

# **Diffraction at the LHC**

**(and elsewhere)**

***Wilhelm und Else Heraeus Physics  
Summer School***

***“Diffractive and electromagnetic processes at high  
energies”***

***Heidelberg, September 2-6, 2013***

**László L. Jenkovszky**  
[jenk@bitp.kiev.ua](mailto:jenk@bitp.kiev.ua)

# Diffraction in optics and in HEP

Necessary condition for **diffraction** (deviation from geometrical optics):  
 $kR^2 \gg 1$ , where  $k = 2\pi/\lambda$ ,  $\lambda$  is the wave length and  $R$  is the size of the obstacle (or hole) is

Fraunhofer diffrr.:  $kR^2/D \ll 1$ ,

Frenel diffrr.:  $kR^2/D \approx 1$ , where  $D$  is the distance between the source and the detector. The case  $kR^2/D \gg 1$  corresponds to linear optics.

In high-energy, say,  $> 1\text{GeV}$ , experiments, *Fraunhofer* diffraction dominates: the obstacle, hole =detector is of  $1\text{Fm}$ , while the distance between the source and the detector is practically infinite. (N.B.: At the LHC, however, Fresnel diffraction may occur in the Coulomb region.) At the LHC,  $\sqrt{14}\text{TeV}$ ,  $R \approx 1\text{Fm}$ ,  $D 1\text{cm}$ , hence  $kR^2/D \approx 10^{-6}$ , (compared with  $\sqrt{50}\text{GeV} \rightarrow kR^2/D \approx 10^{-9}$  at the ISR (CERN)).

**Diffraction extends in a huge span of wavelengths !**

# Plan

1. Historical introduction;
2. Definition: Diffraction= a) Pomeron exchange; b) Rapgap;  
How many Pomerons? One (but complicated!); The Odderon;  
amplitudes and measurables;
3. Regge poles (t-channel) and geometrical (s-channal) models;
4. Unitarity, impact parameter, eikonal, U-matrix, gap survival;
5. Elastic scattering, the dip-bump phenomenon, black disc limit;
6. Resonance-Regge duality, the background, two-component duality;
7. SDD, DDD, CED, factorization relations;
8. The pPX vertex and DIS (HERA), triple Regge limit;
9. Duality and FMSR;
10. Diffraction at the LHC: Pomeron (>95%) dominance!

## References

- P. Collins: *An introduction to Regge poles in HEP*, ~1970;
- S. Donnachie, G. Dosch, P. Landshoff, and
- O. Nachtmann: *Pomeron physics and QCD*;
- V. Barone and E. Predazzi;
- L. Jenkovszky (see below)

## Diffractional Scattering of Fast Deuterons by Nuclei

A. I. AKHIEZER AND A. G. SITENKO

*Physical Technical Institute, Academy of Sciences of Ukrainian S.S.R., Kharkov University, U.S.S.R.*

(Received July 5, 1956; revised manuscript received January 7, 1957)

The elastic scattering cross section  $\sigma_e$  and the diffractional disintegration cross section  $\sigma_d$  for fast deuterons incident on absolutely black nuclei are determined, and the energy spectrum of the disintegration products is found. For  $R \gg R_d \gg \lambda$  [where  $R$  and  $R_d$  are the radii of the nucleus and of the deuteron, respectively, and  $\lambda$  is  $(2\pi)^{-1}$  times the wavelength of the deuteron], the cross sections are  $\sigma_e = \pi R^2 + \frac{2}{3}\pi(1 - \ln 2)RR_d$  and  $\sigma_d = \frac{1}{3}\pi(2 \ln 2 - \frac{1}{2})RR_d$ .

The total cross section for all processes (including the stripping and the absorption of the deuteron) is  $\sigma_t = 2\pi R^2 + \pi RR_d$ .

The disintegration cross section for fast deuterons, taking into account the diffraction and the Coulomb interaction, is found. If the nucleus is absolutely black and if  $R \gg R_d$ , there is no interfer-

ence between the diffractional disintegration and the disintegration due to the Coulomb interaction. If in this case  $n = Ze/v$  (where  $v$  is the velocity of the deuteron), the disintegration section due to the Coulomb interaction is a small correction to the diffractional disintegration cross section. If  $n \gg 1$  and  $E \gg B$  ( $E$  is the energy of the deuteron and  $B$  is the height of the Coulomb barrier), the disintegration cross section can also be found; in this case it is determined mainly by the Coulomb interaction and is  $\sigma_f = (4\pi/3)n^2R_d^2 \ln(R_d/\lambda)$ .

Expressions are found for the elastic scattering cross section for a deuteron, taking into account the semitransparency of the nu-

### I

**T**HE absorption of particles incident on a nucleus causes an additional perturbation of the incident wave and leads to additional elastic scattering not connected with formation of a compound nucleus. In the case of point particles (such as neutrons) with a wavelength small compared to the dimensions of the nucleus, this elastic scattering is similar to the diffraction of light by an absolutely black sphere, and is referred to as a diffractional scattering.

The diffractional scattering of deuterons must possess certain specific features. In addition to elastic scattering similar to the diffractional scattering of point particles, in the case of deuterons there must take place also a diffractional disintegration. Indeed, owing to the small binding energy of a deuteron, a comparatively small change in its momentum during the scattering may cause the disintegration of the deuteron to occur far from the nucleus. This diffractional disintegration of a deuteron, together with stripping, leads to the liberation of the neutron and the proton, i.e., increases the yield of neutrons arising during the interaction of fast deuterons with nuclei.<sup>1</sup>

The diffractional scattering of point particles by an absorbing nucleus

$\psi_{\mathbf{k}} = (1/L) \exp(i\mathbf{k} \cdot \mathbf{r})$ , where  $L$  is the normalization length and  $\mathbf{k}$  and  $\mathbf{r}$  are the projections of the wave vector and the radius vector of the particle on the plane perpendicular to the  $z$  axis. (The functions  $\psi_{\mathbf{k}}$  are normalized by  $\int \psi_{\mathbf{k}} \psi_{\mathbf{k}'}^* d\mathbf{r} = \delta_{\mathbf{k}\mathbf{k}'}$ .) The wave function  $\psi_0 = 1/L$  corresponds to the incident particles.

The presence of an absorbing nucleus leads to the disappearance of a part of this function at  $\rho \leq R$  (where  $R$  is the radius of the nucleus). The diffractional picture due to this absorption can be obtained by developing the remaining part of the wave function, which can be written as  $\Psi = \Omega(\rho)\psi_0$ , where

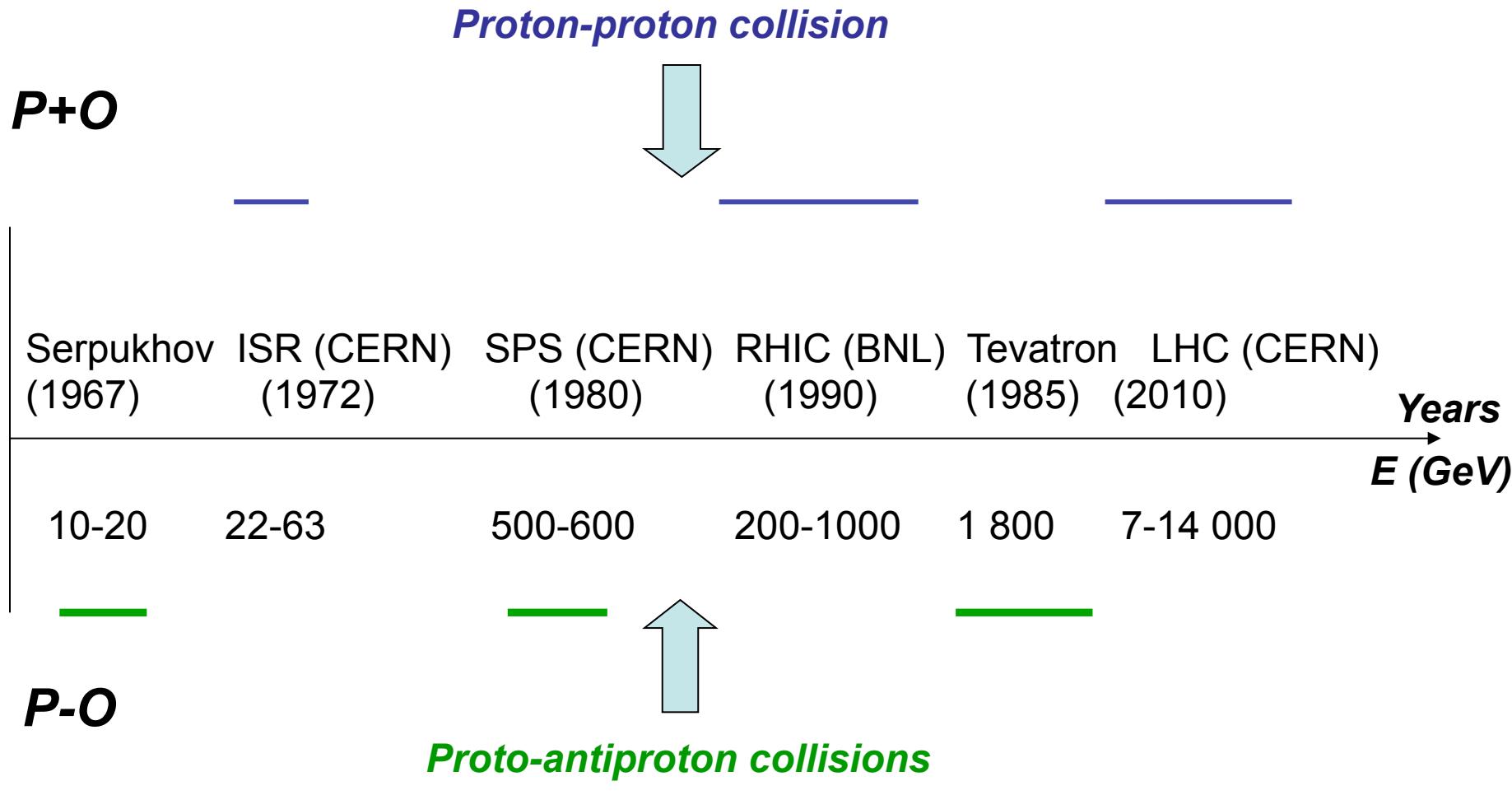
$$\Omega(\rho) = \begin{cases} 0, & \rho \leq R \\ 1, & \rho > R \end{cases}$$

into a series of functions  $\psi_{\mathbf{k}}$ :

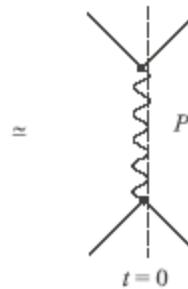
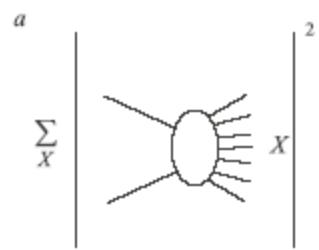
$$\Psi = \Omega(\rho)\psi_0 = \sum_{\mathbf{k}} a_{\mathbf{k}} \psi_{\mathbf{k}}. \quad (1)$$

The probability of diffractional scattering of the particle into the interval  $d\mathbf{k}$  of the wave vector  $\mathbf{k}$  is connected with  $a_{\mathbf{k}}$  by relation  $dw = |a_{\mathbf{k}}|^2 L^2 d\mathbf{k} / (2\pi)^2$ , and the corresponding differential cross section of scattering is equal to

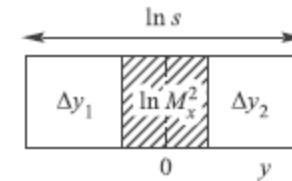
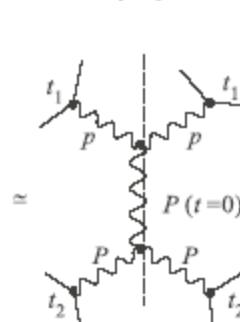
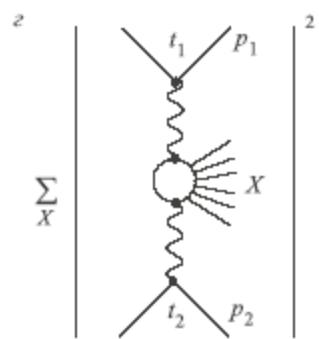
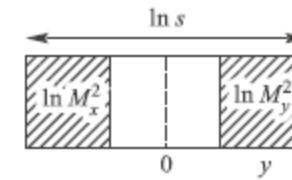
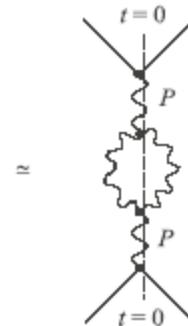
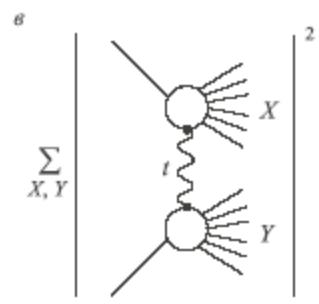
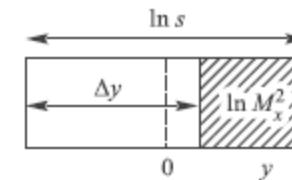
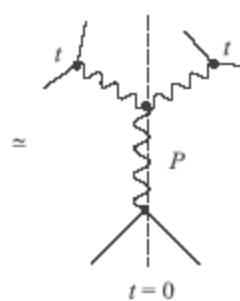
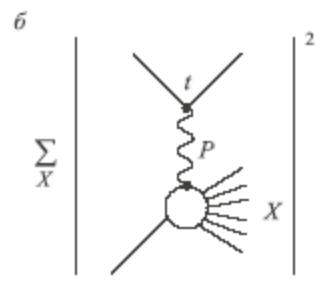
# Accelerators, energy ranges, years (scatch)



**P-Pomeron; O-odderon**



■ Рождение частиц  
□ Зазор (gap)



$$\sigma_t(s) = \frac{4\pi}{s} \text{Im} A(s, t=0); \quad \frac{d\sigma}{dt} = \frac{\pi}{s^2} |A(s, t)|^2; \quad n(s);$$

$$\sigma_{el} = \int_{t_{min} \approx -s/2 \approx \infty}^{t_{thr. \approx 0}} \frac{d\sigma}{dt} dt; \quad \sigma_{in} = \sigma_t - \sigma_{el}; \quad B(s, t) = \frac{d}{dt} \ln\left(\frac{d\sigma}{dt}\right);$$

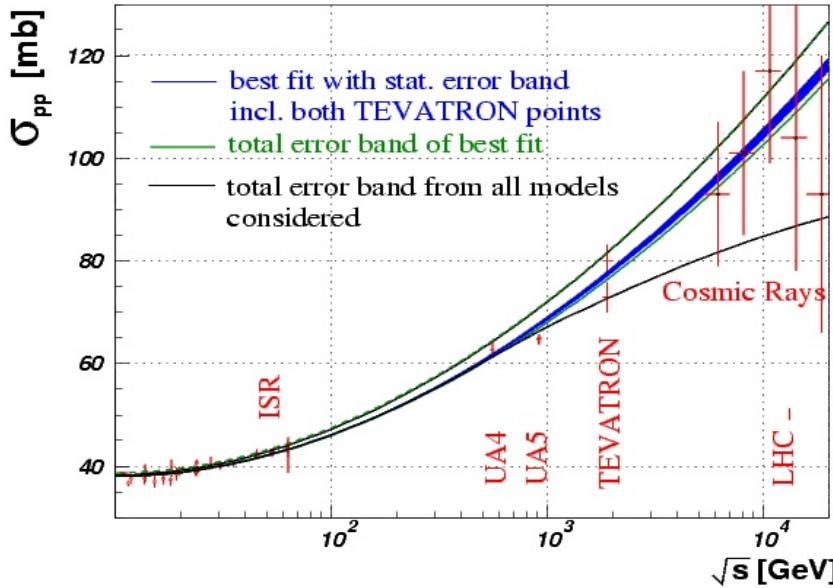
$$A_{pp}^{p\bar{p}}(s, t) = P(s, t) \pm O(s, t) + f(s, t) \pm \omega(s, t) \rightarrow_{LHC} P(s, t) \pm O(s, t),$$

where  $P$ ,  $O$ ,  $f$ .  $\omega$  are the Pomeron, odderon and non-leading Reggeon contributions.

<b>a(0)\C</b>	<b>+</b>	<b>-</b>
<b>1</b>	<b>P</b>	<b>O</b>
<b>1/2</b>	<b>f</b>	<b>ω</b>

**NB: The S-matrix theory (including Regge pole) is applicable to asymptotically free states only (not to quarks and gluons)!**

# Total Cross-Section



$$\sigma_{tot} \propto (\log s)^\gamma$$

$\sigma_{tot}(\text{LHC}) \sim 110 \text{ mb } (\gamma=2; \text{best-fit})$   
 $\sigma_{tot}(\text{LHC}) \sim 95 \text{ mb } (\gamma=1)$

$$\sigma_{tot} = \frac{16\pi}{1 + \rho^2} \times \frac{(dN / dt)|_{t=0}}{N_{el} + N_{inel}}$$

Luminosity-independent measurement via optical-theorem → simultaneous evaluation of forward elastic and inelastic rate (TOTEM)

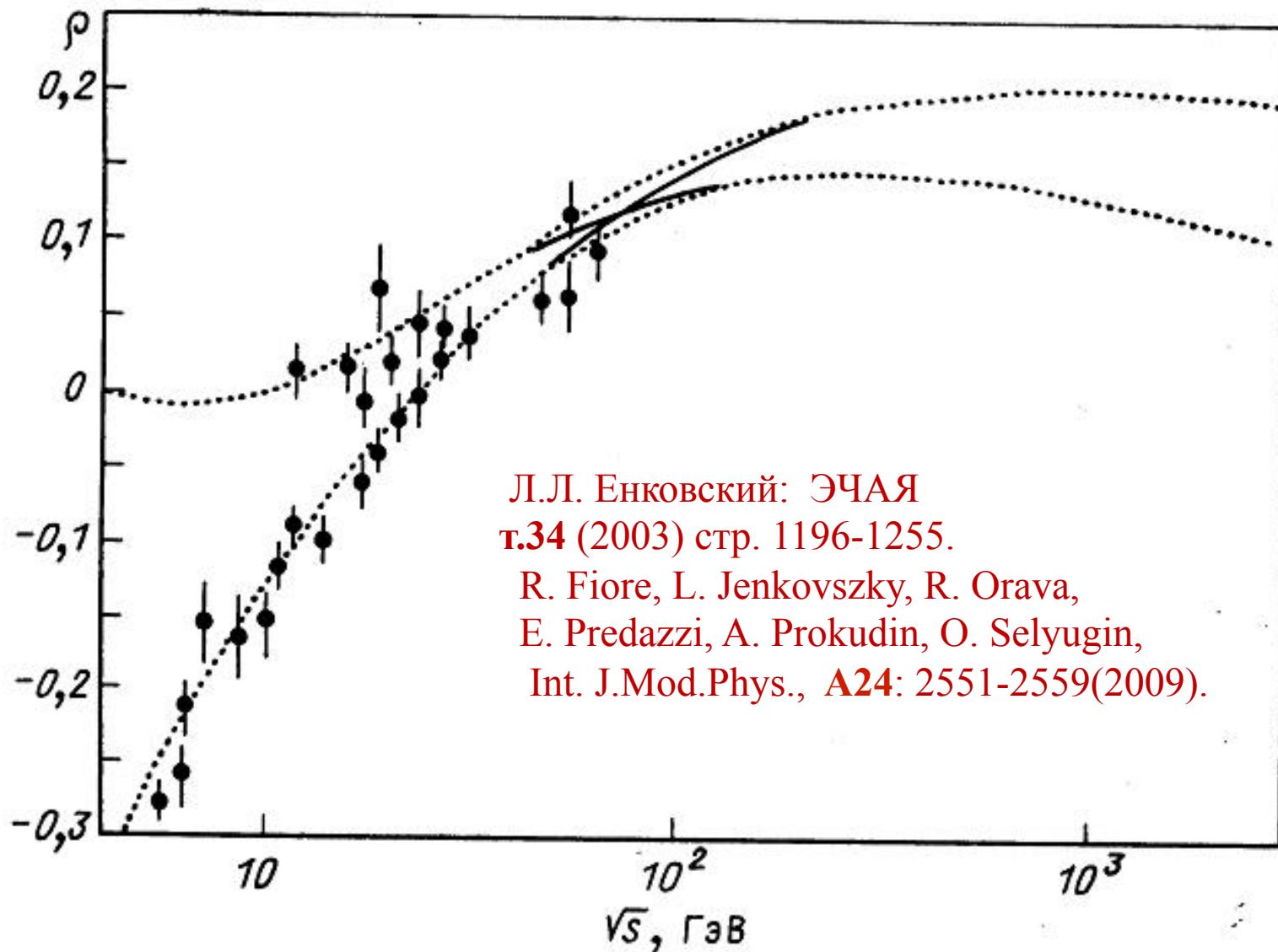
Inversely:

$$L\sigma_{tot} = N_{elastic} + N_{inelastic}$$

$$\begin{aligned}
 & (\sigma_{tot} + dN/dt|_{t=0}) \quad \xrightarrow{\hspace{2cm}} \quad (\Delta L/L \gg \sim 2 \Delta\sigma_{tot}/\sigma_{tot}) \\
 & (L + dN/dt|_{t=0}) \quad \quad \quad \quad \quad \quad \quad (\Delta\sigma_{tot}/\sigma_{tot} \gg \sim \frac{1}{2} \Delta L/L)
 \end{aligned}$$

