High-energy soft reactions:

a model with tensor pomeron and vector odderon

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$C = +1$ and $C = -1$ exchanges

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1 Introduction

Examples of high-energy soft reactions:

- elastic scattering

$$p + p \longrightarrow p + p$$

$$\bar{p} + p \longrightarrow \bar{p} + p$$

$$\pi + p \longrightarrow \pi + p$$

- photoproduction

$$\gamma + p \longrightarrow \rho^0 + p$$

$$\gamma + \gamma \longrightarrow \rho^0 + \rho^0$$

- central production

$$p + p \longrightarrow p + \text{meson} + p$$

$$\sqrt{s} \to \infty, \qquad \sqrt{|t|} \lesssim 1\ \mathrm{GeV}$$

In QCD: neither pure short nor pure long distance regime, difficult to treat.

The physics of exchanges, regge regime

pomeron $\mathbb{P}$

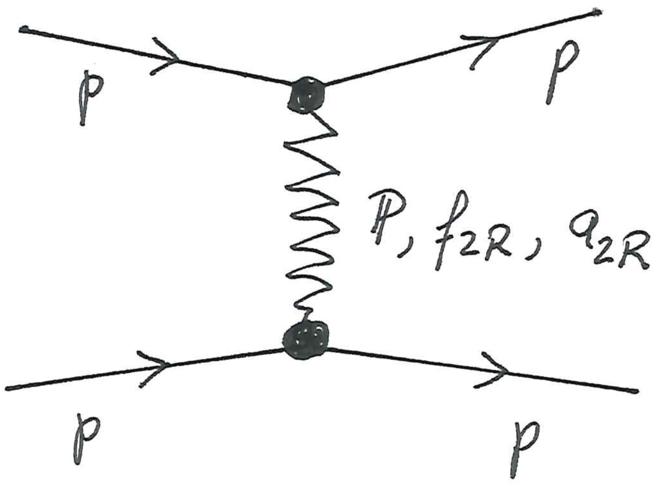
reggeons f_{2R}, a_{2R}, ω_R, ρ_R

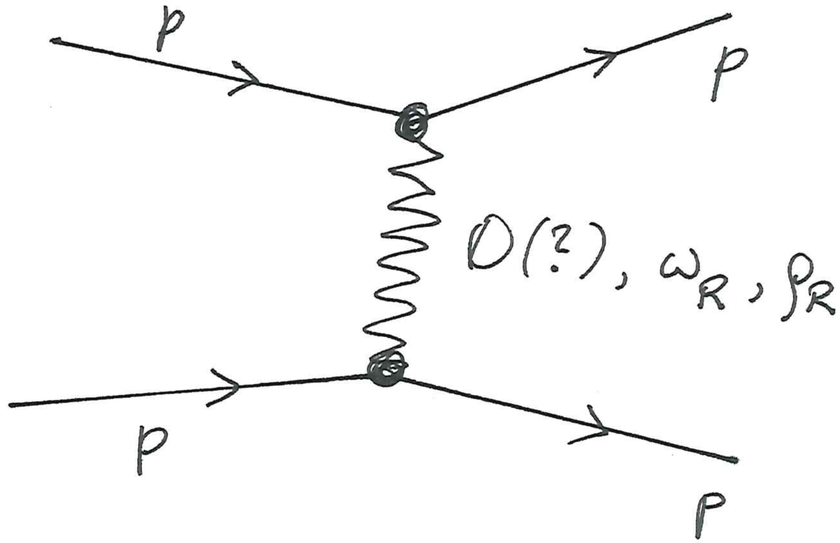
odderon (?) $\mathbb{O}$

- Our aim is to give simple rules, compatible with QFT, for calculating such exchange amplitudes.

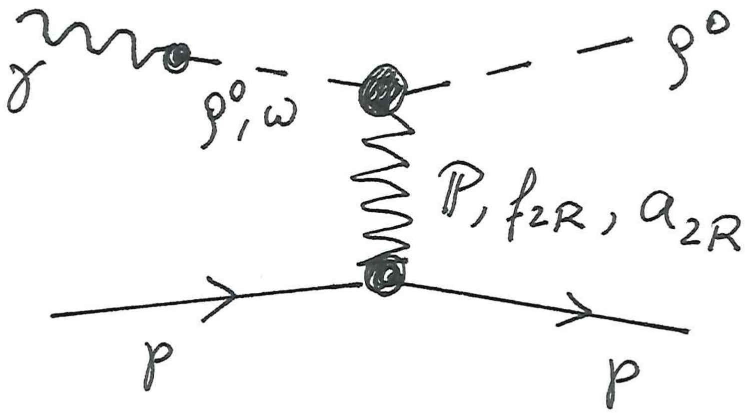
Examples :

- $p + p \longrightarrow p + p$





- $\gamma + p \longrightarrow \rho^0 + p$



We need a <u>list of effective propagators and vertices</u>: $\mathbb{P}$ propagator, $\mathbb{P}pp$ vertex etc.

We tried to make a marriage between

<u>QFT</u> and <u>Regge theory</u>.

Unexpected results:

- $\mathbb{P}$ as an effective rank-two tensor exchange.
- Relations between particle-particle-particle and reggeon-particle-particle vertices.
- Insight into the meaning of the vector-meson-dominance (VMD) relations.

