BPM for Co-propagating Beams

Igor Pinayev, BNL, Upton, NY 11973, USA

Motivation

The energy recovery linacs (ERL) can be used as driver for the light sources with ultra-low emittance. The ERLs have co-propagating beams inside the same vacuum vessel. These beams can have different trajectories, which should be distinguished by beam position monitors (BPM).

Proposed Method

For ERL the time delay between accelerated and decelerated bunches is fixed by design and it becomes possible to employ the phase of the pick-up (PUE) signal to extract information on the position of each beam.

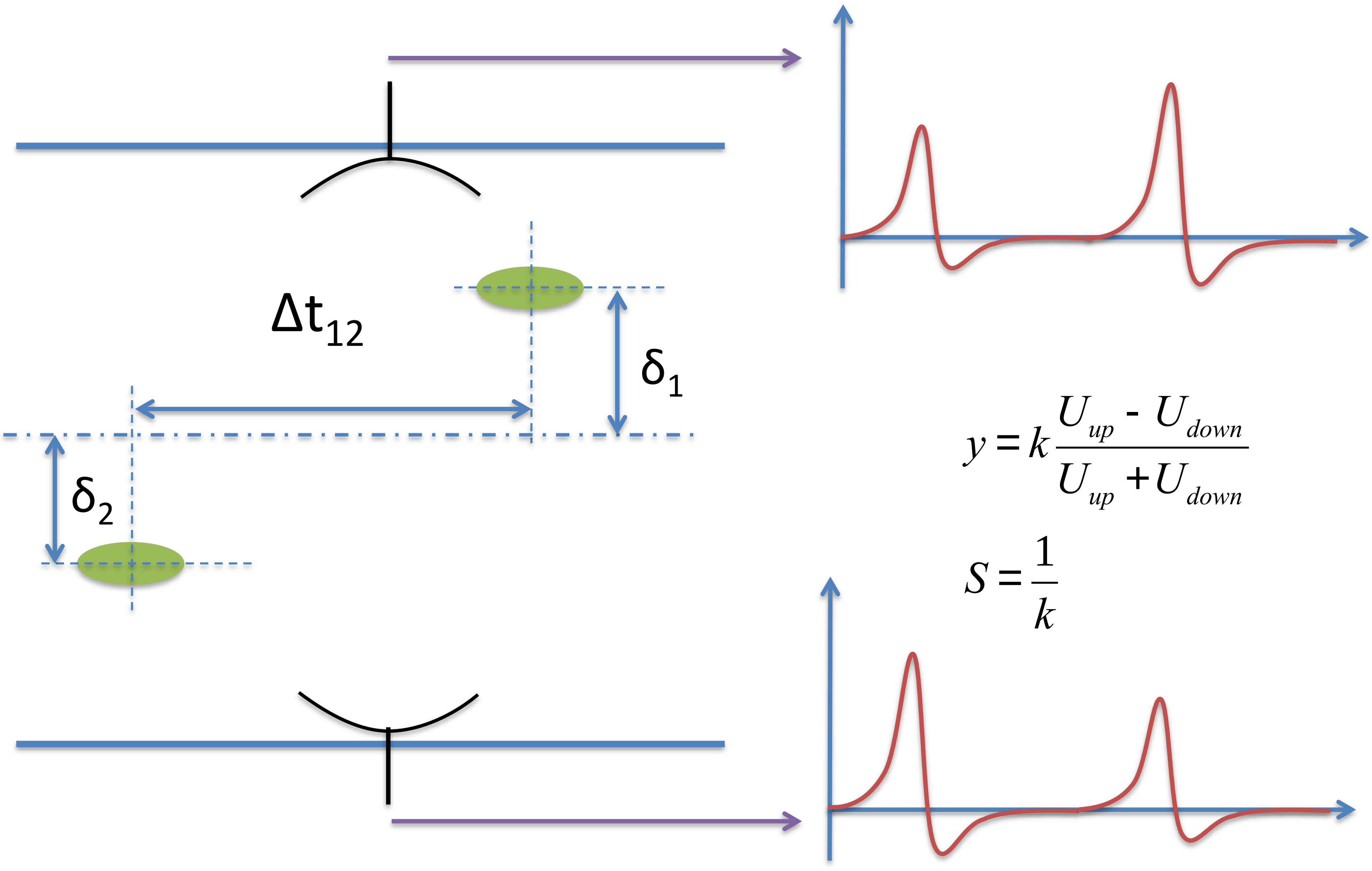
If bunches, separated by a flyby time Δt_{12}, have different positions then each PUE sees different longitudinal "center of gravity" and a phase shift between two signals appears.

