

# *Space-time picture of ultrarelativistic nuclear collisions*

## **I. Introduction**

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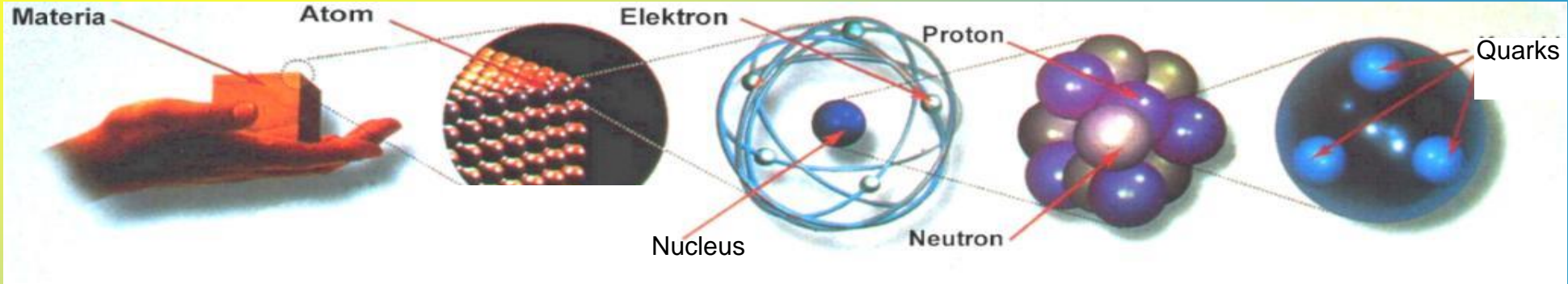
Winter School “Standard Model, QCD and Relativistic Heavy Ion Collisions”  
”

Skieampen Norway 2-12 January 2018

# **Part 1**

## **Elementary Introduction**

# The structure of the matter and spatial scales



$\sim 10^{-1}$  m

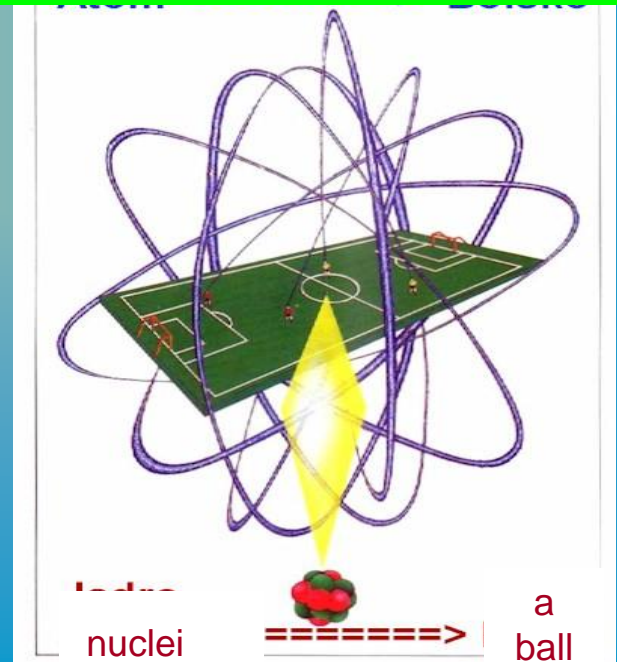
$\sim 10^{-9}$  m (Nanophysics)

$\sim 10^{-15}$  m = 1 fm = 1 Fm

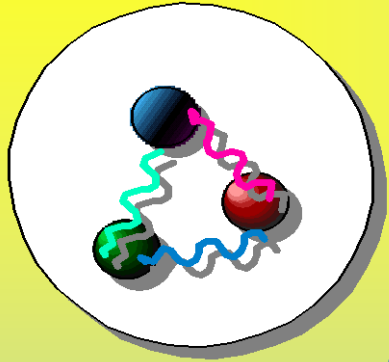
Seed  $\implies$  the Earth

Atom  $\implies$  the seed

Femtophysics



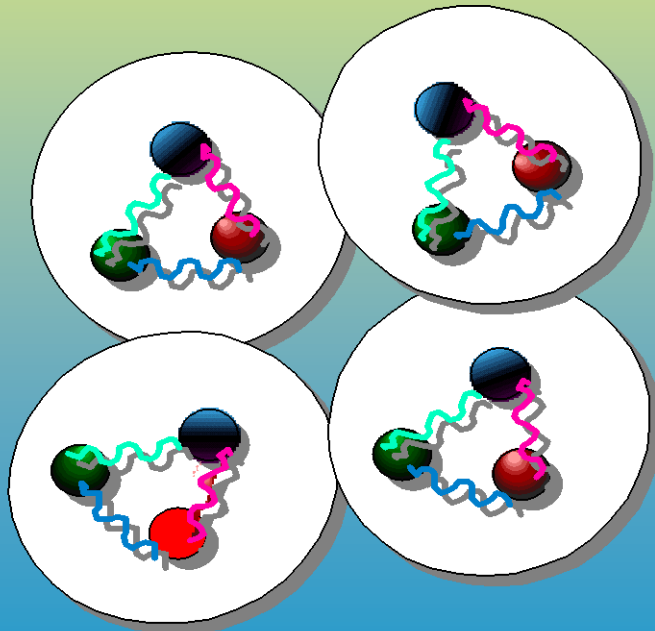
# Nucleon (baryon)



«Confinement» of quarks in hadrons

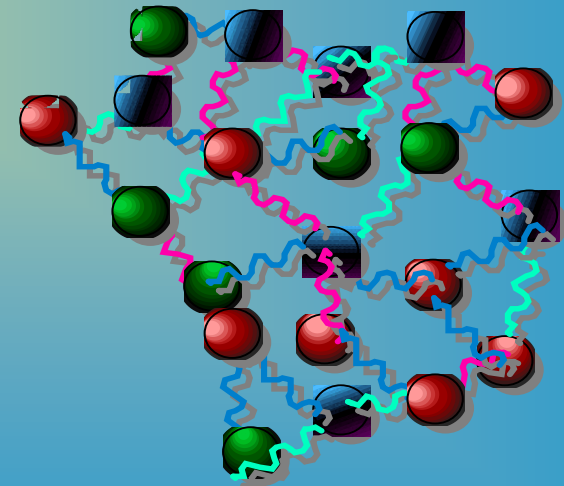
confinement

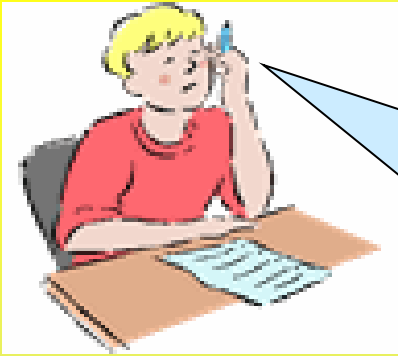
Hadronic matter



deconfinement

Quark-gluon matter (QGP)

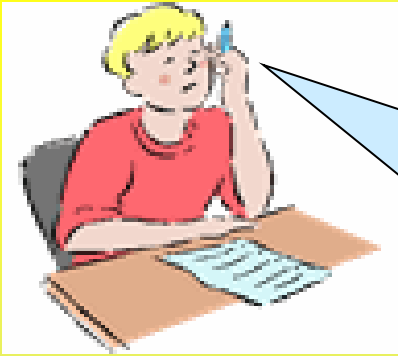




Why does the confinement happen?...  
What is the difference between QCD and QED?

Electrons  $\rightarrow$  quarks,  
Photons  $\rightarrow$  gluons.  
But gluons carries  
color charge.  
This is the essence!



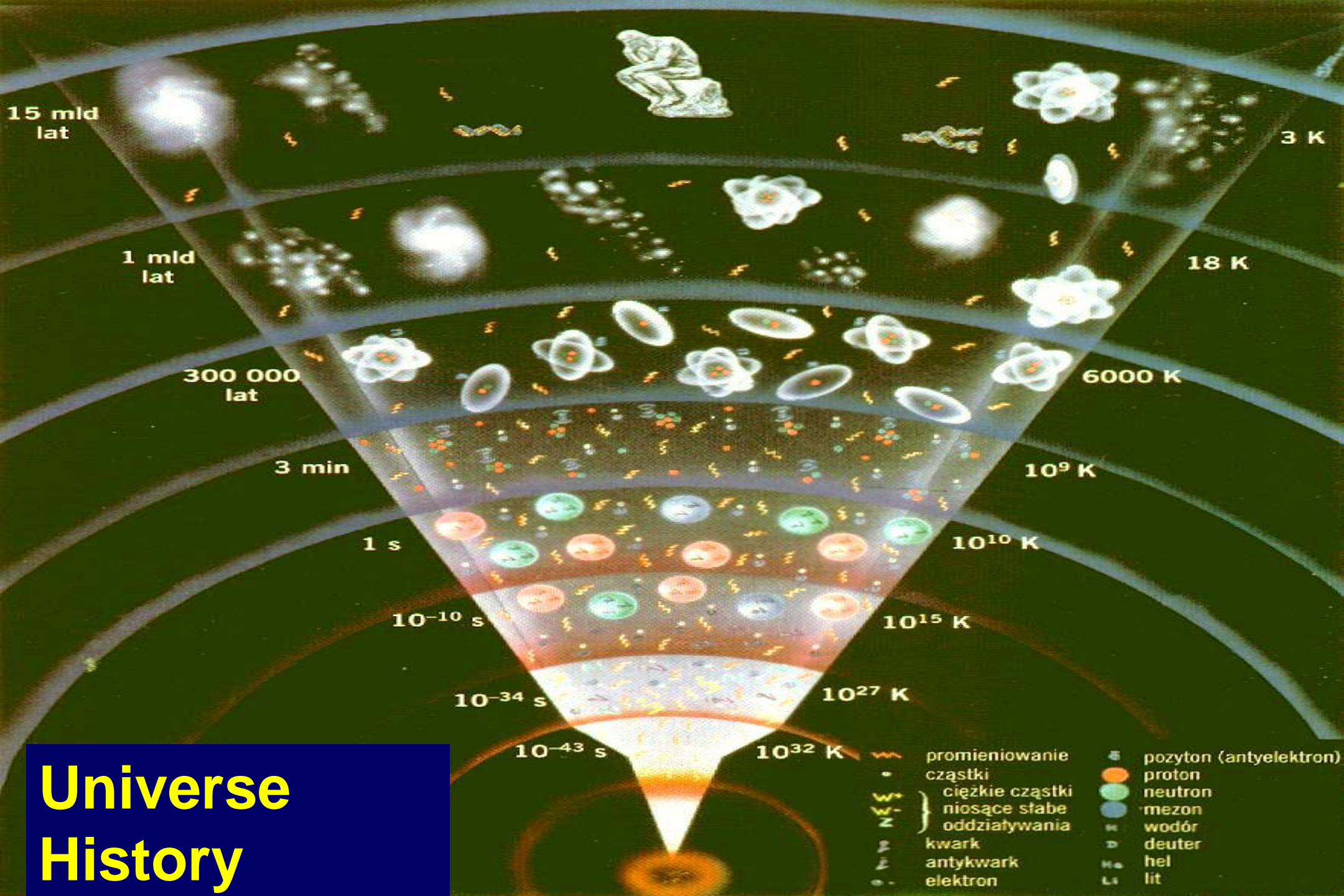


Is it possible to observe free quarks?