Some references which were used in our work:

Donnachie, Dosch, Landshoff, O.N. :
"Pomeron Physics and QCD", CUP, 2002

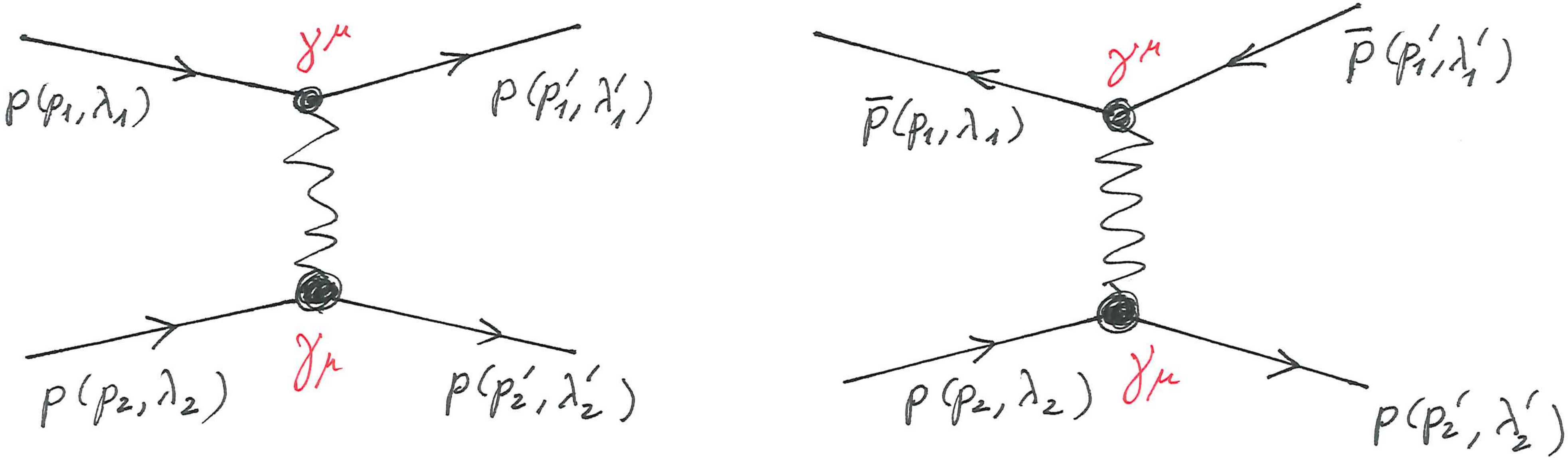
Collins; "An Introduction to Regge Theory and High Energy Physics", CUP, 1977

Close, Donnachie, Shaw: "Electromagnetic interactions and hadronic structure" CUP, 2007

plus many others.

2 The pomeron

Example: pp and $\bar{p}p$ scattering

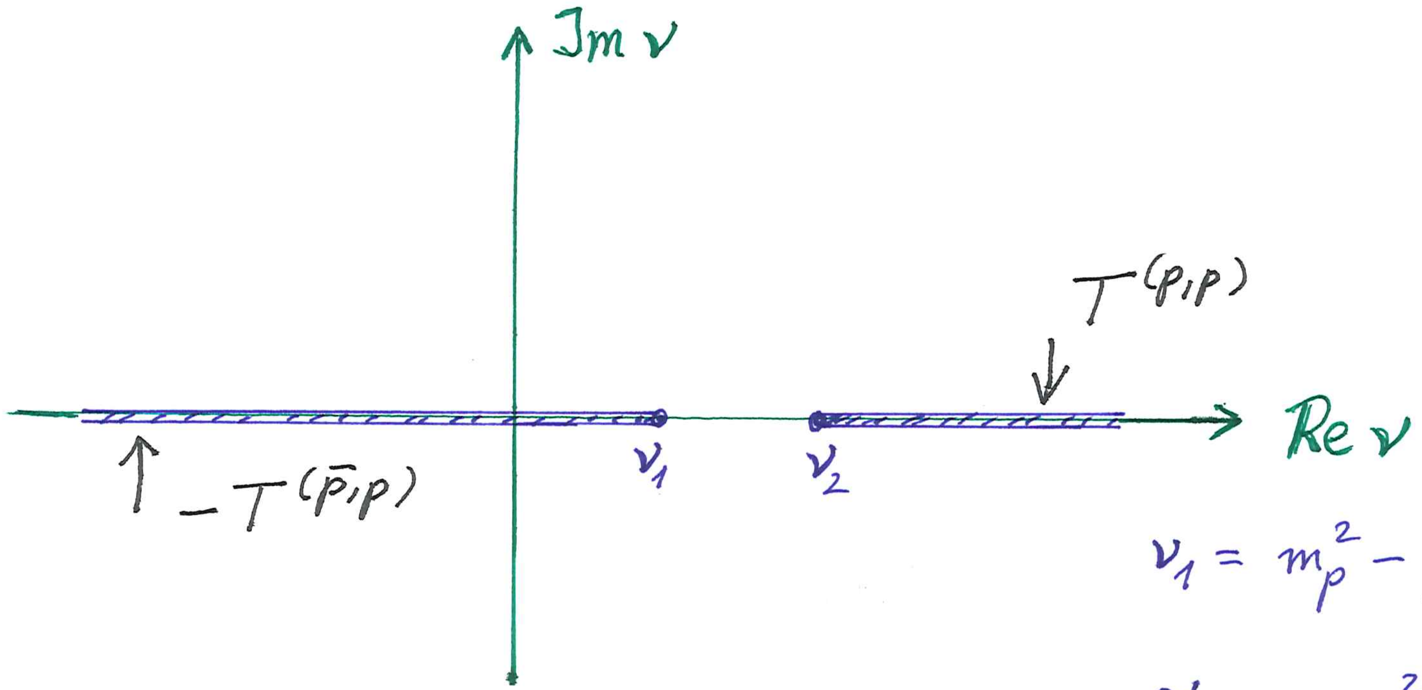


$T^{(p,p)}(\nu, t)\ \gamma^\mu \otimes \gamma_\mu$ $\qquad$ $T^{(\bar{p},p)}(\nu, t)\ \gamma^\mu \otimes \gamma_\mu$

