# Lattice QCD and the Hadron Spectrum

Mike Peardon
School of Mathematics, Trinity College Dublin, Ireland

Heidelberg — 5th September 2013

# Overview

- Essentials
    - QCD and the lattice
    - Quark and gluon actions
    - The path integral and Monte Carlo
- Spectroscopy on the lattice
    - Correlation functions
    - Variational method
    - Making measurements
    - Spin
- Measurements
    - Glueballs
    - Charmonium
    - Light quarks
    - Scattering, phase shifts and resonances

# A constituent picture of hadrons

- QCD has **quarks** (in six flavours) and **gluons**
- **The confinement conjecture:** fields of the QCD lagrangian must be combined into colourless combinations: the **mesons** and **baryons**

A constituent model

| constituents | | | quark model label |
|---|---|---|---|
| $3 \otimes \bar{3}$ | = | $\mathbf{1} \oplus 8$ | meson |
| $3 \otimes 3 \otimes 3$ | = | $\mathbf{1} \oplus 8 \oplus 8 \oplus 10$ | baryon |
| $8 \otimes 8$ | = | $\mathbf{1} \oplus 8 \oplus 8 \oplus 10 \oplus 10$ | glueball |
| $\bar{3} \otimes 8 \otimes 3$ | = | $\mathbf{1} \oplus 8 \oplus 8 \oplus 8 \oplus 10 \oplus 10$ | hybrid |
| ⋮ | | ⋮ | ⋮ |

- QCD does not always respect this constituent labelling! There can be strong mixing.

# Lattice Hadron Spectroscopy

- Significant experimental effort hoping to understand light hadron and charm spectroscopy
  - Are there resonances that don't fit in the quark model?
  - Are there gluonic excitations in this spectrum?
  - What structure does confinement lead to?
  - How do resonances decay?
- To use LQCD to address these questions means:
  - identifying continuum properties of states
  - computing scattering and resonance widths
- To acheive this we need
  - Techniques that give statistical precision
  - Spin identification
  - Control over extrapolations ($m_q \to 0, V \to \infty, a \to 0$.

# Essential properties of QCD

- To discretise theory and write useful lattice representation, important to do best possible job of respecting symmetries of theory.
- Symmetries define universality classes and ensure approach to continuum as we (try to) take $a \to 0$

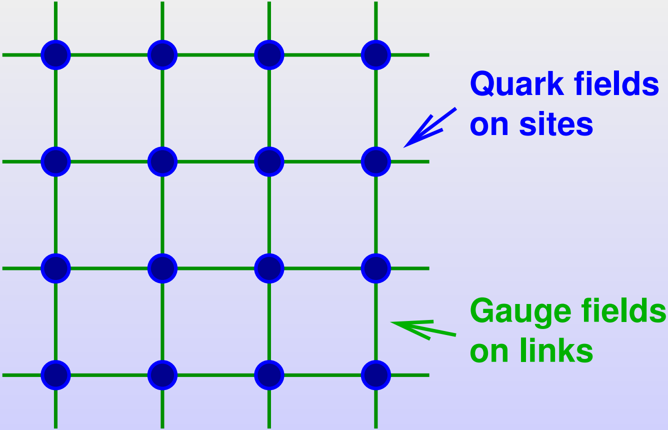
**Symmetries of QCD**

- Poincaré invariance (Lorentz and translation invariance)
- Gauge invariance - $SU(3)$ gauge group
- Discrete symmetries: parity, time-reversal, charge conjugation
- (Near) chiral symmetry (for massless quarks).
- (Near) flavour symmetry (for mass-degenerate quarks).

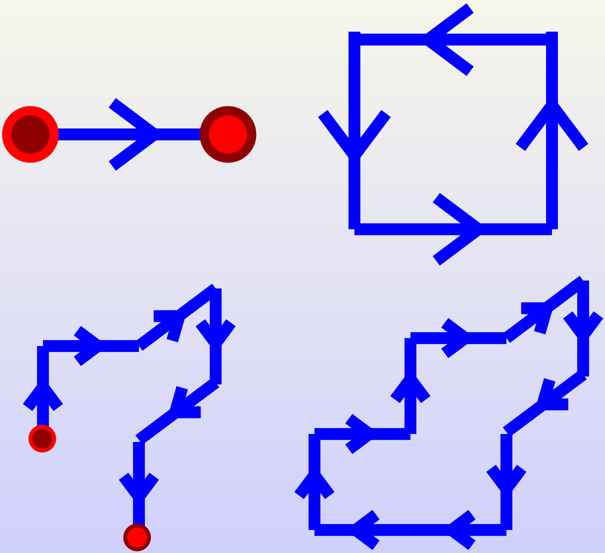
- The QCD path integral is written in terms of the two fundamental fields, the quarks and the gluons.

# Wilson's big idea...

- Wilson realised that ensuring gauge invariance means the gluon fields have to be given special treatment:



# Lattice gauge invariants



# Lattice action - the gluons

- To define a path integral, we also need an action
- The simplest gauge invariant function of the gauge link variables alone is the plaquette (the trace of a path-ordered product of links around a $1 \times 1$ square).

$$S_G[U] = \frac{\beta}{N_c} \sum_{x,\mu<\nu} \mathrm{ReTr}\ \left(1 - U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)\right)$$

  This is the Wilson gauge action
- A path integral for the Yang-Mills theory of gluons would be

$$Z_{YM} = \int \prod_{\mu,x} \mathcal{D}U_\mu(x) e^{-S_G[U]}$$

- The coupling constant, $g$ appears in $\beta = \frac{2N_c}{g^2}$
- No need to fix gauge; the gauge orbits can be trivially integrated over and the group manifold is compact.