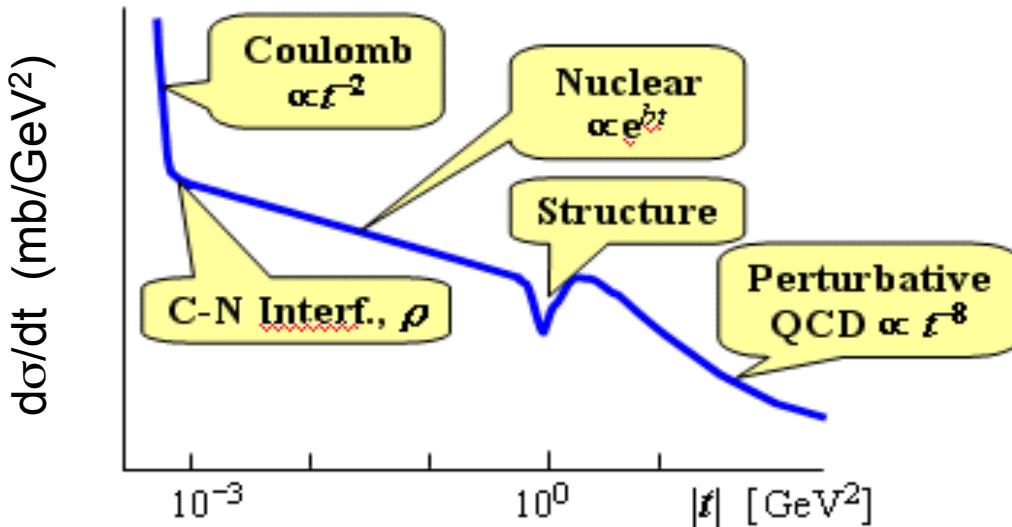
- elastic rate down to  $|t|=10^{-3}$  GeV $^2$  to keep extrapolation error small (1-2%)
- Sufficient  $\eta$  coverage to access  $N_{el}+N_{inel}$

$$L\sigma_{tot}^2 = \frac{16\pi}{1 + \rho^2} \times \frac{dN}{dt} \Big|_{t=0}$$



# Elastic Scattering

$\sqrt{s} = 14 \text{ TeV}$  prediction of BSW model



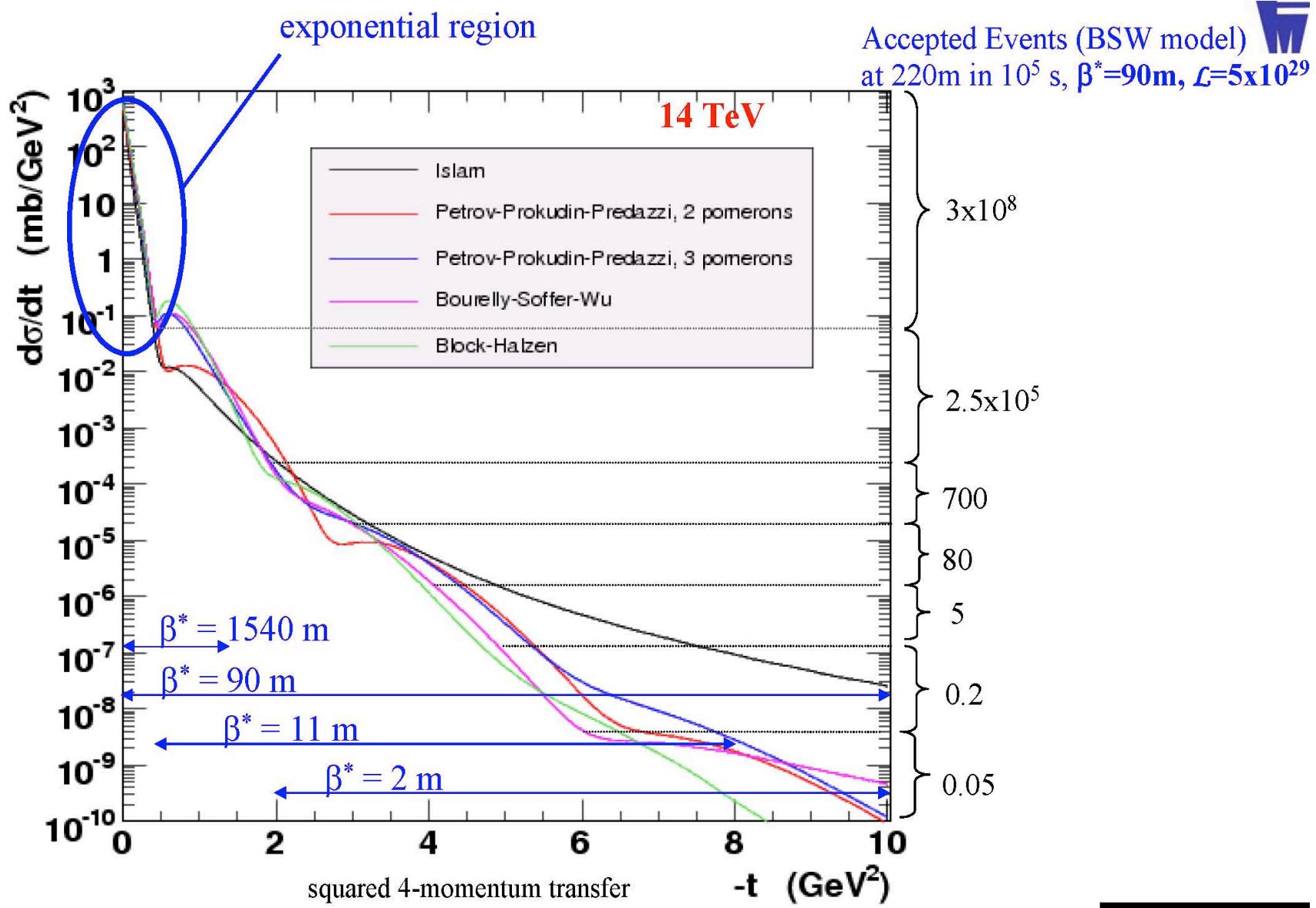
momentum transfer  $-t \sim (p\theta)^2$   
 $\theta$  = beam scattering angle  
 $p$  = beam momentum

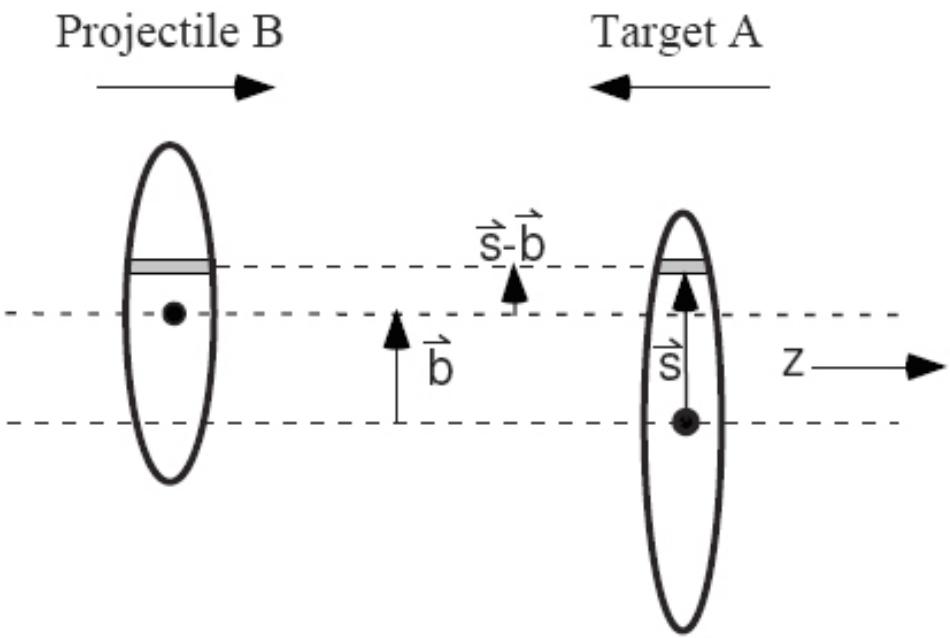
$$\rho = \frac{\operatorname{Re}(f_{el}(t))}{\operatorname{Im}(f_{el}(t))}_{t \rightarrow 0}$$

$$\left. \frac{dN}{dt} \right|_{t=CNI} = L\pi |f_C + f_N|^2 \approx L\pi \left| -\frac{2\alpha_{\text{EM}}}{|t|} + \frac{\sigma_{\text{tot}}}{4\pi} (i + \rho) e^{-\frac{b|t|}{2}} \right|^2$$

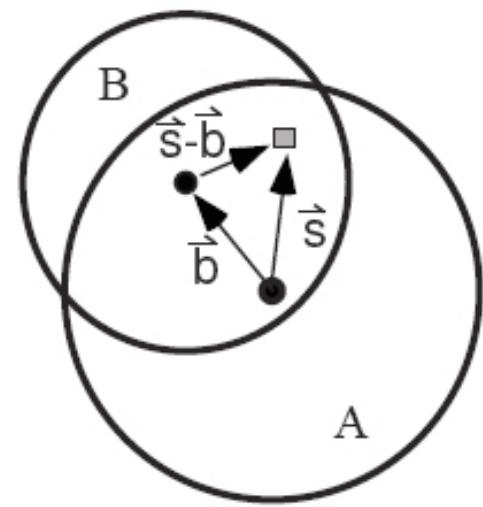
$L$ ,  $\sigma_{\text{tot}}$ ,  $b$ , and  $\rho$   
from FIT in CNI  
region (UA4)

CNI region:  $|f_C| \sim |f_N| \rightarrow @ \text{LHC: } -t \sim 6.5 \cdot 10^{-4} \text{ GeV}^2; \theta_{\min} \sim 3.4 \mu\text{rad}$   
 $(\theta_{\min} \sim 120 \mu\text{rad} @ \text{SPS})$





a) Side View

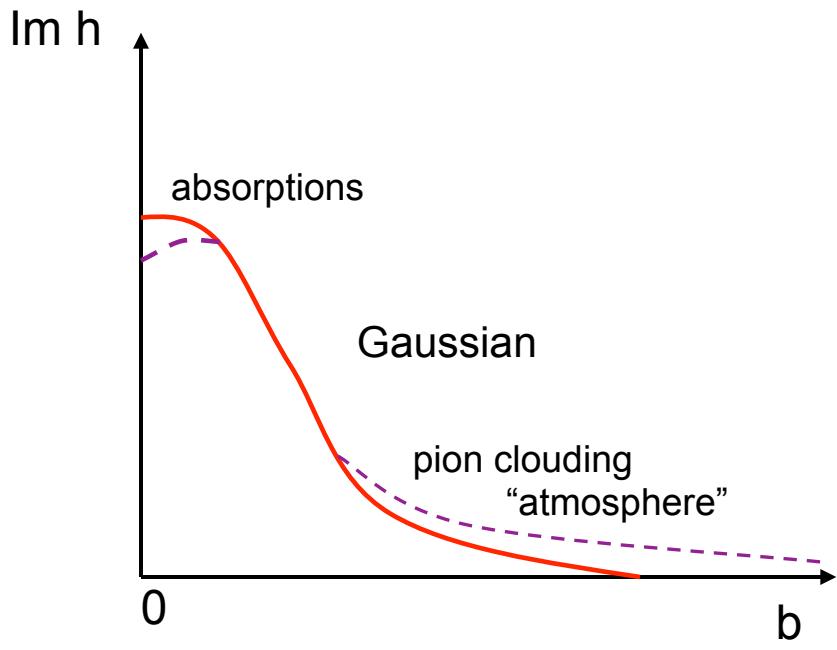
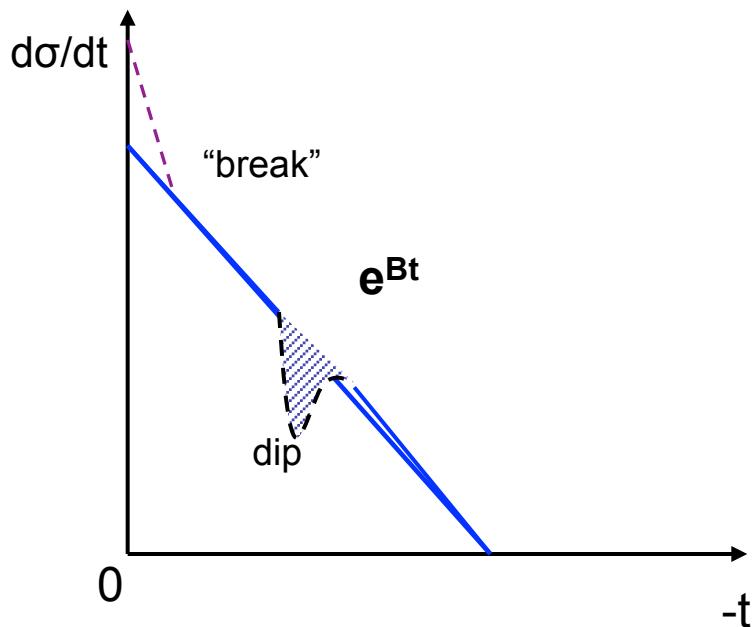


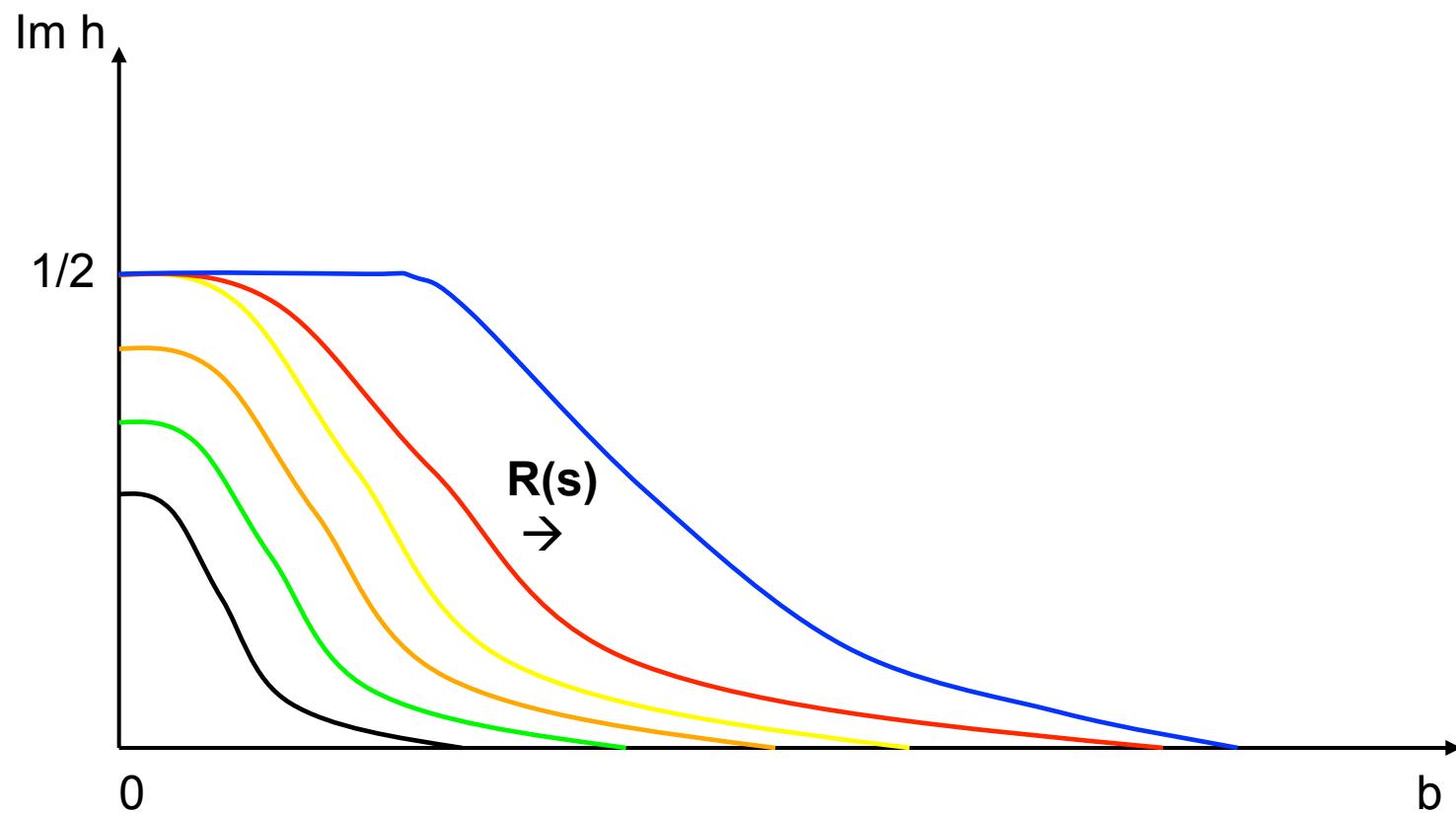
b) Beam-line View

# Geometrical scaling (GS), saturation and unitarity

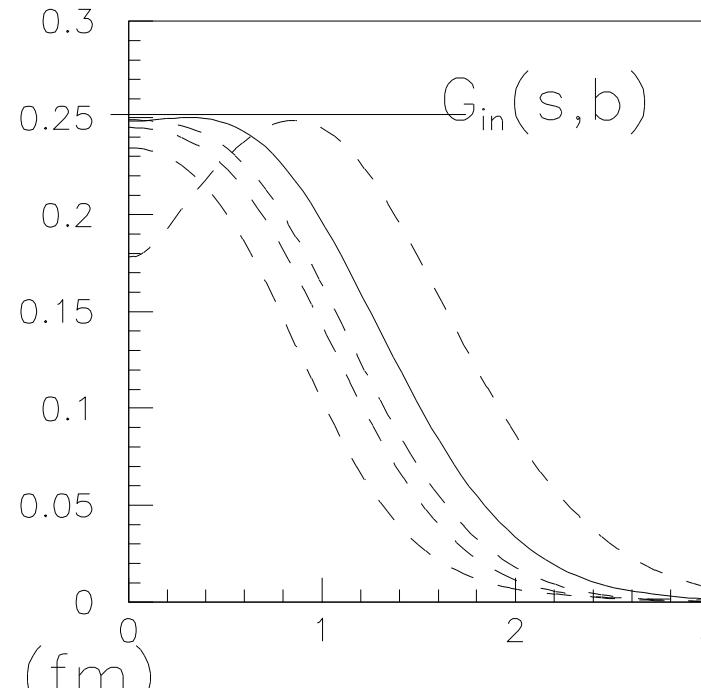
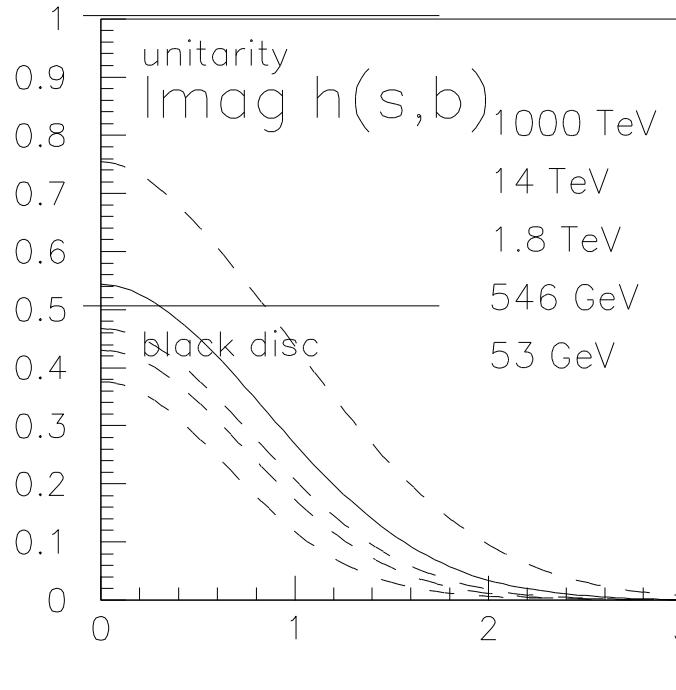
1. On-shell (hadronic) reactions ( $s, t, Q^2 = m^2$ );

$t \leftrightarrow b$  transformation:  $h(s, b) = \int_0^\infty d\sqrt{-t} \sqrt{-t} A(s, t)$   
and dictionary:





# Black disc limit



**P. Desgrolard, L.L. Jenkovszky and B.V. Struminsky,  
Z.Phys. C; Yad. Fizika, 1995.**

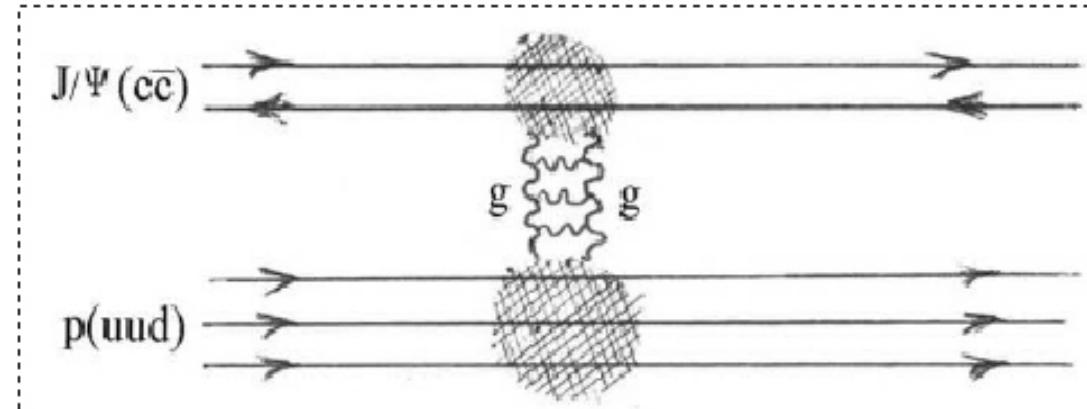
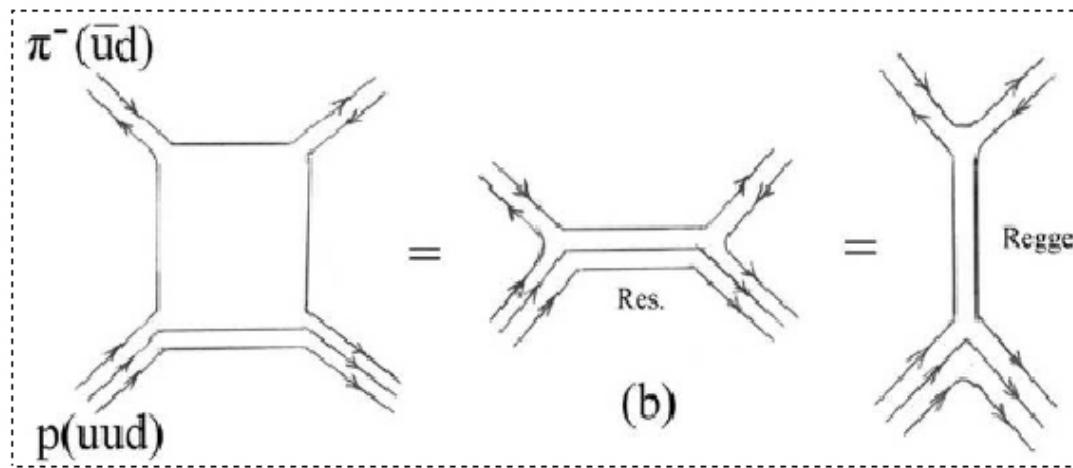
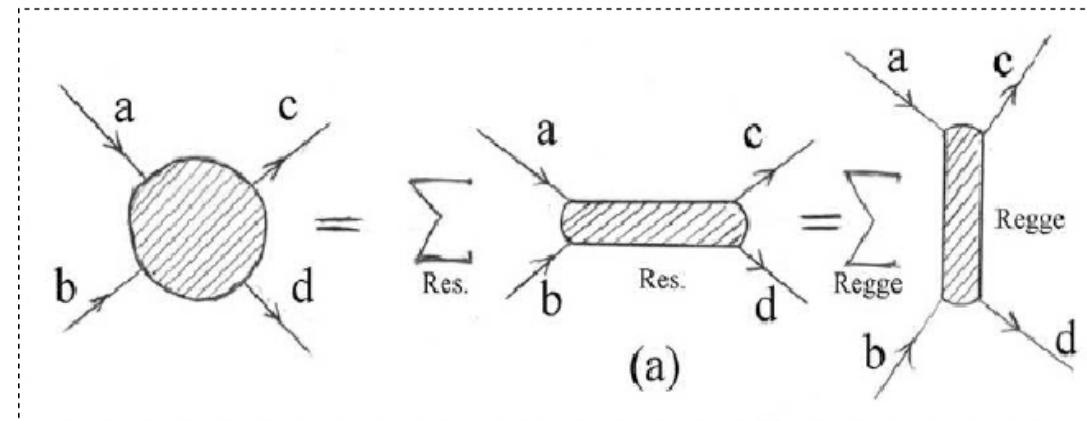


TABLE I: Two-component duality

$\text{Im}A(a + b \rightarrow c + d) =$	$R$	Pomeron
$s$ -channel	$\sum A_{Res}$	Non-resonant background
$t$ -channel	$\sum A_{Regge}$	Pomeron ( $I = S = B = 0; C = +1$ )
Duality quark diagram	Fig. 1b	Fig. 2
High energy dependence	$s^{\alpha-1}, \alpha < 1$	$s^{\alpha-1}, \alpha \geq 1$

The  $(s, t)$  term of a dual amplitude is

$$D(s, t) = c \int_0^1 dx \left(\frac{x}{g_1}\right)^{-\alpha(s')-1} \left(\frac{1-x}{g_2}\right)^{-\alpha(t')-1},$$

where  $s$  and  $t$  are the Mandelstam variables, and  $g_1, g_2$  are parameters,  $g_1, g_2 > 1$ . For simplicity, we set  $g_1 = g_2 = g_0$ .

