Building the Standard Model (SM)
Historical and Qualitative Aspects

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Some Pre-Standard Model Physics

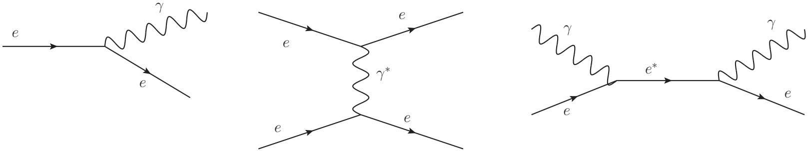


Figure: Quantum Electrodynamics (QED) illustrated in terms of Feynman diagrams

- Ca. 1965 QED was the only successful QFT
  Verified to many orders in perturbation theory
- Example : In *ee*-scattering the *virtual* photon correspond to the Coulomb potential

- Weak and strong interactions were poorly understood theoretically. Some models existed.
- Gravity, electromagnetism, Strong interactions, Newton's laws, Schrödinger eqs. *conserve* C*-*, P*-*, T*-symmetries separately !* *This was believed to be true in all types of interactions*
- Reminder:
  C = charge conjugation : Particle $\rightarrow$ Anti-particle
  P = Parity(mirror) transformation:$\vec{r} \rightarrow -\vec{r}$ ($\vec{r}$ = position vector))
  For axial vector : $\vec{L} = \vec{r} \times \vec{p} \rightarrow +\vec{L}$
  (Remember: H-atom wave funct. has parity (-1)^{l})

T = Time reversal: $t \rightarrow -t$ (Remember : In quantum mechanics the T-operator is anti-unitarian)

- First shock: (Lee and Yang 1956 : Explained experiments !)

Parity symmetry is broken in weak interaction

- "$\theta - \tau$-puzzle : Decays $(\theta\,,\,\tau) \to\ 2\,\pi$ *and* $(\theta\,,\,\tau) \to\ 3\,\pi$
  Actually: θ and τ is the *same particle*, the K-meson (kaon). π and K are pseudo-scalar paprticles (0^-).
- Most known weak process (decay): β-decay: $n \to p\, e^- \overline{\nu_e}$
- Lepton number L_e conserved:
  $L(e^-) \;=\; L(\nu_e) = \;+1$ and $L(e^+) \;=\; L(\overline{\nu_e}) = \;-1$
- For massive fermions and anti-fermions: In total four degrees of freedom. BUT: For massless fermions/anti-fermions: Two degrees of freedom Choice: Lefthanded particles and righthanded antiparticles, i.e $(\nu_e)_L$ and $(\overline{\nu_e})_R$

- Simplest decays of pions: $\pi^+ \to \mu^+ \nu_\mu$ and $\pi^- \to \mu^- \overline{\nu_\mu}$
  Lepton number L_μ conserved:
  $L(\mu^-) = (\nu_\mu) = +1$ and $L(\mu^+) = (\overline{\nu_\mu}) = -1$
- $CP[\pi^+ \to \mu^+ (\nu_\mu)_L] = C[\pi^+ \to \mu^+ (\nu_\mu)_R]$
  $= \pi^- \to \mu^- (\overline{\nu_\mu})_R$.
  In opposite order: $CP[\pi^+ \to \mu^+ (\nu_\mu)_L] = P[\pi^- \to \mu^+ (\overline{\nu_\mu})_L]$
  $= \pi^- \to \mu^- (\overline{\nu_\mu})_R$.
- Conclusion :In this process CP-symmetri (combined C and P) is apparently satisfied ? (-see blackboard ?)- Or ?
- If so: This would be a symmtry between left-handed matter and right-handed anti-matter.

$SU(3)_F$ flavor symmetry

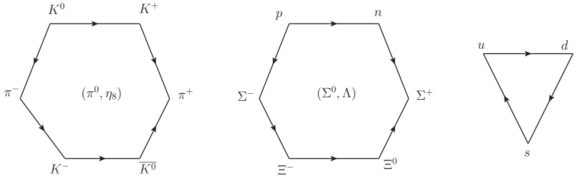


Figure: Pseudoscalars (0^-) and baryons($\frac{1}{2}^-$) put in octet (8-plet) representation of $SU(3)$. Making order i hadronic particles. BUT elementary triplet (3-plet) representation ("quarks") *apparently* not realized ?!?