# Lattice action - the gluons

- A Taylor expansion in $a$ shows that

$$
\begin{aligned}
S_G[U] &= \frac{\beta}{N_c} \sum_{x,\mu<\nu} \mathrm{ReTr}\ \left(1 - U_\mu(x) U_\nu(x+\hat{\mu}) U^\dagger_\mu(x+\hat{\nu}) U^\dagger_\nu(x)\right) \\
&= \int d^4x\ -\frac{1}{4} \mathrm{Tr}\ F_{\mu\nu} F_{\mu\nu} + \mathcal{O}(a^2)
\end{aligned}
$$

- All terms proportional to odd powers in the lattice spacing vanish because the lattice action preserves a discrete parity symmetry.
- The action is also invariant under a charge-conjugation symmetry, which takes $U_\mu(x) \to U^*_\mu(x)$.
- We have kept almost all of the symmetries of the Yang-Mills sector, but broken the $SO(4)$ rotation group down to the discrete group of rotations of a hypercube.

# Lattice actions - the quarks

- Continuum action:

$$S_Q = \int d^4x \bar{\psi}(\gamma_\mu D_\mu + m)\psi$$

- When $m = 0$, the action has an extra, chiral symmetry:

$$\psi \longrightarrow \psi^{(\chi)} = e^{i\alpha\gamma_5}\psi, \bar{\psi} \longrightarrow \bar{\psi}^{(\chi)} = \bar{\psi}e^{i\alpha\gamma_5}$$

- Central difference:

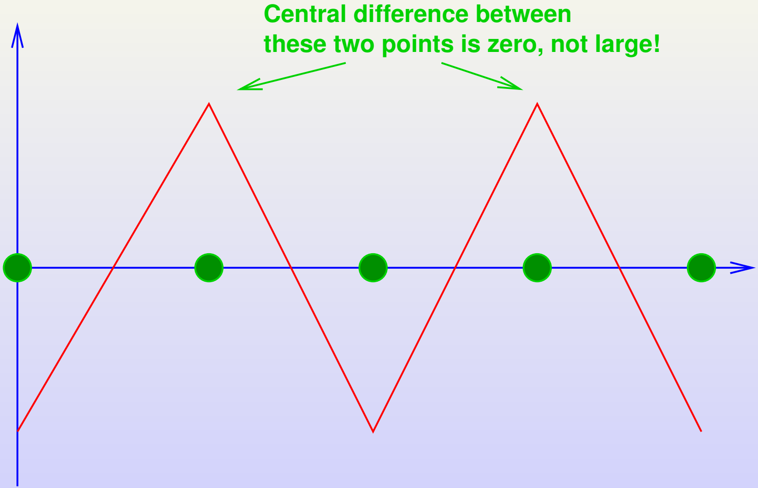
$$\partial_\mu \psi(x) = \frac{1}{2a}\left(\psi(x+\hat{\mu}) - \psi(x-\hat{\mu})\right)$$

- Can be made gauge covariant by including the gauge links:

$$D_\mu \psi(x) = \frac{1}{2a}\left(U_\mu(x)\psi(x+\hat{\mu}) - U_\mu(x-\hat{\mu})\psi(x-\hat{\mu})\right)$$

- **BUT** on closer inspection - more minima to action. With no gauge fields and $\psi(x) = e^{ikx}$ with $k = \{\pi, 0, 0, 0\}$ or $\{\pi, \pi, 0, 0\}$ or $\{\pi, \pi, \pi, 0\}$ or $\ldots$.

# Lattice doubling



# Lattice actions - the quarks (3)

- This is the (in)famous **doubling problem**.

**The Nielson-Ninomiya "no-go" theorem**

There are **no** chirally symmetric, local, translationally invariant doubler-free fermion actions on a regular lattice.

- To put quarks on the lattice, more symmetry must be broken or else a theory with extra flavours of quarks must be simulated.
- A number of solutions are used, each with their advantages and disadvantages.
- The most commonly used are:
  - Wilson fermions
  - Kogut-Susskind (staggered) fermions
  - Ginsparg-Wilson fermions (overlap, domain wall, perfect...)
  - Twisted mass

# QCD on the computer - Monte Carlo integration

- Finite lattice with $a \neq 0$, number of degrees of freedom is finite.
- The path integral is “ordinary” high-dimensional integral. Can be estimated stochastically by Monte Carlo.
- Variance reduction is crucial. Can only be done effectively if theory simulated in Euclidean space-time metric.
- No useful importance sampling weight for the theory in Minkowski space.
- Euclidean path-integral:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \;\; \mathcal{O}[U, \bar{\psi}, \psi] \, e^{-S[U, \bar{\psi}, \psi]}$$

- $e^{-S}$ varies enormously; sample only the tiny region of configuration space that contributes significantly.

# Dynamical quarks in QCD

- Monte Carlo with $N_f = 2$ degenerate quarks. Quark fields obey a grassmann algebra – difficult to manipulate in the computer.
- Quark action is bilinear; integrals done analytically:

$$Z_Q[U] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \;\; e^{-\sum_f \bar{\psi}_f M[U] \psi} = \det M^{N_f}[U]$$

- Including the gauge fields:

$$Z = \int \mathcal{D}U \;\; Z_Q[U] e^{-S_G[U]} = \int \mathcal{D}U \;\; \det M^{N_f}[U] e^{-S_G[U]}$$

- For $N_f = 2 \det M^2 \geq 0$ so can be included in importance sampling (but expensive).
- Using $M^\dagger = \gamma_5 M \gamma_5$, $\det M^2$ is re-written

$$Z_Q[U] = \int \mathcal{D}\phi \mathcal{D}\phi^* e^{-\phi^* [M^\dagger M]^{-1} \phi}$$

# Dynamical quarks in QCD

- Requires applying inverse of $M$ — very large matrix, so takes a lot of computer time.
- Where most computing power in lattice simulations goes; computing the effect of the quark fields acting on the gluons in the Monte Carlo updates.
- Alternative: quenched approximation to QCD. Ignore fermion path integral completely - unphysical approximation so effects are hard to quantify.
- Inversion is needed again in the measurement stage too;

$$\langle \psi(x)\bar{\psi}(y)\rangle = M^{-1}[U](x,y)$$

# Markov Chain Monte Carlo

- How is the configuration space sampled?
- All techniques use a **Markov process**: stochastic transition taking current state of the system randomly to a new state, such that probability of jump is independent of past states/
- Ergodic (positive recurrent, irreducible) Markov chains have unique stationary distributions; build the Markov process so it has our importance sampling distribution as its stationary state.
- If this can be done, then the sequence of configurations generated by the process is our importance sampling ensemble!
- Almost all algorithms exploit **detailed balance** to achieve this.