1. Regge behavior,  $s \rightarrow \infty$ ,  $t = \text{const}$  :  $D(s, t) \sim s^{\alpha(t)-1}$ ;
2. Threshold behavior,  $s \rightarrow s_0$  :  $D(s, t) \sim \sqrt{s_0 - s} [\text{const} + \ln(1 - s_0/s)]$ ;

### 3. Direct-channel poles:

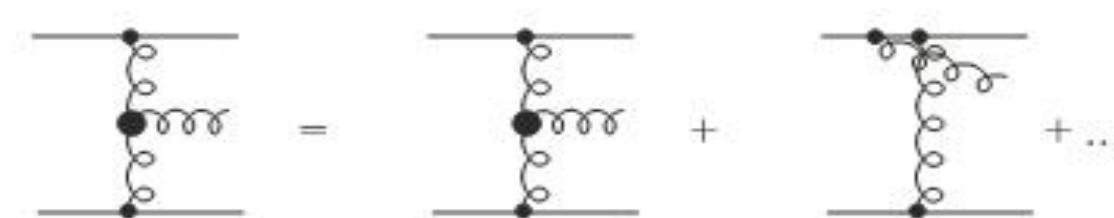
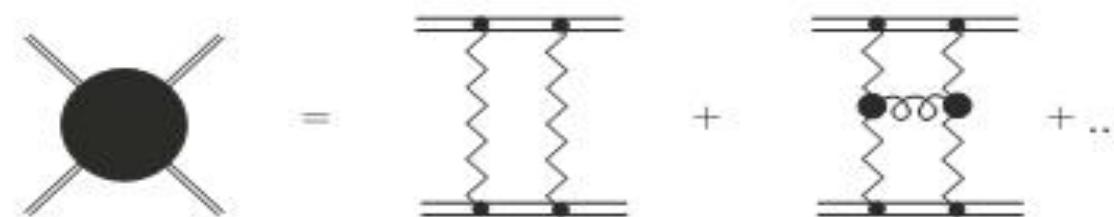
$$D(s, t) = \sum_{n=0}^{\infty} g^{n+1} \sum_{l=o}^n \frac{[-s\alpha'(s)]^l C_{n-l}(t)}{[n - \alpha(s)]^{l+1}}.$$

Exotic direct-channel trajectory:  $\alpha(s) = \alpha(0) + \alpha_1(\sqrt{s_0} - \sqrt{s_0 - s})$ .

"GOLDEN" diffraction reaction:  $J/\Psi p-$  scattering: By VMD, photoproduction is reduced to elastic hadron scattering:

$$D(\gamma p - Vp) = \sum \frac{e}{f_V} D(Vp - Vp).$$

R. Fiore, L.L. Jenkovszky, E.A. Kuraev, A.I. Lengyel, and Z.Z. Tarics, *Predictions for high-energy  $p\bar{p}$  and  $\bar{p}p$  scattering from a finite sum of gluon ladders*, Phys. Rev. D81, #5 (2010) 056005; arXiv0911.2094/hep-ph



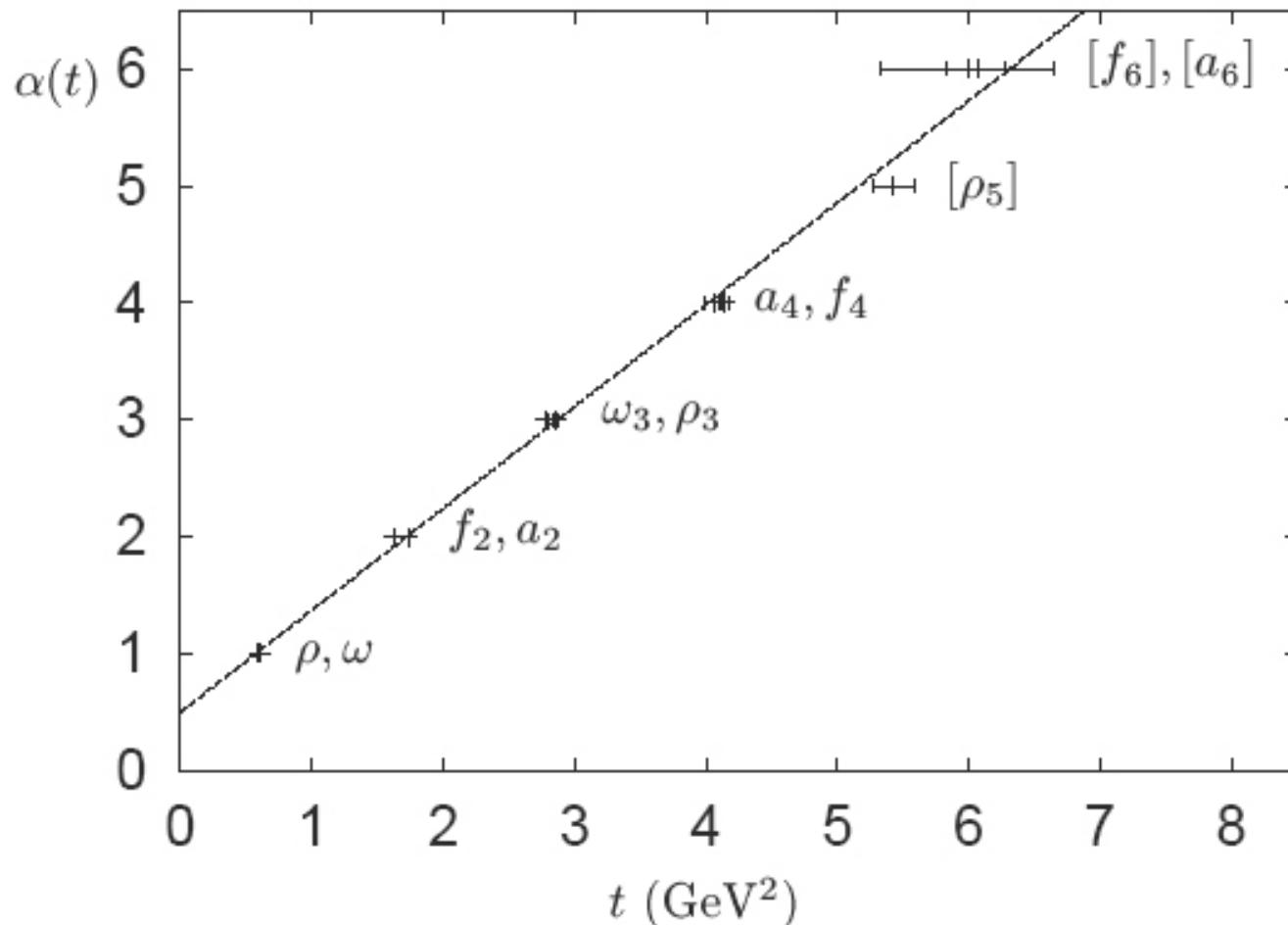
$$\sigma_t^{(P)}(s) = \sum_{i=0}^N f_i \theta(s - s_0^i) \theta(s_0^{i+1} - s), \quad (1)$$

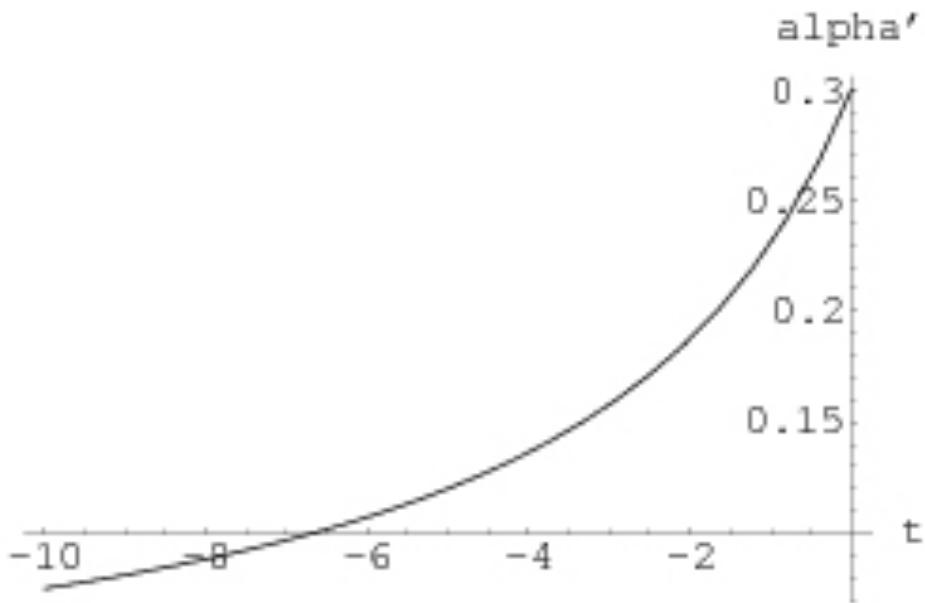
where

$$f_i = \sum_{j=0}^i a_{ij} L^j, \quad (2)$$

# Linear particle trajectories

Plot of spins of families of particles against their squared masses:





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The slope of the cone for a single pole is:  
 $B(s, t) \sim \alpha'(t) \ln s$ . The Regge residue  $e^{b\alpha(t)}$  with a logarithmic trajectory  $\alpha(t) = \alpha(0) - \gamma \ln(1 - \beta t)$ , is identical to a form factor (geometrical model).

## The Pomeron trajectory

The Pomeron trajectory has threshold singularities, the lowest one being due to the two-pion exchange, required by the  $t$ -channel unitarity. There is a constrain (Barut, Zwanziger; Gribov) from the  $t$ - channel unitarity, by which

$$\Im\alpha(t) \sim (t - t_0)^{\Re\alpha(t_0)+1/2}, \quad t \rightarrow t_0,$$

where  $t_0$  is the lightest threshold. For the Pomeron trajectory it is  $t_0 = 4m_\pi^2$ , and near the threshold:

$$\alpha(t) \sim \sqrt{4m_\pi^2 - t}. \quad (1)$$

Representative examples of the Pomeron trajectories: 1) Linear; 2) With a square-root threshold, required by  $t$ -channel unitarity and accounting for the small- $t$  “break” as well as the possible “Orear”,  $e^{\sqrt{-t}}$  behavior in the second cone; and 3) A logarithmic one, anticipating possible “hard effects” at large  $|t|$   $|t| < 8 \text{ GeV}^2$ .

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P} t, \quad (\text{TR.1})$$

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P + \alpha_{1P} t - \alpha_{2P} \left( \sqrt{4\alpha_{3P}^2 - t} - 2\alpha_{3P} \right), \quad (\text{TR.2})$$

$$\alpha_P \equiv \alpha_P(t) = 1 + \delta_P - \alpha_{1P} \ln(1 - \alpha_{2P} t). \quad (\text{TR.3})$$

$$A_{pp}^{p\bar{p}}(s, t) = P(s, t) \pm O(s, t) + f(s, t) \pm \omega(s, t) \rightarrow_{LHC} P(s, t) \pm O(s, t),$$

where  $P$  is the Pomeron contribution and  $O$  is that of the Odderon.

$$P(s, t) = i \frac{as}{bs_0} (r_1^2(s) e^{r_1^2(s)[\alpha_P(t)-1]} - \epsilon r_2^2(s) e^{r_2^2(s)[\alpha_P(t)-1]}),$$

where  $r_1^2(s) = b + L - \frac{i\pi}{2}$ ,  $r_2^2(s) = L - \frac{i\pi}{2}$  with  $L \equiv \ln \frac{s}{s_0}$ ;  $\alpha_P(t)$  is the Pomeron trajectory and  $a, b, s_0$  and  $\epsilon$  are free parameters.

## Does GS imply saturation? Not necessarily!

$ImH(s, b) = |h(s, b)|^2 + G_{in}(s, b)$ , ( $h$  is associated with the "opacity"), Here from:  $0 \leq |h(s, b)|^2 \leq \Im h(s, b) \leq 1$ . The Black Disc Limit (BDL) corresponds to  $\Im h(s, b) = 1/2$ , provided  $h(s, b) = i(1 - \exp[i\omega(s, b)])/2$ , with an imaginary eikonal  $\omega(s, b) = i\Omega(s, b)$ .

There is an alternative solution, that with the "minus" sign in  $h(s, b) = [1 \pm \sqrt{1 - 4G_{in}(s, b)}]/2$ , giving (S.Troshin and N.Tyurin (Protvino)):  $h(s, b) = \Im u(s, b)/[1 - iu(s, b)]$ ,

# EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH



TOTEM 2011-01  
22 June 2011

CERN-PH-EP-2011-101  
26 June 2011

## Elastic pp Scattering at the LHC at $\sqrt{s} = 7$ TeV.

The TOTEM Collaboration

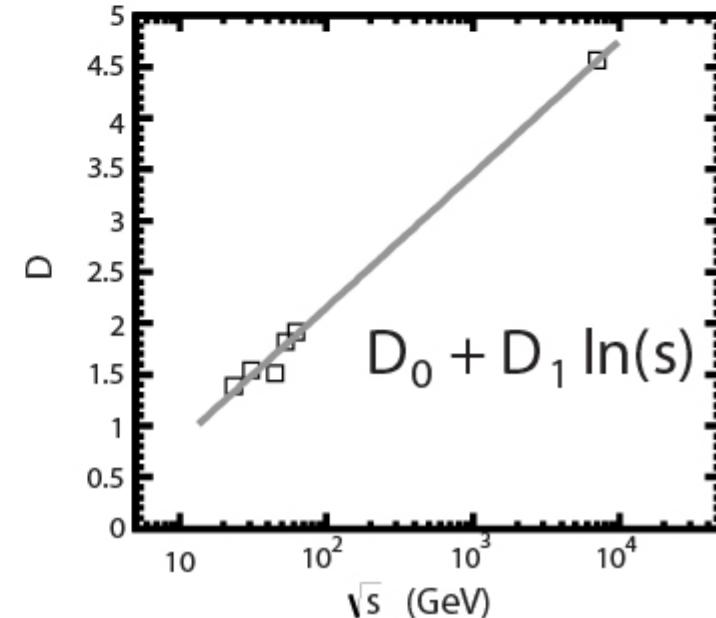
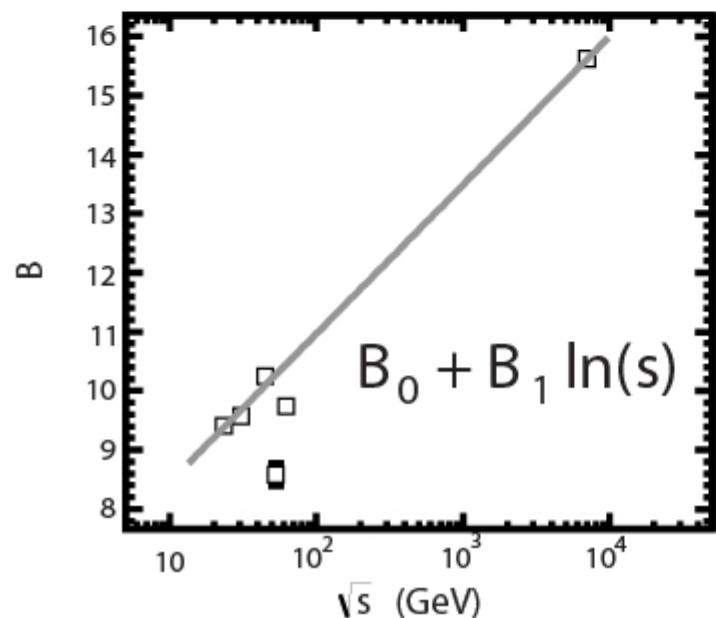
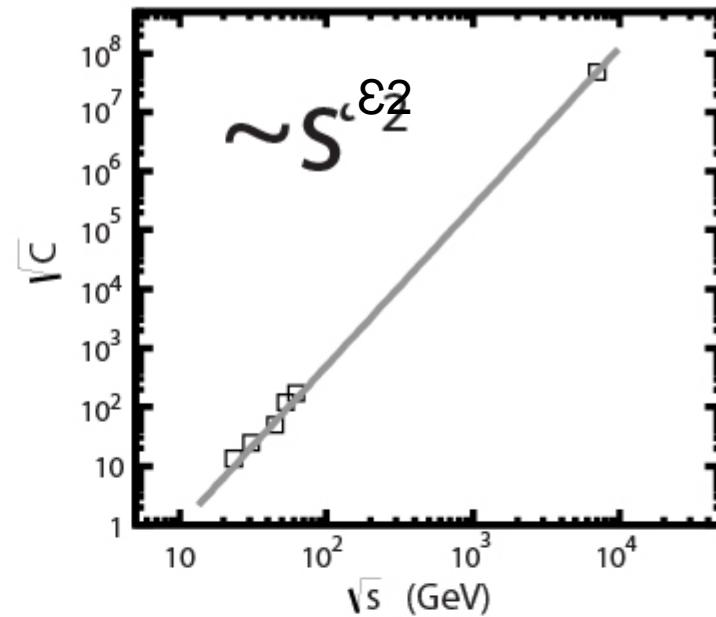
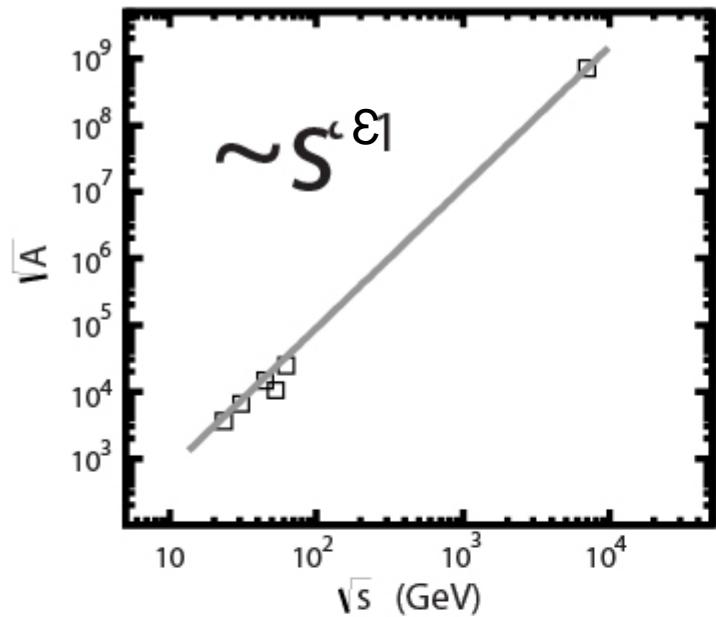
G. Antchev\*, P. Aspell<sup>8</sup>, I. Atanassov<sup>8,\*</sup>, V. Avati<sup>8</sup>, J. Baechler<sup>8</sup>, V. Berardi<sup>5b,5a</sup>, M. Berretti<sup>7b</sup>,  
M. Bozzo<sup>6b,6a</sup>, E. Brücken<sup>3a,3b</sup>, A. Buzzo<sup>6a</sup>, F. Cafagna<sup>5a</sup>, M. Calicchio<sup>5b,5a</sup>, M. G. Catanesi<sup>5a</sup>,  
C. Covault<sup>9</sup>, M. Csand<sup>4†</sup>, T. Cs rg  <sup>4</sup>, M. Deile<sup>8</sup>, E. Dimovasili<sup>8</sup>, M. Doubek<sup>1b</sup>, K. Eggert<sup>9</sup>,  
V. Eremin<sup>‡</sup>, F. Ferro<sup>6a</sup>, A. Fiergolski<sup>§</sup>, F. Garcia<sup>3a</sup>, S. Giani<sup>8</sup>, V. Greco<sup>7b,8</sup>, L. Grzanka<sup>8,¶</sup>, J. Heino<sup>3a</sup>,  
T. Hilden<sup>3a,3b</sup>, M. Janda<sup>1b</sup>, J. Ka par<sup>1a,8</sup>, J. Kopal<sup>1a,8</sup>, V. Kundr t<sup>1a</sup>, K. Kurvinen<sup>3a</sup>, S. Lami<sup>7a</sup>,  
G. Latino<sup>7b</sup>, R. Lauhakangas<sup>3a</sup>, T. Leszko<sup>§</sup>, E. Lippmaa<sup>2</sup>, M. Lokaj  ek<sup>1a</sup>, M. Lo Vetere<sup>6b,6a</sup>,  
F. Lucas Rodr  guez<sup>8</sup>, M. Macri<sup>6a</sup>, L. Magaletti<sup>5b,5a</sup>, G. Magazz  <sup>7a</sup>, A. Mercadante<sup>5b,5a</sup>, M. Meucci<sup>7b</sup>,  
S. Minutoli<sup>6a</sup>, F. Nemes<sup>4,†</sup>, H. Niewiadomski<sup>8</sup>, E. Noschis<sup>8</sup>, T. Novak<sup>4,||</sup>, E. Oliveri<sup>7b</sup>, F. Oljemark<sup>3a,3b</sup>,  
R. Orava<sup>3a,3b</sup>, M. Oriunno<sup>8\*\*</sup>, K.  sterberg<sup>3a,3b</sup>, A.-L. Perrot<sup>8</sup>, P. Palazzi<sup>8</sup>, E. Pedreschi<sup>7a</sup>,  
J. Pet  j  rvi<sup>3a</sup>, J. Proch  zka<sup>1a</sup>, M. Quinto<sup>5a</sup>, E. Radermacher<sup>8</sup>, E. Radicioni<sup>5a</sup>, F. Ravotti<sup>8</sup>,  
E. Robutti<sup>6a</sup>, L. Ropelewski<sup>8</sup>, G. Ruggiero<sup>8</sup>, H. Saarikko<sup>3a,3b</sup>, A. Santroni<sup>6b,6a</sup>, A. Scribano<sup>7b</sup>,  
G. Sette<sup>6b,6a</sup>, W. Snoeys<sup>8</sup>, F. Spinella<sup>7a</sup>, J. Sziklai<sup>4</sup>, C. Taylor<sup>9</sup>, N. Turini<sup>7b</sup>, V. Vacek<sup>1b</sup>, J. Welti<sup>3a,b</sup>,  
M. V  tek<sup>1b</sup>, J. Whitmore<sup>10</sup>.