For a processing unit, utilizing signal processing at frequency ω, and small displacements of the first and the second bunches δ_1 and δ_2 ($S\delta_1, S\delta_2 \ll 1$, where S=1/k is a sensitivity coefficient) we can write the linearized equations:

$$U_{up} = U_1(1 + S\delta_1)\sin\omega(t + \Delta t_{12}/2) + U_2(1 + S\delta_2)\sin\omega(t - \Delta t_{12}/2)$$

$$U_{down} = U_1(1 - S\delta_1)\sin\omega(t + \Delta t_{12}/2) + U_2(1 - S\delta_2)\sin\omega(t - \Delta t_{12}/2)$$

Two pick-up electrodes



$$y = k\frac{U_{up} - U_{down}}{U_{up} + U_{down}}$$

$$S = \frac{1}{k}$$

When both bunches have equal charges we can re-write as

$$U_{up} = U_0 \cos\frac{\omega\Delta\tau_{12}}{2}\left(2 + S\left(\delta_1 + \delta_2\right)\right)\sin\omega t + U_0 \sin\frac{\omega\Delta\tau_{12}}{2} S\left(\delta_1 - \delta_2\right)\cos\omega t$$

$$U_{down} = U_0 \cos\frac{\omega\Delta\tau_{12}}{2}\left(2 - S\left(\delta_1 + \delta_2\right)\right)\sin\omega t - U_0 \sin\frac{\omega\Delta\tau_{12}}{2} S\left(\delta_1 - \delta_2\right)\cos\omega t$$

For the small displacement we can neglect the second order terms

$$U_{up} \approx 2U_0 \cos\left(\omega\Delta\tau_{12}/2\right)\left[1 + \frac{\delta_1 + \delta_2}{2}\right]$$

$$U_{down} \approx 2U_0 \cos\left(\omega\Delta\tau_{12}/2\right)\left[1 - \frac{\delta_1 + \delta_2}{2}\right]$$

$$\hat{y} = k\frac{U_{up} - U_{down}}{U_{up} + U_{down}} = \frac{\delta_1 + \delta_2}{2}$$

Amplitude processing gives average position of the beams.

$$\varphi_{up} \approx \frac{S\left(\delta_1 - \delta_2\right)}{2}\tan\left(\omega\Delta\tau_{12}/2\right)$$

$$\varphi_{down} \approx -\frac{S\left(\delta_1 - \delta_2\right)}{2}\tan\left(\omega\Delta\tau_{12}/2\right)$$

$$\Delta\varphi = \varphi_{up} - \varphi_{down} = S\left(\delta_1 - \delta_2\right)\tan\left(\omega\Delta\tau_{12}/2\right)$$

Phase shift gives information on the difference in the orbits (with the same sensitivity as delta over sum).

Four Pick-up Electrodes

$$x_{ave} = k_x \frac{U_B - U_A + U_C - U_D}{U_A + U_B + U_C + U_D}$$

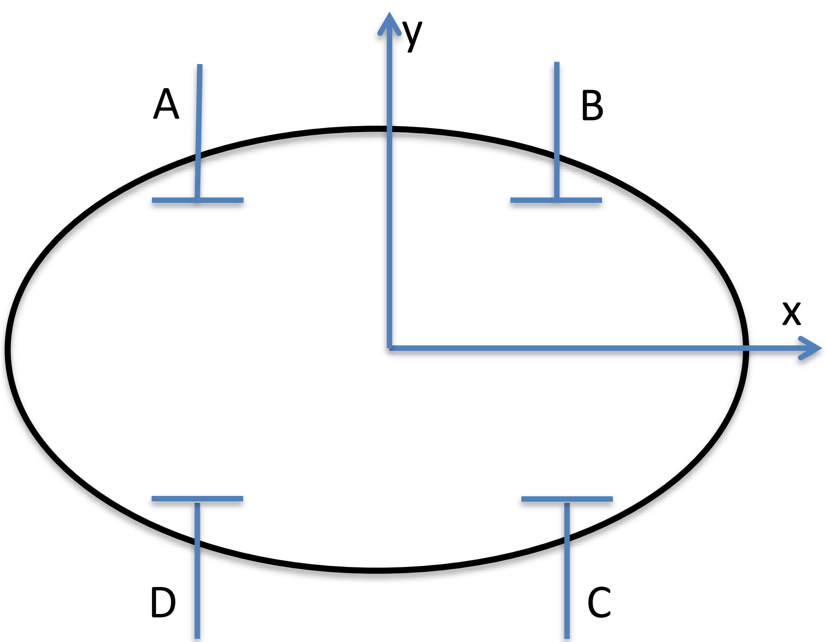
$$y_{ave} = k_y \frac{U_A - U_C + U_B - U_D}{U_A + U_B + U_C + U_D}$$