Such  
happened in the Early  
Universe!



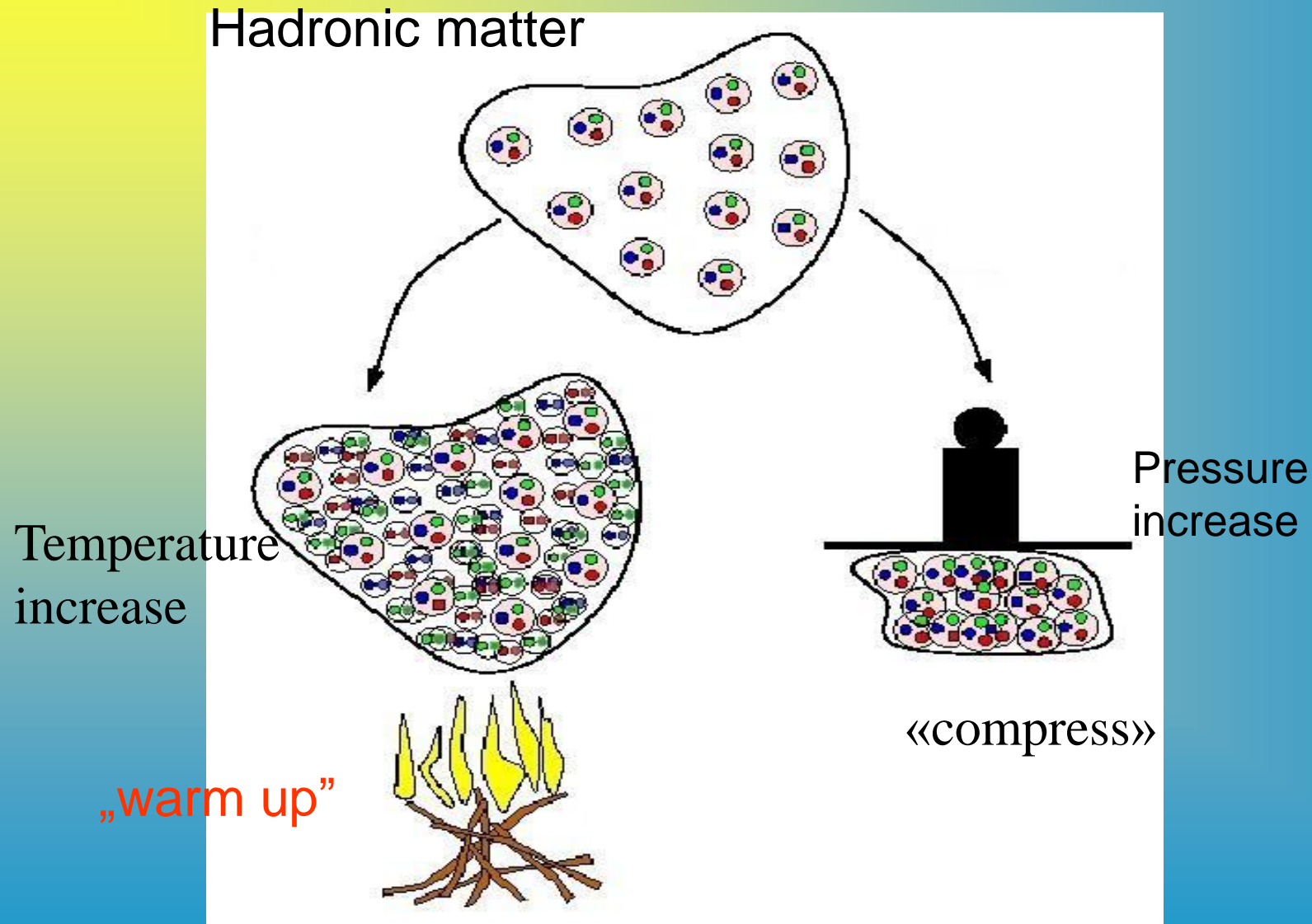




# Universe History

- |                                                                                       |                             |                                                                                       |                        |
|---------------------------------------------------------------------------------------|-----------------------------|---------------------------------------------------------------------------------------|------------------------|
|  | promieniowanie              |  | pozyton (antyelektron) |
|  | cząstki                     |  | proton                 |
|  | ciężkie cząstki             |  | neutron                |
|  | niosące słabe oddziaływania |  | mezon                  |
|  | kwark                       |  | wodór                  |
|  | antykwar                    |  | deuter                 |
|  | elektron                    |  | hel                    |
|                                                                                       |                             |  | lit                    |

# Is it possible to get Quark-Gluon Plasma in experiment?

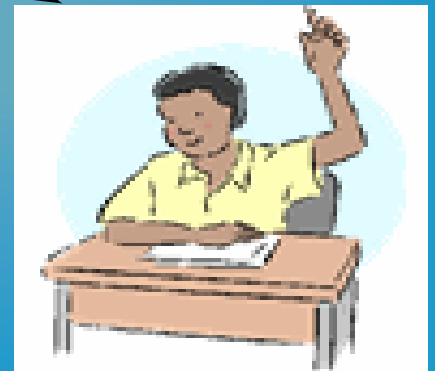


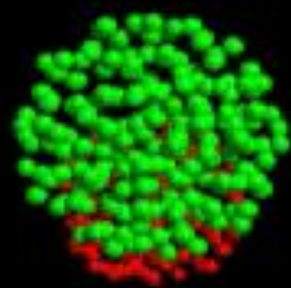




How does it possible to “compress” and “warm up” hadrons: protons and neutrons”?

Really, how do create the pressure higher than in the neutron stars, and temperature in billion times higher than inside the Sun?



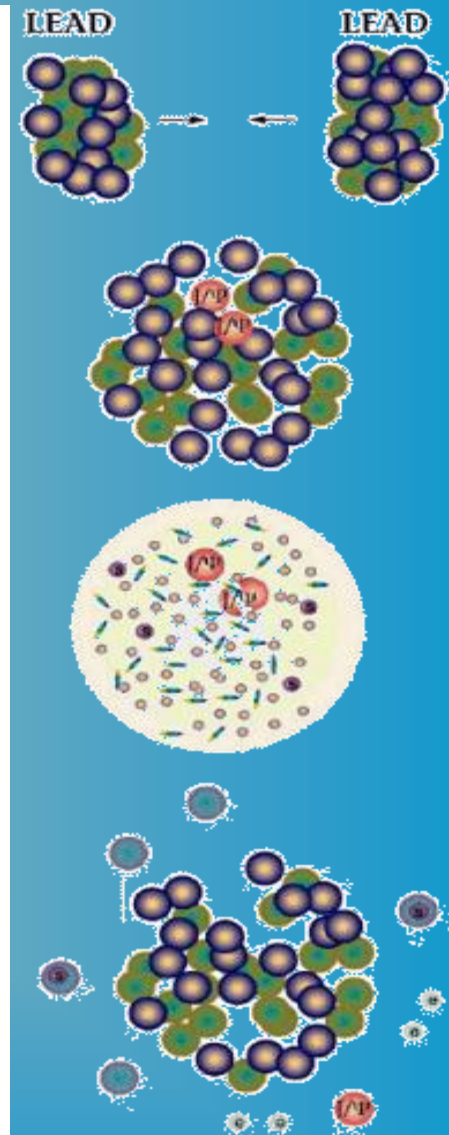
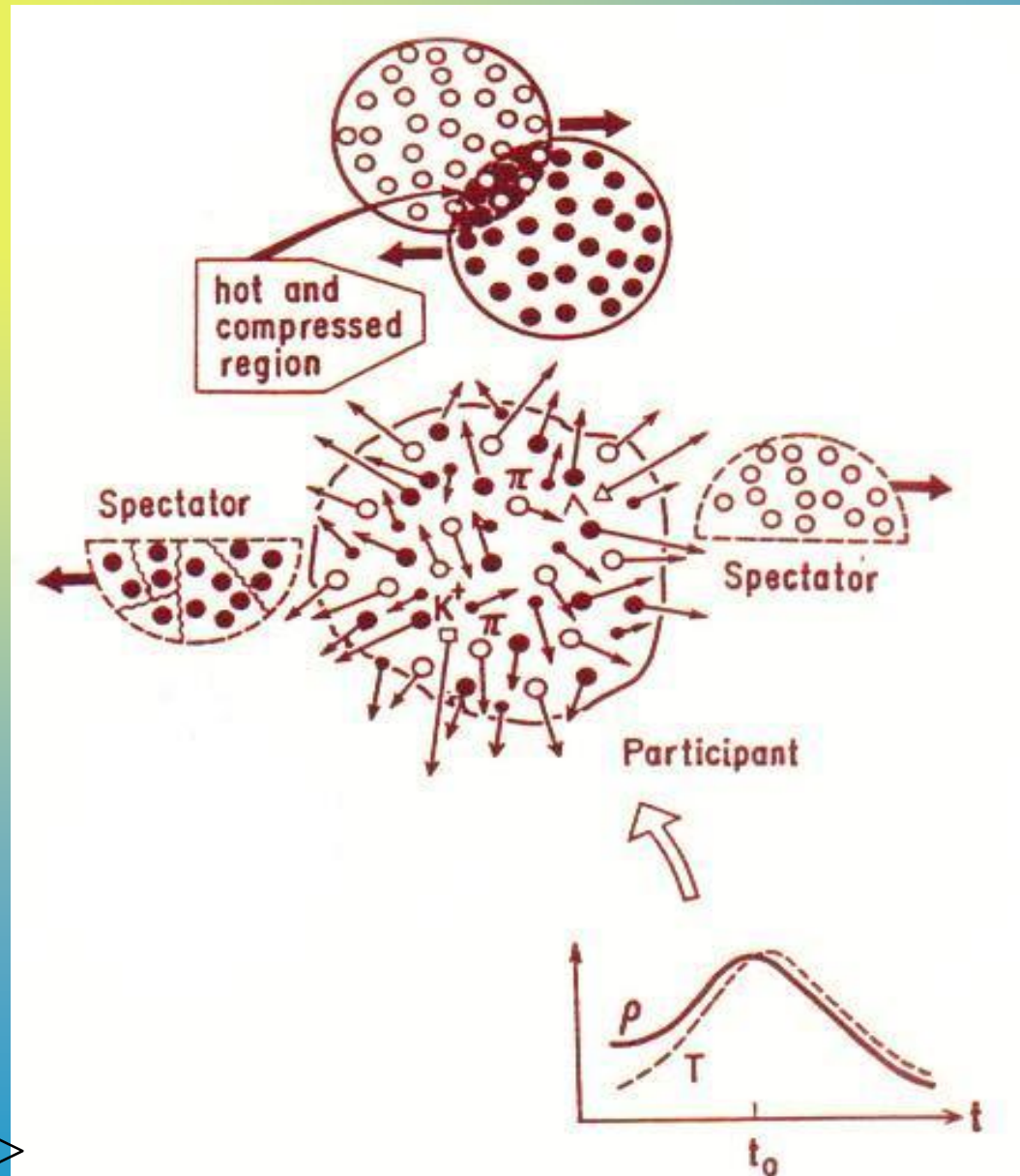