Phillips and Barger in 1973 [ ], right after its first observation at the ISR. Their formula reads

$$\frac{d\sigma}{dt} = |\sqrt{A} \exp(Bt/2) + \sqrt{C} \exp(Dt/2 + i\phi)|^2, \quad (1)$$

where  $A$ ,  $B$ ,  $C$ ,  $D$  and  $\phi$  are determined independently at each energy.

[\*\*L.Jenkovszky, A. Lengyel, D. Lontkovskyi:  
The Pomeron and Odderon in elastic, inelastic and total cross-sections,  
hep-ph/056014, International Journal of Modern Physics.\*\*](#)



L. Jenkovszky and D. Lontkovskiy: In the Proc. of the Crimean (2011) Conference;  
also presented at the Russian-Spanish Dual Meeting in Barcelona, November, 2011:

Parameter name	Value	Uncertainty
	$\chi^2/\text{NDF}$	<b>4.60434</b>
$\sqrt{A}$	+2.214549e+001	+2.344779e-001
$B_0$	+1.391529e+000	+7.408365e-003
$B_1$	+3.032435e-001	+1.525472e-003
$\sqrt{C}$	+6.952456e+003	+1.596950e+001
$D_0$	+9.766352e+000	+1.234181e-002
$D_1$	+4.648188e-001	+9.965043e-003
$\epsilon_1$	+1.358585e+000	+1.243930e-003
$\epsilon_2$	+1.057306e+000	+1.830864e-003
$\phi$	+3.511842e+000	+3.713867e-003

**T a b l e 2:** Parameters obtained from the fit to the  $pp$  data

$P$  and  $f$  (second column) have positive  $C$ -parity, thus entering in the scattering amplitude with the same sign in  $pp$  and  $\bar{p}p$  scattering, while the Odderon and  $\omega$  (third column) have negative  $C$ -parity, thus entering  $pp$  and  $\bar{p}p$  scattering with opposite signs, as shown below:

$$A(s, t)^{\bar{p}p}_{pp} = A_P(s, t) + A_f(s, t) \pm [A_\omega(s, t) + A_O(s, t)], \quad (1)$$

where the symbols  $P$ ,  $f$ ,  $O$ ,  $\omega$  stand for the relevant Regge-pole amplitudes and the super(sub)script, evidently, indicate  $\bar{p}p(pp)$  scattering with the relevant choice of the signs in the sum.

$$\begin{aligned} A_P(s, t) &= \frac{d}{d\alpha_P} \left[ e^{-i\pi\alpha_P/2} G(\alpha_P) \left( s/s_0 \right)^{\alpha_P} \right] = \\ &e^{-i\pi\alpha_P(t)/2} \left( s/s_0 \right)^{\alpha_P(t)} \left[ G'(\alpha_P) + \left( L - i\pi/2 \right) G(\alpha_P) \right]. \end{aligned}$$

The Pomeron is a dipole in the  $j$ -plane

$$A_P(s, t) = \frac{d}{d\alpha_P} \left[ e^{-i\pi\alpha_P/2} G(\alpha_P) \left( s/s_0 \right)^{\alpha_P} \right] = \quad (1)$$

$$e^{-i\pi\alpha_P(t)/2} \left( s/s_0 \right)^{\alpha_P(t)} \left[ G'(\alpha_P) + (L - i\pi/2) G(\alpha_P) \right].$$

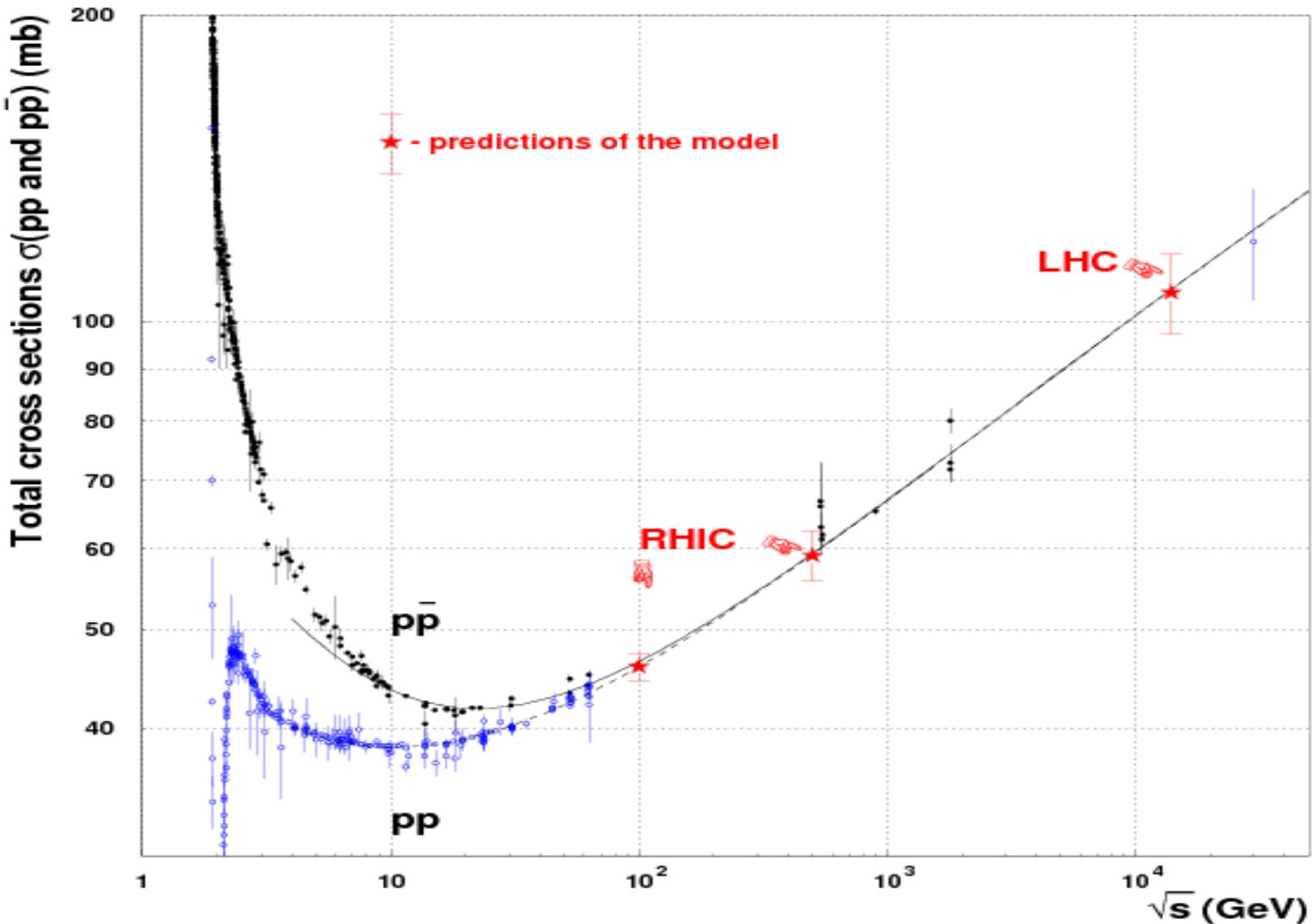
Since the first term in squared brackets determines the shape of the cone, one fixes

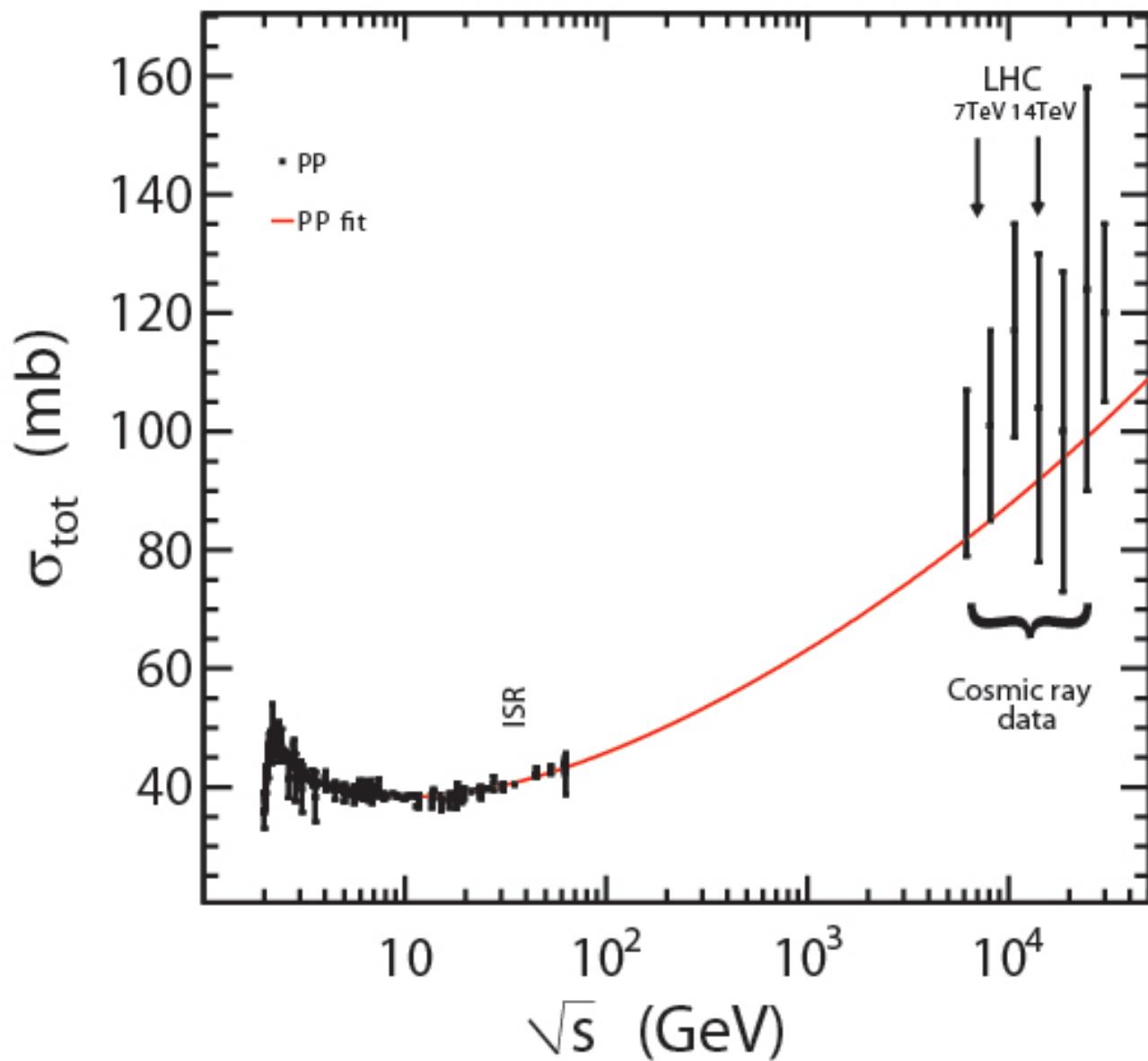
$$G'(\alpha_P) = -a_P e^{b_P[\alpha_P-1]}, \quad (2)$$

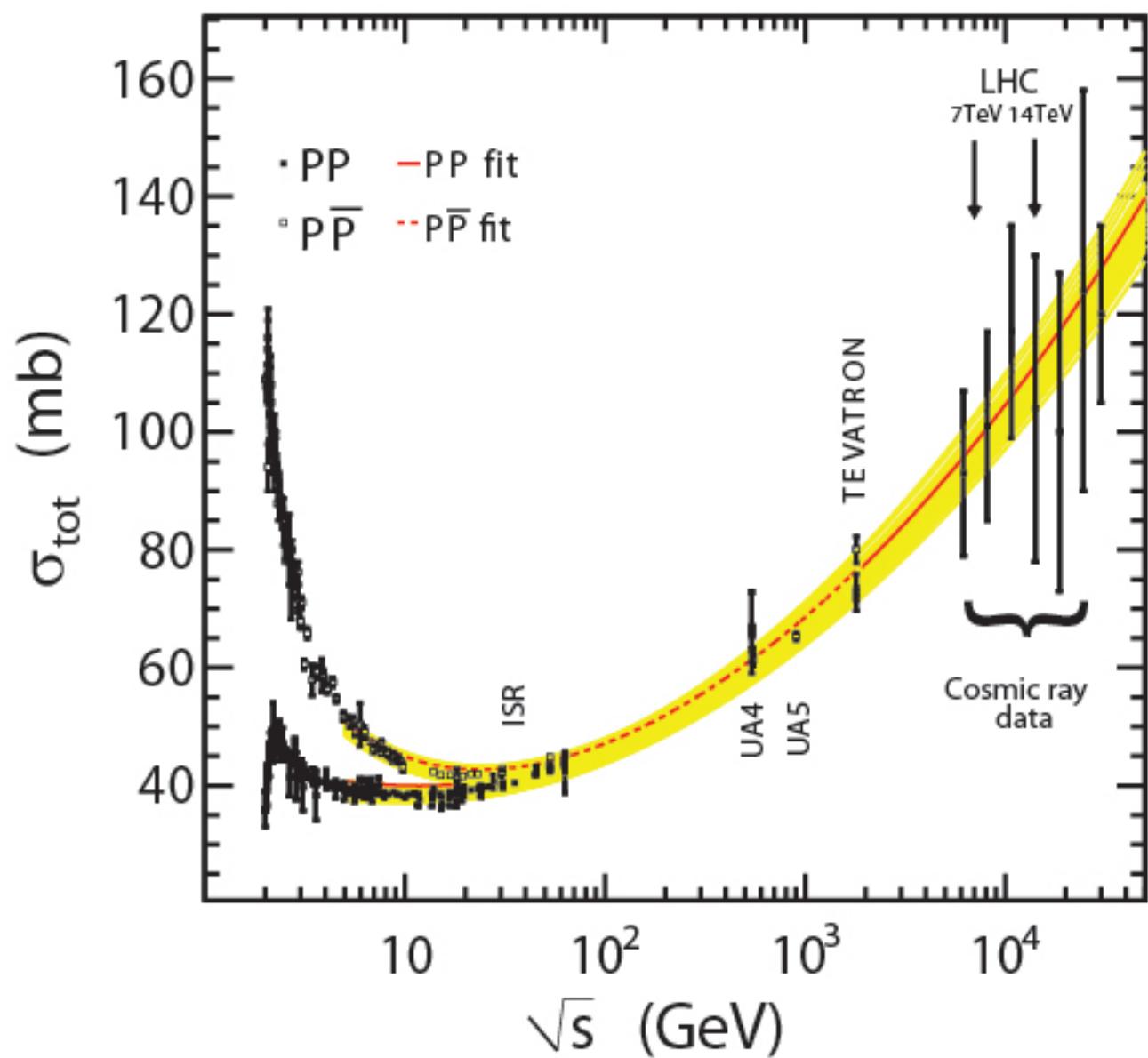
where  $G(\alpha_P)$  is recovered by integration, and, as a consequence, the Pomeron amplitude can be rewritten in the following “geometrical” form

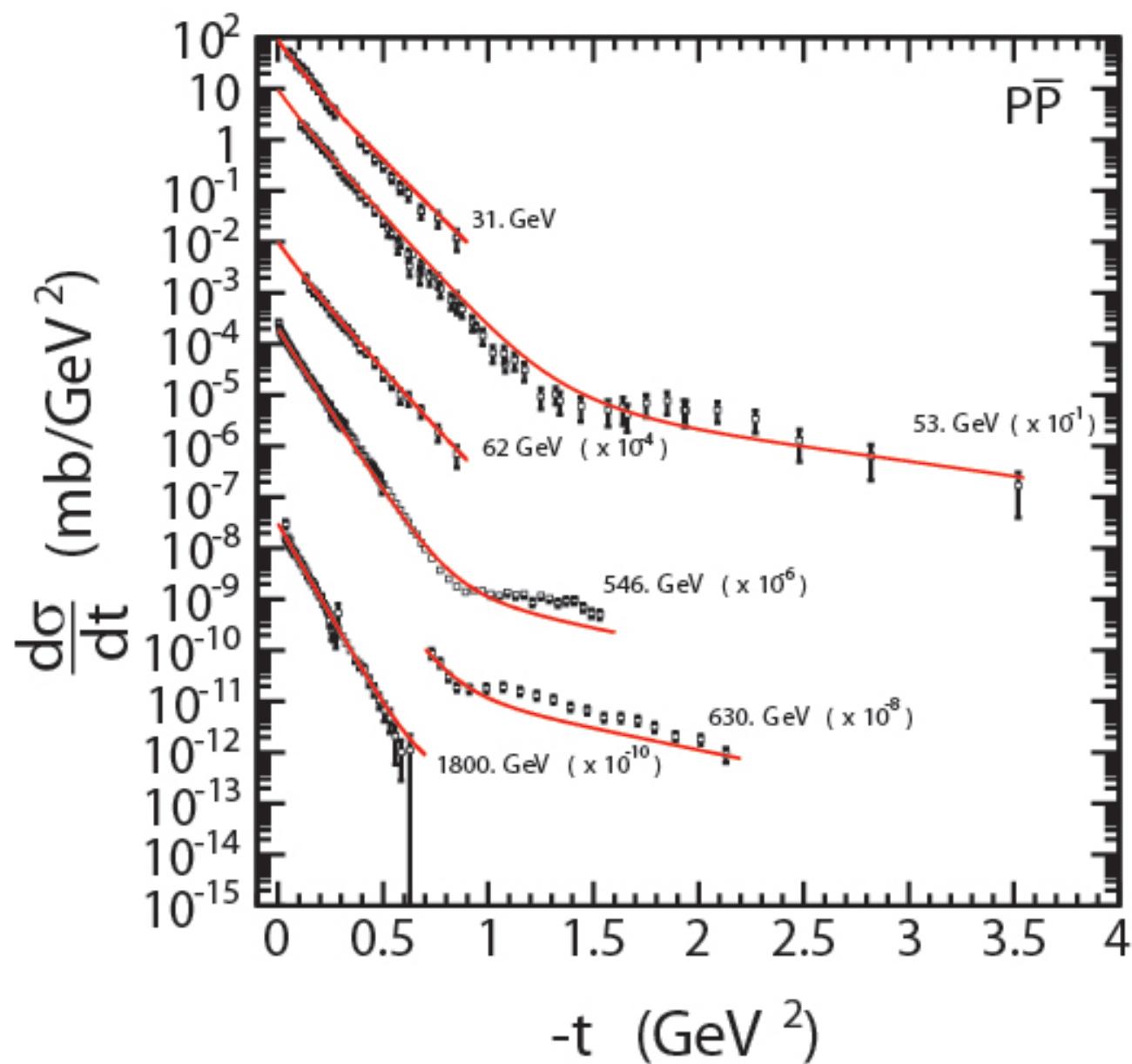
$$A_P(s, t) = i \frac{a_P}{b_P} \frac{s}{s_0} [r_1^2(s) e^{r_1(s)[\alpha_P-1]} - \varepsilon_P r_2^2(s) e^{r_2(s)[\alpha_P-1]}], \quad (3)$$

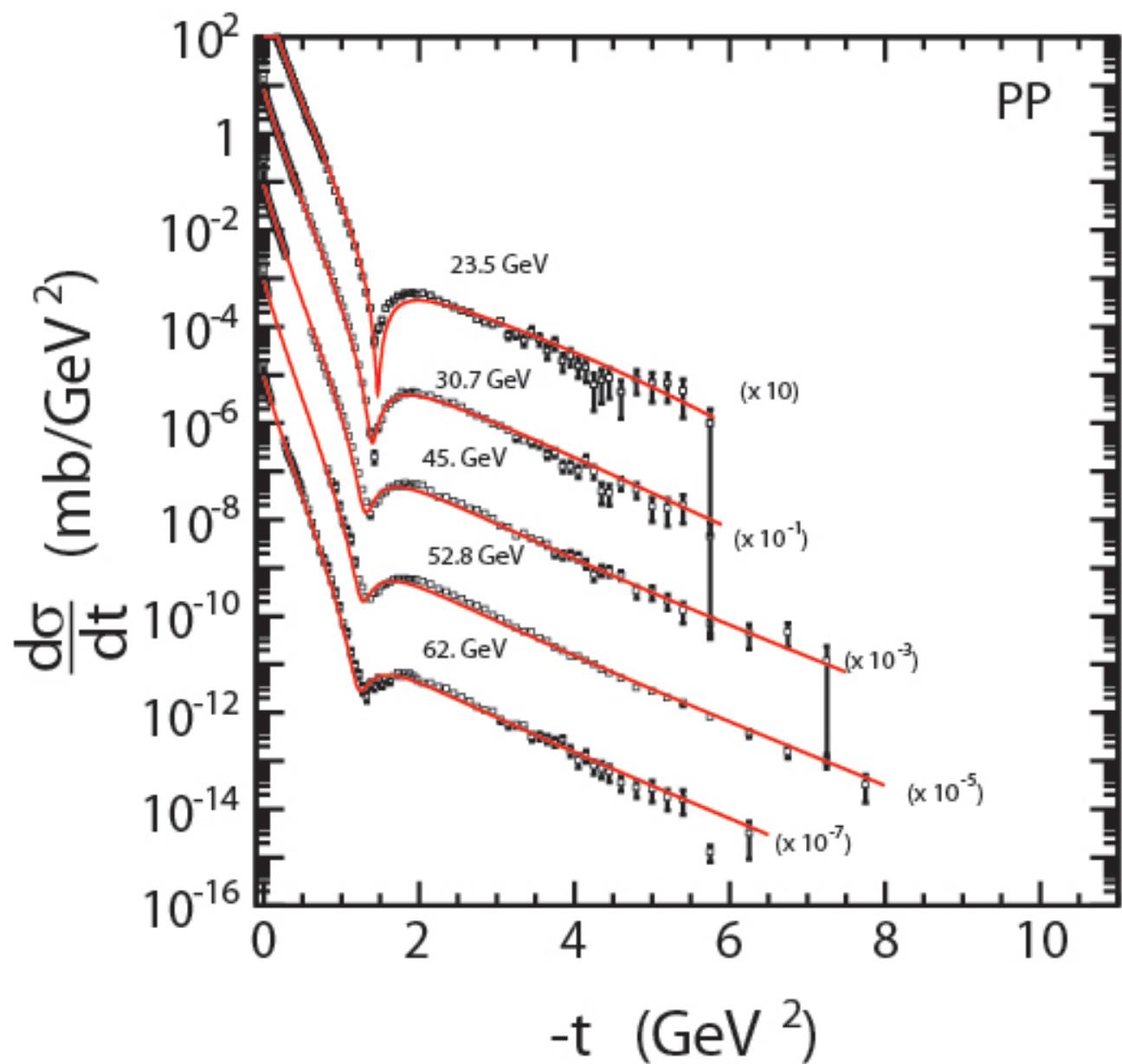
where  $r_1^2(s) = b_P + L - i\pi/2$ ,  $r_2^2(s) = L - i\pi/2$ ,  $L \equiv \ln(s/s_0)$ .