$s = (p_1 + p_2)^2, \quad t = (p_1 - p_1')^2, \quad u = (p_1 - p_2')^2, \quad \nu = \frac{1}{4}(s-u)$

$\lambda_i, \lambda_i' \in \{1/2, -1/2\}$: helicities

From QFT : $\exists \; A(\nu, t)$ analytic function of ν for fixed t



$$\nu_1 = m_p^2 - \frac{1}{2} m_\pi^2 - \frac{1}{4} t$$

$$\nu_2 = m_p^2 + \frac{1}{4} t$$

$$T^{(\bar{p},p)}(\nu,t) = -\lim_{\varepsilon \to 0} A(-\nu - i\varepsilon, t) \qquad T^{(p,p)}(\nu,t) = \lim_{\varepsilon \to 0} A(\nu + i\varepsilon, t)$$

Power law ansatz – suggested by regge theory – for the crossing-even contribution to $A(\nu, t)$ respecting the analyticity structure:

$$A^{\mathbb{P}}(\nu, t) \propto \nu \left[(\nu_2 - \nu)(\nu - \nu_1) \right]^{\frac{1}{2}(\alpha_{\mathbb{P}}(t) - 2)},$$

pomeron trajectory:

$$\alpha_{\mathbb{P}}(t) = 1 + \varepsilon_{\mathbb{P}} + \alpha'_{\mathbb{P}} t,$$

$$\varepsilon_{\mathbb{P}} = 0.0808, \qquad \alpha'_{\mathbb{P}} = 0.25 \text{ GeV}^{-2}$$

Fit parameters: see Donnachie & Landshoff (DL), Nucl. Phys. 1986, P.L. 1992

Note that analyticity determines the phase of $A^{\mathbb{P}}(\nu, t)$.

With suitable real factors we get the DL-pomeron ansatz for pp and $\bar{p}p$ scattering:

For $s \to \infty$:

- $$\langle p(1'), p(2') | \mathcal{T} | p(1), p(2) \rangle \Big|_{\mathbb{P}} \to$$
$$i \left[3\beta_{\mathbb{P}NN} F_1(t) \right]^2 (-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1} \underbrace{\bar{u}(1') \gamma^\mu u(1) \, \bar{u}(2') \gamma_\mu u(2)}$$
$$\to 2s \, \delta_{\lambda_1' \lambda_1} \delta_{\lambda_2' \lambda_2}$$

- $$\langle \bar{p}(1'), p(2') | \mathcal{T} | \bar{p}(1), p(2) \rangle \Big|_{\mathbb{P}} \to$$
$$i \left[3\beta_{\mathbb{P}NN} F_1(t) \right]^2 (-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1} \underbrace{\bar{v}(1) \gamma^\mu v(1') \, \bar{u}(2') \gamma_\mu u(2)}$$
$$\to 2s \, \delta_{\lambda_1' \lambda_1} \delta_{\lambda_2' \lambda_2}$$

$F_1(t)$: electromagnetic Dirac form factor of the proton,

$3\beta_{\mathbb{P}NN} = 3 \times 1.87 \text{ GeV}^{-1}$ $\mathbb{P}$-nucleon coupling const.

- Does the $\gamma^\mu \otimes \gamma_\mu$ structure of these amplitudes imply that <u>$\mathbb{P}$ exchange</u> is effectively a <u>vector exchange</u>?

The answer from <u>QFT is clearly NO</u>!

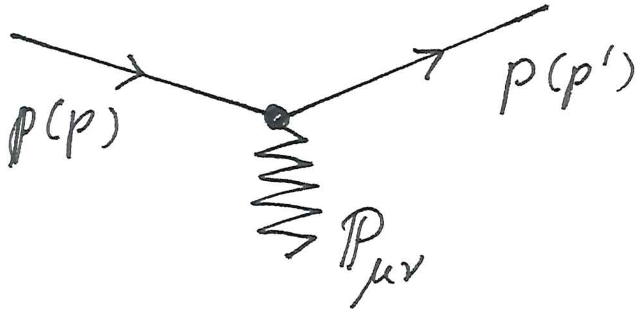
A vector must couple with opposite signs to p and $\bar{p}$.

But $\mathbb{P}$ couples equally to p and $\bar{p}$.

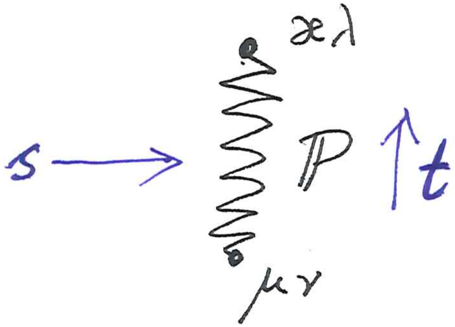
- A way out of this dilemma:
  Write $\mathbb{P}$ exchange as <u>effective tensor exchange</u>.

  A tensor, like in gravity, couples equally to particles and antiparticles.