# Relativistic heavy ion collisions

At the start =====>

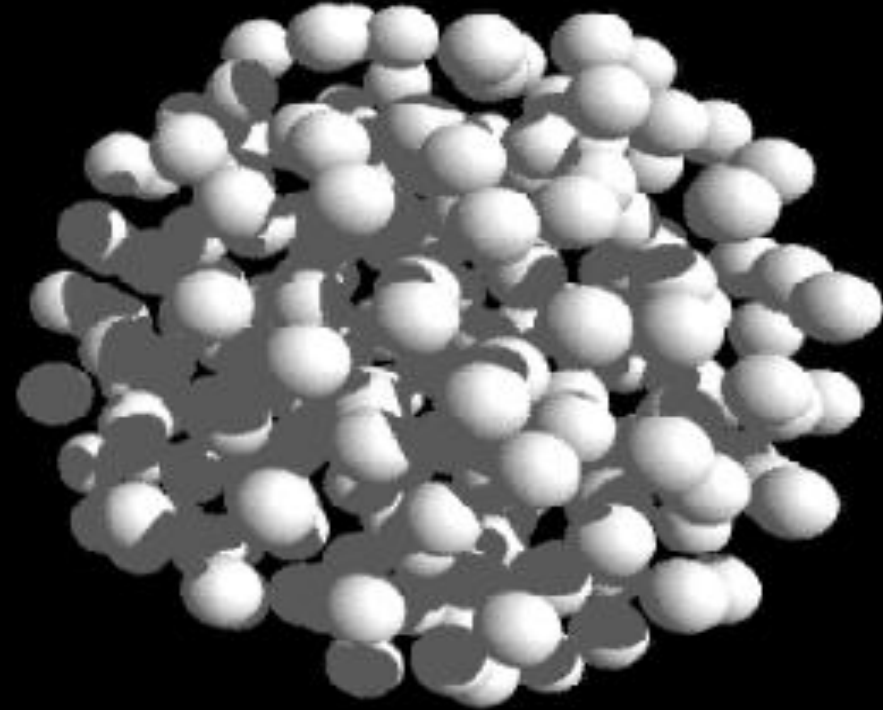
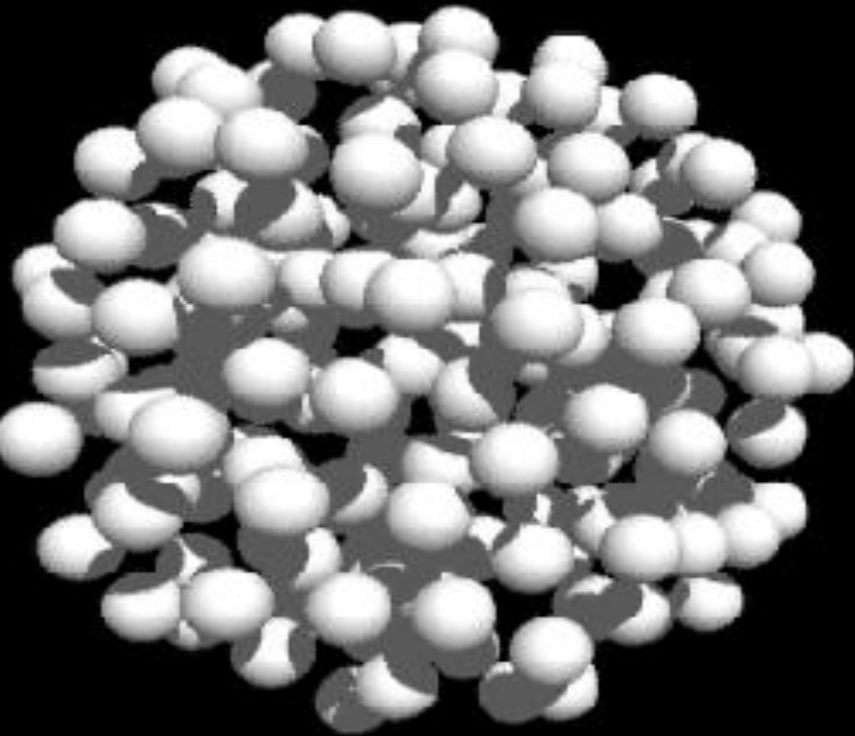
At the end =====>

Temporal dependence  
of the pressure and  
temperature =====>



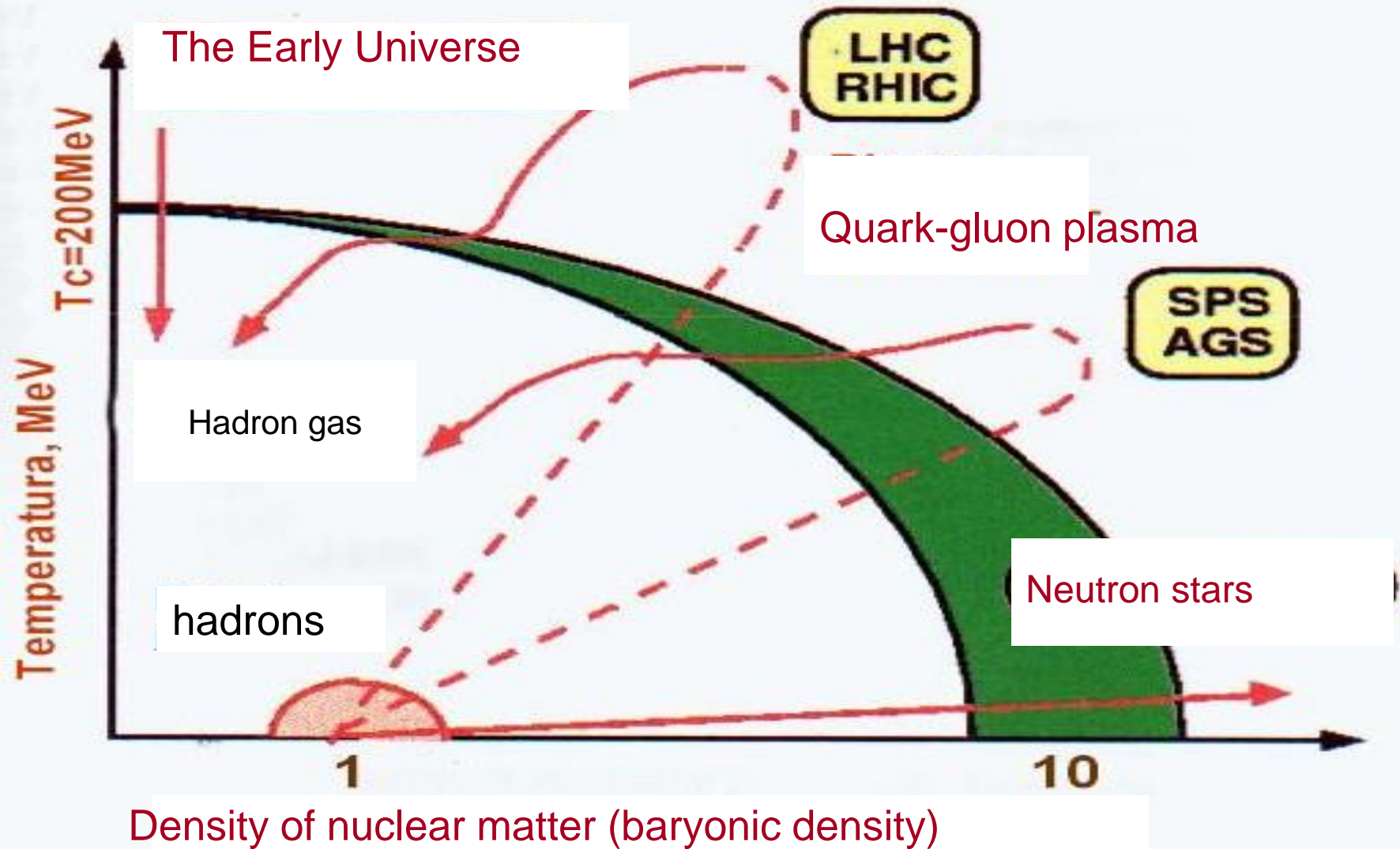
60 GeV/A

$t = -0.22$  fm/c



Computer simulation of A+A collisions

# How to study QGP ?







How does it become possible to create the Universe using a few hundreds colliding protons ?