# Variational method in Euclidean QFT

- Ground-state energies found from $t \to \infty$ limit of:

**Euclidean-time correlation function**

$$
\begin{aligned}
C(t) &= \langle 0 | \; \Phi(t) \quad \Phi^\dagger(0) \; | 0 \rangle \\
&= \sum_{k,k'} \langle 0 | \; \Phi | k \rangle \langle k | e^{-\hat{H}t} | k' \rangle \langle k' | \Phi^\dagger \; | 0 \rangle \\
&= \sum_{k} |\langle 0 | \; \Phi | k \rangle|^2 \quad e^{-E_k t}
\end{aligned}
$$

- So $\lim_{t\to\infty} C(t) = Z e^{-E_0 t}$
- Variational idea: find operator $\Phi$ to maximise $C(t)/C(t_0)$ from sum of basis operators $\Phi = \sum_a v_a \phi_a$

[C. Michael and I. Teasdale. NPB215 (1983) 433]

[M. Lüscher and U. Wolff. NPB339 (1990) 222]

# Excitations

## Variational method

**If we can measure** $C_{ab}(t) = \langle 0|\phi_a(t)\phi_b^\dagger(0)|0\rangle$ for all $a, b$ and solve generalised eigenvalue problem:

$$\mathbf{C}(t)\,\underline{v} = \lambda \mathbf{C}(t_0)\,\underline{v}$$

then

$$\lim_{t-t_0\to\infty} \lambda_k = e^{-E_k t}$$

For this to be practical, we need:

- a 'good' basis set that **resembles the states** of interest
- **all elements** of this correlation matrix measured

[see Blossier et.al. JHEP 0904 (2009) 094]

# Fermions in the path integral

- In path integral, fermions are represented using **Grassmann** algebra.

$$\int d\eta = 0, \quad \int d\eta\, \eta = 1, \quad \eta^2 = 0$$

- Higher dimensions - anticommutation rule:

$$\eta_i \eta_j = -\eta_j \eta_i$$

- Expensive to manipulate directly by computer ...

# Fermions in the path integral

- In QCD the action is (usually) bilinear.
- Consider computing a correlation function for the $\rho$-meson in 2-flavour QCD:

$$C_\rho(t_1, t_0) = \frac{\int \mathcal{D}U \mathcal{D}\bar{\psi}\mathcal{D}\psi \; \bar{\psi}_u \gamma_i \psi_d(t_1) \; \bar{\psi}_d \gamma_i \psi_u(t_0) \;\; e^{-S_G[U] + \bar{\psi}_f M_f[U] \psi_f}}{\int \mathcal{D}U \mathcal{D}\bar{\psi}\mathcal{D}\psi \; e^{-S_G[U] + \bar{\psi}_f M_f[U] \psi_f}}$$

- Integrate the grassmann fields analytically, giving:

$$C_\rho(t_1, t_0) = \frac{\int \mathcal{D}U \; \mathrm{Tr}\, \gamma_i M_d^{-1}(t_1, t_0) \gamma_i M_u^{-1}(t_0, t_1) \;\; \det M^2[U] \; e^{-S_G[U]}}{\int \mathcal{D}U \;\; \det M^2[U] \; e^{-S_G[U]}}$$

- Fermions in lagrangian $\rightarrow$ fermion determinant
- Fermions in measurement $\rightarrow$ propagators

# Fermions in the path integral

- With more insertions, need **Wick's theorem**
- Example — four field insertions:

$$\langle \psi_i \bar{\psi}_j \psi_k \bar{\psi}_l \rangle$$

- and the pairwise contraction can be done in two ways:

$$\psi_i \bar{\psi}_j \psi_k \bar{\psi}_l \quad \text{and} \quad \psi_i \bar{\psi}_j \psi_k \bar{\psi}_l$$

- ...giving the propagator combination

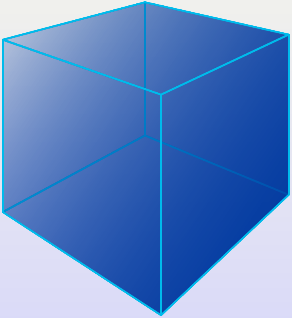
$$M^{-1}_{ij} M^{-1}_{kl} - M^{-1}_{jk} M^{-1}_{il}$$

- the minus-sign comes from the anti-commutation needed in the second term.
- More fields means more combinations
- This is important in (eg.) isoscalar meson spectroscopy.

# A tale of two symmetries

- Continuum: states classified by $J^P$ irreducible representations of $O(3)$.

$O(3)$



$O_h$

- Lattice regulator breaks $O(3) \to O_h$
- Lattice: states classified by $R^P$ **“quantum letter” labelling irrep of** $O_h$

# Irreps of $O_h$

- $O$ has 5 conjugacy classes (so $O_h$ has 10)
- Number of conjugacy classes = number of irreps
- Schur: $24 = 1^2 + 1^2 + 2^2 + 3^2 + 3^2$
- These irreps are labelled $A_1, A_2, E, T_1, T_2$

| | $E$ | $8C_3$ | $6C_2$ | $6C_4$ | $3C_2$ |
|---|---|---|---|---|---|
| $A_1$ | 1 | 1 | 1 | 1 | 1 |
| $A_2$ | 1 | 1 | -1 | -1 | 1 |
| $E$ | 2 | -1 | 0 | 0 | 2 |
| $T_1$ | 3 | 0 | -1 | 1 | -1 |
| $T_2$ | 3 | 0 | 1 | -1 | -1 |