- Pseudoscalars:
  $\pi^+ = (u\bar{d}), \pi^0 = (u\bar{u} - u\bar{u})/\sqrt{2}, \pi^- = (d\bar{u}), K^+ = (u, \bar{s}), K^0 = (d, \bar{s}), K^- = (s, \bar{u}), \overline{K^0} = (s, \bar{d}), \eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$
- Baryons : $p = (uud), n = (ddu), \Sigma^+ = (uus), \Sigma^0 = (uds), \Sigma^- = (dds), \Lambda = (uds), \Xi^0 = (uss), \Xi^- = (dss)$

- Second shock 1964 (experiments by Fitch and Cronin)

CP-symmetry is broken in weak interaction

- The neutral K-mesons K^0 and $\overline{K^0}$ are degenerate in mass. CP eigenstates: $|K_\pm> = (|K^0> \mp |\overline{K^0}>)/\sqrt{2}$, and $CP|K_\pm> = \pm|K_\pm>$.
- Physical states $K_S = K_+$ (shortlived) and $K_L = K_-$ (longlived) *If* CP-symmetry were fulfilled, then we should see $K_S \to 2\pi$ and $K_L \to 3\pi$. (2 π has CP-sym = + ; while 3 π may have CP-sym = -
  But this was not exact some 2 to 3 permille of K_L decayed to 2π !
- Explanation. $K_L \simeq K_- + \epsilon K_+$, with $|\epsilon| \simeq 2.6 \times 10^{-3}$.

Trying to build a theory for weak interactions

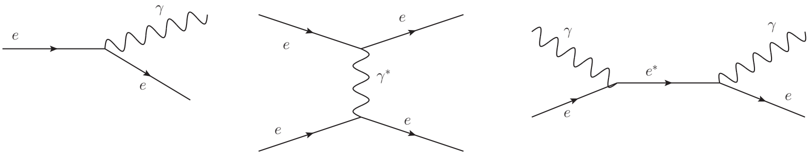


Figure: Some inspiration from QED?

Recall electron-electron scattering (q = momentum transfer via virtual photon)

$$Ampl(ee) \, = \, \frac{e^2}{q^2} j^\mu(e) \, j_\mu(e) \; , \quad j^\mu(e) \, = \bar{\psi} \, \gamma^\mu \, \psi$$

Note that $j^\mu(e)$ is a pure four vector current.

Fermi "theory"

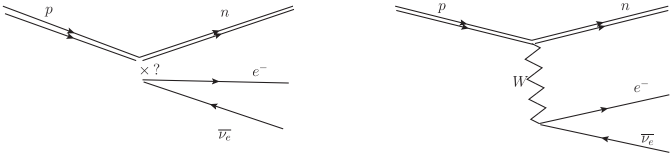


Figure: Fermi-diagram for beta-decay. Hypothetical extension with W

Fermi theory- a product of two *left-handed* currents:

$$\mathcal{L}_F = 4\frac{G_F}{\sqrt{2}} j(W,N)^\mu j(W,l)_\mu \ , \quad j(W)_\mu = \frac{1}{2}(j_\mu^V - j_\mu^A)$$

If the two currents were mediated by a heavy weak boson W, then

$$\frac{g_W^2}{q^2 - M_W^2} = -4\frac{G_F}{\sqrt{2}} \quad ; \ |q^2| << M_W^2 \quad , G_F \simeq \frac{10^{-5}}{m_p^2}$$

Note $j^A_\mu = \psi\,\gamma_\mu\,\gamma_5\,\psi$, such that $j(W)_\mu$ is left-handed :

$$j(W)_\mu = \psi\gamma_\mu\,P_L\,\psi\ , \quad P_{L,R} \equiv \frac{1}{2}\,(1 \mp \gamma_5\,)$$

where $P_{L,R}$ are the projectors in Dirac space:

$$(P_L)^2 = P_L\ ,\ (P_R)^2 = P_R\ ,\ P_L \cdot P_R = P_R \cdot P_L = 0\,, P_L + P_R = 1$$

A Dirac field can be split in two parts:

$$\psi = \psi_L + \psi_R\ ,\ \psi_L = P_L\,\psi\ ,\ \psi_R = P_R\,\psi$$

ψ_L = left spinning (left screw)particle.
ψ_R = right spinning (right screw) particle.