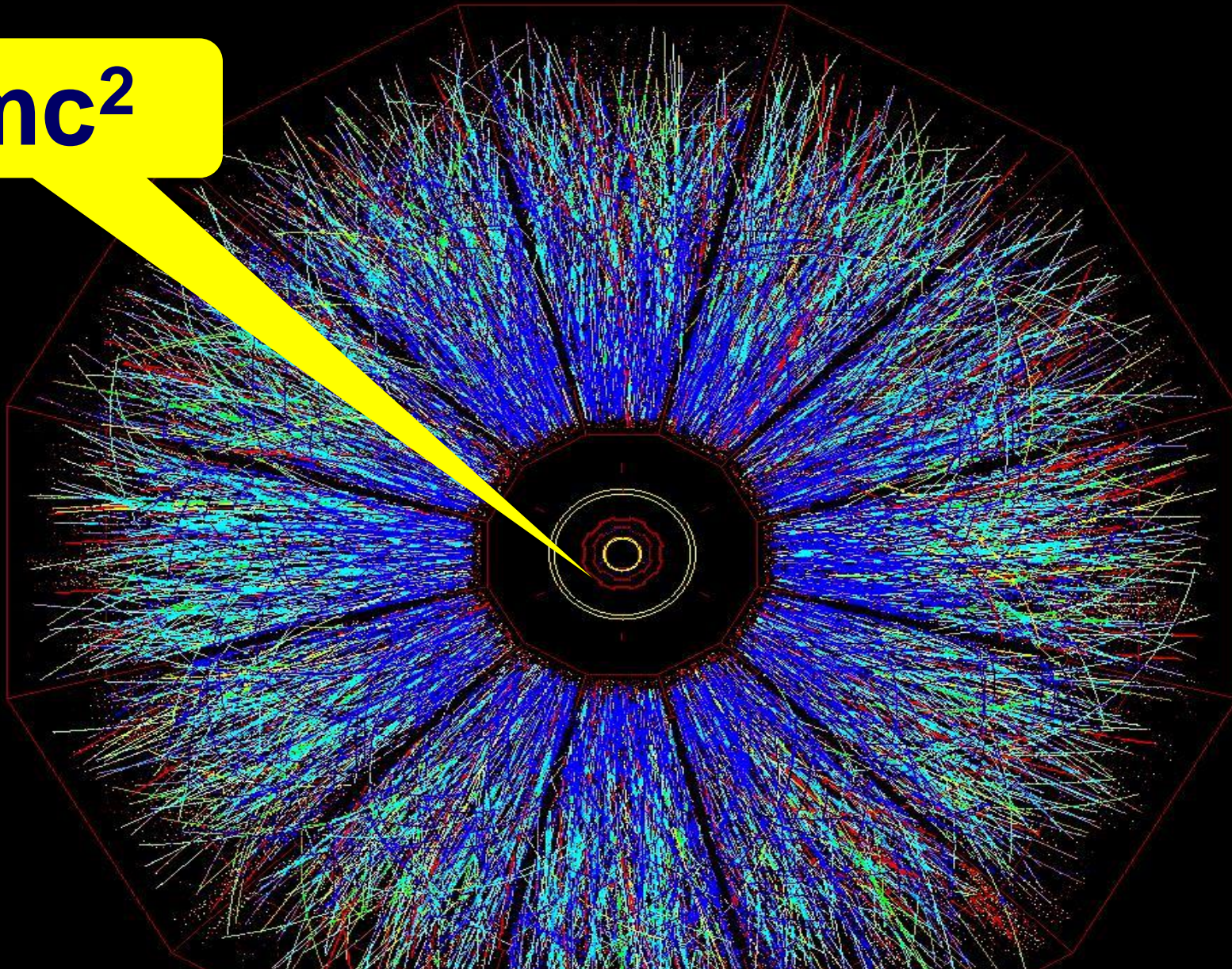
$$E=mc^2 !$$

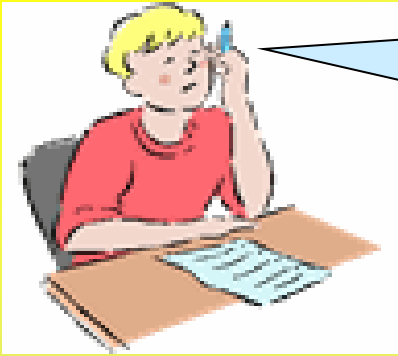


The result of collision ( $^{197}\text{Au} + ^{197}\text{Au}$ ) at the CMS energy : 200 GeV per nucleon pair , experiment BNL STAR

400  $\rightarrow$  4000

$$E=mc^2$$





How does it work in practice?

The two things are necessary:  
1. Accelerator (Collider)  
2. Detector.

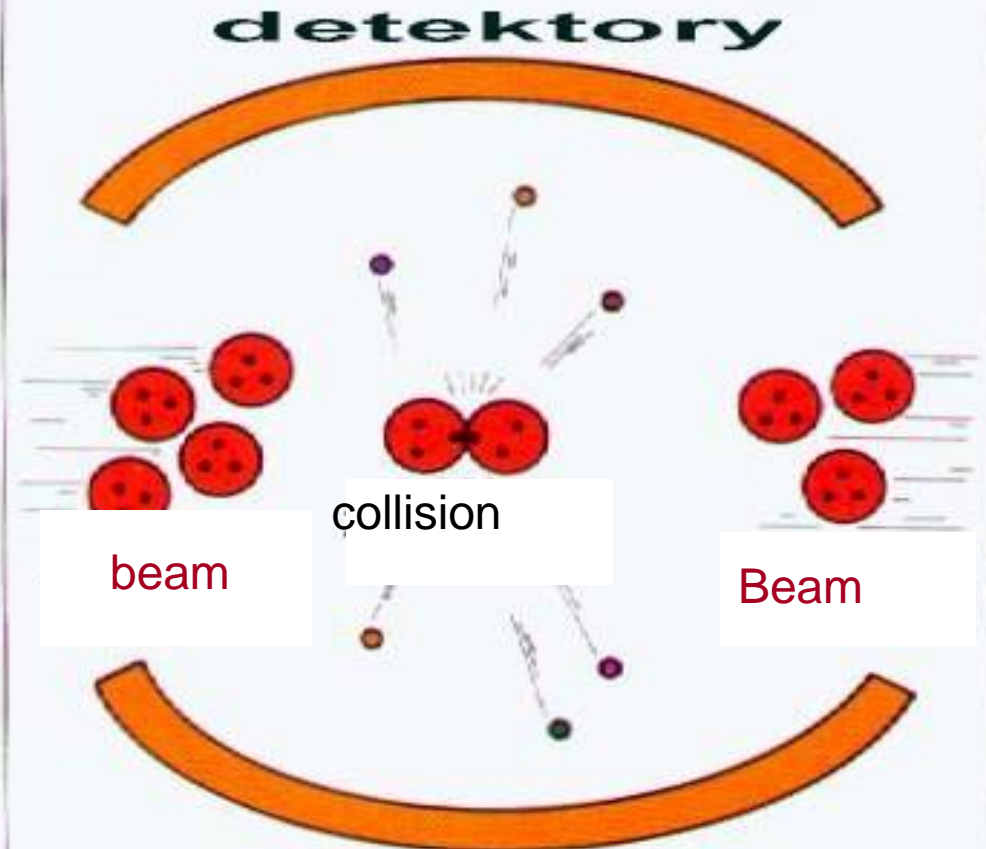
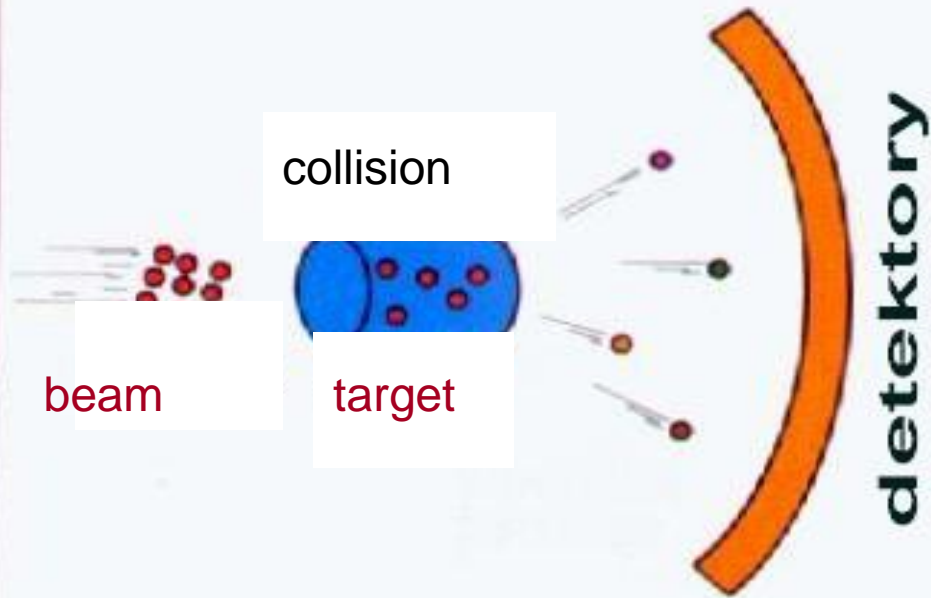