# Spin on the lattice

- $O_h$ has 10 irreps: $\{A_1^{g,u}, A_2^{g,u}, E^{g,u}, T_1^{g,u}, T_2^{g,u}, \}$, where $\{g, u\}$ label even/odd parity.
- Link to continuum: subduce representations of $O(3)$ into $O_h$

| | $A_1$ | $A_2$ | $E$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|
| $J = 0$ | 1 | | | | |
| $J = 1$ | | | | 1 | |
| $J = 2$ | | | 1 | | 1 |
| $J = 3$ | | 1 | | 1 | 1 |
| $J = 4$ | 1 | | 1 | 1 | 1 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

- Enough to search for degeneracy patterns in the spectrum?

$$4 \equiv 0 \oplus 1 \oplus 2$$

# Operator basis — derivative construction

- A closer link to the continuum is needed
- Start with continuum operators, built from $n$ derivatives:

$$\Phi = \bar{\psi}\,\Gamma\left(D_{i_1} D_{i_2} D_{i_3} \ldots D_{i_n}\right)\psi$$

- Construct irreps of $SO(3)$, then subduce these representations to $O_h$
- Now replace the derivatives with lattice finite differences:

$$D_j\psi(x) \rightarrow \frac{1}{a}\left(U_j(x)\psi(x+\hat{\jmath}) - U_j^{\dagger}(x-\hat{\jmath})\psi(x-\hat{\jmath})\right)$$

# Example: $J^{PC} = 2^{++}$ meson creation operator

- Need more information to discriminate spins. Consider continuum operator that creates a $2^{++}$ meson:

$$\Phi_{ij} = \bar{\psi}\left(\gamma_i D_j + \gamma_j D_i - \frac{2}{3}\delta_{ij}\gamma \cdot D\right)\psi$$

- Lattice: Substitute gauge-covariant lattice finite-difference $D_{latt}$ for $D$
- A reducible representation:

$$\Phi^{T_2} = \{\Phi_{12}, \Phi_{23}, \Phi_{31}\}$$

$$\Phi^{E} = \left\{\frac{1}{\sqrt{2}}(\Phi_{11} - \Phi_{22}), \frac{1}{\sqrt{6}}(\Phi_{11} + \Phi_{22} - 2\Phi_{33})\right\}$$

- Look for signature of continuum symmetry:

$$\langle 0|\Phi^{(T_2)}|2^{++(T_2)}\rangle = \langle 0|\Phi^{(E)}|2^{++(E)}\rangle$$

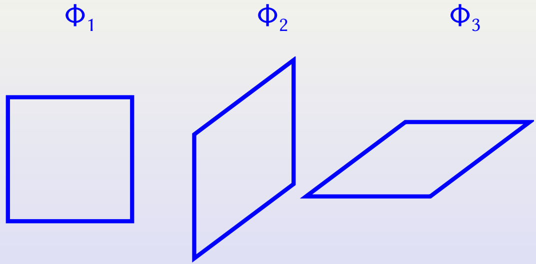
# Glueballs

# Creation operators: glueballs

- To measure the correlation functions, we need to measure appropriate creation operators on our ensemble.
- The operators should be functions of the fields on a time-slice and transform irreducibly according to an irrep of $O_h$ (as well as isospin, charge conjugation etc.)
- First example: the glueball. An appropriate operator would be a gauge invariant function of the gluons alone: a closed loop trace.
- Link smearing greatly improved ground-state overlap.
- Apply smoothing filters to the links to extract just slowly varying modes that then have better overlap with the lowest states.

# Creation operators: glueballs

- What do operators that transform irreducibly under $O_h$ look like?

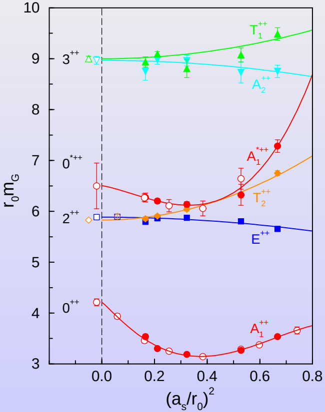


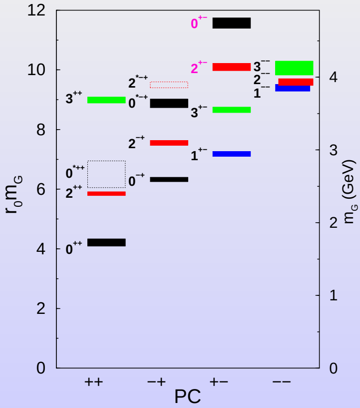

- Can make three operators by taking linear combinations of these loops.
- They form two irreducible representations ($A_1^g$ and $E_g$).

$$
\begin{array}{ccccccc}
\Phi_{A_1^g} & = & \Phi_1 & + & \Phi_2 & + & \Phi_3 \\
\hline
\Phi_{E^g}^{(1)} & = & \Phi_1 & - & \Phi_2 & & \\
\Phi_{E^g}^{(2)} & = & \frac{1}{\sqrt{3}}(\Phi_1 & + & \Phi_2 & - & 2\Phi_3)
\end{array}
$$

# Creation operators: glueballs

- After running simulations at more than one lattice spacing, a **continuum extrapolation** ($a \to 0$) can be attempted.
- The expansion of the action can suggest the appropriate choice of extrapolating function.