Effective $\mathbb{P}NN$ vertex and $\mathbb{P}$ propagator:

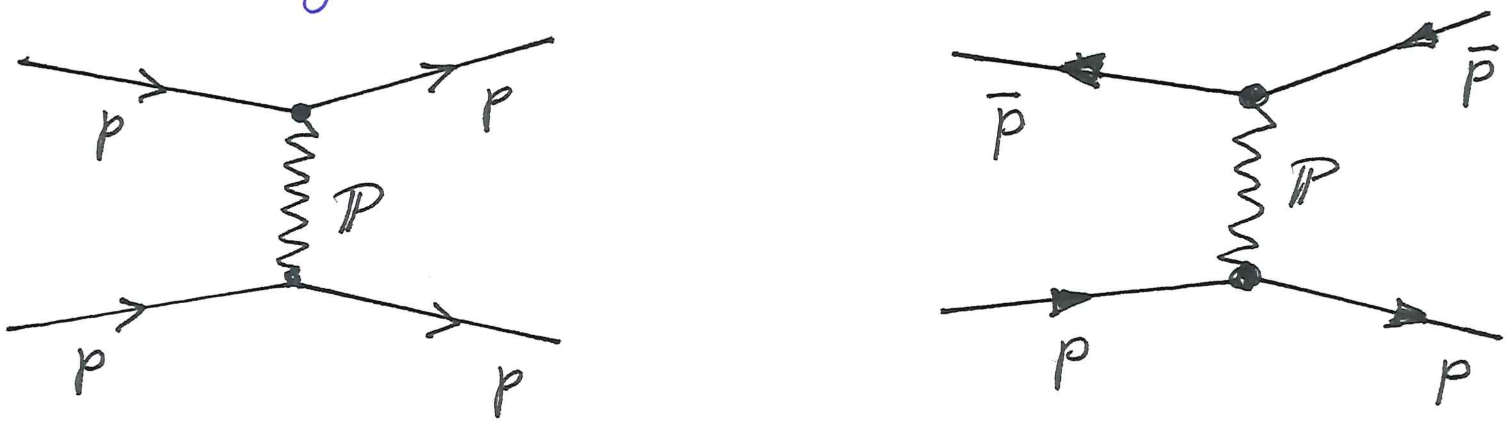


$$-i\,3\beta_{\mathbb{P}NN}\,F_1(t)\left[\frac{1}{2}\gamma_\mu(p'+p)_\nu+\frac{1}{2}\gamma_\nu(p'+p)_\mu-\frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right]$$



$$\frac{1}{4s}\left(-is\alpha'_{\mathbb{P}}\right)^{(\alpha_{\mathbb{P}}(t)-1)}\left[g_{\mu\kappa}g_{\nu\lambda}+g_{\mu\lambda}g_{\nu\kappa}-\frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right]$$

From the diagrams



we get now with the <u>standard rules of QFT</u> the DL ansatz for the pp and $\bar{p}p$ amplitudes.

- What have we gained? Very much, it turns out, when we consider reactions involving <u>vector particles</u>: examples:

$$\gamma + p \longrightarrow \rho^0 + p,$$

$$\gamma + \gamma \longrightarrow \rho^0 + \rho^0.$$

The pomeron as coherent sum of spin $2+4+6+\ldots$ exchanges: $\quad \nu = (p_1+p_1', p_2+p_2')/4$,

$$\langle p(p_1'), p(p_2') | T | p(p_1), p(p_2)\rangle \big|_{\mathbb{P}}$$

$$= A^{\mathbb{P}}(\nu, t)\; \bar{u}(p_1')\gamma^{\mu_1} u(p_1)\, \bar{u}(p_2')\gamma_{\mu_1} u(p_2)$$

$$\xrightarrow[\nu\to\infty]{} f(t)\, (4\alpha_{\mathbb{P}}'^{2}\nu^2)^{\frac{1}{2}(\alpha_{\mathbb{P}}(t)-2)}$$

$$2\alpha_{\mathbb{P}}'\nu\; \bar{u}(p_1')\gamma^{\mu_1} u(p_1)\, \bar{u}(p_2')\gamma_{\mu_1} u(p_2)$$

$$\propto \underbrace{(4\alpha_{\mathbb{P}}'^{2}\nu^2)^{\frac{1}{2}(\alpha_{\mathbb{P}}(t)-2)}}_{\text{function of } \nu^2}\; \bar{u}(p_1')\left[\gamma^{\mu_1}(p_1'+p_1)^{\mu_2} + (\mu_1 \leftrightarrow \mu_2)\right] u(p_1)$$

$$\bar{u}(p_2')\left[\gamma_{\mu_1}(p_2'+p_2)_{\mu_2} + (\mu_1 \leftrightarrow \mu_2)\right] u(p_2)$$

tensor ⊗ tensor

$$\left(4\alpha_{\mathbb{P}}'^{2}\nu^{2}\right)^{\frac{1}{2}(\alpha_{\mathbb{P}}(t)-2)} = \frac{1}{\Gamma\left(1-\frac{1}{2}\alpha_{\mathbb{P}}(t)\right)} \int_0^\infty d\tau \, \tau^{-\frac{1}{2}\alpha_{\mathbb{P}}(t)} \exp\left(-4\alpha_{\mathbb{P}}'^{2}\nu^{2}\tau\right)$$

$$= \frac{1}{\Gamma\left(1-\frac{1}{2}\alpha_{\mathbb{P}}(t)\right)} \int_0^\infty d\tau \, \tau^{-\frac{1}{2}\alpha_{\mathbb{P}}(t)} \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \left(-4\alpha_{\mathbb{P}}'^{2}\tau\right)^{n-1} \nu^{2n-2}$$