# The two ways to realize A+A collisions

Collision beam - target

Beam-beam collisions



Accelerator

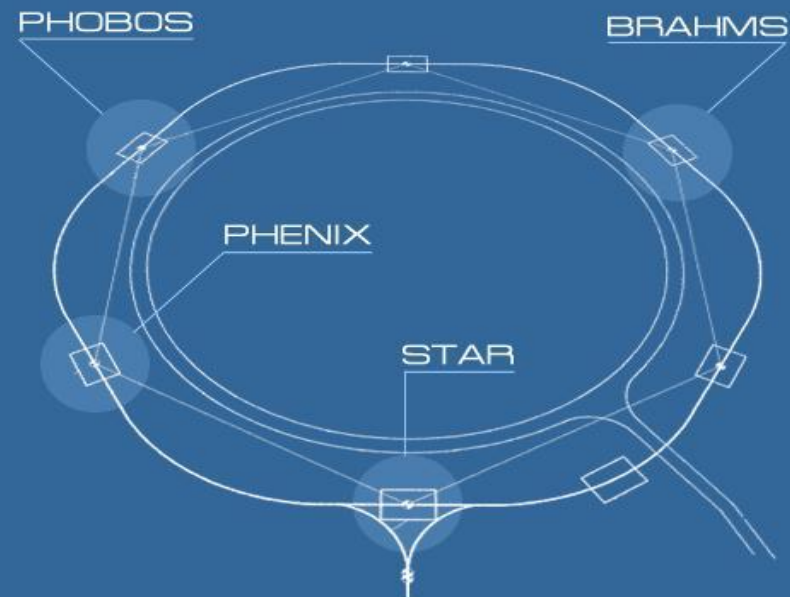
Collider

# Brookhaven National Laboratory, Long Island (USA)

## RHIC Relativistic Heavy-Ion Collider

# RHIC

relativistic heavy ion collider



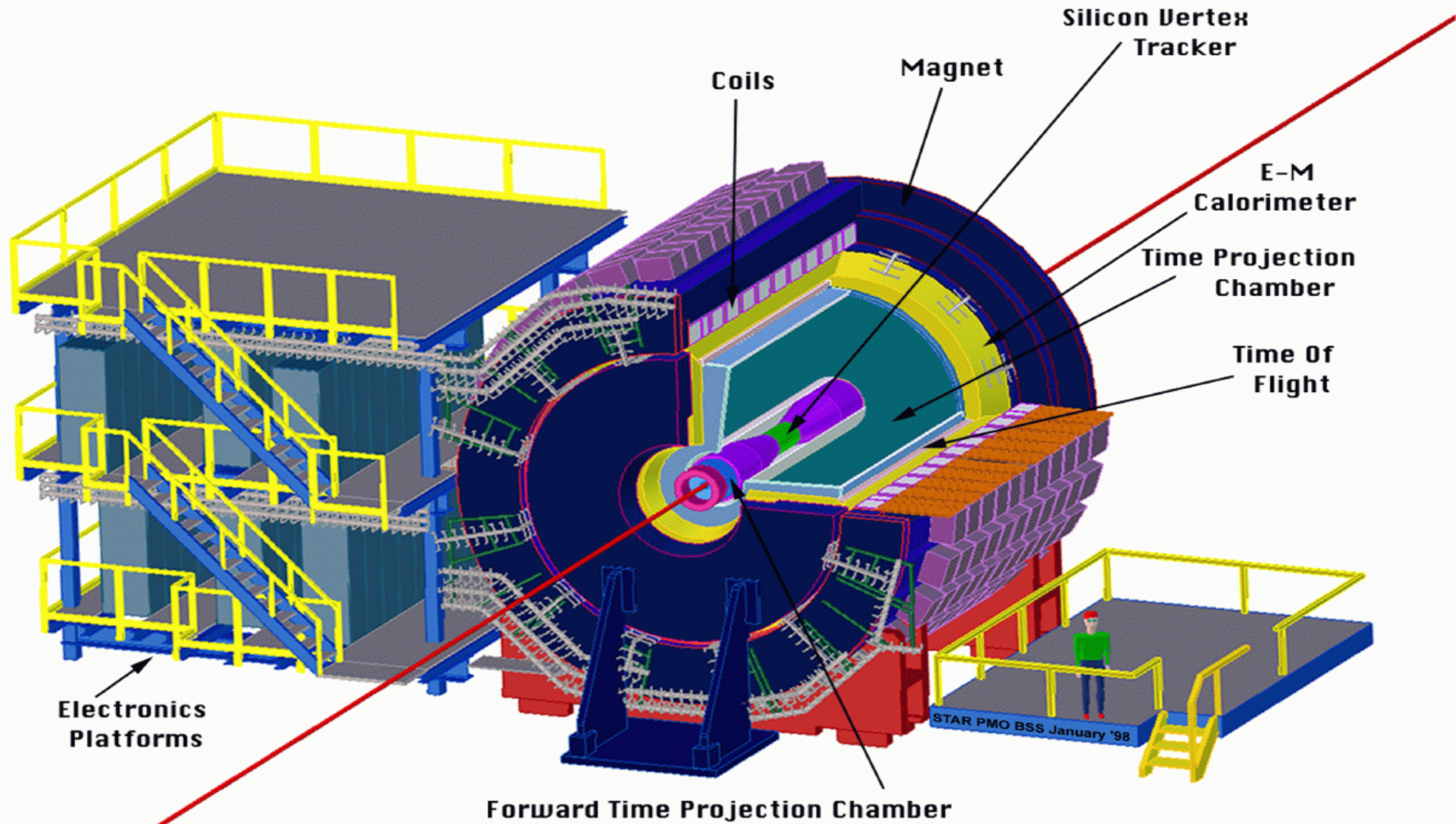
### The Experiments

RHIC's 2.4 mile ring has six intersection points where its two rings of accelerating magnets cross, allowing the particle beams to collide. The collisions produce the fleeting signals that, when captured by one of RHIC's experimental detectors, provide physicists with information about the most fundamental workings of nature.

If RHIC's ring is thought of as a clock face, the four current experiments are at 6 o'clock (STAR), 8 o'clock (PHENIX), 10 o'clock (PHOBOS) and 2 o'clock (BRAHMS). There are two additional intersection points at 12 and 4 o'clock where future experiments may be placed. Visit any experiment by clicking on it.