- Note: The helicity operator $S_p = (1/2)\,\vec{\Sigma}\cdot\vec{p}/|\vec{p}|$ has the same effect as $(1/2)\gamma_5$ for massless Dirac particles.
- Note that if the neutrino ν_e is purely lefthanded, then the current for $\nu_e \to e^-$ is automatically lefthanded:

$$j(W,e)_\mu = \overline{\psi_e}\,\gamma_\mu\,\psi_{\nu_e} = \overline{\psi_e}\,\gamma_\mu\,(\psi_{\nu_e})_L = \overline{\psi_e}\,\gamma_\mu\,P_L\psi_{\nu_e}$$

Note that $\overline{\psi_L} = (P_L\psi)^\dagger\gamma_0 = \overline{\psi}\,P_R$, such that:

$$\overline{\psi_L}\,\gamma_\mu\,\psi_L = \overline{\psi}\,\gamma_\mu\,P_L\psi \;,\; \overline{\psi_R}\,\gamma_\mu\,\psi_R = \overline{\psi}\,\gamma_\mu\,P_R\psi \;,\; \overline{\psi}\,\psi = \overline{\psi_L}\,\psi_R\,\overline{\psi_R}\,\psi_L$$

We observe that symbolically

$$(V-A)^2 = (V\,V + A\,A) - (V\,A + A\,V)$$

where the last term violates parity

Trying to build a theory for strong interactions

Phenomeological models, Regge poles,...π- nucleon interactions

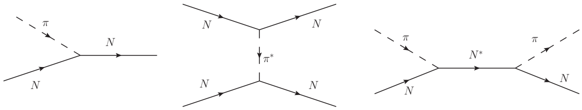


Figure: Some π-Nucleon Feynman diagrams

Interaction Lagrangian for strong interactions? -not sucessful!

$$\mathcal{L}_{\pi N} \; = \; G_{\pi N} \, \bar{N} \, (\tau_i \, \Phi_i) \, \gamma_5 \, N \;\; , \;\; N \; = \; \begin{pmatrix} p \\ n \end{pmatrix}$$

where N is the nucleon doublet, Φ the pion triplet and τ the Pauli matrices which here represents isospin.
$G_{\pi N}$ too big to make perturbative theory(expansion) valid!
Chiral perturbation theory (χPT)- (see later).

An elektron-proton collision

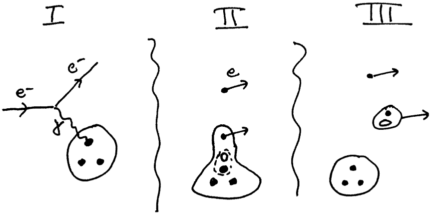


Figure: Elektron-proton collision at high energy (at Stanford Linear Accellerator Center, iCalifornia, in 1969) $e\,p \rightarrow e\,N\,\pi$

What is seen: The elektron cannot shoot loose a quark, but a *meson* ($q - \bar{q}$-pair). A quark cannot be free! The “color”-charge is confined ! (bound to a baryon or a meson.)

The proton is not elementary- The parton model

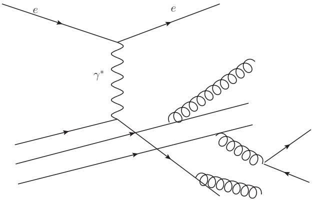


Figure: Collision $ep \to eX$ in the Parton Model

Cross setion contain the structure functions $f_p(x)$, and depend only on the momentumfraction of the scattered parton,- called *Scaling*
$x \equiv p_{part}/p_{proton} = (-q^2)/(2M_p\nu)$, where $\nu = E_e - E_e'$
Intepretation of data: At high energies, the proton consist of "sea-quarks" = *quarks* and *anti-quarks* and *gluons* an addition to the *valence quarks uud*.

Picture of a proton

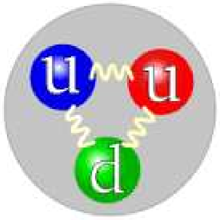


Figure: Quarks have "Color". The "color"-forces bind the protonet together. The proton has two *u*-type quarks and a *d*-type quark. Plus *quantum fluctuations* -with additional (anti-)quarks and gluons- depending on the energy

The color forces are so strong that the quarks could not come out of the proton.
The proton (-and the neutron) are color nuetral.