$$\langle p(p_1'), p(p_2') | \mathcal{T} | p(p_1), p(p_2) \rangle \Big|_{\mathbb{P}} \xrightarrow[\nu\to\infty]{} \tilde{f}(t) \int_0^\infty d\tau \, \tau^{-\frac{1}{2}\alpha_{\mathbb{P}}(t)}$$

$$\sum_{n=1}^{\infty} (-\tau)^{n-1} \left(\frac{1}{2}\alpha_{\mathbb{P}}'\right)^{2n-1} \frac{1}{(n-1)!\,[(2n)!]^2}$$

$$\bar{u}(p_1')\left[\gamma^{\mu_1}(p_1'+p_1)^{\mu_2}\cdots(p_1'+p_1)^{\mu_{2n}} + \text{perm.}\right] u(p_1)$$

$$\bar{u}(p_2')\left[\gamma_{\mu_1}(p_2'+p_2)_{\mu_2}\cdots(p_2'+p_2)_{\mu_{2n}} + \text{perm.}\right] u(p_2)$$

coherent sum of spin 2 + 4 + 6 + ... exchanges

A frequently asked question:

- why is the pomeron not spin 0 exchange?

A structure

$$\left(\bar{u}(p_1')\, u(p_1)\right)\left(\bar{u}(p_2')\, u(p_2)\right)$$

gives for high-energy small-angle scattering s-channel helicity conserving and violating amplitudes of the same size.

Experiment sees, however, helicity conservation.

In theory we understand this as being in essence due to the helicity conserving quark-gluon coupling.

3 Effective propagators and vertices for $C = 1$ & $C = -1$ exchanges

Our aim: give a list of propagators & vertices.

If you want to calculate the amplitude for a specific process, e.g.

$$\gamma + p \longrightarrow \rho^0 + p$$

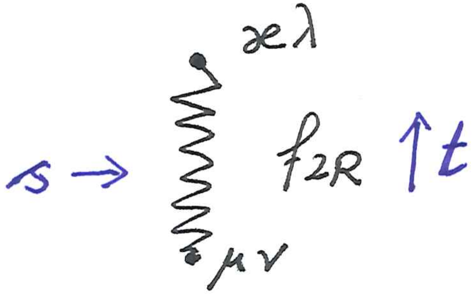
proceed as follows:

- draw the relevant diagrams
- combine propagators and vertices according to the rules of QFT
- get a result fitting the data perfectly !?

Some highlights from our list:

- all $C = 1$ exchanges, $\mathbb{P}$, $f_{2\mathbb{R}}$, $a_{2\mathbb{R}}$, are represented as rank-two-tensor exchanges.

Example: $f_{2\mathbb{R}}$ effective propagator:



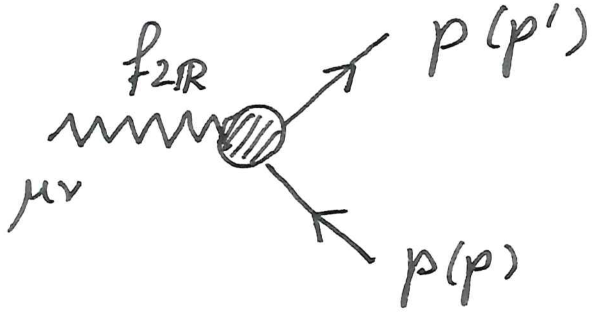
$$\frac{1}{4s}\left(-is\alpha'_{\mathbb{R}_+}\right)^{\alpha_{\mathbb{R}_+}(t)-1}$$

$$\left[g_{\mu\varkappa}g_{\nu\lambda}+g_{\mu\lambda}g_{\nu\varkappa}-\frac{1}{2}g_{\mu\nu}g_{\varkappa\lambda}\right]$$

$$\alpha_{\mathbb{R}_+}(t) = \alpha_{\mathbb{R}_+}(0) + \alpha'_{\mathbb{R}_+}\, t$$

$\alpha_{\mathbb{R}_+}(0) = 0.5475$ $\qquad\qquad$ $\alpha'_{\mathbb{R}_+} = 0.9\ \mathrm{GeV}^{-2}$

$f_{2\mathbb{R}}\, pp$ vertex



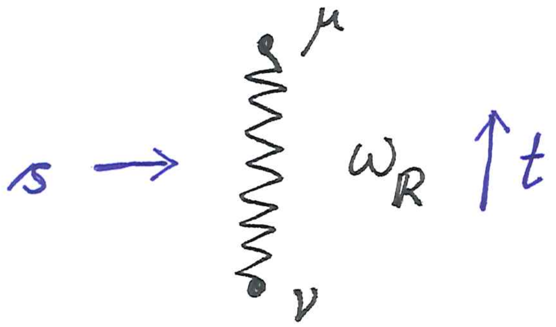
$$-i\, g_{f_{2\mathbb{R}} pp} \frac{1}{M_0} F_1\left[(p'-p)^2\right]$$

$$\left\{ \frac{1}{2} \gamma_\mu (p'+p)_\nu + \frac{1}{2} \gamma_\nu (p'+p)_\mu - \frac{1}{4} g_{\mu\nu} (\not{p}' + \not{p}) \right\}$$

$M_0 \equiv 1\,\mathrm{GeV}$, $\quad g_{f_{2\mathbb{R}} pp} = 11.04$

- all $C=-1$ exchanges, $\mathbb{O}(?)$, $\omega_{\mathbb{R}}$, $\rho_{\mathbb{R}}$, are represented as vector exchanges.