# STAR Detector



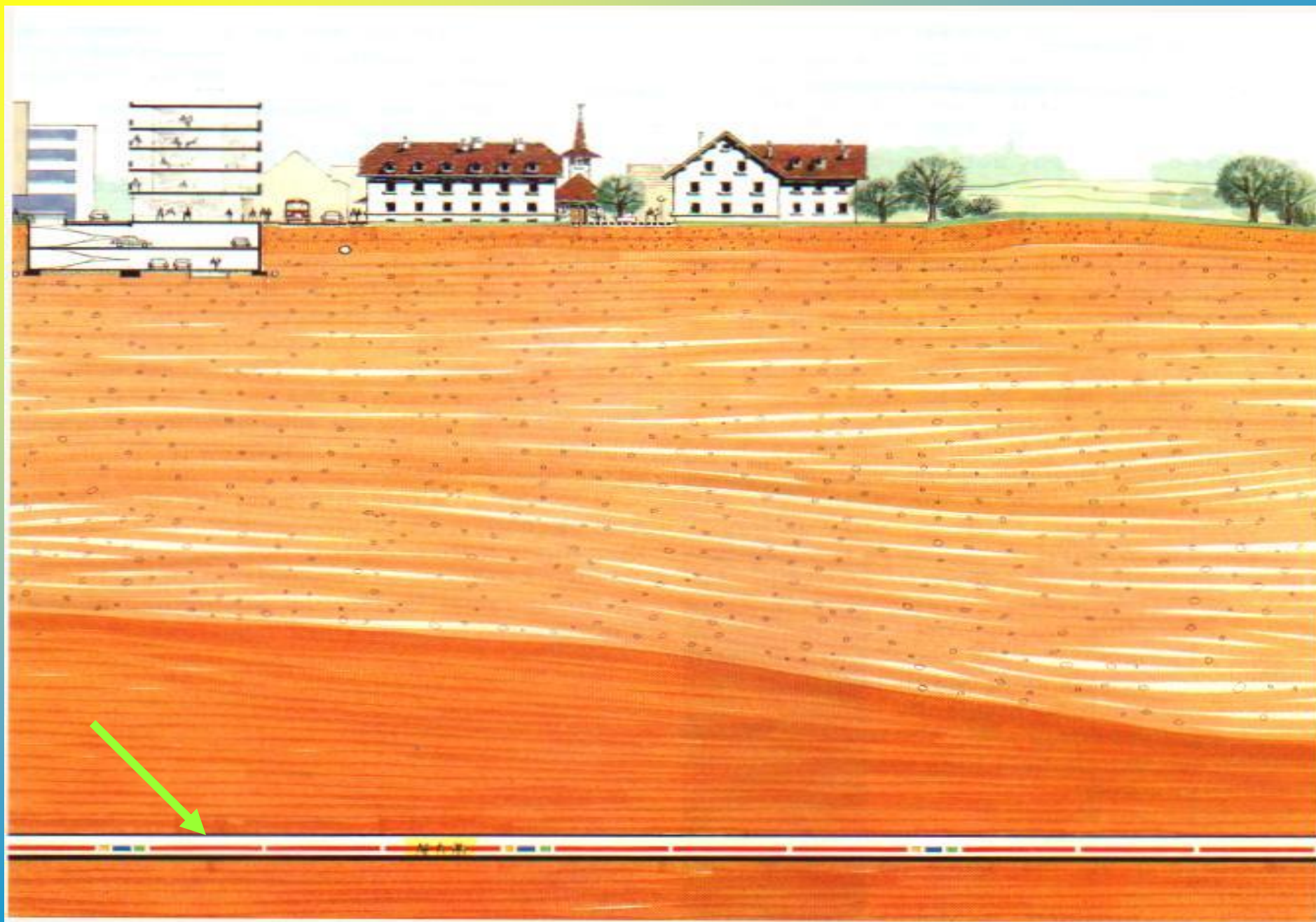


# CERN





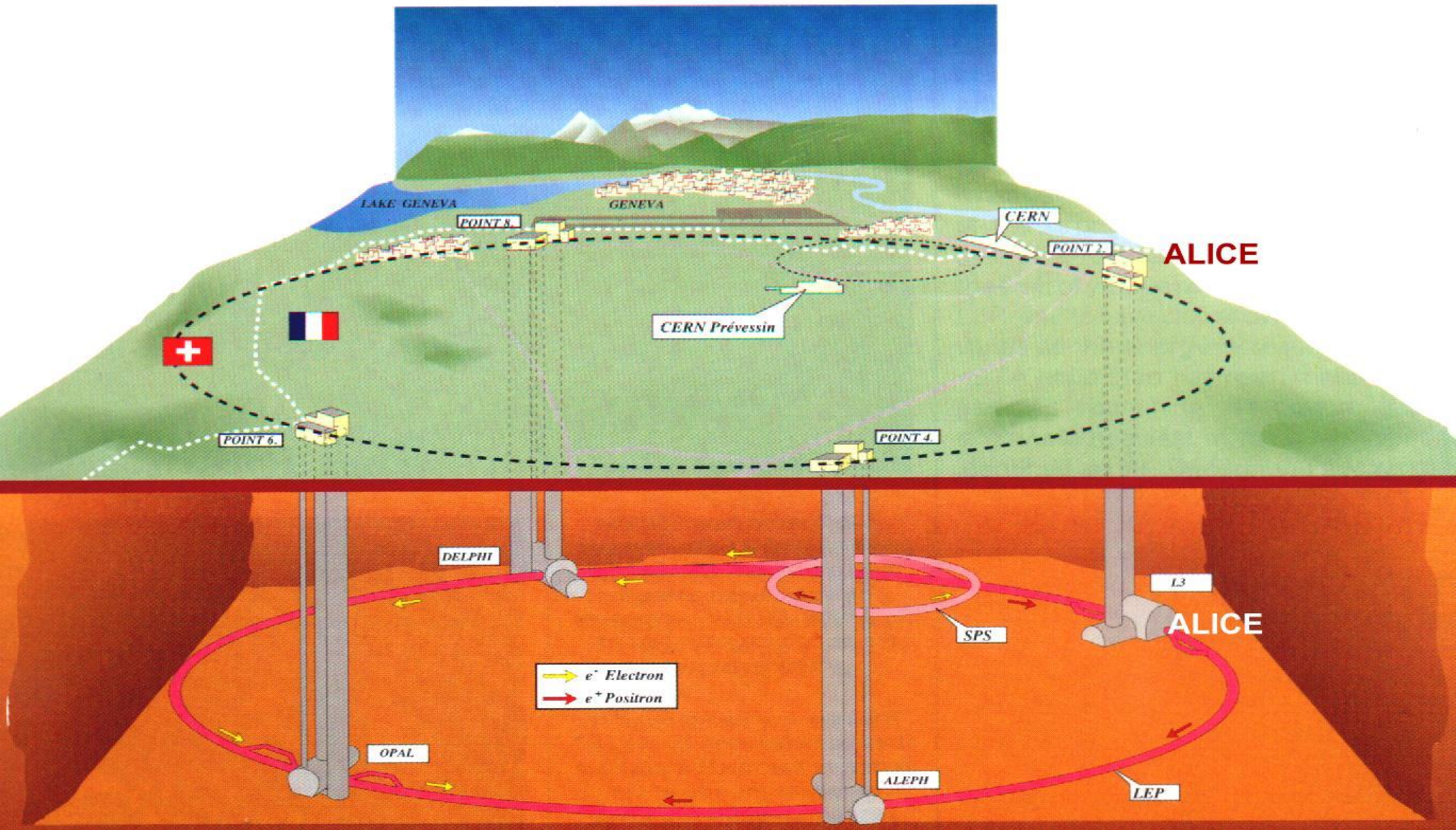
# CERN: on- ... and under- ground



ok. 100m



# CERN – underground tunnel LEP/LHC





# Large Hardon Collider

L=27km

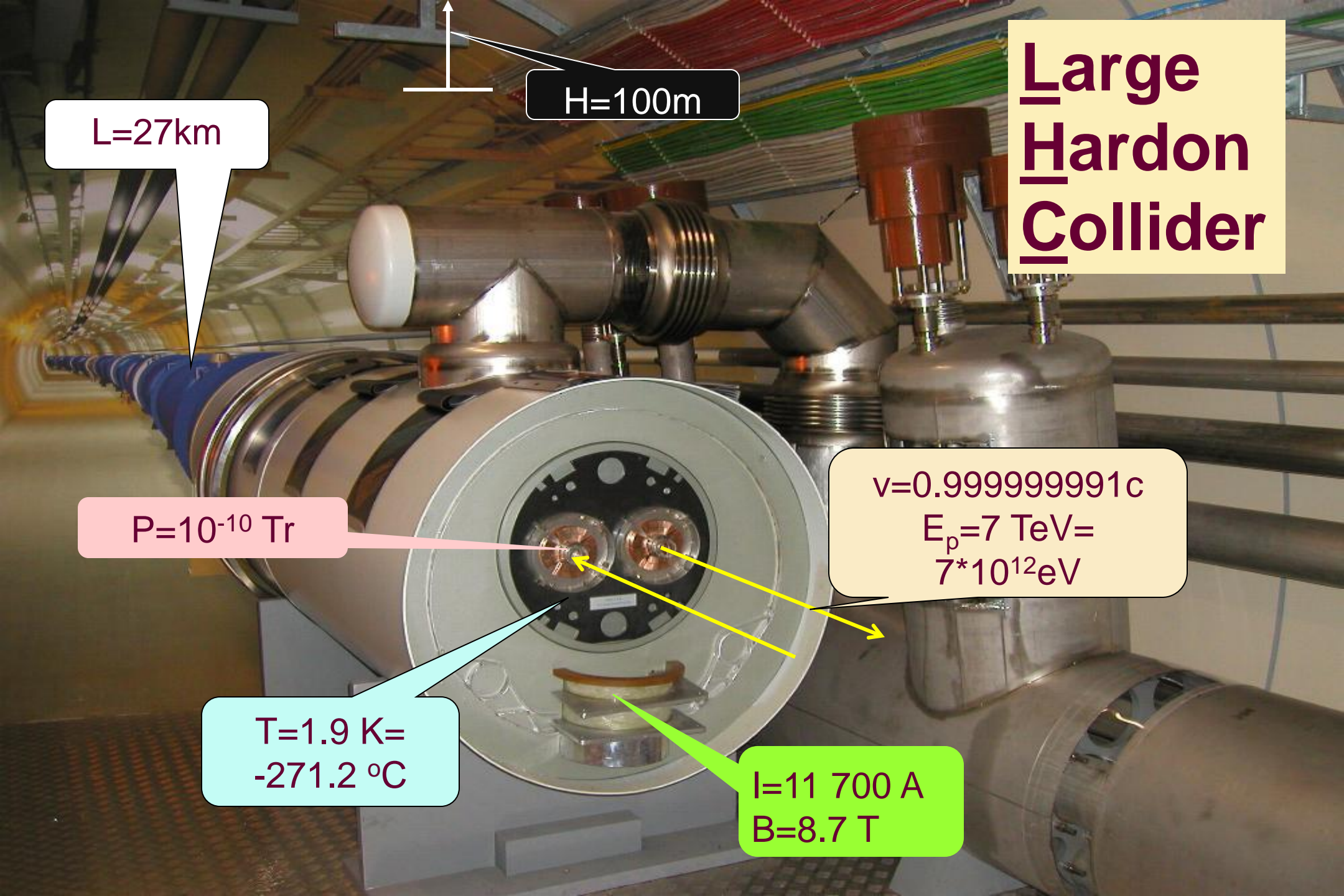
H=100m

$P=10^{-10}$  Tr

T=1.9 K=  
-271.2 °C

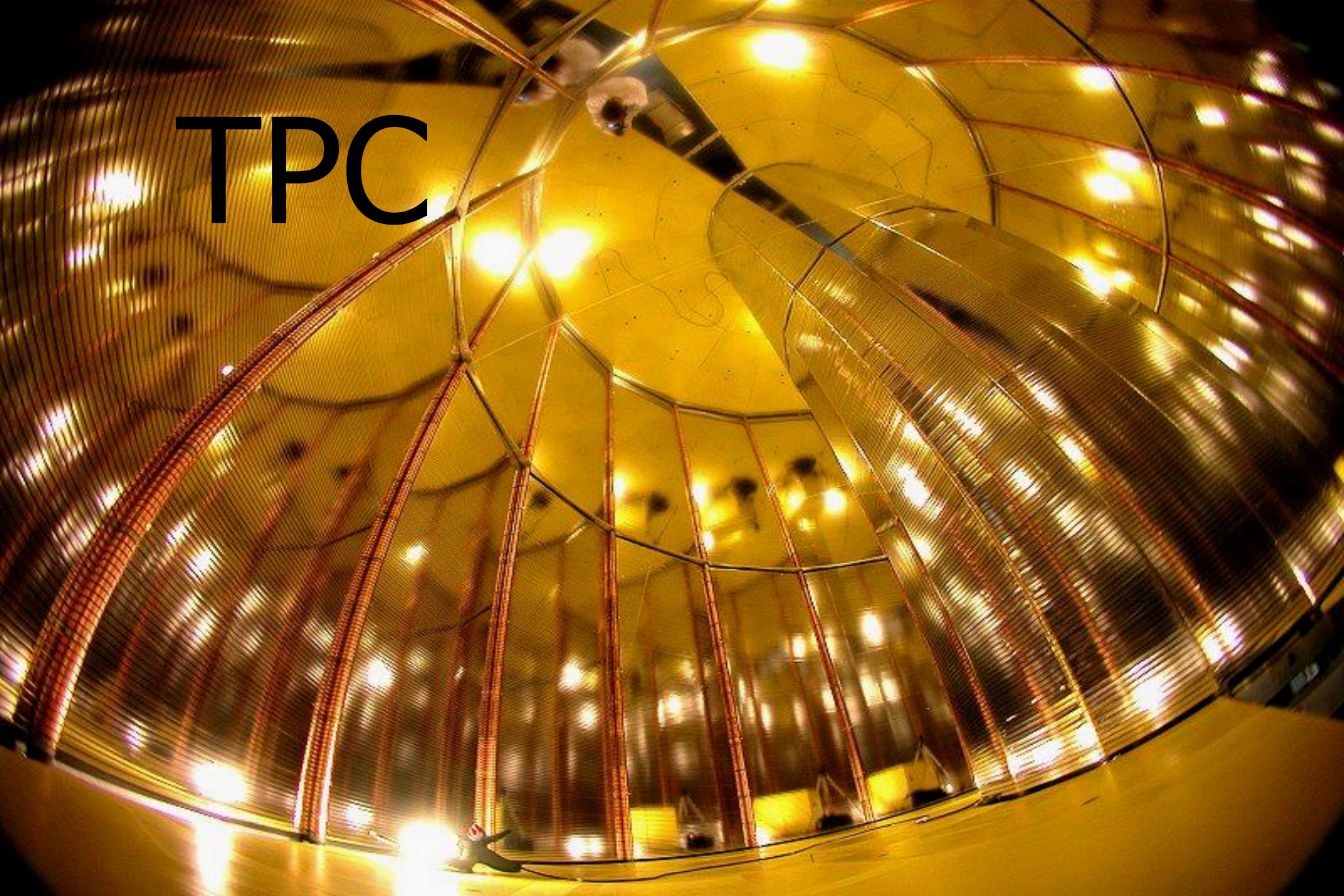
$v=0.9999999991c$   
 $E_p=7$  TeV=  
 $7 \cdot 10^{12}$  eV