Nuclear forces stronger than electric !

Figure: Attractive nuclear forces

Two protons feels the “tail” of quark-gluon-forces, in spite of elektric repulsion
Nuclear forces are much stronger than elektromagnetic forces
-at short distance

Weak int. : The birth of a symmtry

- 1967 Weinberg: “A model of leptons” (Similar ideas by Glashhow and Salam)
- A gauge theory of $SU(2)_L \times U(1)_Y$ for two lepton families :
  $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ and e_R , $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$ and μ_R
- W_μ^i of $SU(2)_L$ and B_μ of $U(1)_Y$ combines to $W_\mu^{(\pm)}, Z_\mu, A_\mu$.
- Quark families: $\begin{pmatrix} u \\ d \end{pmatrix}_L$ and u_R , d_R , $\begin{pmatrix} ?? \\ s \end{pmatrix}_L$ and $??, s_R$
- Cabibbo mixing: $d \rightarrow d_\theta = d\, cos\theta \, + s\, sin\theta$ and $s \rightarrow s_\theta = s\, cos\theta \, - d\, sin\theta$ to explain the beta-decay of Σ^0: $\Sigma^0 \rightarrow p\, e^- \, \overline{\nu_e}$. Here $\theta = \theta_C$ = Cabibbo-angle. Ratio of Σ^0 and n beta decay amplitudes $=$ $tan\theta_C$. Exp.: $sin\theta_C \simeq 0.225$

- GIM 1970: Postulate existence of fourth quak c (=*charm*)
- Now two quark doublets: $\begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad , \quad \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$
  and four singlets $u_R, (d_\theta)_R, (s_\theta)_R, c_R$
- The 1974 revolution: Found Ψ-particles. Qualitative behaviour of decay-spectrum like positronium $e^+ e^-$. Intepreted as $c\,\bar{c}$.
- After this, the quark picture was accepted among (almost) all particle physicists

Later: three families: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad , \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad , \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$

$$\begin{pmatrix} u \\ d_{CKM} \end{pmatrix}_L \quad , \quad \begin{pmatrix} c \\ s_{CKM} \end{pmatrix}_L \quad , \quad \begin{pmatrix} t \\ b_{CKM} \end{pmatrix}_L$$

-plus all the right-handed singlets.

$$d_{CKM} = V_{ud}\, d + V_{us}\, s + V_{ub}\, b$$
$$s_{CKM} = V_{cd}\, d + V_{cs}\, s + V_{cb}\, b$$
$$b_{CKM} = V_{td}\, d + V_{ts}\, s + V_{tb}\, b$$

where (d, s, b) are the physical quarks (with definite mass). Now neutrinos are known to have tiny masses, which means that there is a mixing also in the neutrino sector.

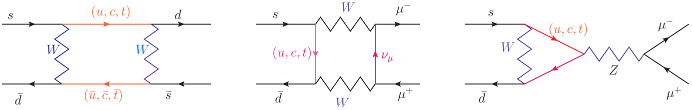


Figure: Quark diagram for $\overline{K^0} \to K^0$ determiing $(m_L - m_S)$ and ϵ , and diagrams for $\overline{K^0} \to \mu^+\mu^-$. Found from these processes that $m_c < 2$ GeV

Decays of kaons (K-mesons) have played an important role in understanding of the SM. Some of the elements of V_{CKM} are complex!

$\Rightarrow$ CP-violation !

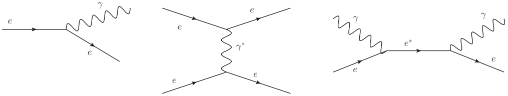
Quantum Field Theory

$$\mathcal{L}_{QED} = \overline{\psi}(\gamma^\mu \, i\, D_\mu - m)\,\psi \; - \; \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \;\; , \;\; i\, D_\mu \equiv i\partial_\mu - eA_\mu$$

- ψ , $\overline{\psi}$ and A_μ are field operators.
- One might say that the particles e^- , e^+ , γ are exitations of the fields. In QFT: the number of particles are not conserved, only the total energy and the (electric) charge.
- Particles might appear, live for a very short time and disappear, as *quantum fluctuations*.