Example: $\omega_{\mathbb{R}}$ exchange, effective propagator:

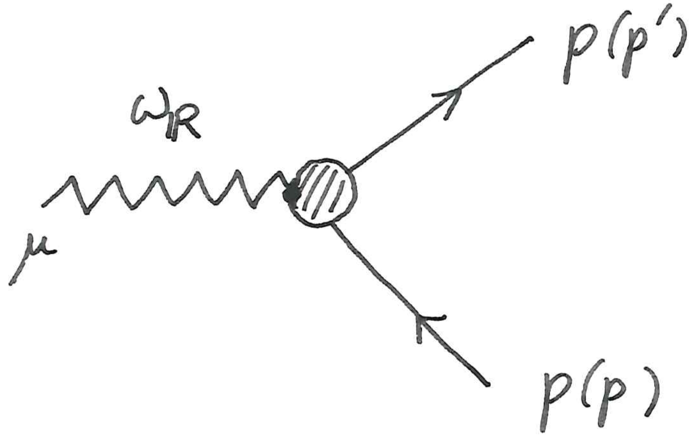


$$i g_{\mu\nu} (M_-)^{-2} \left(-is\alpha'_{\mathbb{R}_-}\right)^{\alpha_{\mathbb{R}_-}(t)-1}$$

$$\alpha_{\mathbb{R}_-}(t) = \alpha_{\mathbb{R}_-}(0) + \alpha'_{\mathbb{R}_-}\, t$$

$$\alpha_{\mathbb{R}_-}(0) = 0.5475\,, \quad \alpha'_{\mathbb{R}_-} = 0.9\ \mathrm{GeV}^{-2}\,, \quad M_- = 1.41\ \mathrm{GeV}$$

$\omega_{\mathbb{R}}\, p\, p$ vertex



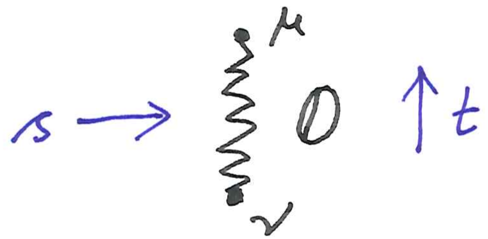
$$i\Gamma_\mu^{(\omega_{\mathbb{R}} pp)}(p', p) =$$

$$-i\, g_{\omega_{\mathbb{R}} pp} F_1\left[(p'-p)^2\right] \gamma_\mu \,,$$

$$g_{\omega_{\mathbb{R}} pp} = 8.65$$

Odderon exchange: parameters still to be determined by exp.

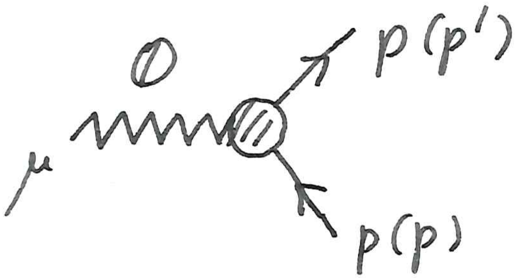
- effective propagator:

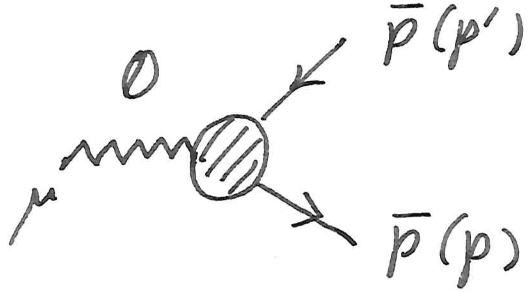


$$-i g_{\mu\nu} \frac{\eta_{\mathbb{O}}}{M_0^2} \left(-i s \alpha'_{\mathbb{O}}\right)^{\alpha_{\mathbb{O}}(t)-1} ,$$

$$\eta_{\mathbb{O}} = \pm 1 , \quad M_0 = 1\,\text{GeV}$$

- ONN vertex:



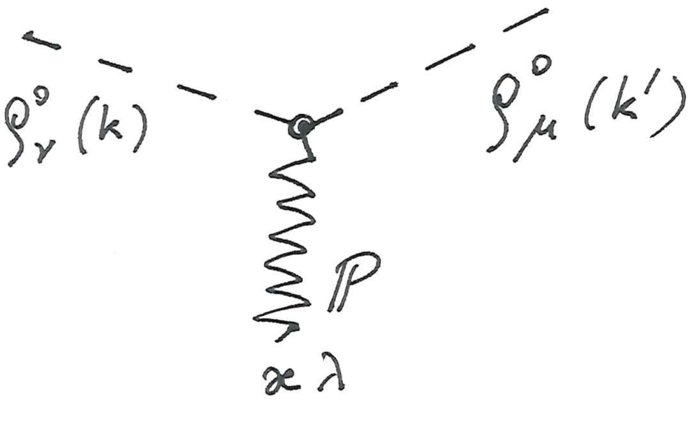


$$i\Gamma_\mu^{(\mathbb{O}pp)}(p',p) = -i\Gamma_\mu^{(\mathbb{O}\bar{p}\bar{p})}(p',p) = -i\,3\beta_{\mathbb{O}pp} M_0 F_1(t)\,\gamma_\mu$$

The vector ansatz for $\mathbb{O}$ automatically gives minus sign for the coupling of proton versus antiproton.

- all vertices respect standard QFT rules.