I=11 700 A  
B=8.7 T





TPC



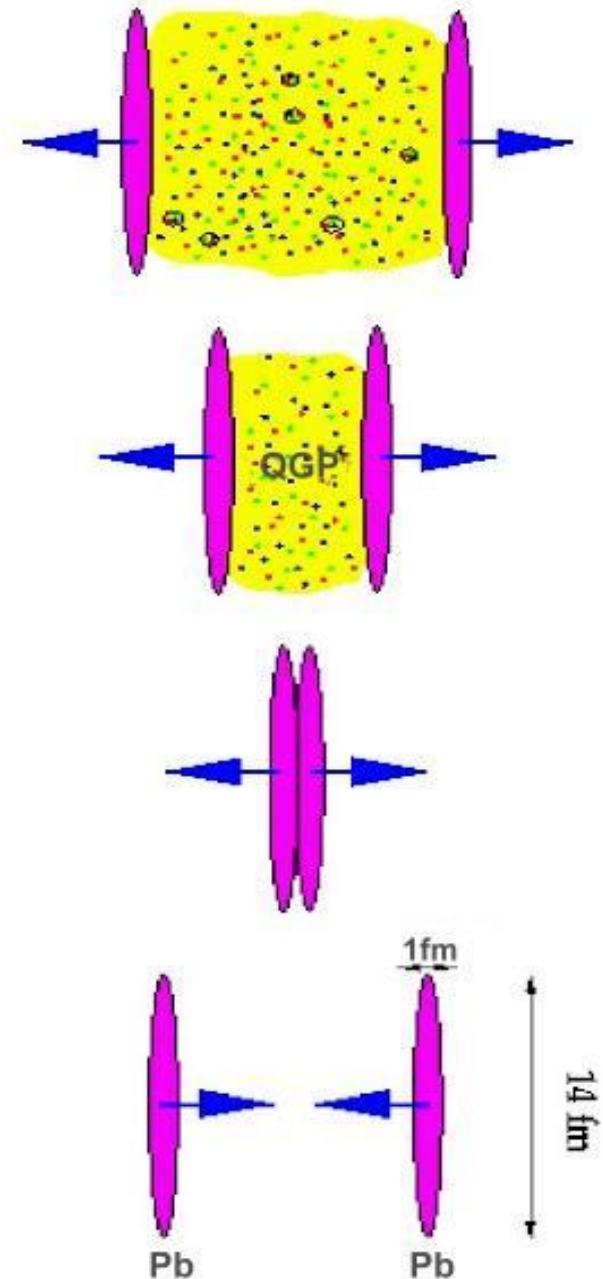
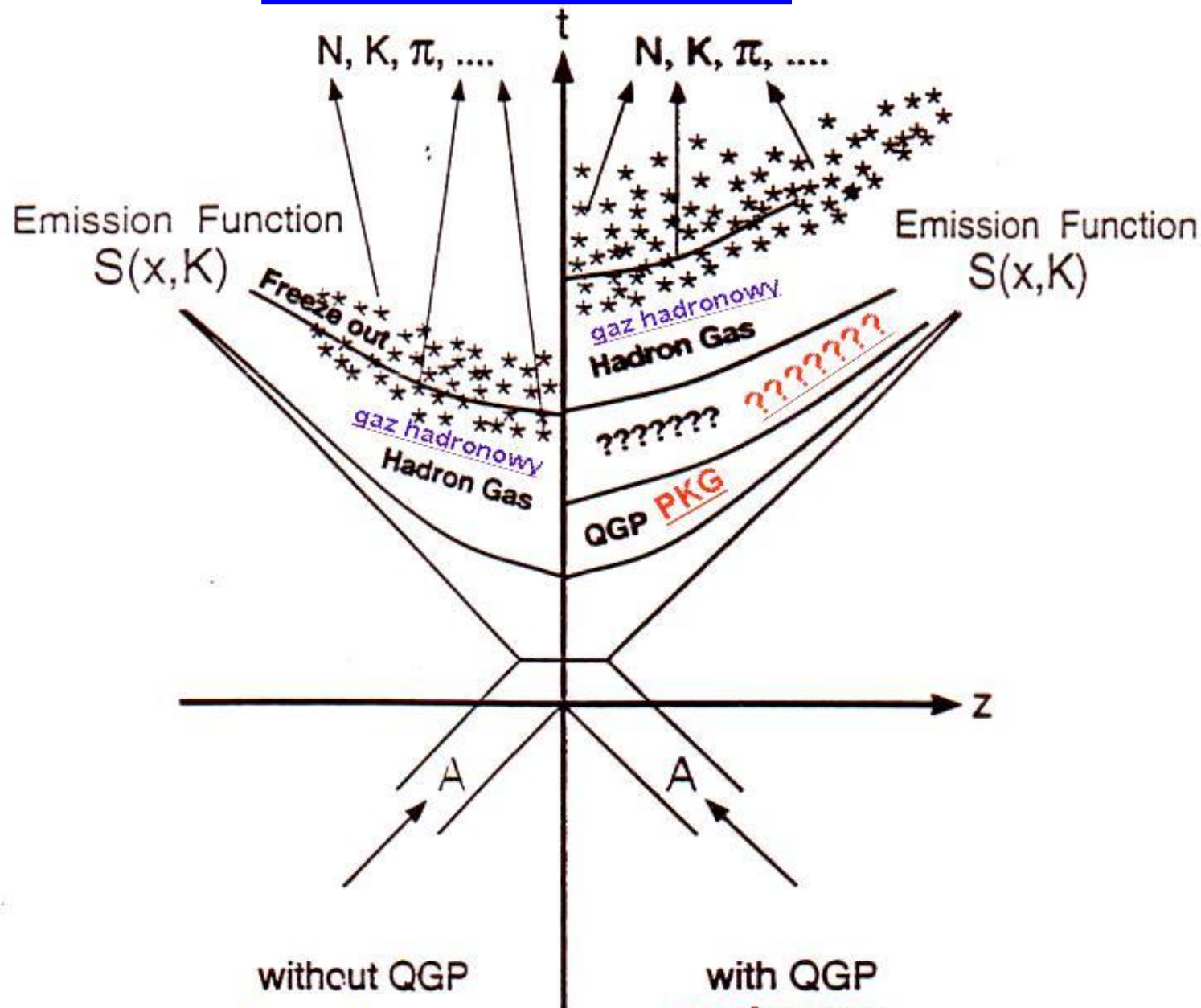




05.25.2007

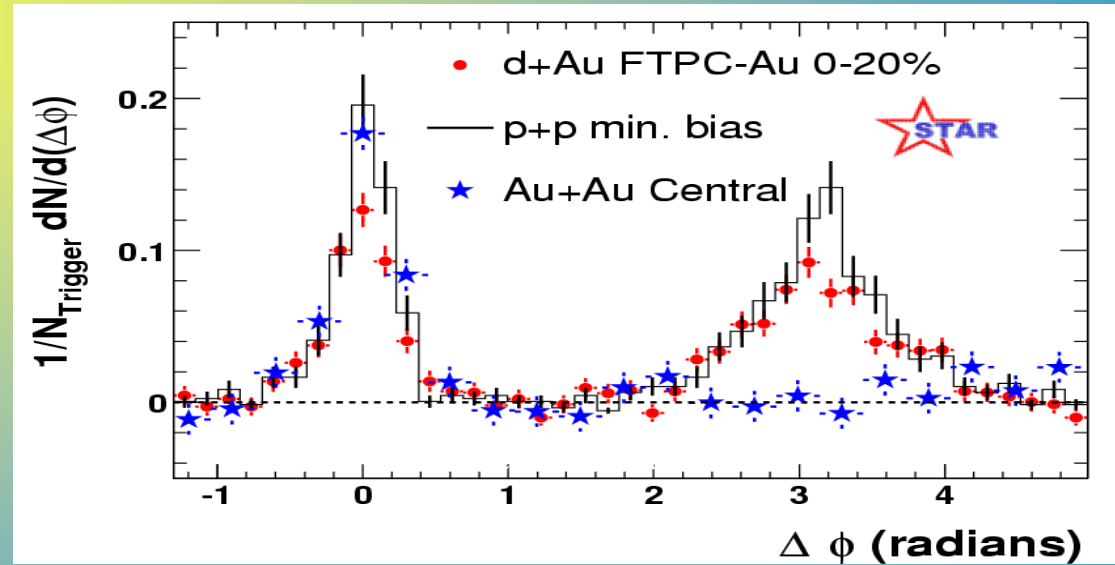
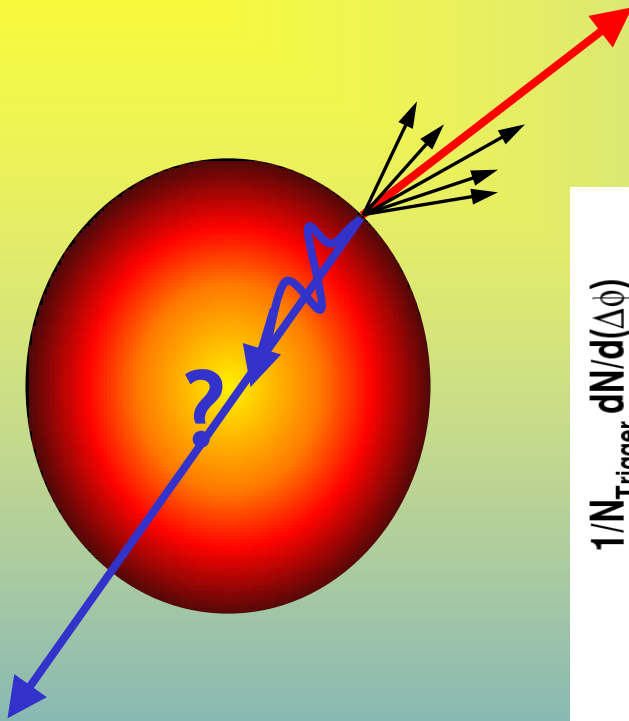


# Two scenarios



# Jet quenching as a signature of very dense matter

Phys. Rev. Lett. 91, 072304 (2003).



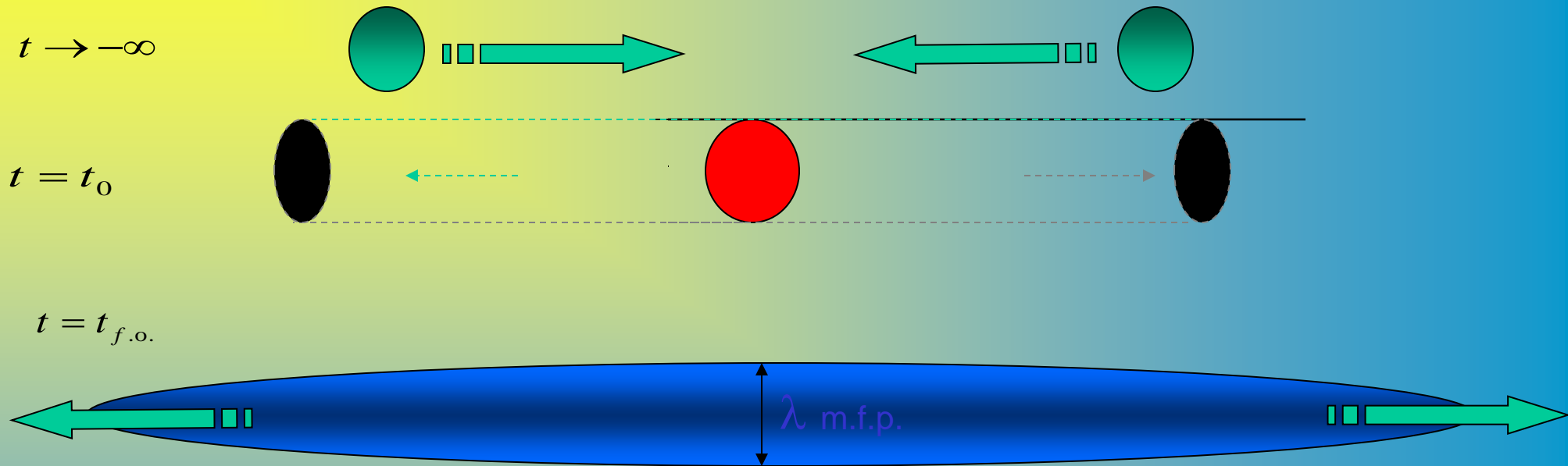
"... was observed *jet quenching* predicted to occur in a hot deconfined environment 100 times dense than ordinary nuclear matter" (BNL RHIC, June 2003).