The field tensor $F_{\mu\nu}$ can be defines by

$$[i\, D_\mu, i\, D_\nu] = -ieF_{\mu\nu} \;\; , \quad F_{\mu\nu} \; = \; (\partial_\mu A_\nu \; - \; \partial_\nu A_\mu)$$



Yang-Mills theory (1954 !) has generic form: ($iD_\mu \equiv i\partial_\mu - gt^a A^a_\mu$)

$$\mathcal{L}_{YM} = \bar{f}\,\gamma^\mu\,(iD_\mu - m_f)\,f - \frac{1}{4}F^{a,\mu\nu}\,F^a_{\mu\nu}$$

The field tensor has an extra term - to obtain gauge invariance!:

$$[i\,D_\mu, i\,D_\nu] = -igt^a F^a_{\mu\nu}\,,\ \ F^a_{\mu\nu} = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + g\,f^{abc} A^b_\nu A^c_\nu$$

- The field f has n components (i.e n Dirac fields). t^a are generators ($n \times n$-matrices) of gauge group (say $SU(n)$)
- Important: Vector bosons interact among themselves! We obtain triple and quartic vertices.
- Came “too early”. No use for it ? No known massless vector bosons, except the photon. Y.-M. -just a mathematical exercise?
- Gauge invariance is the “Guiding principle”

Gauge transformations

Gauge transformation in QED:

$$\psi(x) \;\rightarrow\; \psi(x)' \;=\; e^{i\alpha(x)}\;\psi(x)\;,\; A_\mu(x) \rightarrow A_\mu(x)' \;=\; A_\mu(x) \;-\; \frac{1}{e}\,\partial_\mu\alpha(x)$$

For Yang-Mills theory

$$f(x) \;\rightarrow\; f(x)' \;=\; U(x)\,f(x) \quad ; \; U(x) \,\in\, SU(n)(-\text{forist.})$$

Gauge field transf. in the non-Abelian case. Rotation among fields:
(Here $A(x)_\mu \equiv t^a A(x)^a_\mu\,,\,)$:

$$A_\mu \;\rightarrow A'_\mu \;=\; U\,A_\mu\,U^\dagger \;+\; \frac{i}{g}\,(\partial_\mu U)\;U^\dagger,$$

$$F_{\mu\nu} \;\rightarrow\; F'_{\mu\nu} \;=\; U\,F_{\mu\nu}\,U^\dagger \;;\; U \;=\; U(x) \;=\; e^{it^a\alpha^a(x)}$$

$\alpha^a(x)$ is a set of real functions.

$SU(2_L \times U(1)_Y$ theory for first lepton doublet

$$\chi_L \equiv \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \text{ and } e_R \quad \text{, Four gauge bosons } W^i_\mu \text{ , } i = 1,2,3, \text{ and } B_\mu$$

$$\mathcal{L}_{EW1l} = \overline{\chi_L}\,\gamma^\mu \left(i\partial_\mu - g\,\frac{1}{2}\tau^i W^i_\mu - \frac{1}{2}\,g\,Y_L\,B_\mu \right) \chi_L$$
$$+\,\overline{e_R}\gamma^\mu \left(i\partial_\mu - \frac{1}{2}\,g\,Y_R\,B_\mu \right) e_R - \frac{1}{4} W^{i,\mu\nu}\,W^i_{\mu\nu} - \frac{1}{4} B^{\mu\nu}\,B_{\mu\nu}$$
$$+(\text{sim. for the other fermions}) + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- All particles are apparently massless- to have *gauge invariance !*
- Masses of $W^\pm$ and Z-bosons come from $\mathcal{L}_{Higgs}$.
- Fermion masses are coming from $\mathcal{L}_{Yukawa}$

The Higgs sector

Lagrangian for the Higgs doublet ϕ :

$$\mathcal{L}_{Higgs} = \left(iD_\alpha^{(\phi)} \, \phi \right)^\dagger \left(iD^\alpha_{(\phi)} \, \phi \right) - V(\phi)$$

$$V(\phi) \equiv \mu^2 \left(\phi^\dagger \, \phi \right) + \lambda \left(\phi^\dagger \, \phi \right)^2$$

Coupling $\lambda > 0$. Complex Higgs doublet:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \; ; \; iD_\alpha^{(\phi)} = i\partial_\alpha - g \, \frac{\tau^k}{2} W_\alpha^k - \frac{g'}{2} Y_\phi \, B_\alpha$$

For $\mu^2 > 0$, we have a theory for spin zero particles with mass μ.