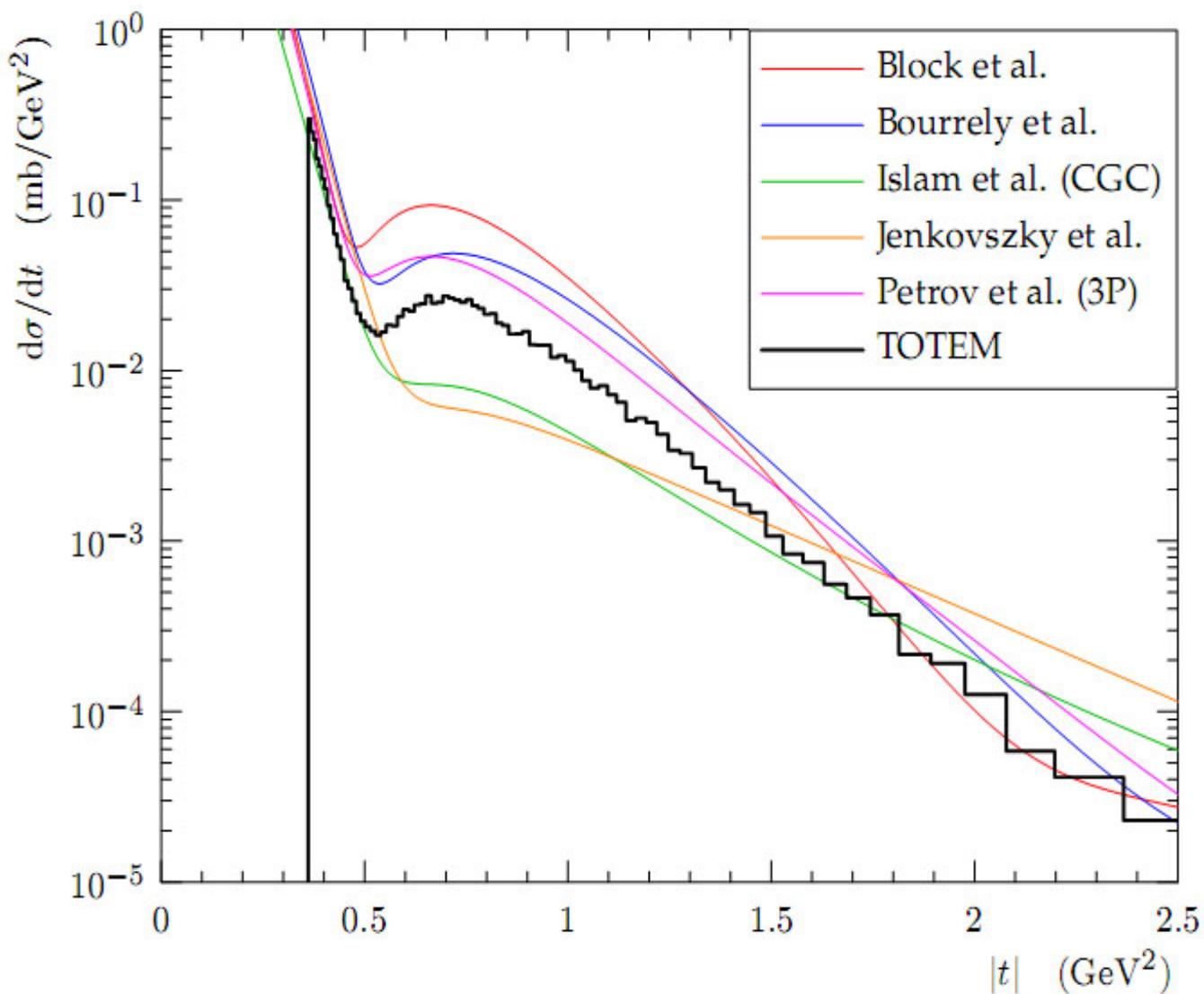


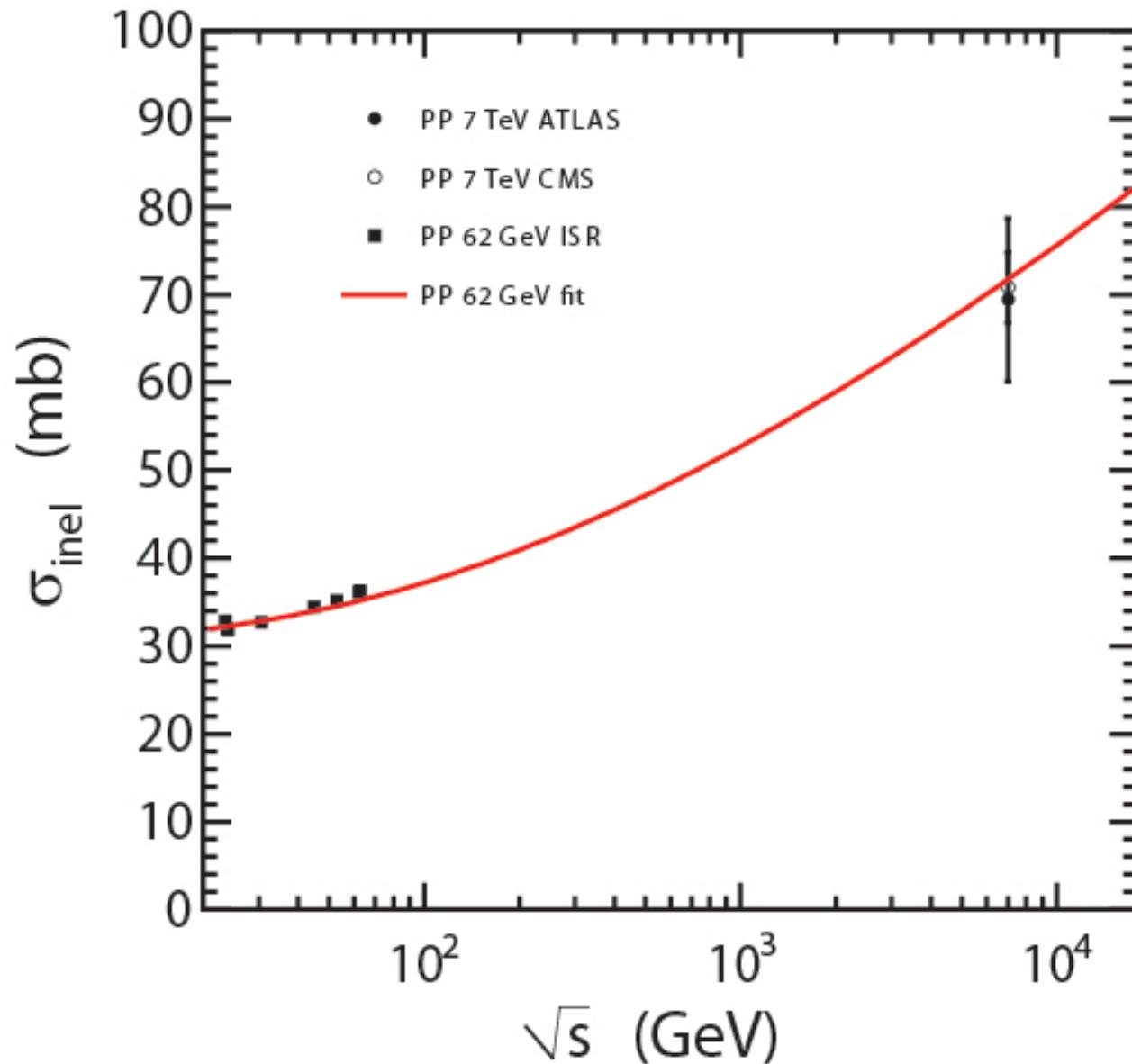


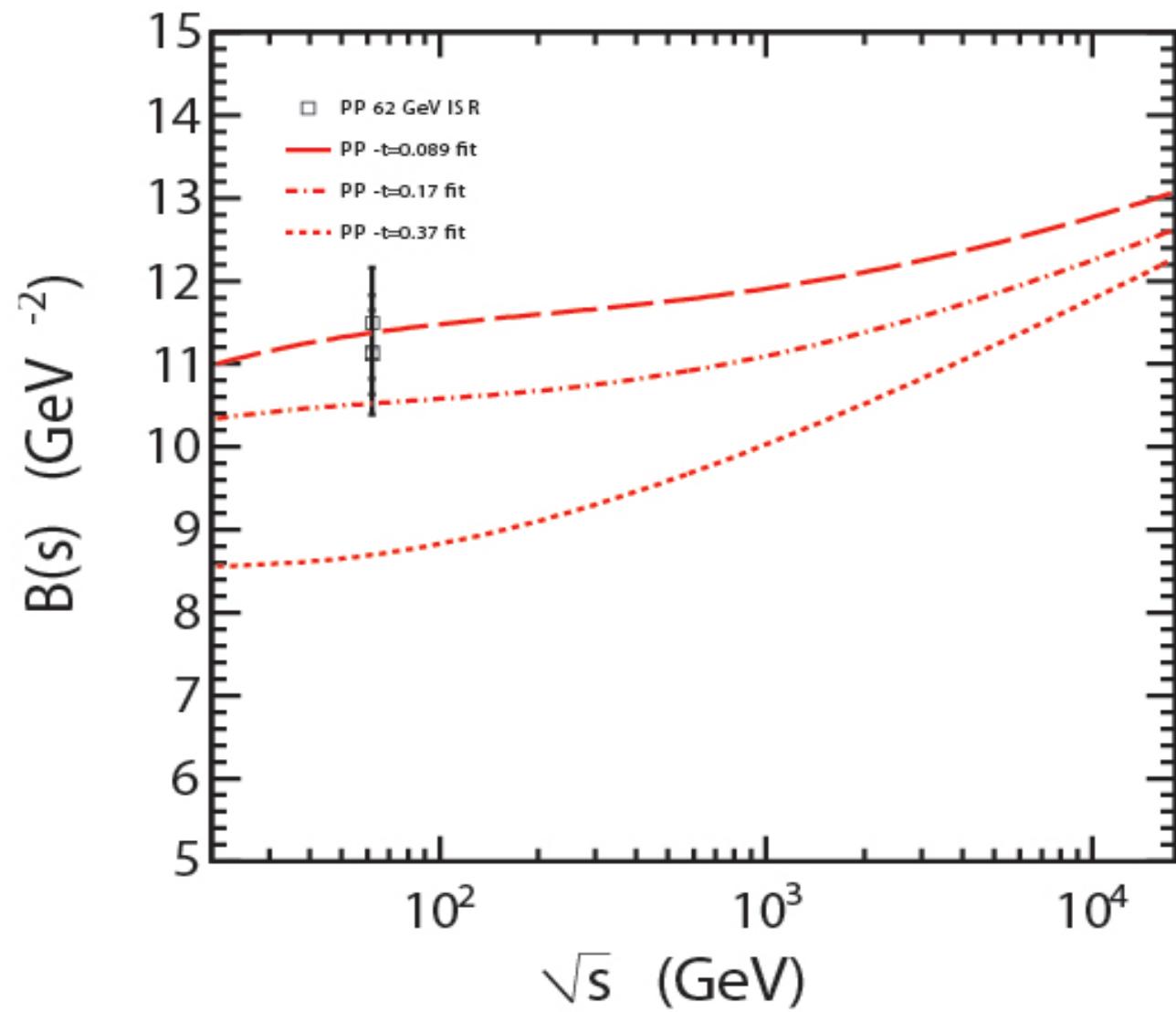


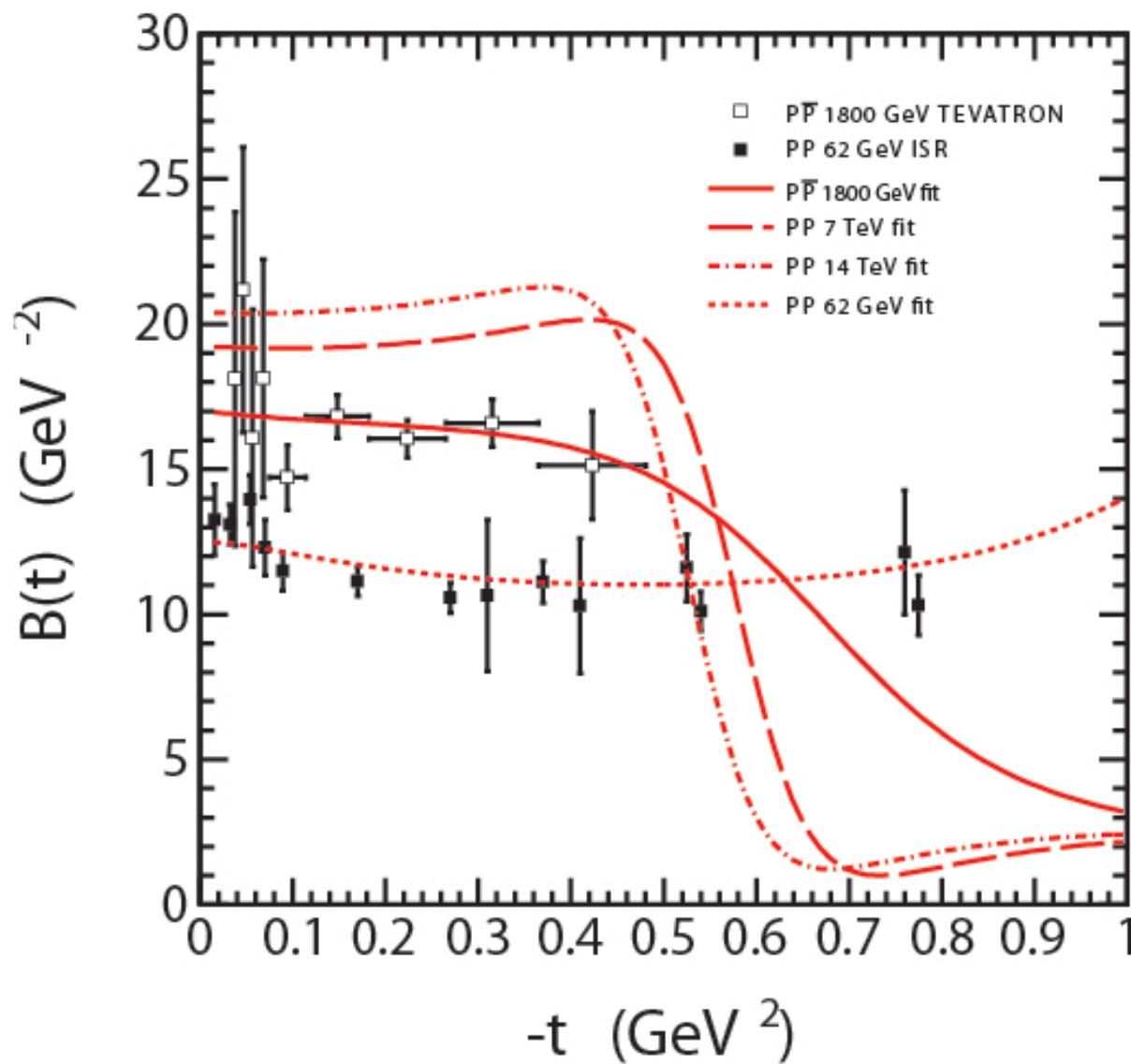


# CERN LHC, TOTEM Collab., June 26, 2011:









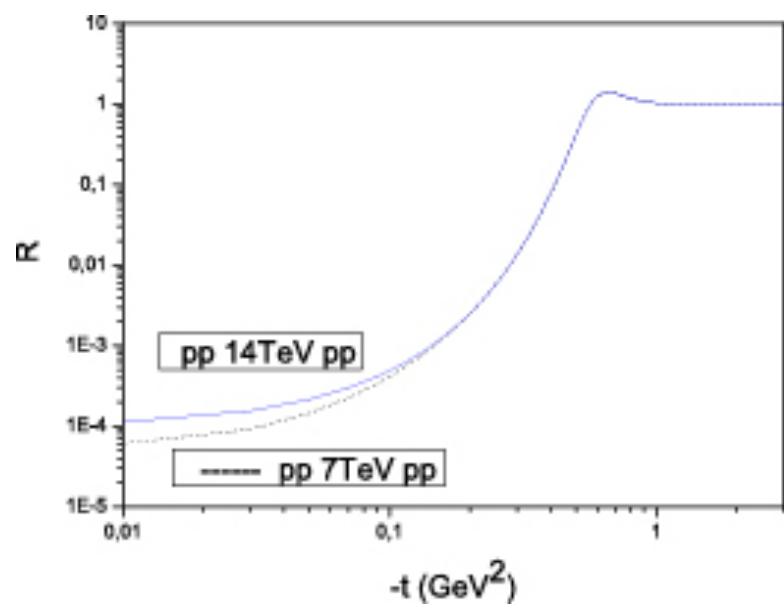
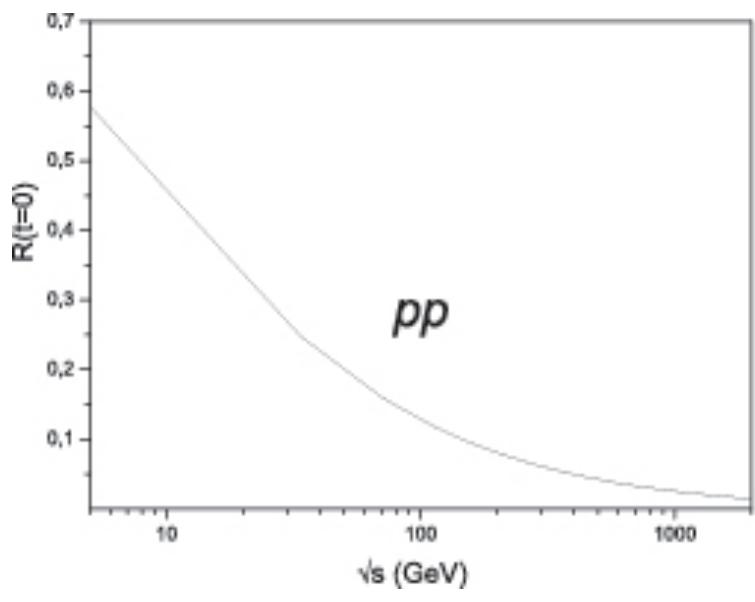
Energy variation of the relative importance of the Pomeron with respect to contributions from the secondary trajectories and the Odderon:

$$R(s, t = 0) = \frac{\Im m(A(s, t) - A_P(s, t))}{\Im A(s, t)}, \quad (1)$$

where the total scattering amplitude  $A$  includes the Pomeron contribution  $A_P$  plus the contribution from the secondary Reggeons and the Odderon.

Starting from the Tevatron energy region, the relative contribution of the non-Pomeron terms to the total cross-section becomes smaller than the experimental uncertainty and hence at higher energies they may be completely neglected, irrespective of the model used.

$$R(s, t) = \frac{|(A(s, t) - A_P(s, t)|^2}{|A(s, t)|^2}. \quad (2)$$



# Low-mass diffraction dissociation at the LHC

L. Jenkovszky, O. Kuprash, J. Lamsa, V. Magas, and R., Orava:  
Dual-Regge approach to high-energy, low-mass DD at the LHC,  
Phys. Rev. D83(2011)0566014; hep-ph/11-11.0664.