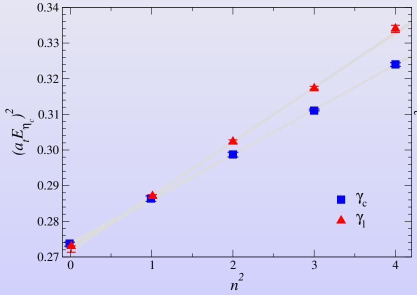


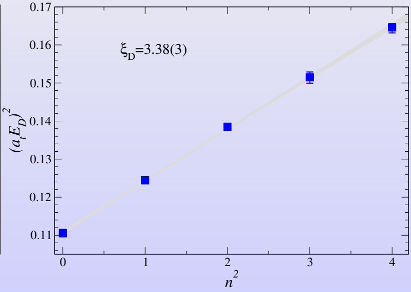



# Charmonium and $D/D_s$

# Dispersion relations - $\eta_c$ and D mesons

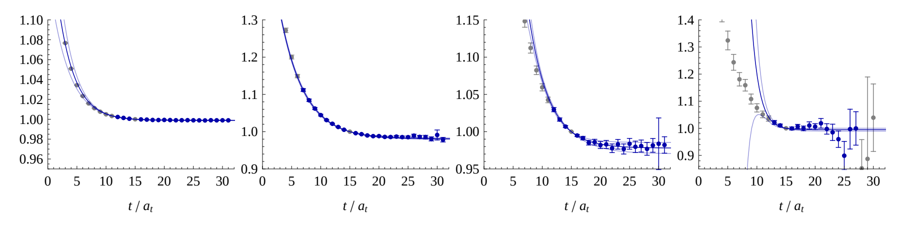
- Action parameters for charm quark tuned to ensure dispersion relation for $\eta_c$ is relativistic
- Using these tuned parameters, D meson also has relativistic dispersion relation



# Fits to $\lambda_k(t)$

- Variational basis, so can access excited states
- Fit $\lambda_k(t)$ to one or two exponentials
- Second exponential to stabilise some fits - value not used
- Plots show $\lambda_k(t) \times e^{E_k(t-t_0)}$



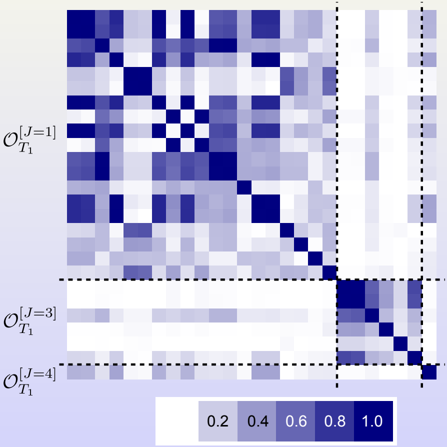

- Data from $T_1^{--}$ channel ($J = 1, 3, 4, \ldots$)

# Subduction of derivative-based operators

- $T_1^{--}$ variational basis
- 26 operators, up to $D_i D_j D_k$
- Correlation matrix at $t/a_t = 5$, normalised:

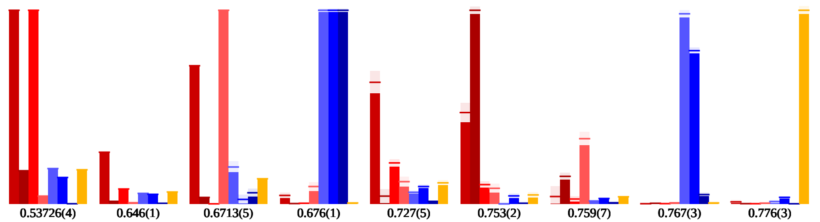
$$Q_{ij} = \frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}$$

- Reasonable **spin separation** seen



# Spin identification

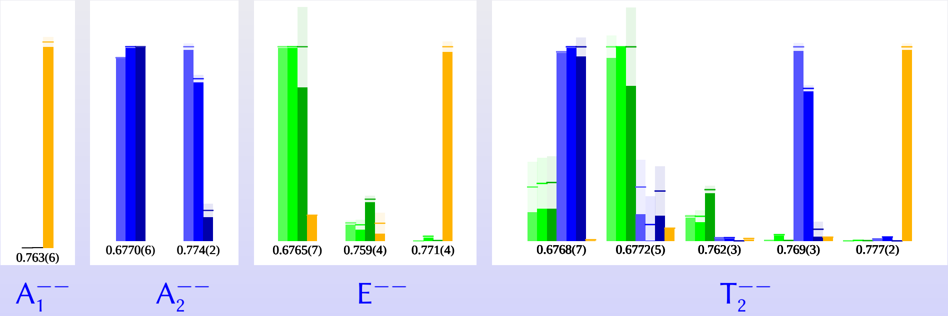
- Using $Z = \langle 0|\Phi|k\rangle$, helps to identify continuum spins
- For high spins, can look for agreement between irreps
- Data below for $T_1^{--}$ irrep, colour-coding is **Spin 1**, **Spin 3** and **Spin 4**.




- Can help identify glue-rich states, using operators with $[D_i, D_j]$

# …the rest of the spin-4 state

- All polarisations of the spin-4 state are seen
- Spin labelling: **Spin 2**, **Spin 3** and **Spin 4**.

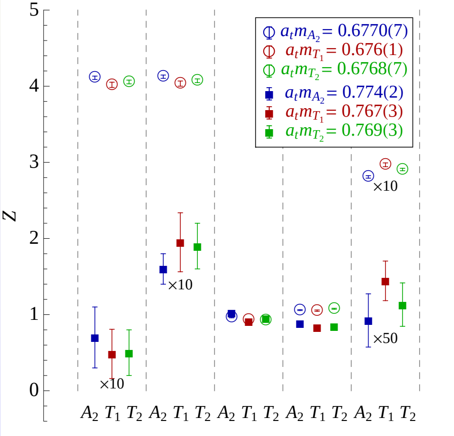


# Identifying spin - operator overlaps

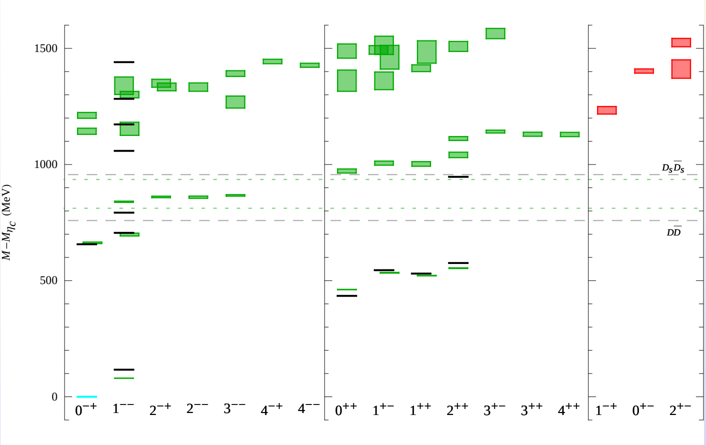
- Example — $3^{--}$ continuum
- Look for remnant of continuum symmetry:

$$\langle 0|\Phi^{[J=3]}_{A_2^{--}}|k\rangle = \langle 0|\Phi^{[J=3]}_{T_1^{--}}|k\rangle = \langle 0|\Phi^{[J=3]}_{T_2^{--}}|k\rangle$$

- Can identify two spin-3 states.