$$V_A = V_0\left(1 - S_x\delta_{x1} + S_y\delta_{y1}\right)\sin\omega\left(t + \Delta t_{12}/2\right) + V_0\left(1 - S_x\delta_{x2} + S_y\delta_{y2}\right)\sin\omega\left(t - \Delta t_{12}/2\right)$$

$$V_B = V_0\left(1 + S_x\delta_{x1} + S_y\delta_{y1}\right)\sin\omega\left(t + \Delta t_{12}/2\right) + V_0\left(1 + S_x\delta_{x2} + S_y\delta_{y2}\right)\sin\omega\left(t - \Delta t_{12}/2\right)$$

$$V_C = V_0\left(1 + S_x\delta_{x1} - S_y\delta_{y1}\right)\sin\omega\left(t + \Delta t_{12}/2\right) + V_0\left(1 + S_x\delta_{x2} - S_y\delta_{y2}\right)\sin\omega\left(t - \Delta t_{12}/2\right)$$

$$V_D = V_0\left(1 - S_x\delta_{x1} - S_y\delta_{y1}\right)\sin\omega\left(t + \Delta t_{12}/2\right) + V_0\left(1 - S_x\delta_{x2} - S_y\delta_{y2}\right)\sin\omega\left(t - \Delta t_{12}/2\right)$$



$$x_{diff} = \frac{k_x}{\tan\left(\omega\Delta t_{12}/2\right)}\left[\left(\varphi_B - \varphi_D\right) - \left(\varphi_A - \varphi_C\right)\right]$$

$$y_{diff} = \frac{k_y}{\tan\left(\omega\Delta t_{12}/2\right)}\left[\left(\varphi_A - \varphi_C\right) + \left(\varphi_B - \varphi_D\right)\right]$$

Alternative Approach for 4 PUEs

Use auxiliary variables

$$u = \frac{U_A - U_C}{U_A + U_C}$$

$$v = \frac{U_B - U_D}{U_B + U_D}$$

Since $U_A+U_C \approx U_B+U_D$

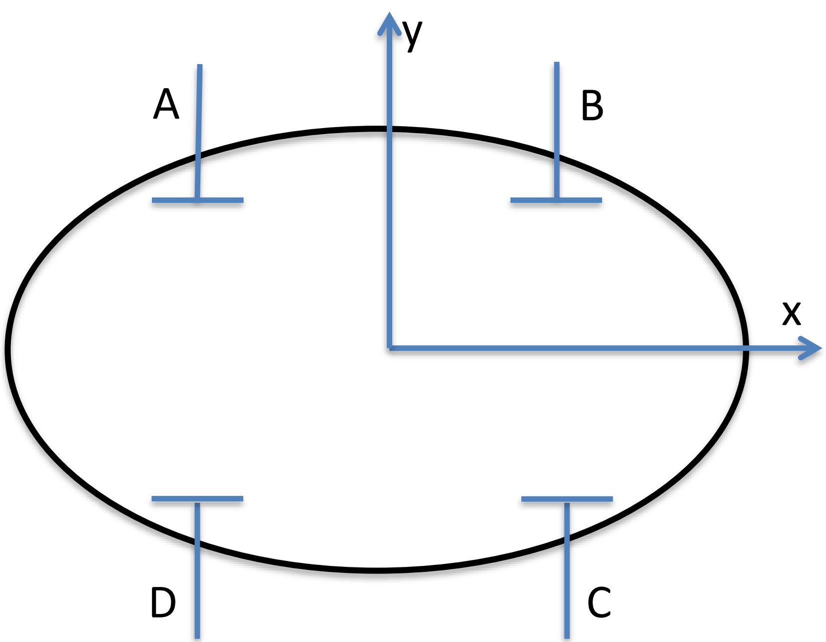
$$x_{ave} \approx \frac{k_x}{2}(v - u)$$

$$y_{ave} \approx \frac{k_y}{2}(u + v)$$

Each variable can be processed in the similar manner as for two pick-up electrodes and *X* and *Y* can be calculated after.

$$x_{diff} = \frac{k_x}{\tan(\omega\Delta t_{12}/2)}\left[(\varphi_B - \varphi_D) - (\varphi_A - \varphi_C)\right]$$