L. Jenkovszky, O. Kuprash, J. Lamsa and R. Orava: hep-ph/11063299,  
Mod. Phys. Letters A. **26**(2011) 1-9, August 2011;

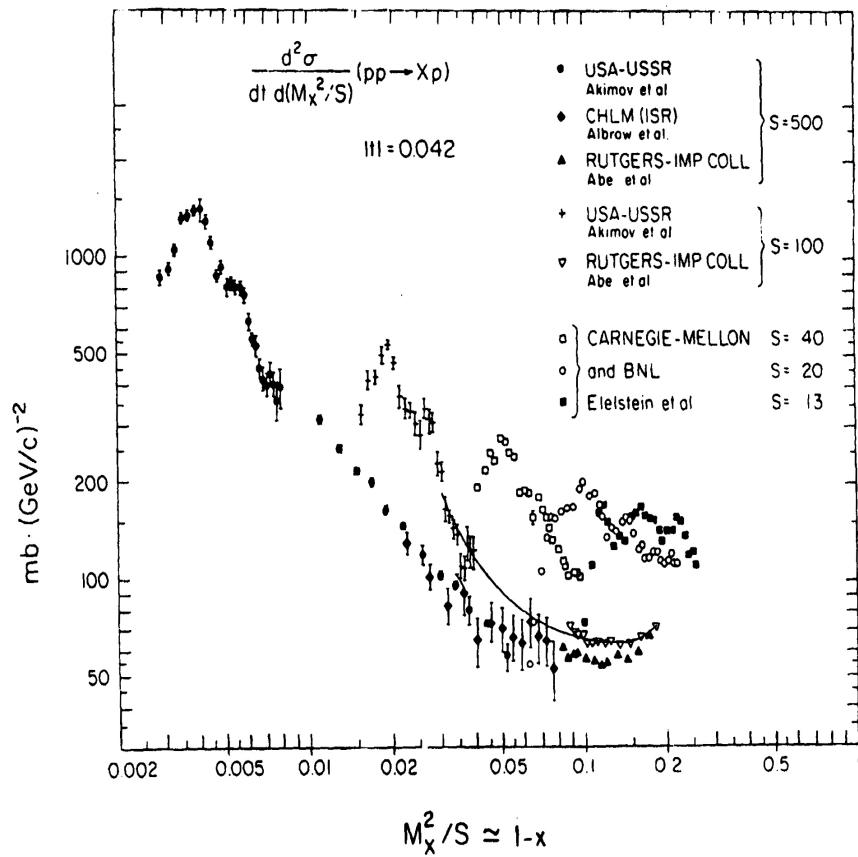
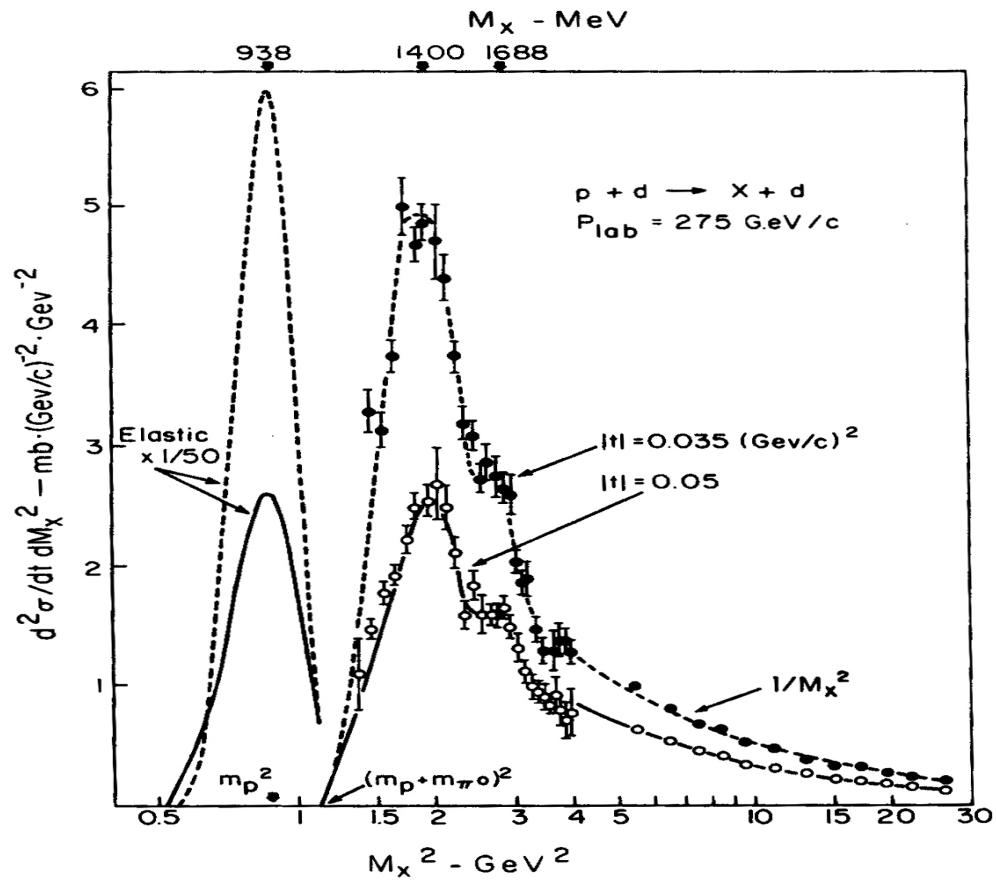
L. Jenkovszky, O. Kuprash, Risto Orava, A. Salii, arXiv:**1211.584**,  
Low missing mass, single- and double diffraction dissociation at the LHC

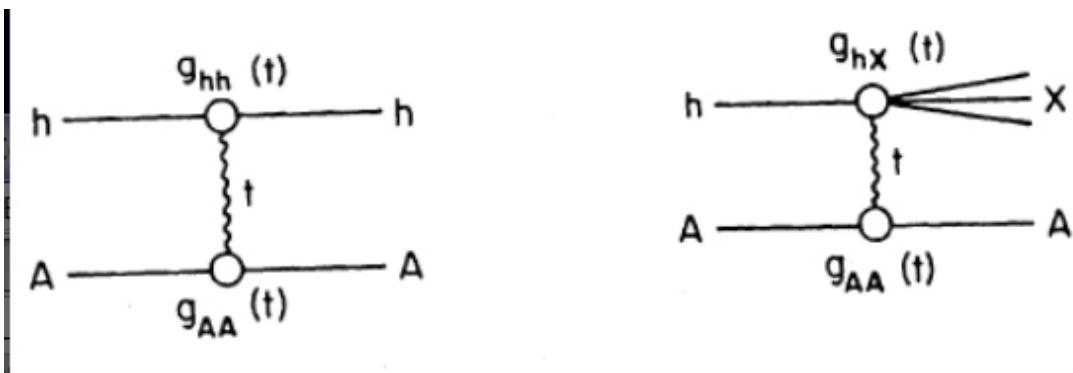
Experimentally, diffraction dissociation in proton-proton scattering was intensively studied in the '70-ies at the Fermilab and the CERN ISR. In particular, double differential cross section  $\frac{d\sigma}{dt dM_X^2}$  was measured in the region  $0.024 < -t < 0.234 \text{ (GeV/c)}^2$ ,  $0 < M^2 < 0.12s$ , and  $(105 < s < 752) \text{ GeV}^2$ , and a single peak in  $M_X^2$  was identified.

Low-mass single diffraction dissociation (SDD) of protons,  $pp \rightarrow pX$  as well as their double diffraction dissociation (DDD) are among the priorities at the LHC. For the CMS Collaboration, the SDD mass coverage is presently limited to some 10 GeV. With the Zero Degree Calorimeter (ZDS), this could be reduced to smaller masses, in case the SDD system produces very forward neutrals, i.e. like a  $N^*$  decaying into a fast leading neutron. Together with the T2 detectors of TOTEM, SDD masses down to 4 GeV could be covered.

While high-mass diffraction dissociation receives much attention, mainly due to its relatively easy theoretical treatment within the triple Reggeon formalism and successful reproduction of the data, this is not the case for low-masses, which are beyond the range of perturbative quantum chromodynamics (QCD). The forthcoming measurements at the LHC urge a relevant theoretical understanding and treatment of low mass DD, which essentially has both spectroscopic and dynamic aspects. The low-mass,  $M_X$  spectrum is rich of nucleon resonances. Their discrimination is a difficult experimental task, and theoretical predictions of the appearance of the resonances depending on  $s$ ,  $t$  and  $M$  is also very difficult since, as mentioned, perturbative QCD, or asymptotic Regge pole formula are of no use here. Below we concentrate on single diffraction dissociation; generalization to DDD is straightforward.

# FNAL

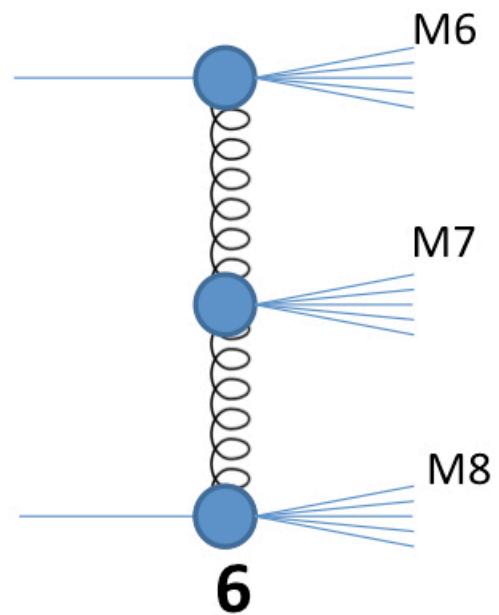
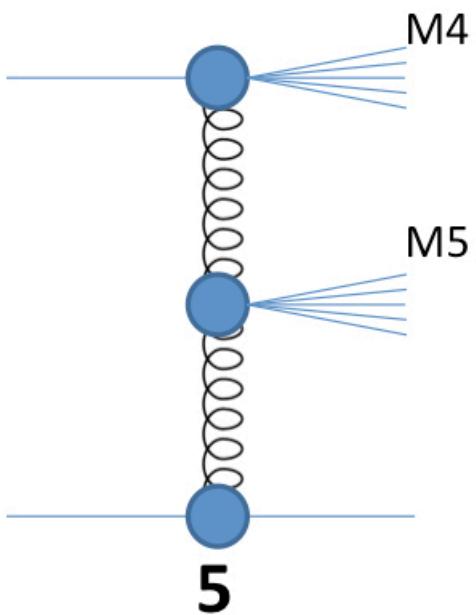
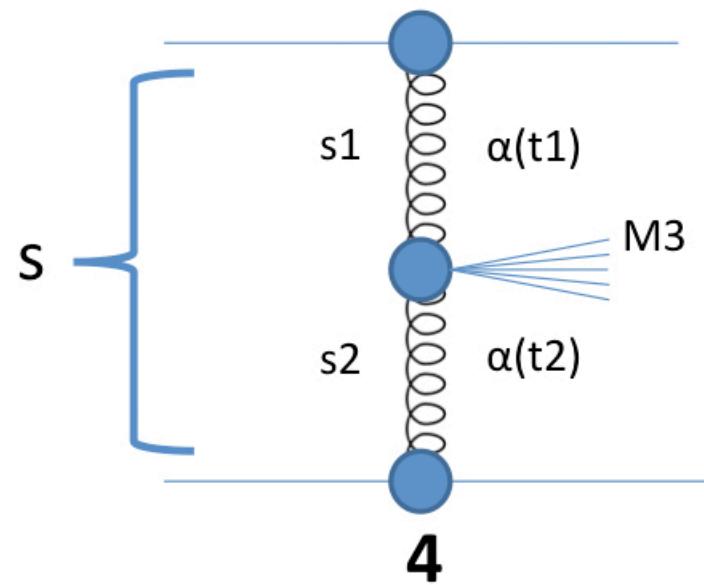
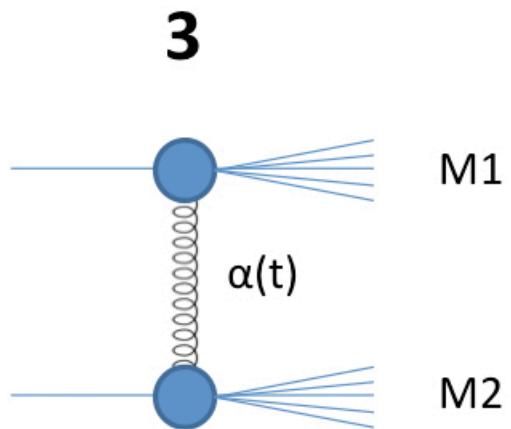
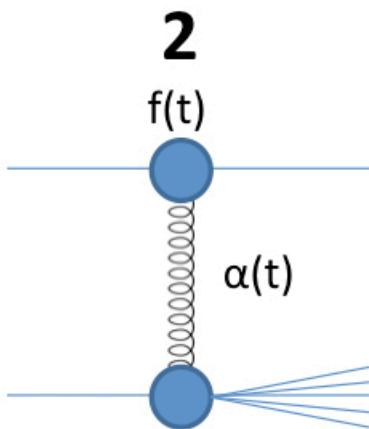
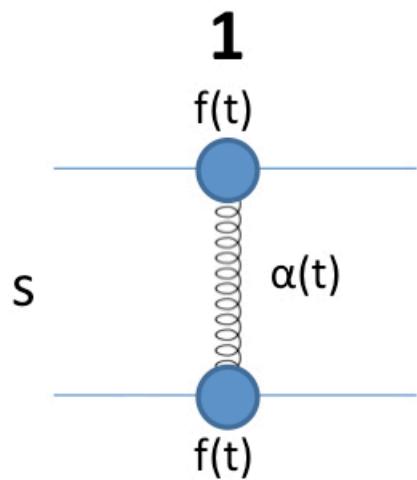




$$\frac{d^2\sigma}{dt dx} = \left| \begin{array}{c} h \\ | \\ \text{wavy line} \\ | \\ p \end{array} \right|^2 = \begin{array}{c} h \\ | \\ \text{wavy line} \\ | \\ p \end{array} \xrightarrow{t=0} \begin{array}{c} h \\ | \\ \text{wavy line} \\ | \\ p \end{array} = \begin{array}{c} h \\ | \\ \text{wavy line} \\ | \\ p \end{array}$$

$$\sigma_{\text{tot}} = \left| \begin{array}{c} h \\ | \\ \text{circle} \\ | \\ p \end{array} \right|^2 = \begin{array}{c} h \\ | \\ \text{circle} \\ | \\ p \end{array} = \begin{array}{c} h \\ | \\ \text{circle} \\ | \\ p \end{array} = \begin{array}{c} h \\ | \\ \text{wavy line} \\ | \\ p \end{array}$$

The first row shows the differential cross-section  $d^2\sigma/dt dx$  as a square of a Feynman diagram with a wavy line and a vertex at  $t=0$ . The second row shows the total cross-section  $\sigma_{\text{tot}}$  as a square of a Feynman diagram with a circle and a vertex at  $t=0$ .



## Factorization (nearly perfect at the LHC!)

$$(g_1 g_2)^2 = \frac{(g_1 f_1)^2 (f_1 g_2)^2}{(f_1 f_2)^2}.$$

Hence

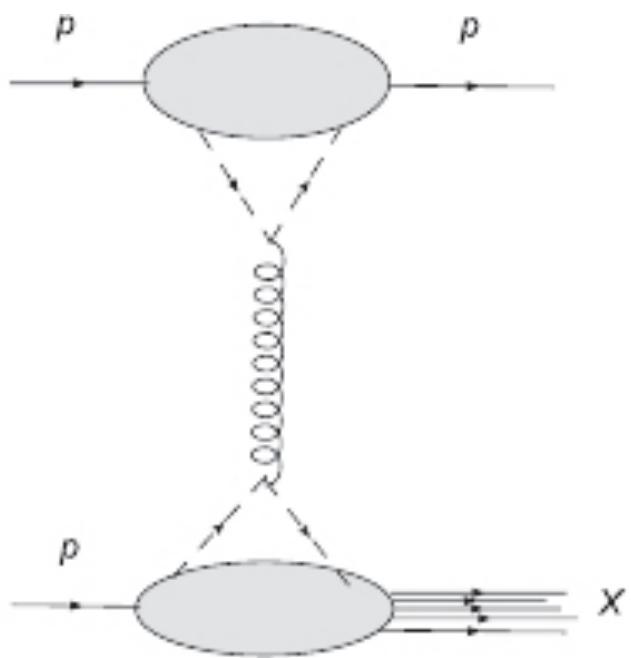
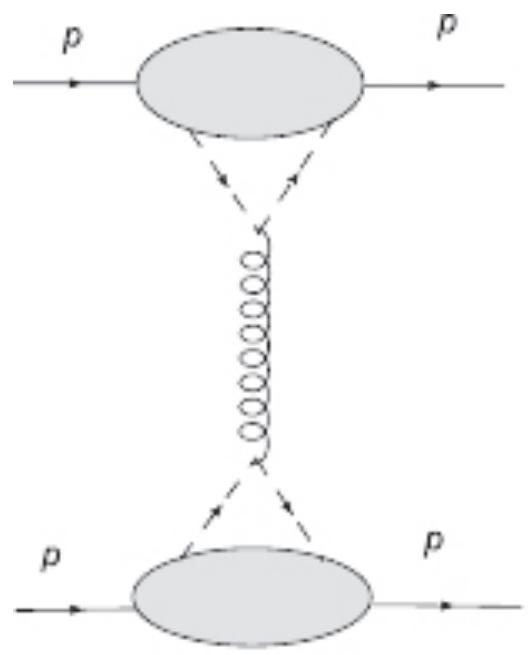
$$\frac{d^3 \sigma}{dt dM_1^2 dM_2^2} = \frac{d^2 \sigma_1}{dt dM_1^2} \frac{d^2 \sigma_2}{dt dM_2^2} \frac{d\sigma_{el}}{dt}.$$

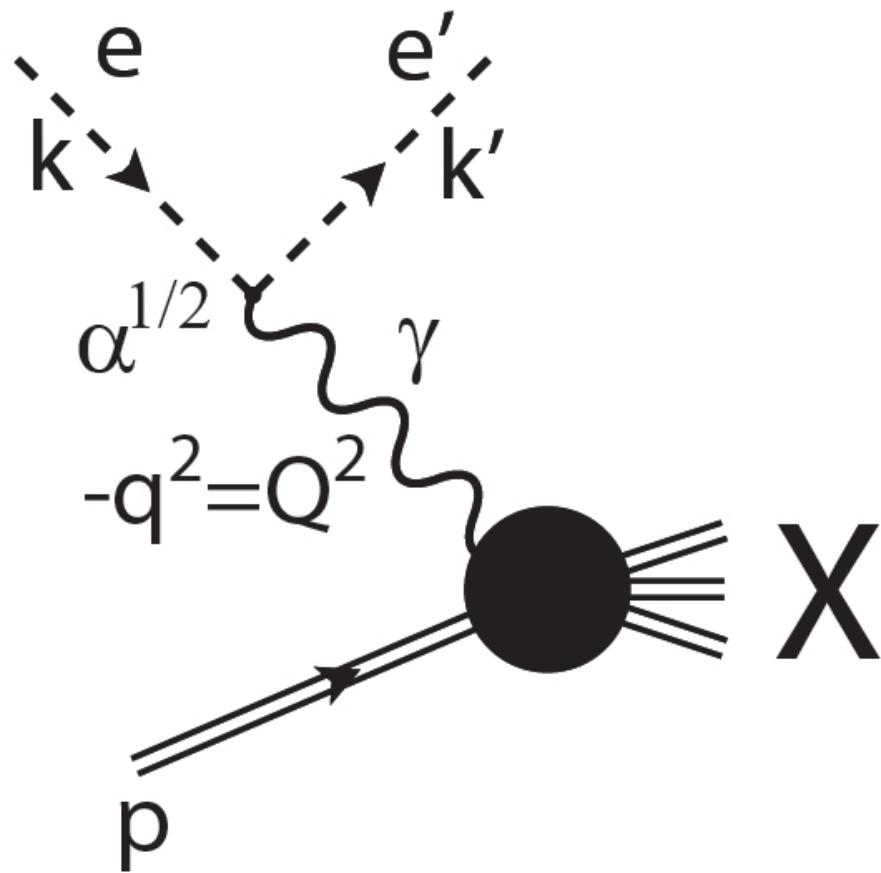
Assuming exponential cone,  $t^{bt}$  and integrating in  $t$ , one gets

$$\frac{d^2 \sigma_{DD}}{dM_1^2 dM_2^2} = k \frac{1}{\sigma_{el}} \frac{d\sigma_1}{dM_1^2} \frac{d\sigma_2}{dM_2^2},$$

where  $k = r^2/(2r - 1)$ ,  $r = b_{SD}/b_{el}$ .

Further integration in  $M^2$  yields  $\sigma_{DD} = k \frac{\sigma_{SD}^2}{\sigma_{el}}$ .





$$\left| \sum_X \frac{q}{p} X \right|^2 = \sum_X \frac{q}{p} X X = \text{Unitarity}_{t=0} = \sum_R R R = \text{Veneziano duality} = \sum_{\text{Res}} \text{Res Res}$$

The equation shows the decomposition of the annihilation amplitude into a sum over intermediate states  $X$  (labeled  $\frac{q}{p} X$ ), followed by a sum over resonance states  $R$ . The label "Unitarity  $t=0$ " indicates the application of unitarity constraints, and "Veneziano duality" refers to the equivalence between different representations of the same physical process.

Similar to the case of elastic scattering, the double differential cross section for the SDD reaction, by Regge factorization, can be written as

$$\frac{d^2\sigma}{dt dM_X^2} = \frac{9\beta^4 [F^p(t)]^2}{4\pi \sin^2[\pi\alpha_P(t)/2]} (s/M_X^2)^{2\alpha_P(t)-2} \times \\ \left[ \frac{W_2}{2m} \left( 1 - M_X^2/s \right) - mW_1(t+2m^2)/s^2 \right], \quad (1)$$

where  $W_i$ ,  $i = 1, 2$  are related to the structure functions of the nucleon and  $W_2 \gg W_1$ . For high  $M_X^2$ , the  $W_{1,2}$  are Regge-behaved, while for small  $M_X^2$  their behavior is dominated by nucleon resonances. The knowledge of the inelastic form factors (or transition amplitudes) is crucial for the calculation of low-mass diffraction dissociation.

In the LHC energy region it simplifies to:

$$\frac{d^2\sigma}{dt dM_X^2} \approx \frac{9\beta^4 [F^p(t)]^2}{4\pi} (s/M_X^2)^{2\alpha_P(t)-2} \frac{W_2}{2m}. \quad (1)$$

These expressions for SDD do not contain the elastic scattering limit because the inelastic form factor  $W_2(M_X, t)$  has no elastic form factor limit  $F(t)$  as  $M_X \rightarrow m$ . This problem is similar to the  $x \rightarrow 1$  limit of the deep inelastic structure function  $F_2(x, Q^2)$ . The elastic contribution to SDD should be added separately.

The  $pp$  scattering amplitude

$$A(s, t)_P = -\beta^2 [f^u(t) + f^d(t)]^2 \left(\frac{s}{s_0}\right)^{\alpha_P(t)-1} \frac{1 + e^{-i\pi\alpha_P(t)}}{\sin \pi\alpha_P(t)}, \quad (1)$$

where  $f^u(t)$  and  $f^d(t)$  are the amplitudes for the emission of  $u$  and  $d$  valence quarks by the nucleon,  $\beta$  is the quark-Pomeron coupling, to be determined below;  $\alpha_P(t)$  is a vacuum Regge trajectory. It is assumed that the Pomeron couples to the proton via quarks like a scalar photon.

A single-Pomeron exchange is valid at the LHC energies, however at lower energies (e.g. those of the ISR or the SPS) the contribution of non-leading Regge exchanges should be accounted for as well.