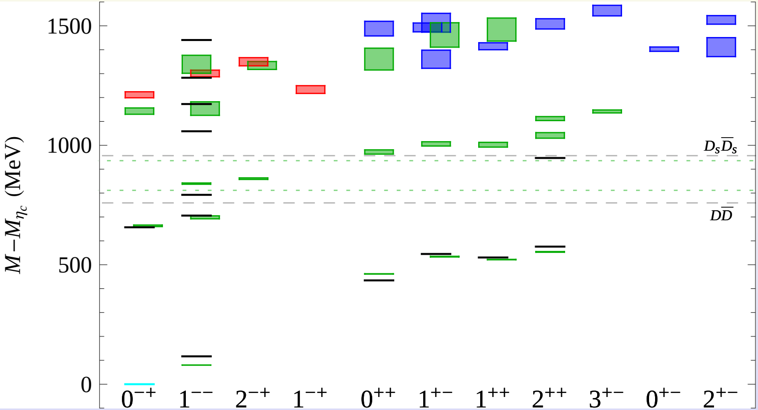
# Excitation spectrum of charmonium




- Quark model: 1S, 1P, 2S, 1D, 2P, 1F, 2D, . . . all seen.
- Not all fit quark model: spin-exotic (and non-exotic) hybrids seen

[Liu et.al. arXiv:1204.5425]

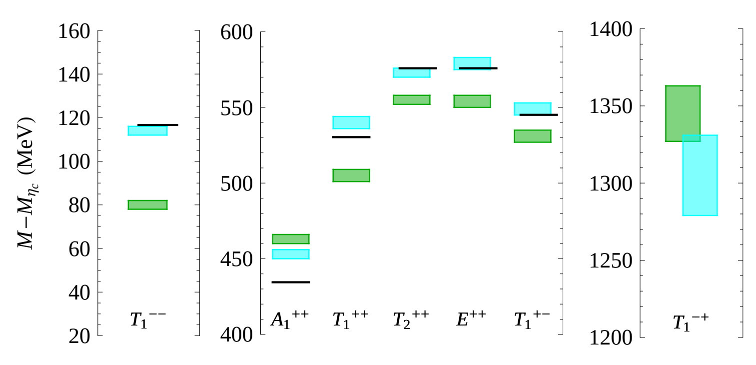
# Gluonic excitations in charmonium?




- See states created by operators that excite intrinsic gluons
- two- and three-derivatives create states in the open-charm region.

[Liu et.al. arXiv:1204.5425]

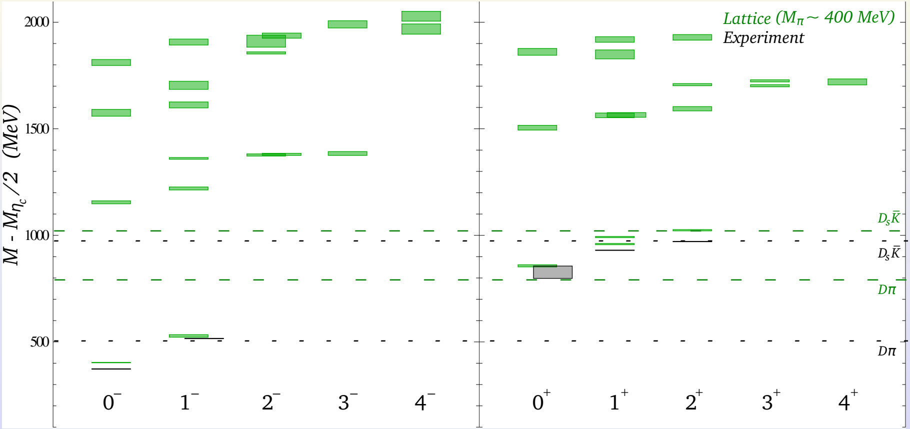
# Lattice artefacts in charmonium




- Hyperfine structure sensitive to lattice artefacts. Boost co-efficient of action term to suppress these.
- green $\rightarrow$ light blue. Shifts are $\approx 40$ MeV.

[Liu et.al. arXiv:1204.5425]

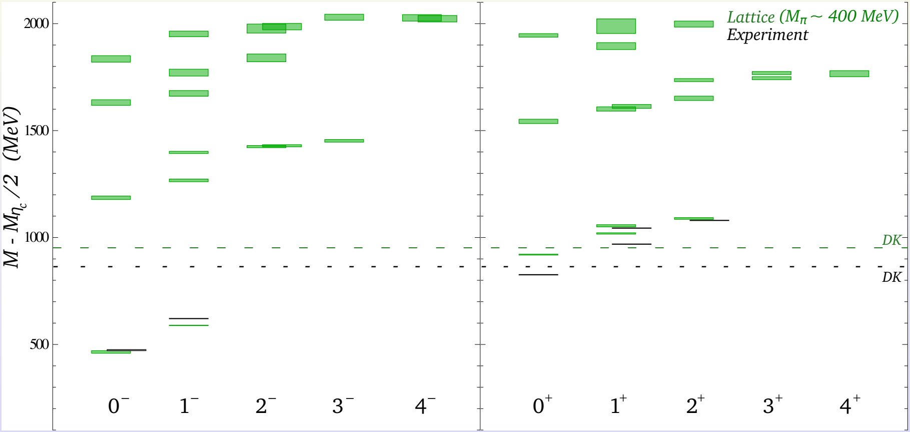
# Excitation spectrum of D




- Subtract $\frac{1}{2}M_{\eta_c}$ to reduce systematic error
- Thresholds for both physical $M_\pi$ and $M_\pi \approx 400$ MeV