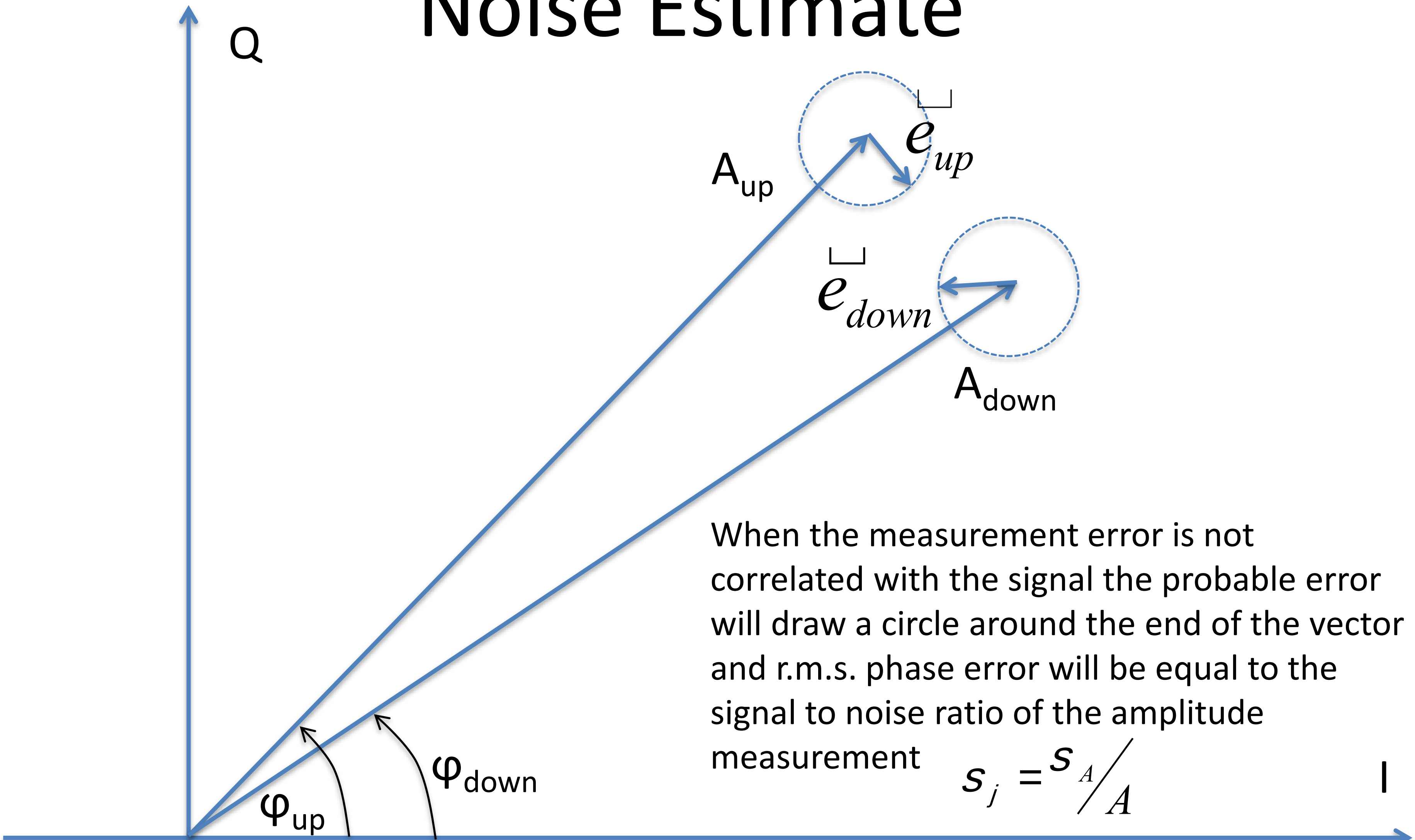
$$y_{diff} = \frac{k_y}{\tan(\omega\Delta t_{12}/2)}\left[(\varphi_A - \varphi_C) + (\varphi_B - \varphi_D)\right]$$



Two channel processing can be beneficial for conventional BPM electronics:

1) Match two pairs of channels is easier than four channels
2) Switching channels can be done with at least twice higher rate. Phase shift due to the notch filter will affect orbit feedback system less.