Thus, the unpolarized elastic  $pp$  differential cross section is

$$\frac{d\sigma}{dt} = \frac{[3\beta F^p(t)]^4}{4\pi \sin^2[\pi\alpha_P(t)/2]} (s/s_0)^{2\alpha_P(t)-2}. \quad (2)$$

The final expression for the double differential cross section reads:

$$\begin{aligned}
& \frac{d^2\sigma}{dt dM_X^2} = \\
& A_0 \left( \frac{s}{M_X^2} \right)^{2\alpha_P(t)-2} \frac{x(1-x)^2 [F^p(t)]^2}{(M_x^2 - m^2) \left( 1 + \frac{4m^2x^2}{-t} \right)^{3/2}} \times \\
& \sum_{n=1,3} \frac{[f(t)]^{2(n+1)} Im \alpha(M_X^2)}{(2n + 0.5 - Re \alpha(M_X^2))^2 + (Im \alpha(M_X^2))^2}. \tag{1}
\end{aligned}$$

# SD and DD cross sections

$$\frac{d^2\sigma_{SD}}{dt dM_x^2} = F_p^2(t) F(x_B, t) \frac{\sigma_T^{Pp}(M_x^2, t)}{2m_p} \left( \frac{s}{M_x^2} \right)^{2(\alpha(t)-1)} \ln \left( \frac{s}{M_x^2} \right)$$

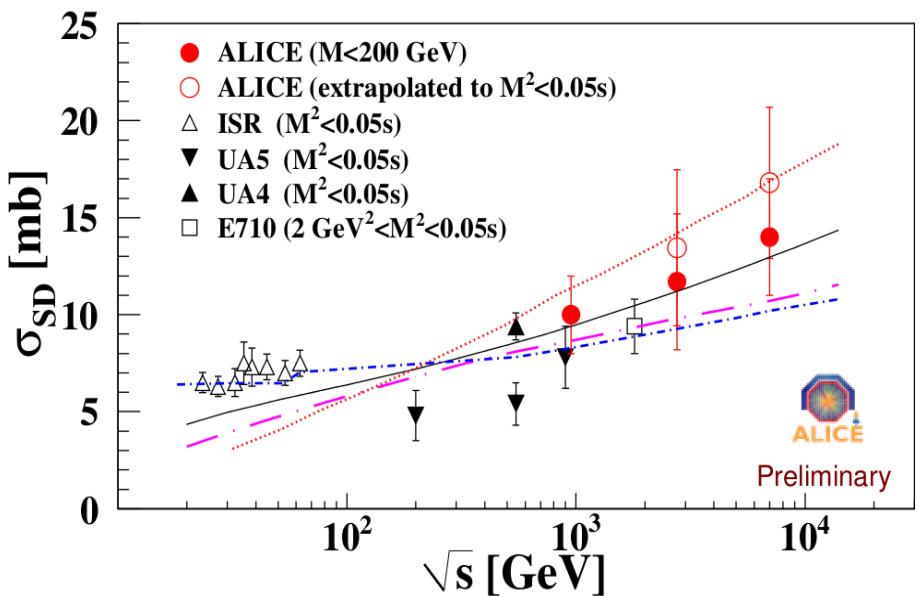
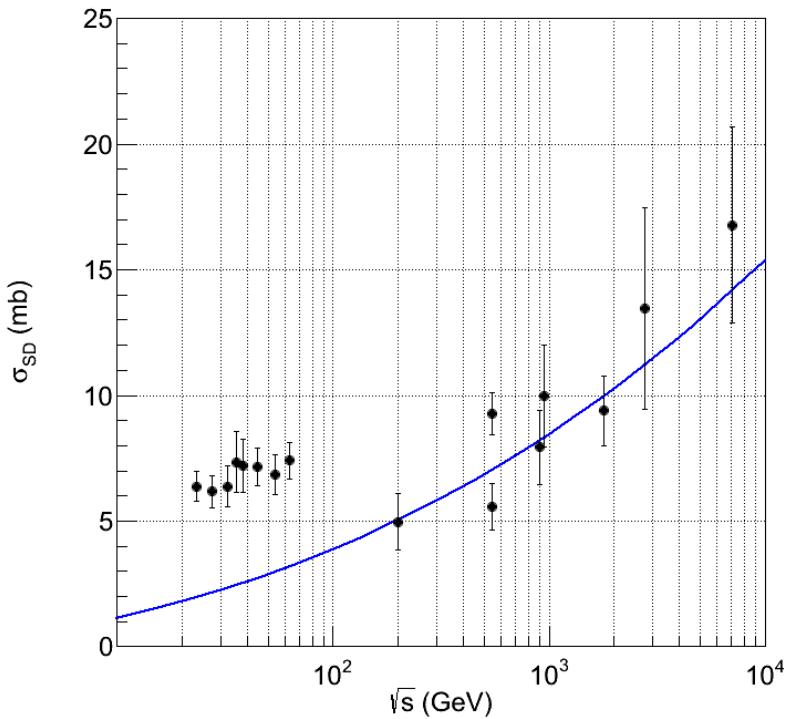
$$\frac{d^3\sigma_{DD}}{dt dM_1^2 dM_2^2} = C_n F^2(x_B, t) \frac{\sigma_T^{Pp}(M_1^2, t)}{2m_p} \frac{\sigma_T^{Pp}(M_2^2, t)}{2m_p}$$

$$\times \left( \frac{s}{(M_1 + M_2)^2} \right)^{2(\alpha(t)-1)} \ln \left( \frac{s}{(M_1 + M_2)^2} \right)$$

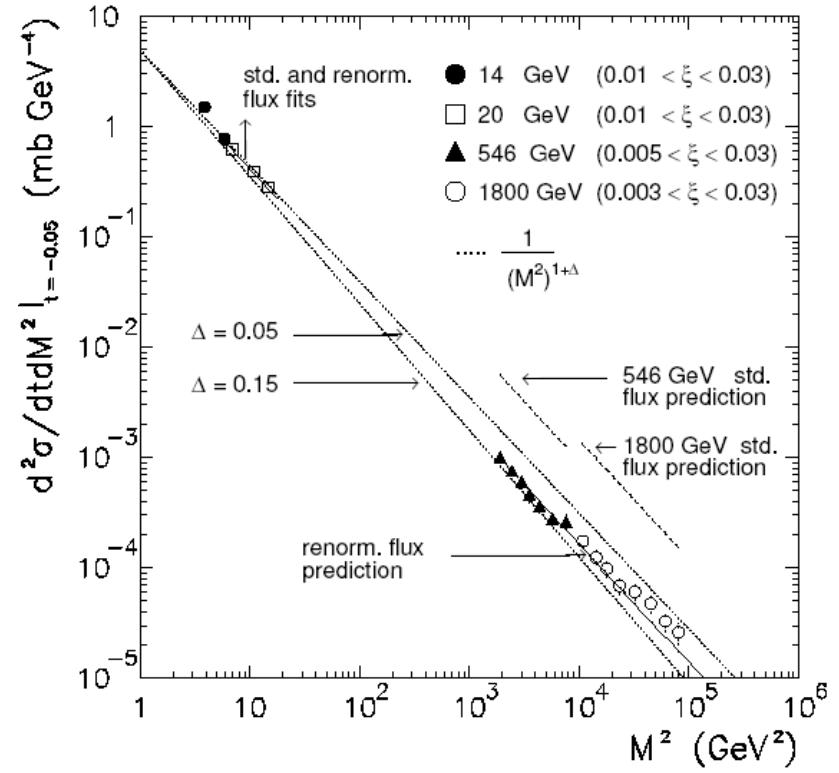
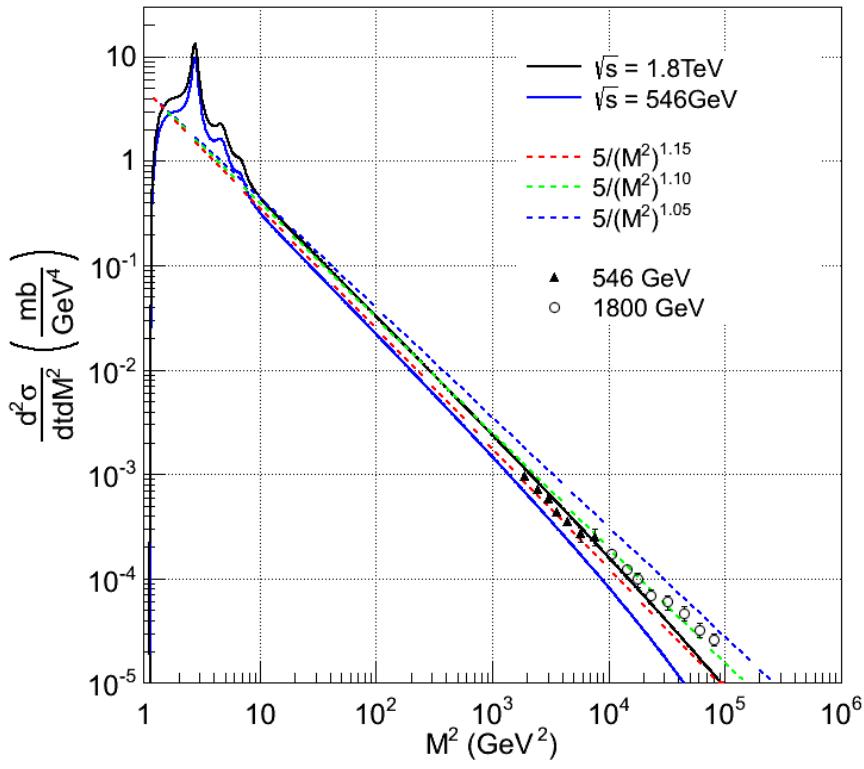
## “Reggeized (dual) Breit-Wigner” formula:

$$\begin{aligned}
& \sigma_T^{Pp}(M_x^2, t) = \text{Im } A(M_x^2, t) = \frac{A_{N^*}}{\sum_n n - \alpha_{N^*}(M_x^2)} + Bg(t, M_x^2) = \\
& = A_n \sum_{n=0,1,\dots} \frac{[f(t)]^{2(n+1)} \text{Im } \alpha(M_x^2)}{(2n + 0.5 - \text{Re } \alpha(M_x^2))^2 + (\text{Im } \alpha(M_x^2))^2} + B_n e^{b_{in}^{bg} t} (M_x^2 - M_{p+\pi}^2)^\epsilon \\
& F(x_B, t) = \frac{x_B(1-x_B)}{(M_x^2 - m_p^2) (1 + 4m_p^2 x_B^2 / (-t))^{3/2}}, \quad x_B = \frac{-t}{M_x^2 - m_p^2 - t} \\
& F_p(t) = \frac{1}{1 - \frac{t}{0.71}}, \quad f(t) = e^{b_{in} t} \\
& \alpha(t) = \alpha(0) + \alpha' t = 1.04 + 0.25t
\end{aligned}$$

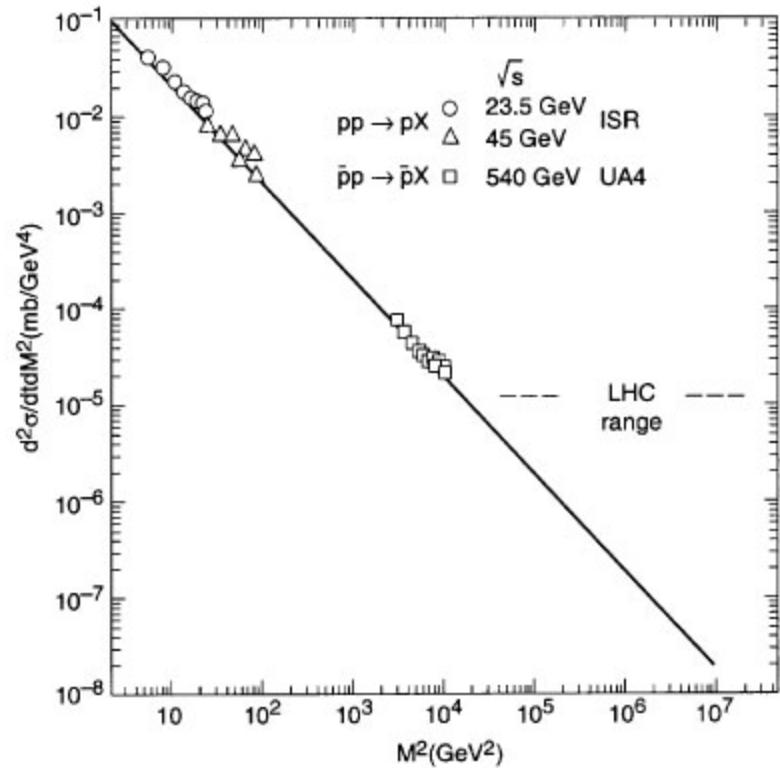
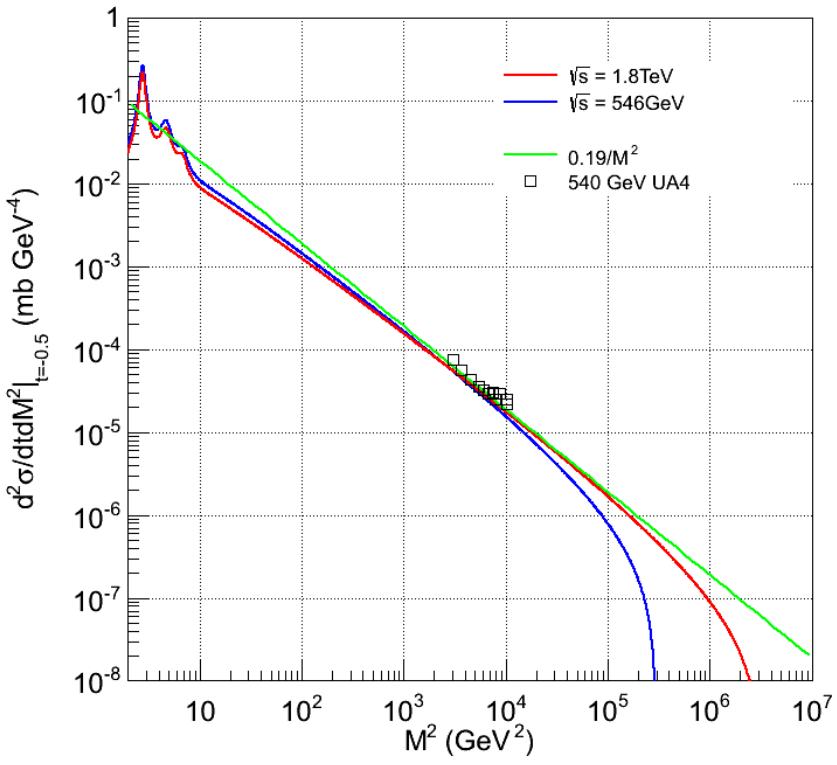
# SDD cross sections vs. energy.



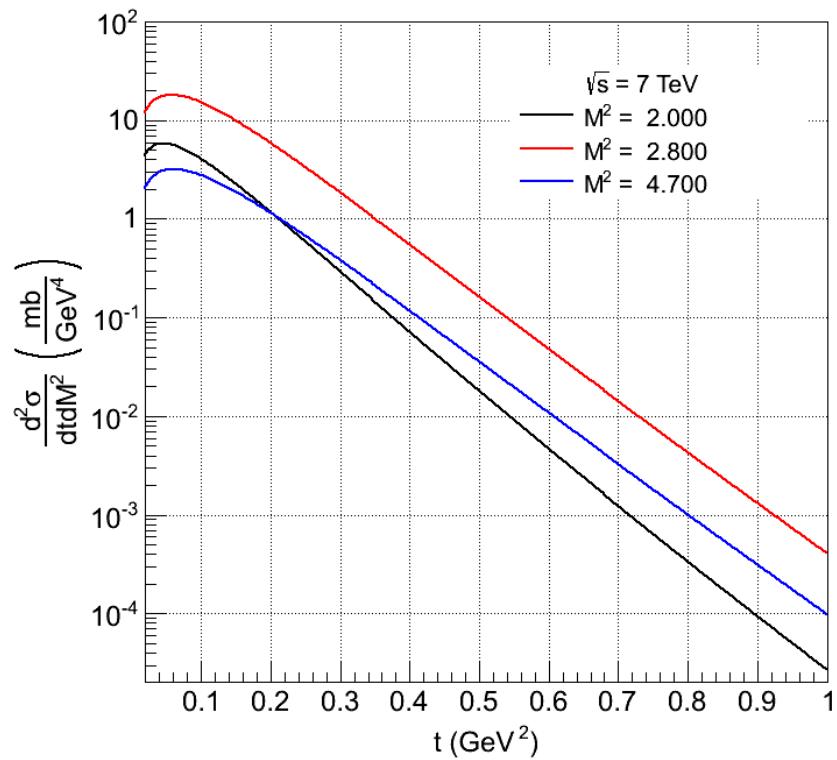
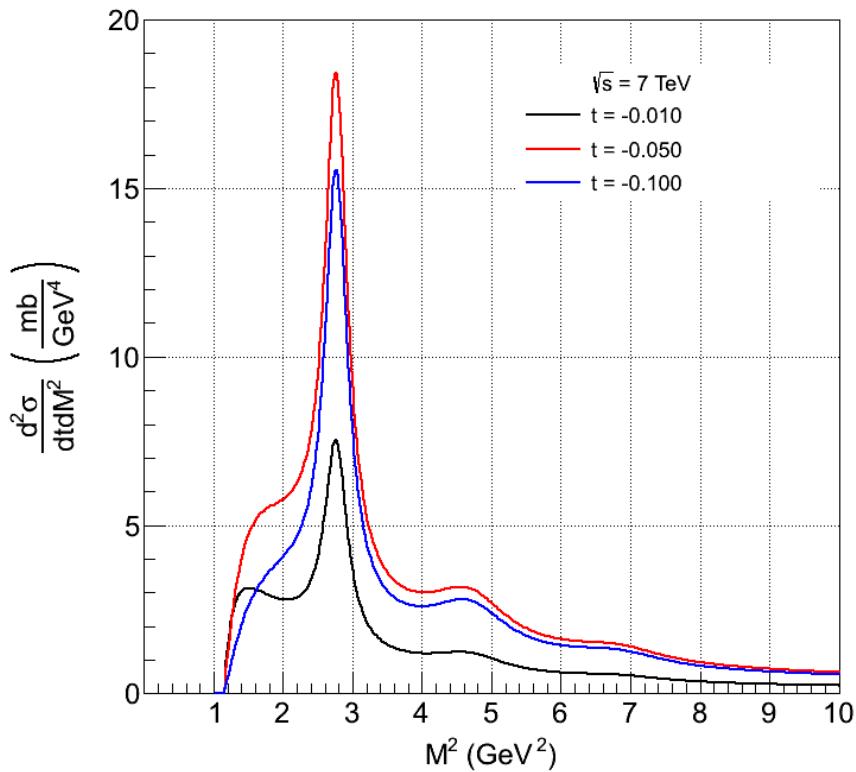
# Approximation of background to reference points ( $t=-0.05$ )



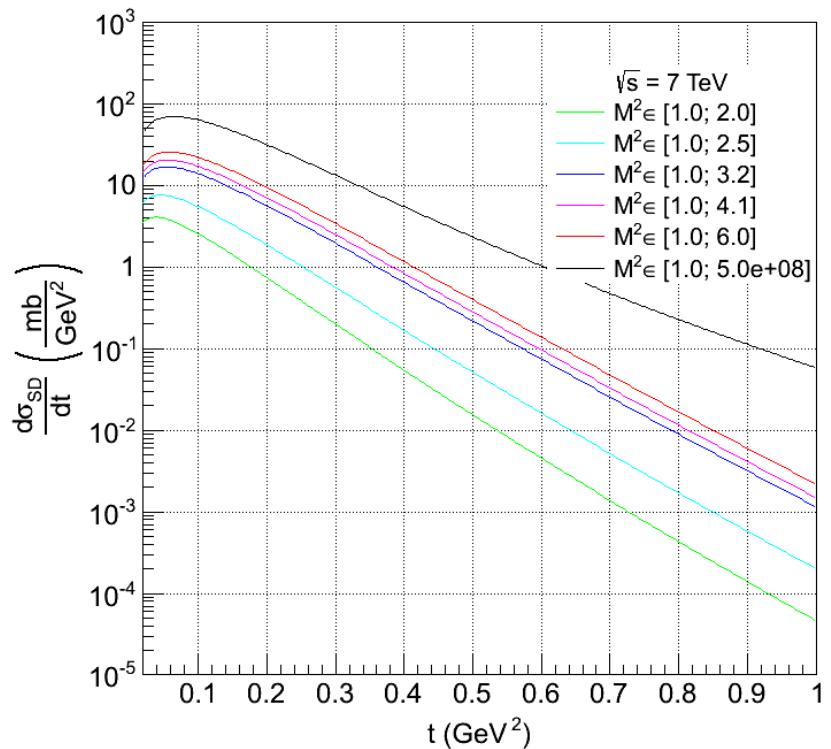
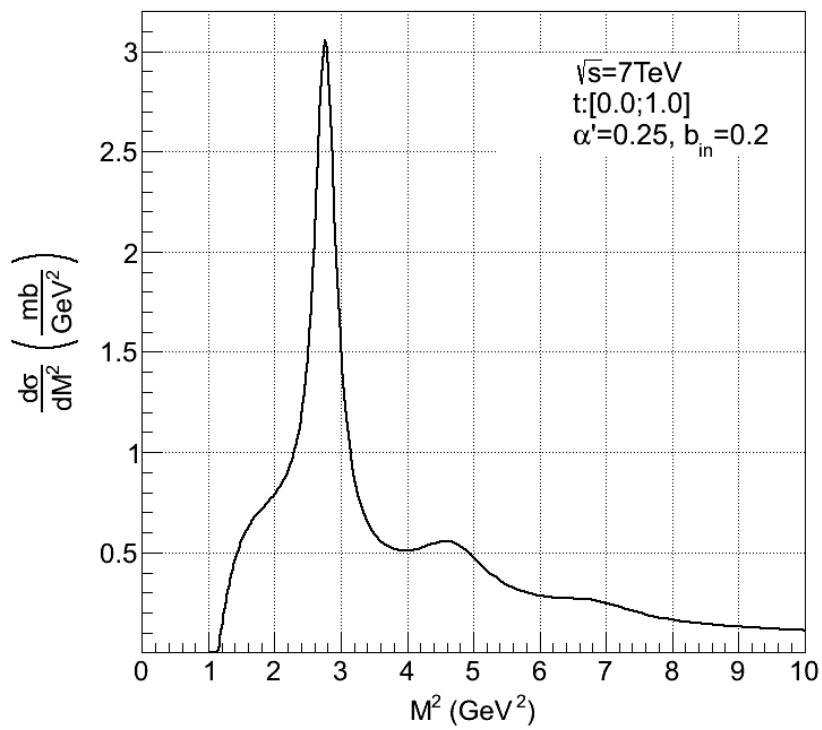
# Approximation of background to reference points ( $t=-0.5$ )



