \frac{dN^{\rm eff}_{A_1}(\omega)}{d\omega} \sigma(\gamma A_2 \to f A_2; 2\omega\sqrt{s}) + (1 \leftrightarrow 2)$$

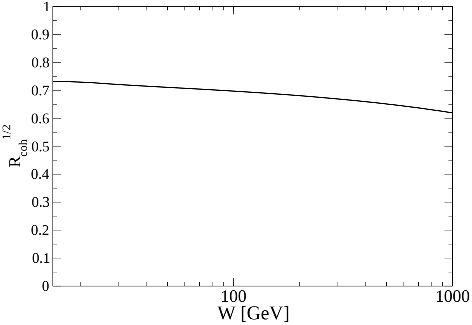
$$dN^{eff} = \int d^2\boldsymbol{b}\, S^2_{el}(\boldsymbol{b}) dN(\omega, \boldsymbol{b})$$

- $dN(\omega)$= Weizsäcker-Williams flux
- survival probability:

$$S^2_{el}(\boldsymbol{b}) = \exp\Big(-\sigma_{NN} T_{A_1 A_2}(\boldsymbol{b}) \Big) \sim \theta(|\boldsymbol{b}| - (R_1 + R_2))$$

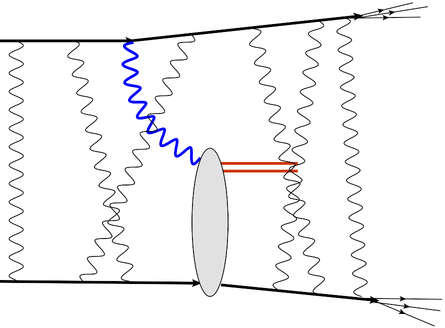
rapidity distributions

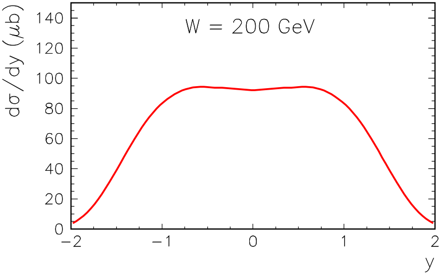




- $y = \pm 3.1 \to W \sim 20\,\mathrm{GeV} \to x \sim 2 \cdot 10^{-2}$ $y = 0 \to W \sim 92\,\mathrm{GeV} \to x \sim 10^{-3}$
- the putative "gluon shadowing": $R_G = \sqrt{R_{\mathrm{coh}}(x \sim 10^{-3})} \sim 0.7$.
- for illustration: $R_G(x, m_c^2) \equiv g_A(x, m_c^2)/(A \cdot g_N(x, m_c^2))$ from popular DGLAP fits:
- EPS09: $R_G(10^{-3}, m_c^2) \sim 0.6$
- EPS08: $R_G(10^{-3}, m_c^2) \sim 0.3$
- Inclusive dijet observables depend on unintegrated nuclear glue nonlinearly.

Few-neutron topological cross sections





$$d\sigma(AA \to V(Xn)(Yn)) = \int d^2\boldsymbol{b}\, d\sigma(AA \to VAA; \boldsymbol{b}) \times P(AA \to (Xn)(Yn); \boldsymbol{b})$$

- for the integrated case Klein and Nystrand estimate a suppresion of 0.55
- exp. result: PHENIX Collab. (2009):

$$\frac{d\sigma(AuAu \to J/\Psi Xn)}{dy}(y=0) = 76 \pm 33 \pm 11\ \mu b$$

See Antoni Szczurek's talk on friday.

Summary

- Glauber theory provides us with a framework to calculate nuclear observables from –physical– free nucleon input.
- When diffractive dissociation in hadron-prton scattering becomes important, the Glauber approach must be extended to include Gribov's inelastic shadowing.
- The method of diffraction scattering eigenstates is of great utility on nuclear targets.
- In a pQCD based picture, color dipoles can be viewed as the diffraction scattering eigenstates. Here the $q\bar{q}$-states correspond to the "low-mass" states and $q\bar{q}g, q\bar{q}gg \dots$ are the high mass states of the triple-Pomeron regime.
- The fraction of events with a nucleus intact is large in Deep Inelastic Scattering!
- In photoproduction of heavy quarkonia, the large quark mass ensures dominance of small dipoles $\to$ pQCD.
- a sensitive probe of the (unintegrated) gluon distribution of the target nucleus. Rescattering/saturation effects entail that the unintegrated glue enters inclusive dijet observables nonlinearly.
- "gluon shadowing" is included via the rescattering of higher $Q\bar{Q}g$ Fock states. The effective "gluon shadowing" ratio $R_G(x, m_c^2) \sim 0.74 \div 0.62$. For $x \sim 10^{-2} \div 10^{-5}$. ALICE data appear to indicate somewhat stronger effect $R_G(10^{-3}, m_c^2) \div 0.6$.
- J/ψ-pair production in via $\gamma\gamma$ fusion in AA is dominated by the "box-diagram" mechanisms. Multiple interactions of the type $(\gamma\mathbb{P} \to J/\psi) \otimes (\gamma\mathbb{P} \to J/\psi)$ may also be important.

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- Diffraction scattering eigenstates for nuclear targets:
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- Color dipole approach:
- V. Barone, E. Predazzi, "High-energy particle diffraction," Berlin, Germany: Springer (2002) 407 p; S. Donnachie, H. G. Dosch, O. Nachtmann and P. Landshoff, "Pomeron physics and QCD,", Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **19** (2002) 1.
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