Example: $\mathbb{P}\rho\rho$ vertex $\left(t = (k' - k)^2 \right)$



$$i F_M(t) \Big[2 a_{\mathbb{P}\rho\rho} \Gamma^{(0)}_{\mu\nu\varkappa\lambda}(k', -k) - b_{\mathbb{P}\rho\rho} \Gamma^{(2)}_{\mu\nu\varkappa\lambda}(k', -k) \Big]$$

$$F_M(t) = \frac{m_0^2}{m_0^2 - t}, \quad m_0^2 = 0.50 \ \mathrm{GeV}^2$$

$a_{\mathbb{P}\rho\rho}$, $b_{\mathbb{P}\rho\rho}$ coupling constants

$$\Gamma^{(0)}_{\mu\nu\varkappa\lambda}(k_1, k_2) = \left[(k_1 k_2)\, g_{\mu\nu} - k_{2\mu} k_{1\nu}\right]$$

$$\left[k_{1\varkappa} k_{2\lambda} + k_{2\varkappa} k_{1\lambda} - \frac{1}{2}(k_1 k_2)\, g_{\varkappa\lambda}\right]$$

$$\Gamma^{(2)}_{\mu\nu\varkappa\lambda}(k_1, k_2) = (k_1 k_2)\left(g_{\mu\varkappa} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\varkappa}\right)$$

$$+ g_{\mu\nu}\left(k_{1\varkappa} k_{2\lambda} + k_{2\varkappa} k_{1\lambda}\right)$$

$$- k_{1\nu} k_{2\lambda}\, g_{\mu\varkappa} - k_{1\nu} k_{2\varkappa}\, g_{\mu\lambda}$$

$$- k_{2\mu} k_{1\lambda}\, g_{\nu\varkappa} - k_{2\mu} k_{1\varkappa}\, g_{\nu\lambda}$$

$$- \left[(k_1 k_2)\, g_{\mu\nu} - k_{2\mu} k_{1\nu}\right] g_{\varkappa\lambda}$$

This $\mathbb{P}\rho\rho$ vertex is obtained from the coupling Lagrangian:

$$\mathcal{L}'_{\mathbb{P}\rho\rho} = \Big[a_{\mathbb{P}\rho\rho} \left(\partial_\varkappa F^{(\rho)}_{\mu\nu} \right) \left(\partial_\lambda F^{(\rho)\mu\nu} \right)$$

$$+ b_{\mathbb{P}\rho\rho} F^{(\rho)}_{\mu\varkappa} F^{(\rho)\mu}{}_{\lambda} \Big] \left(g^{\varkappa\varkappa'} g^{\lambda\lambda'} - \frac{1}{4} g^{\varkappa\lambda} g^{\varkappa'\lambda'} \right) \mathbb{P}_{\varkappa'\lambda'} \,,$$

$$F^{(\rho)}_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu \,.$$

- inclusion of photons using vector meson dominance, VMD

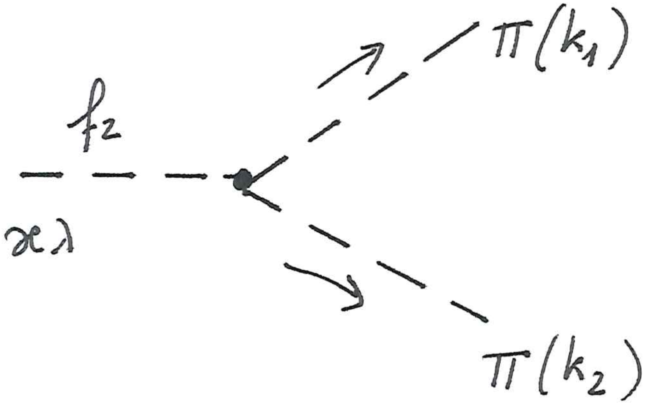
$$-ie\,\frac{m_V^2}{\gamma_V}\,g_{\mu\nu} \qquad V = \rho^0, \omega, \phi$$

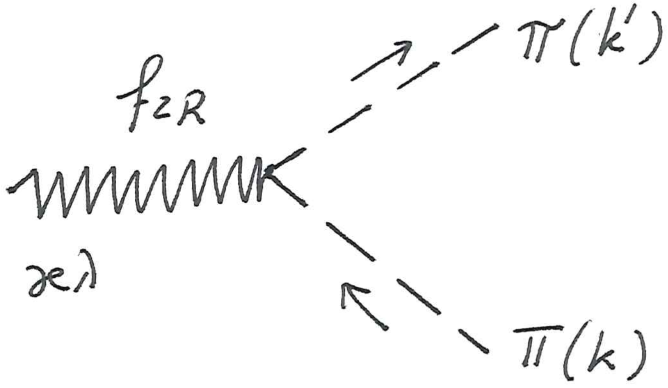
No gauge invariance problems using our QFT vertices!

- relations between particle-particle-particle and reggeon-particle-particle vertices.

Example:

$$f_2\,\pi\,\pi \qquad\qquad f_{2R}\,\pi\,\pi$$





$$-i\,\frac{g_{f_2\pi\pi}}{2M_0}\Big[(k_1-k_2)_\varkappa (k_1-k_2)_\lambda - \frac{1}{4}\, g_{\varkappa\lambda}\,(k_1-k_2)^2\Big]$$

$$M_0 = 1\ \mathrm{GeV}$$

$$g_{f_2\pi\pi} = 9.26 \pm 0.15$$

from $\Gamma(f_2 \to \pi\pi)$

$$-i\,\frac{g_{f_{2R}\pi\pi}}{2M_0}\, F_M\big[(k'-k)^2\big]\Big[(k'+k)_\varkappa (k'+k)_\lambda - \frac{1}{4}\, g_{\varkappa\lambda}\,(k'+k)^2\Big]$$

$$g_{f_{2R}\pi\pi} = 9.30$$

from $\sigma_{tot}(\pi^\pm p)$

4 Comparison with experiment

So far we used mainly data from total cross sections and from some decay reactions to determine the parameters of the model.