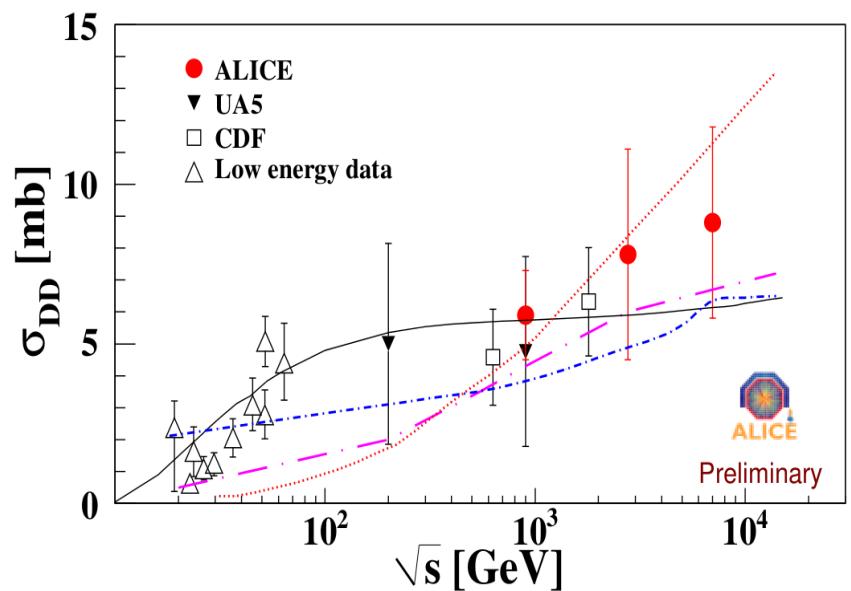
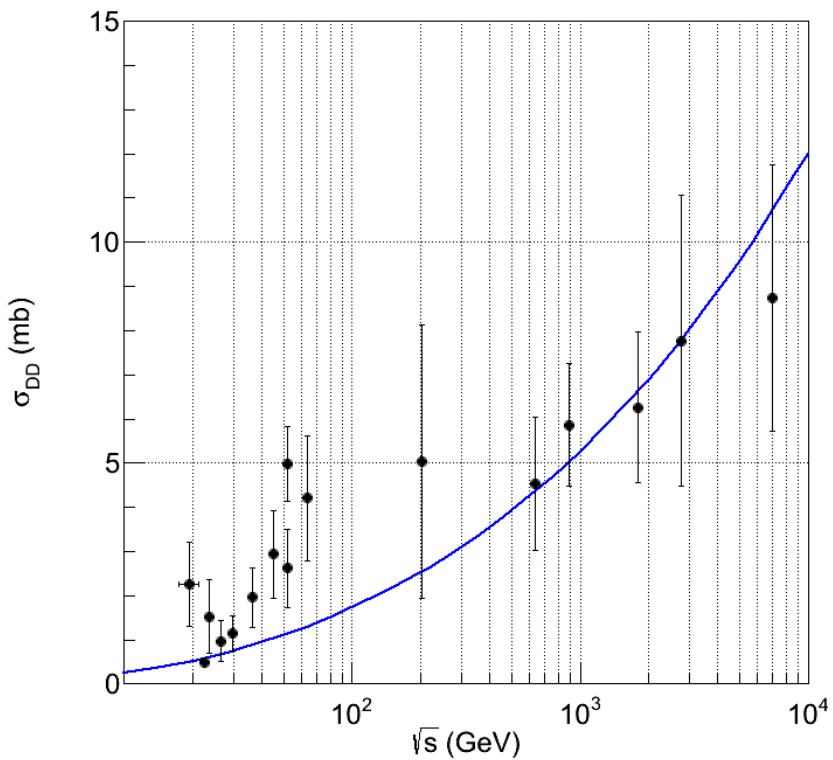
# Double differential SD cross sections



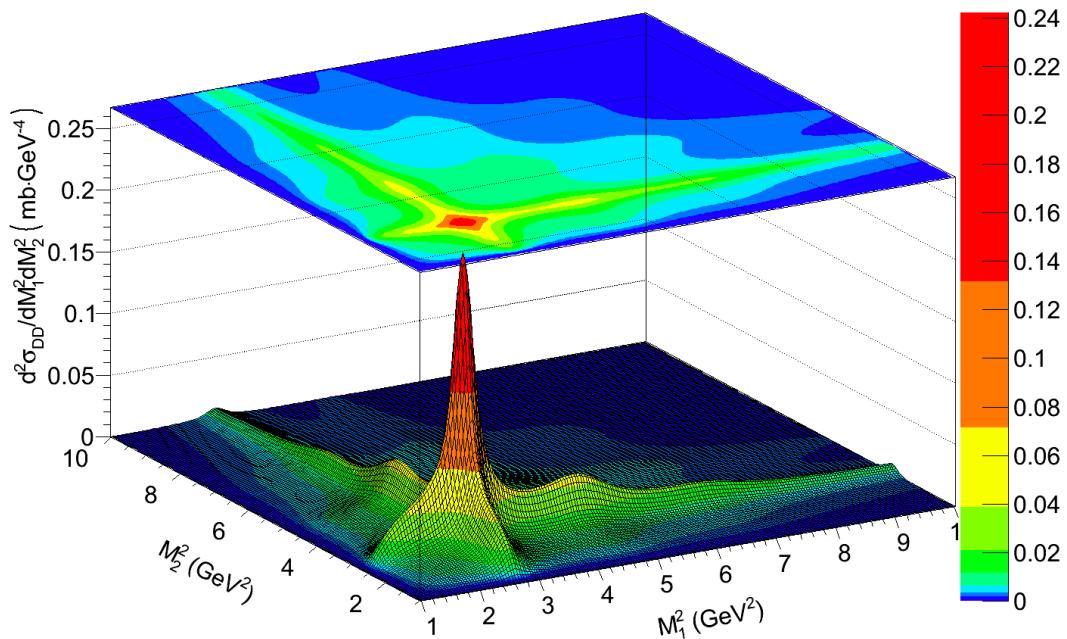
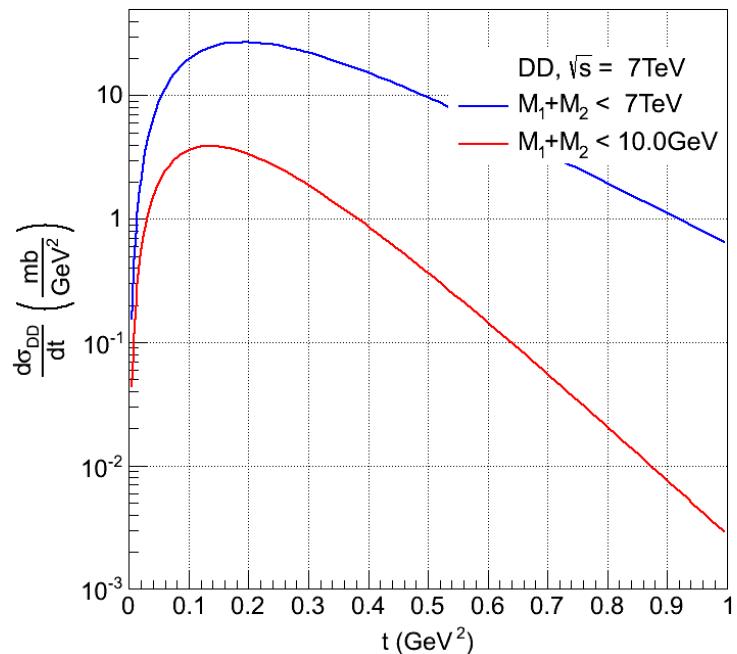
# Single differential integrated SD cross sections



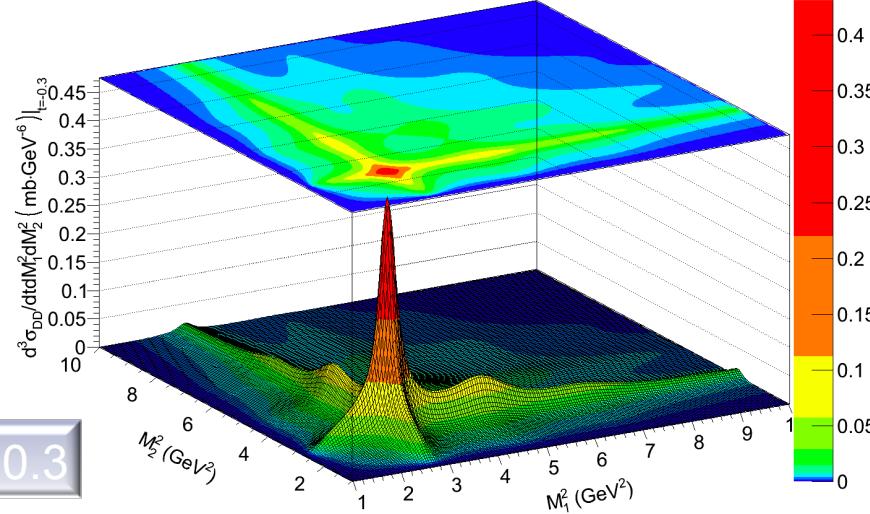
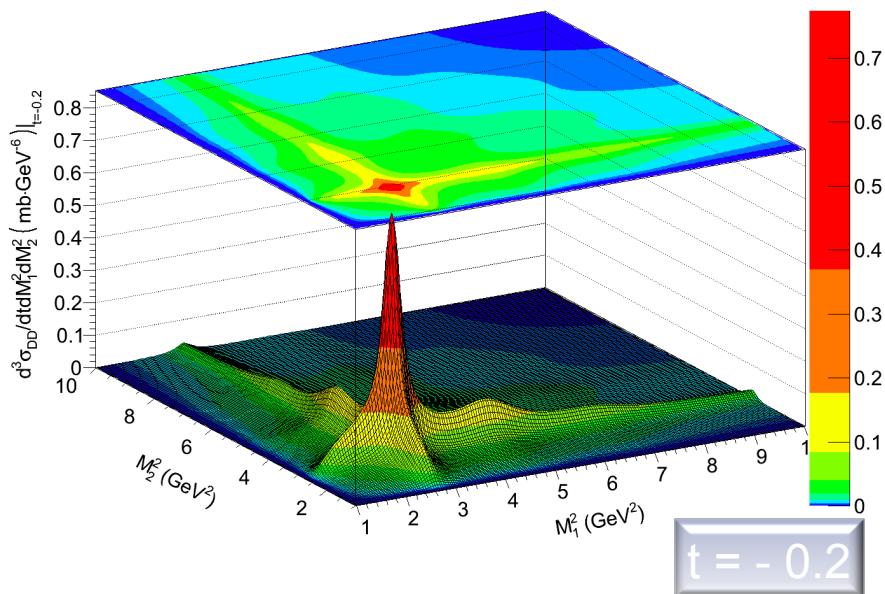
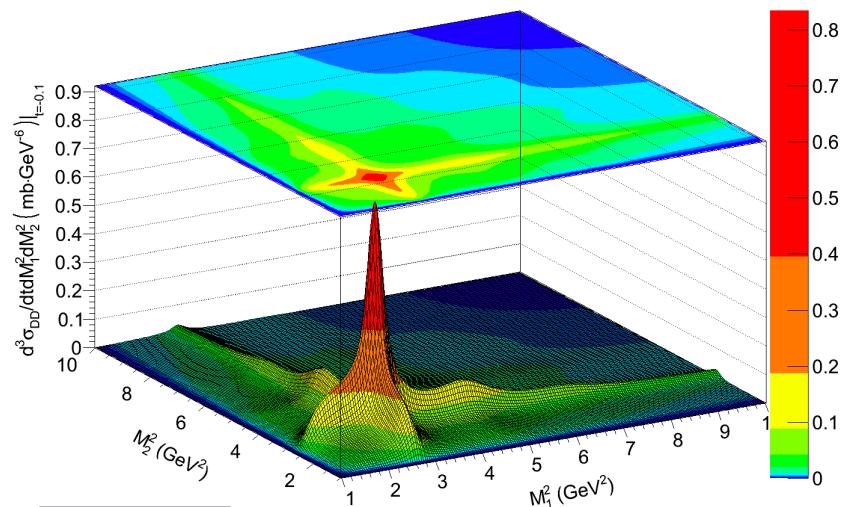
# DDD cross sections vs. energy.



# Integrated DD cross sections



# Triple differential DD cross sections



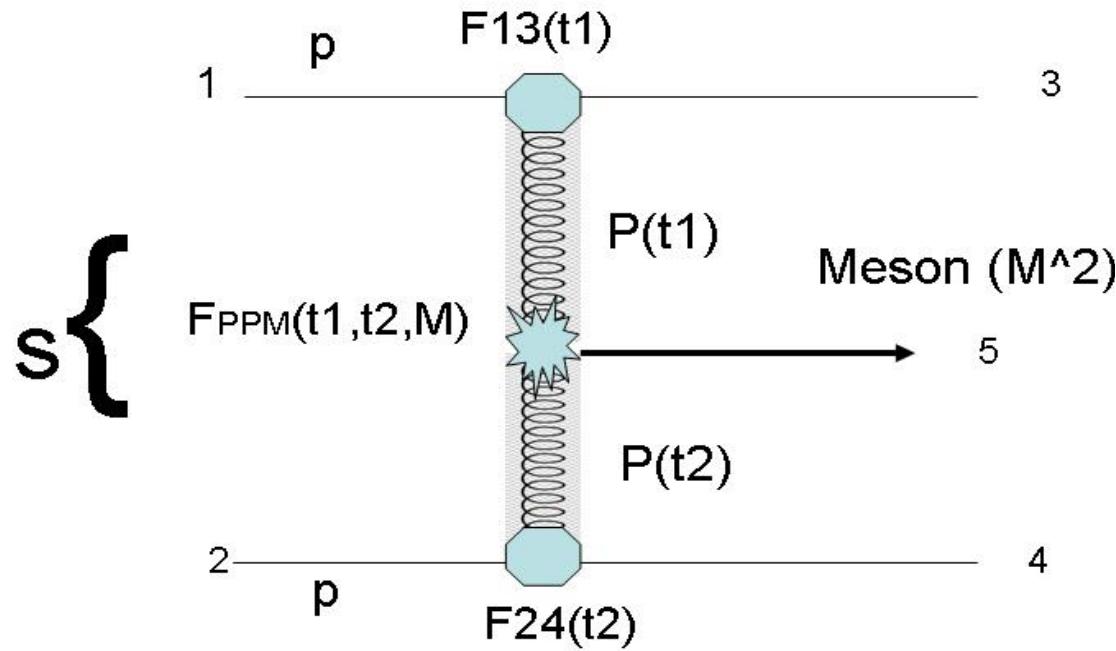
# The parameters and results

$b_{in}$ ( $GeV^{-2}$ )	0.2
$b_{in}^{bg}$ ( $GeV^{-2}$ )	3
$\alpha'$ ( $GeV^{-2}$ )	0.25
$\alpha(0)$	1.04
$\epsilon$	1.03
$A_n$	18.7
$B_n$	8.8
$C_n$	3.79e-2

$\sigma_{SD}$ ( $mb$ )	14.13
$\sigma_{SD}(M < 3.5GeV)$ ( $mb$ )	4.68
$\sigma_{SD}(M > 3.5GeV)$ ( $mb$ )	9.45
$\sigma_{Res}^{SD}$ ( $mb$ )	2.48
$\sigma_{Bg}^{SD}$ ( $mb$ )	9.45
$\sigma_{DD}$ ( $mb$ )	10.68
$\sigma_{DD}(M < 10GeV)$ ( $mb$ )	1.05
$\sigma_{DD}(M > 10GeV)$ ( $mb$ )	9.63

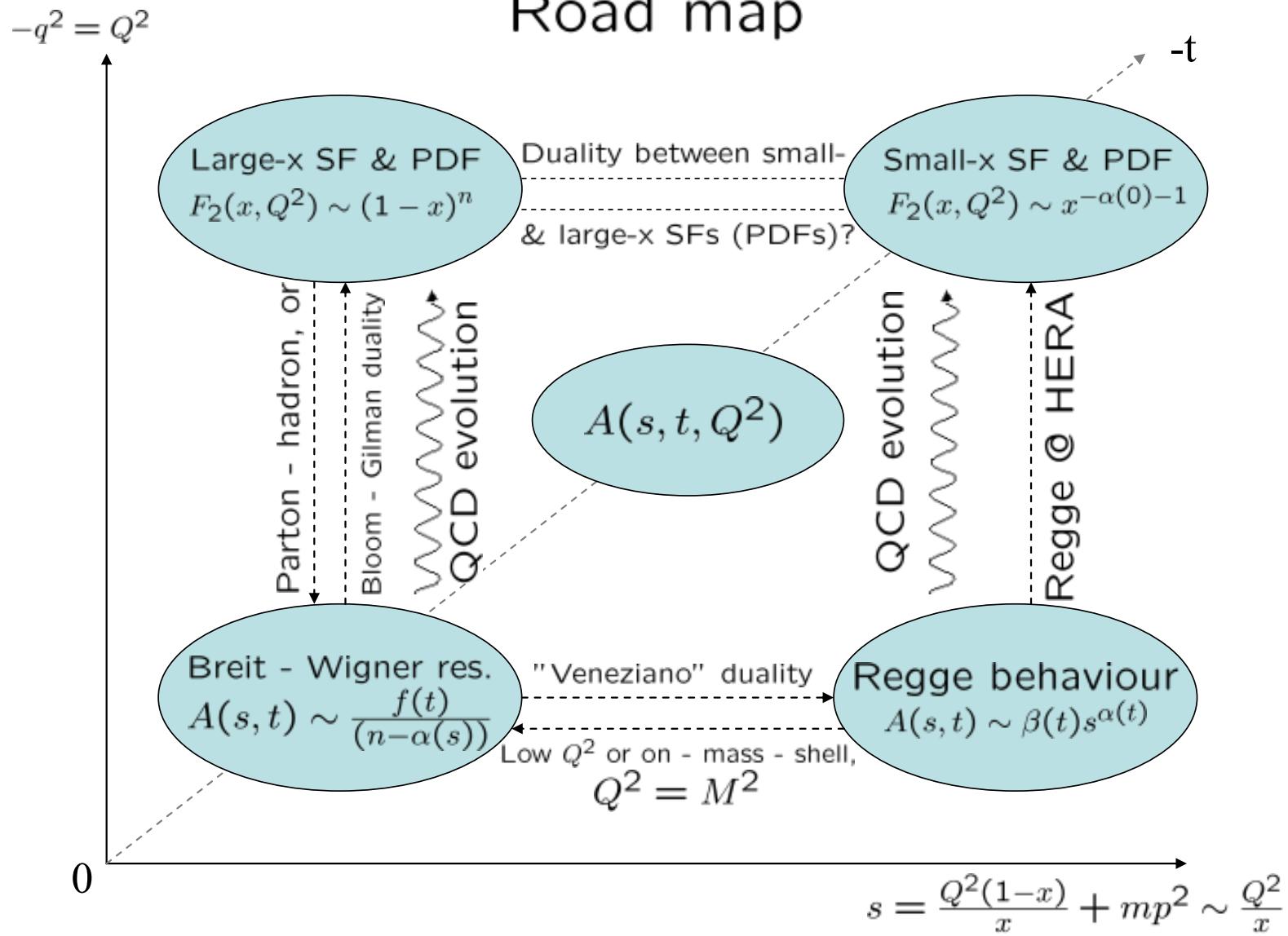
## *Prospects (future plans):*

### 1. Central diffractive meson production (double Pomeron exchange);



### 2. Charge exchange reactions at the LHC (single Reggeon exchange), e.g. $pp \rightarrow n\Delta$ (in collaboration with Oleg Kuprash and Rainer Schicker)

# Road map



The basic object of the theory

$$A(s, t, Q^2 = m^2) \text{ (on mass shell)}$$

$$A(s, t, Q^2)$$

$$\Im m A(s, t = 0, Q^2) \sim F_2 \quad \text{DIS}$$

Reconstruction of the DVCS amplitude from DIS

$$\begin{aligned} F_2 &\sim \Im m A(\gamma^* p \rightarrow \gamma^* p) \Big|_{t=0} \rightarrow \Im m A(\gamma^* p \rightarrow \gamma p) \Big|_{t=0} \\ &\rightarrow A(\gamma^* p \rightarrow \gamma p) \Big|_{t=0} \rightarrow A(\gamma^* p \rightarrow \gamma p) \end{aligned}$$

or

$$\Im m A(\gamma^* p \rightarrow \gamma^* p) \Big|_{t=0} \sim F_2(x_B, Q^2) = x_B q(x_B, Q^2)$$

$$q(x_B, Q^2) \rightarrow q(\xi, \eta, t, x_B, Q^2) \rightarrow$$

$$\rightarrow \xi q(\xi, \eta, t, x_B, Q^2) \stackrel{?}{=} GPD(\xi, \eta, t, x_B, Q^2)$$

## Open problems:

1. Interpolation in energy: from the Fermilab and ISR to the LHC;  
(Inclusion of non-leading contributions);
3. Deviation from a simple Pomeron pole model and breakdown of Regge-factorization;
4. Experimental studies of the exclusive channels ( $p+\pi, \dots$ ) produced from the decay of resonances ( $N^*$ , Roper?,,,) in the nearly forward direction.
5. Turn down of the cross section towards  $t=0?$ !
6. Need for a bank of models. Open an international PROJECT

## **Elastic and total scattering, diffraction in hadron- and lepton-induced reactions:**

А.Н. Валл, Л.Л. Енковский, Б.В. Струминский:  
*Взаимодействие адронов при высоких энергиях*, Физика элементарных частиц и атомного ядра (ЭЧАЯ – Particles and Nuclei) т.19 (1988) стр. 181-223.

Л.Л. Енковский: *Дифракция в адрон-адронных и лептон-адронных процессах при высоких энергиях*, (ЭЧАЯ – Particles and Nuclei) т.34 (2003) стр. 1196-1255.

R. Fiore, L. Jenkovszky, R. Orava, E. Predazzi,  
A. Prokudin, O. Selyugin, *Forward Physics at the LHC;  
Elastic Scattering*, Int. J.Mod.Phys., **A24**: 2551-2559  
(2009).

# Thank you!

László L. Jenkovszky

[jenk@bitp.kiev.ua](mailto:jenk@bitp.kiev.ua)