[Moir et.al. JHEP 1305 (2013) 021]

# Excitation spectrum of $D_s$

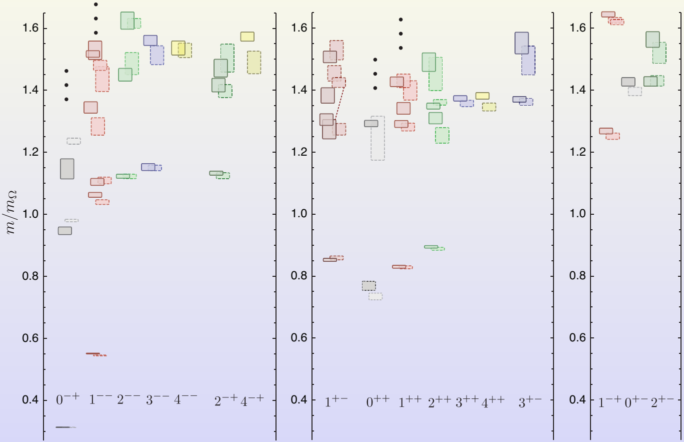



- Subtract $\frac{1}{2}M_{\eta_c}$ to reduce systematic error
- Thresholds for both physical $M_\pi$ and $M_\pi \approx 400$ MeV

[Moir et.al. JHEP 1305 (2013) 021]

# Light quark hadrons

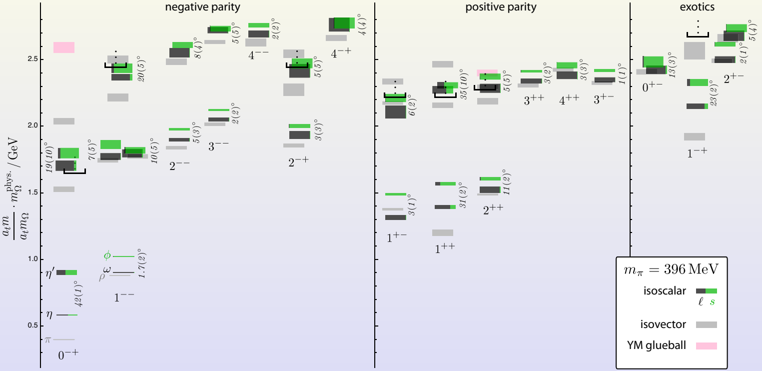
# Isovector meson spectroscopy




- $m_\pi = 400$ MeV
- No 2-meson operators
- Spin-exotic states seen
- Non-exotic hybrids too?

[Dudek et.al. Phys.Rev.D82:034508,2010]

# Isoscalar mesons



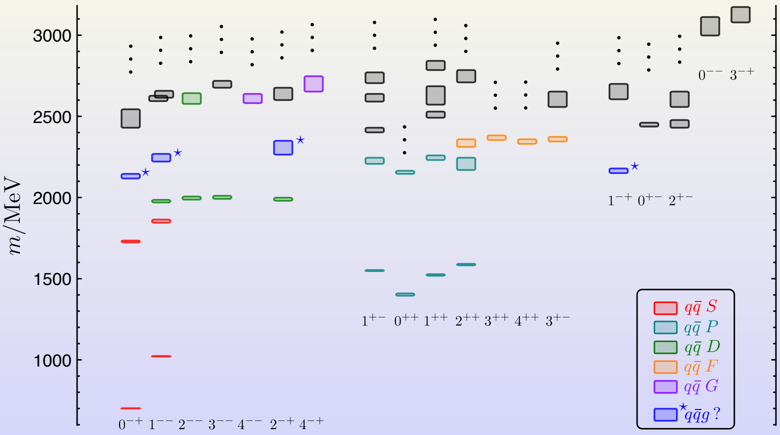

- $m_\pi = 400$ MeV, finite $a$
- No $0^{++}$ data presented
- No glueball or two-meson operators

Statistical precision:

| | |
|---|---|
| $\eta$ | 0.5 % |
| $\eta'$ | 1.9 % |

[Dudek et.al. Phys.Rev.D83:111502,2011]

# Hybrid excitations?

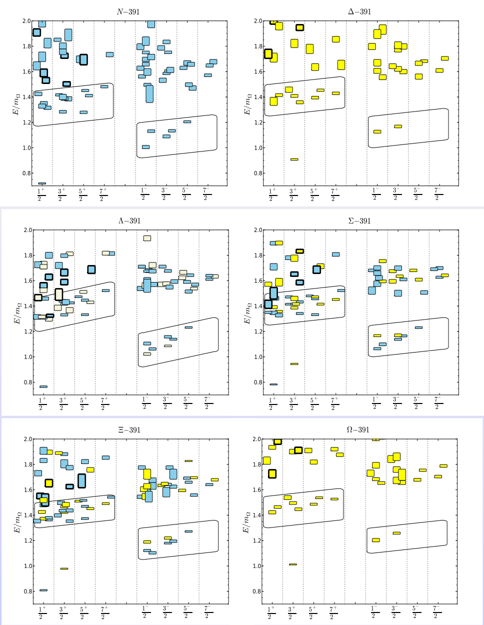



- $m_\pi = 700$ MeV
- Complete hybrid supermultiplet seen

[J.Dudek, Phys.Rev.D84 (2011) 074023]

# Light Baryon Spectra

- Baryon spectra using operators with $SU(3)_F$
- $m_\pi \approx 400$ MeV
- Blue - flavour octet
- Yellow - flavour decuplet
- White - flavour singlet
- Thick boxes - hybrid content

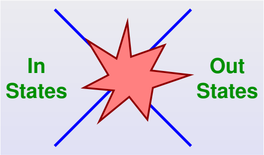


# Scattering

# Scattering

Scattering matrix elements not directly accessible from Euclidean QFT [Maiani-Testa theorem]

- Scattering matrix elements: asymptotic $|\text{in}\rangle, |\text{out}\rangle$ states. $\langle \text{out} \,|e^{i\hat{H}t}|\, \text{in}\rangle \to \langle \text{out} \,|e^{-\hat{H}t}|\, \text{in}\rangle$
- Euclidean metric: project onto ground-state




- **Lüscher's formalism:** information on elastic scattering inferred from **volume dependence** of spectrum
- Requires precise data, resolution of two-hadron and excited states.