## Part 2

# Matter evolution in ultrarelativistic $A+A$ collisions



# Hydrodynamic approach to multiparticle production [Landau, 1953]



**Studying of** (one- and multi- particle) **spectra** versus **IC** and **EoS** one can get, in principle information about earlier partonic stage of evolution: possible formation of QGP or even type of the phase transition.

# Quasi-inertial hydrodynamics

- Hydrodynamic equation

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

energy momentum tensor of perfect fluid

$$p = c_0^2 \epsilon, \quad (0 < c_0^2 = \text{const} < 1)$$

- Coordinates  $(t, x, y, z)$

- Quasi-inertial flows

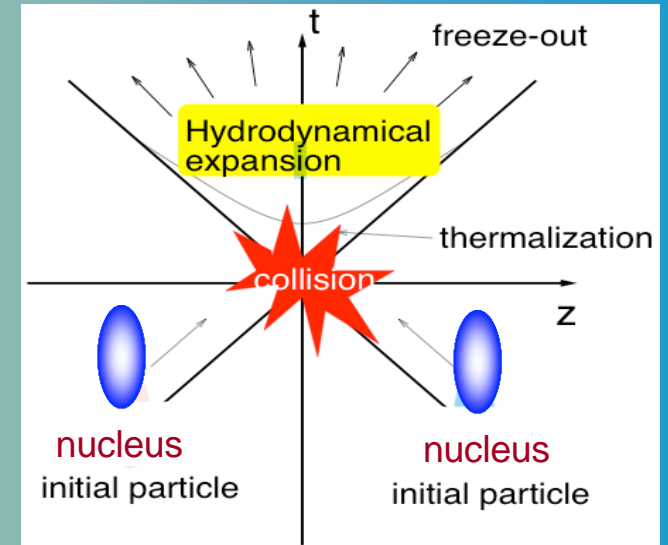
Projection of equation on the direction of 4-velocity

$$u^\nu \partial_\mu T^{\mu\nu}$$

$$u^\nu \partial_\nu u^\mu = 0$$

$$\frac{du^{*\mu}}{dt} = 0$$

$$(\epsilon + p) \partial_\nu u^\nu + u^\nu \partial_\nu \epsilon = 0$$



- Thermodynamic identities:

$$\epsilon + p = Ts + \mu n$$

$$d\epsilon = Tds + \mu dn$$

$$T \partial_k (su^k) + \mu \partial_k (nu^k) = 0$$

Entropy is conserved

$$\partial_k (su^k) \text{ if } \mu = 0$$

or particle number is conserved:

$$\partial_k (nu^k) = 0$$

# (1+1)D boost-invariant hydrodynamic models

## Quasi-inertial Hydrodynamic Equations

$$(\varepsilon + p)\partial_\nu u^\nu + u^\nu \partial_\nu \varepsilon = 0$$

$$u^\mu u^\nu \partial_\nu p - \partial^\mu p = 0$$

- New variables**

$$(\tau, x, y, \eta) : \tau = \sqrt{t^2 - z^2}, \eta = \tanh^{-1}\left(\frac{z}{t}\right) \qquad t = \tau \cosh \eta, z = \tau \sinh \eta$$

- One dimensional boost-invariant approximation:**

$$u_x = u_y = 0; \varepsilon = \varepsilon(\tau)$$

**Solution:**

- Hydro-velocity:**

$$v_z = \frac{z}{t}; \left(u_0 = \frac{t}{\sqrt{t^2 - z^2}}, u_x = 0, u_y = 0, u_z = \frac{z}{\sqrt{t^2 - z^2}}\right) \longrightarrow u^\nu \partial_\nu u^\mu = 0$$

- Quasi-inertiality**

$$(\varepsilon + p)\partial_\nu u^\nu + u^\nu \partial_\nu \varepsilon = 0 \longrightarrow \frac{d\varepsilon}{d\tau} = (1 + c_0^2)\varepsilon(\tau)$$

$$\varepsilon(\tau) = \varepsilon(\tau_0) \left(\frac{\tau_0}{\tau}\right)^{(1+c_0^2)}$$

$$s(\tau) = s(\tau_0) \frac{\tau_0}{\tau}$$

It is so called “Bjorken solution”, in fact, invented by R. Hwa and C. Chiu

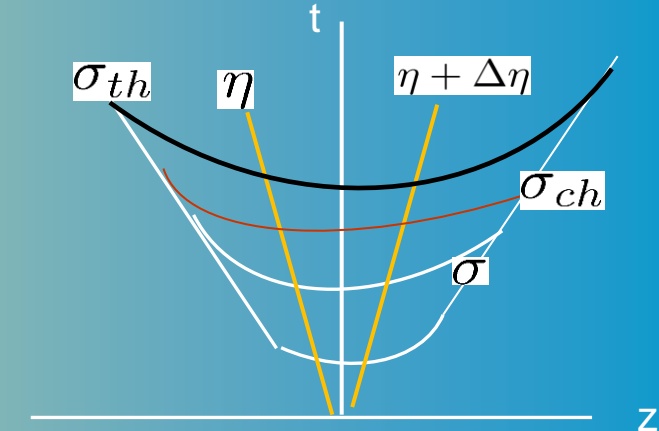
# The basic properties of the boost-invariant solution

$$S(\Delta\eta) = \int_{\eta}^{\eta+\Delta\eta} s(\tau) u^{\mu} d\sigma_{\mu} = s(\tau_0) \tau_0 \Delta\eta \pi R^2$$

$$E(\Delta\eta) = \int_{\eta}^{\eta+\Delta\eta} T^{0\mu}(\tau, \eta) u^{\mu} d\sigma_{\mu} \xrightarrow{c_0^2 \rightarrow 0} \epsilon(\tau_0) \tau_0 2 \sinh(\Delta\eta/2) \pi R^2$$

$$\epsilon_0 = \frac{\langle m_t \rangle}{(R^2 \pi) \tau_0} \frac{dn}{dy}$$

Conception of thermal freeze-out



Cooper-Frye formula for sudden thermal freeze-out  $p^0 \frac{d^3 N}{d^3 p} = \int_{\sigma_{th}} d\sigma_{\mu} p^{\mu} f(x, p)$

Conception of chemical freeze-out

$$N_i = p^0 \frac{d^3 N}{d^3 p} = \int_p \int_{\sigma_{ch}} \frac{d^3 p}{p^0} d\sigma_{\mu} p^{\mu} f\left(\frac{p^{\mu} u_{\mu}(x)}{T_{ch}(x)}, \frac{\mu_{i,ch}(x)}{T_{ch}(x)}\right)$$

Generalization of sudden freeze-out to continuous one:  
Hydro + Cascade models

$$p^0 \frac{d^3 N}{d^3 p} \approx \int_{\sigma(p)} d\sigma_{\mu} p^{\mu} f(x, p)$$

Where  $\sigma(p)$  is piece of hypersurface where the particles with momentum near  $p$  has a maximal emission rate



# Useful formulas 1

At relativistic energies, due to dominant longitudinal motion, it is convenient to substitute the Cartesian coordinates  $t, z$  by the Bjorken ones

$$\tau = (t^2 - z^2)^{1/2}, \quad \eta = \frac{1}{2} \ln \frac{t + z}{t - z}$$

and introduce the the radial vector  $\vec{r} \equiv \{x, y\} = \{r \cos \phi, r \sin \phi\}$ , i.e.:

$$x^\mu = \{\tau \cosh \eta, \vec{r}, \tau \sinh \eta\} = \{\tau \cosh \eta, r \cos \phi, r \sin \phi, \tau \sinh \eta\}.$$

Representing the freeze-out hypersurface by the equation  $\tau = \tau(\eta, r, \phi)$ , the

hypersurface element in terms of the coordinates  $\eta, r, \phi$  becomes

$$d^3\sigma_\mu = \epsilon_{\mu\alpha\beta\gamma} \frac{dx^\alpha dx^\beta dx^\gamma}{d\eta dr d\phi} d\eta dr d\phi, \quad (32)$$

where  $\epsilon_{\mu\alpha\beta\gamma}$  is the completely antisymmetric Levy-Civita tensor in four dimensions with  $\epsilon^{0123} = -\epsilon_{0123} = 1$ . Particulary, for azimuthaly symmetric hypersurface  $\tau = \tau(\eta, r)$ , Eq. (32) yields [12]:

# Useful formulas 2

$$u^\mu(r, \eta) = \gamma(\cosh \eta, v \cos \phi, v \sin \phi, \sinh \eta), \quad (7)$$

where  $\gamma = (1 - v^2)^{-1/2}$ . The element of the hypersurface  $\sigma(x)$  takes the form

$$d\sigma_\mu = \tau(r, \eta) d\eta dr_x dr_y \times \left( \frac{1}{\tau} \frac{d\tau}{d\eta} \sinh \eta + \cosh \eta, -\frac{d\tau}{dr_x}, -\frac{d\tau}{dr_y}, -\frac{1}{\tau} \frac{d\tau}{d\eta} \cosh \eta - \sinh \eta \right). \quad (8)$$

$$p^\mu = (m_T \cosh y, p_T \cos \psi, p_T \sin \psi, m_T \sinh y)$$

$$m_T = \sqrt{m^2 + p_T^2}$$

In Bjorken 1+1 D model:

$$d\sigma_\mu p^\mu = \pi R_T^2 \tau dy \cosh(y - \eta)$$

$$u_\mu p^\mu = m_T \cosh(y - \eta)$$