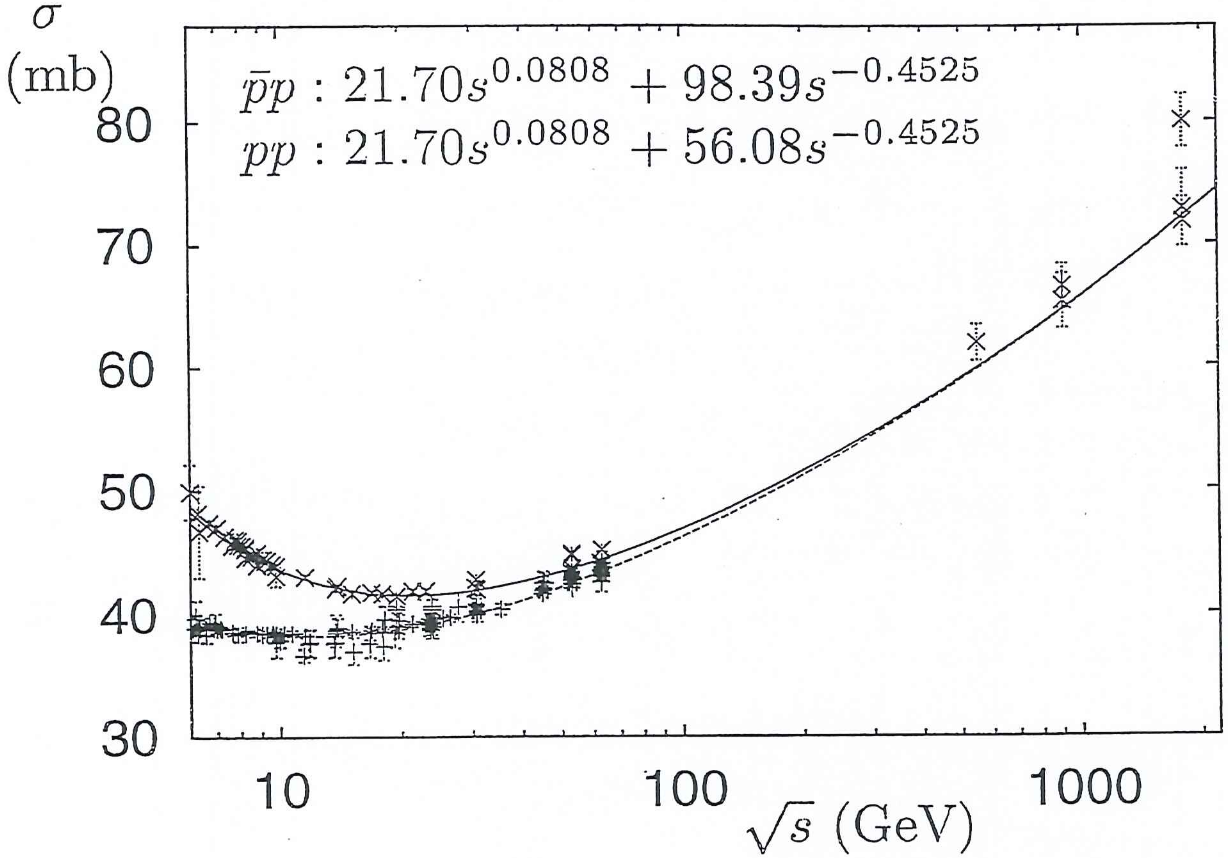
Example: fit to σ_{tot} from DL / Pomeron book:

$$\sigma_{tot}(a,b) = X_{ab}\left(\frac{s}{M_0^2}\right)^{0.0808} + Y_{ab}\left(\frac{s}{M_0^2}\right)^{-0.4525},$$

$a = p, \bar{p}$; $b = p, n$.

$M_0 \equiv 1$ GeV , $X_{ab} \equiv X = 21.70$ mb

a	p	p	$\bar{p}$	$\bar{p}$
b	p	n	p	n
Y_{ab} (mb)	56.08	54.77	98.39	92.71



From "Pomeron physics and QCD" by Donnachie, Dosch, Landshoff, O.N.

From this we extract:

$$\beta_{\mathbb{P}NN}\ ,\quad g_{f_{2R}pp}\ ,\quad g_{a_{2R}pp}\ ,\quad g_{\omega_R pp}\ ,\quad g_{\rho_R pp}$$

- Comparison with data from, e.g.,

$$\gamma + p \longrightarrow \rho^0 + p$$
$$\hookrightarrow \pi^+ + \pi^-$$

would be most welcome (HERA, LHC)

- Comparison with central meson production is being done together with A. Szczurek and P. Lebiedowicz (WA 102 exp.)

5 Conclusions

- We outlined a model for high-energy soft reactions based on QFT plus elements of Regge theory. We give a list of propagators and vertices.
- $C = +1$ exchanges, $\mathbb{P}$, f_{2R}, a_{2R}, are represented as tensors of rank 2.
- $C = -1$ exchanges, $\mathbb{O}$ (?), ω_R, ρ_R, are represented as vectors.
- Comparisons with data would be most welcome: ISR, UA1, UA2, FNAL, HERA, LHC, COMPASS, RHIC

- The model should allow to make predictions for cross sections and distributions of soft reactions in terms of just a few coupling parameters.
- <u>Open problems</u>:
  - Absorption effects?
  - Derivation of our rules from QCD?
  - Form factors

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