Noise Estimate



When the measurement error is not correlated with the signal the probable error will draw a circle around the end of the vector and r.m.s. phase error will be equal to the signal to noise ratio of the amplitude measurement $\sigma_{\varphi} = \sigma_A / A$

M. Znidarcic et al., "Testing of New Hadron Beam Phase and Position Monitor at CIEMAT Laboratory," DIPAC'11, Hamburg, MOPD26, p. 104. For the described system r.m.s. position noise is 3×10^{-4} and phase noise is 1.75×10^{-4} radians

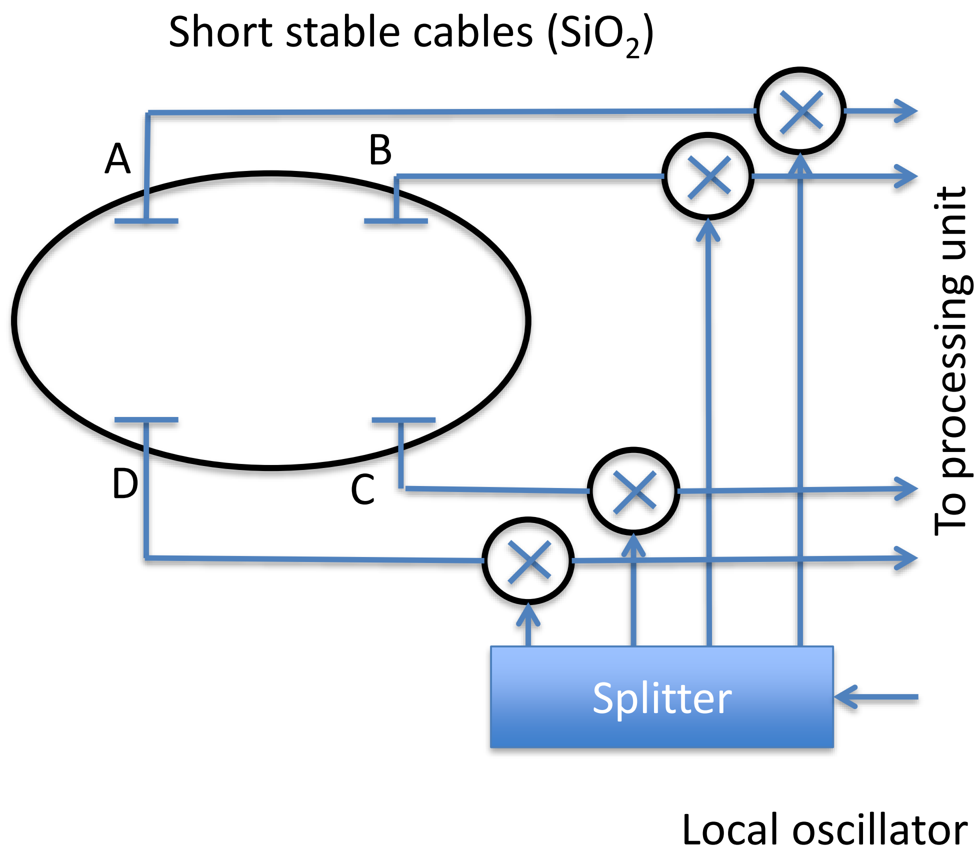
Signal Processing Frequency

The processing frequency is usually equal to the RF frequency. Such a choice allows processing of any fill pattern because all bunches are separated by a multiple of the RF period. In the ERL distance between bunches can be different. It may be as small as half of the RF wavelength because the decelerated bunch should be in the opposite phase vs. accelerated one. Moreover, this distance can differ from the multiple of the half periods due to the shifts in the merge lines. Also, there is no revolution period in the ERL but only a round-trip time. Hence, choice of the processing frequency is somewhat arbitrary and can be used for optimization of the system. There are frequencies that should be avoided: if $\cos\omega\Delta t_{12}=0$ then there is no signal for the average position calculation and with $\sin\omega\Delta t_{12}=0$ phases will be constant and difference in orbits can not be found. Therefore, the processing frequency should be in between close to $\omega = \frac{\pi}{\Delta t_{12}}\left(\frac{1}{4}+N\right)$, where N is an integer.

The system transfer function will not be identical for all channels and a phase shift corresponding to the identical beam positions should be established. It can be done either using an RF generator with splitter or using only bunches being accelerated dumping them at high energy.

Suppression of the Cables Drifts

The temperature changes and other slow processes can change physical length of the cable or phase velocity of the signal in the cable. To suppress the drifts one can lower the processing frequency to the range where the sensitivity to the variations of delays is less. Unfortunately, it also lowers the sensitivity to the position difference due to the tan$\omega\Delta t_{12}/2$ term. To have both high sensitivity to the differential beam position (and low noise) and to suppress influence of drifts we can convert down the PUE signals.



Conclusion

- The proposed usage of the phase information can help to distinguish two beams in the common vacuum vessel
- The coefficients and signal-to-noise ratio are the same as for delta over sum processing
- Choice of the processing frequency is not obvious and can be machine specific (or even path specific)
- Switching of two pairs instead of four channels can simplify regular BPM receiver design and improve quality of the processed signal