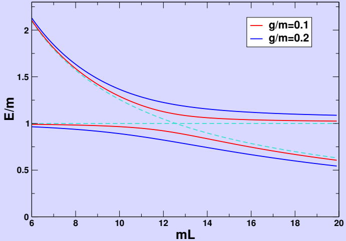
# Hadrons in a finite box: scattering

- On a finite lattice with periodic b.c., hadrons have quantised momenta; $\underline{p} = \frac{2\pi}{L}\left\{n_x, n_y, n_z\right\}$
- Two hadrons with total $P = 0$ have a discrete spectrum
- These states can have same quantum numbers as those created by $\bar{q}\Gamma q$ operators and QCD can mix these

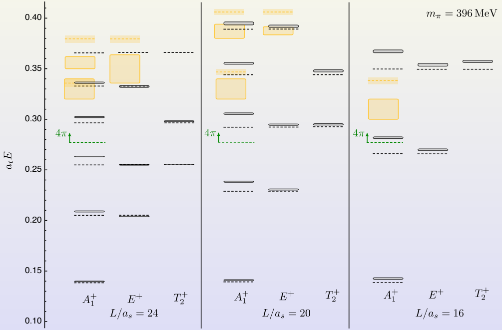
- This leads to shifts in the spectrum in finite volume
- This is the same physics that makes resonances in an experiment
- Lüscher's method - relate elastic scattering to energy shifts

**Toy model**

$$H = \begin{pmatrix} m & g \\ g & \frac{4\pi}{L} \end{pmatrix}$$



# $I = 2$ $\pi - \pi$ phase shift

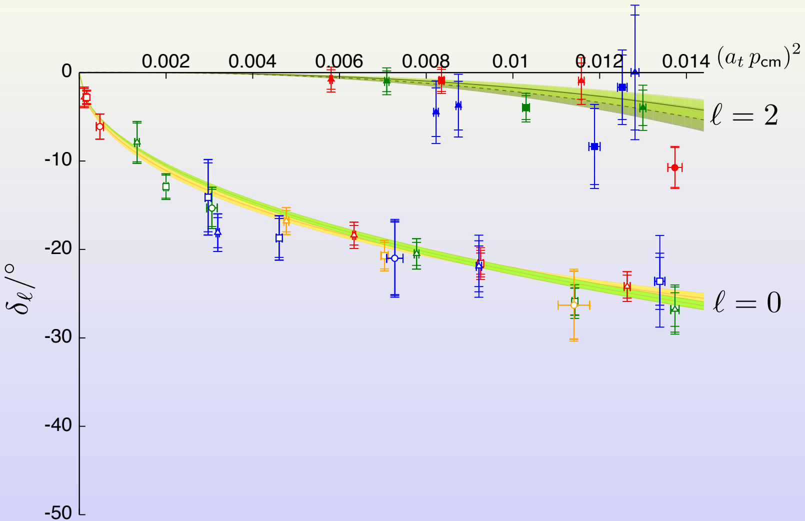



- Lüscher's method: first determine energy shifts as volume changes
- Data for $L = 16a_s, 20a_s, 24a_s$
- Small energy shifts are resolved

- Measured $\delta_0$ and $\delta_2$ ($\delta_4$ is very small)
- $I = 2$ a useful first test - simplest Wick contractions

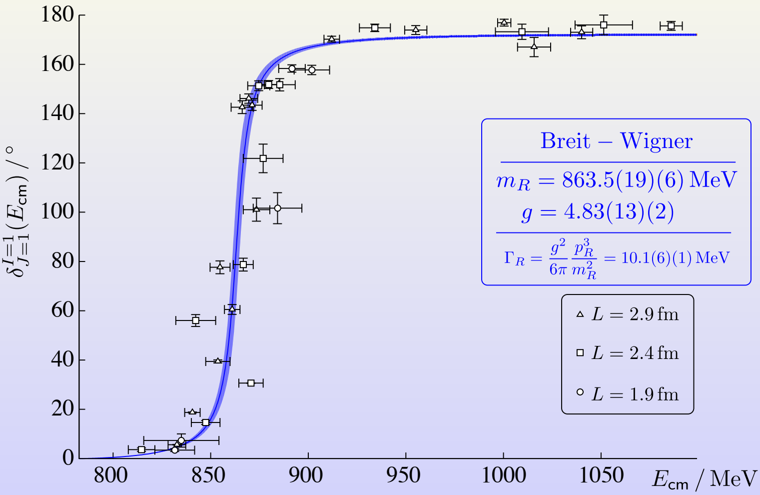
Dudek et.al. [Phys.Rev.D83:071504,2011, arXiv:1203.6041]

# I = 2 $\pi - \pi$ phase shift




Dudek et.al. [Phys.Rev.D83:071504,2011, arXiv:1203.6041]

# $I = 1$ $\pi - \pi$ phase shift (near the $\rho$)




Dudek et.al. [arXiv:1212.0830]

# Summary

- The lattice provides a non-perturbative regularisation of the QCD path integral.
- Modern simulations include light quark dynamics close to the physical up and down mass.
- Excitation spectra can be computed using variational methods
- Scattering can not be directly computed - but can be inferred (below inelastic thresholds) using Lüscher's method.
- Inelastic scattering is a new field...
- Also: Matrix elements, distribution functions, production rates, ...