Lagrangian generating fermion masses $\left(\sim m(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R)\right)$:

$$\mathcal{L}_{Yukawa} \; = \; -\, G_e \, \left(\overline{\chi_L} \, \phi \, e_R \; + \; \overline{e_R} \, \phi^\dagger \, \chi_L \right) \; + \; ....$$

-and extended with similar terms for all doublets...

NB! Up to now: Completely gauge symmetric Lagrangian !

Free parameters: g , g', g_s, μ^2, λ, all G_e's in $\mathcal{L}_{Yukawa}$

Later: Physical intepretation i spesific gauge !

- Via the "Higgs mechanism"

Challenge: Mass term for a vector field (V_μ) has the generic form:

$\mathcal{L}_{Mass} \; = \; m_V^2 \, V_\alpha \, V^\alpha$

Such terms should appear for the $W^\pm$- and Z-bosons.

Mass terms for fermions should also appear in the correct way

Spontanous Symmetry Breaking (SSB)

For $\mu^2 > 0$, the minimum of $V(\phi)$ is at $\phi = 0$, or $\langle 0|\phi|0\rangle = 0$ for the quantum case.
If μ is *assumed* to be imaginary, and $\mu^2 < 0$, V has minimum for a value $|\phi| \neq 0$, SSB (= Vac. has not full sym of dynamical eqs.) will occurr, and the vacuum value of the field ϕ may be taken to be

$$\langle 0|\phi|0\rangle \; = \; \frac{v}{\sqrt{2}} \chi_V \; ; \; v \equiv \sqrt{\frac{-\mu^2}{\lambda}} \; ; \; \chi_V \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

This form of χ_V give a massless photon.
With SSB, a triplet $\xi^j(x)$ of massless neutral (Goldstone) boson fields and one massive neutral Higgs field H with mass $m_H = \sqrt{-2\mu^2}$ appear.

The Higgs mechanism

Goldstone triplet fields ξ^i used as gauge parameters and transformed into three linear combinations $(W^{(\pm)}, Z)$ of W^k_μ and B_μ to *make them massive.* This very special $SU(2)_L \times U(1)_Y$ gauge transformation containing Goldstone fields ξ^j:

$$\phi \to \phi' = U_\xi \, \phi = \frac{1}{\sqrt{2}} (v + H) \, \chi_V$$

This is the "Physical gauge"

H is the physical Higgs boson.
After Higgs-mechanism:
All transformed fields depend on the Goldstone fields ξ^i.
The vector bosons W^i_μ get an additional longitudinal term $\partial_\mu \xi^i$ and thereby obtains a mass

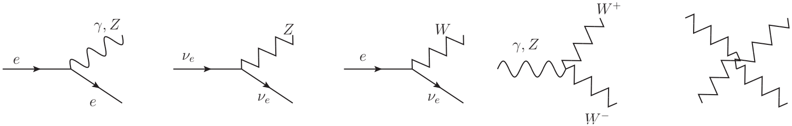


Figure: Electroweak vertices we shold have and some dictated by $SU(2)_L \times U(1)_Y$ gauge symmetry. There are also vertices with the Higgs boson H.

The couplings for the physical bosons $\gamma, Z, W^{\pm}$ will be

$$e \equiv g_\gamma \ = g \, sin\theta_W \ , g_Z \ = \ \frac{g}{cos\theta_W} \ , \ g_W \ = \ \frac{g}{\sqrt{2}} \ , \ tan\theta_W \ = \ \frac{g'}{g}$$

Partial Unification: Three couplings has become two !

Exp. : $sin^2\theta_W \simeq$0.23. From α_{em} and G_F, find $v \simeq$ 246 GeV.

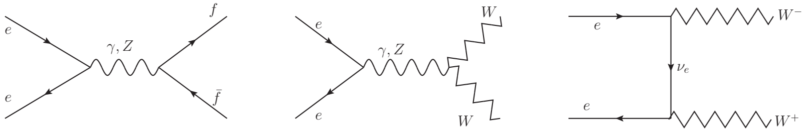


Figure: Examples: $ee \to f\bar{f}$ and $ee \to W^+ W^-$

Masses of W and Z-bosons (The photon γ remains massless)

$$M_W = \frac{1}{2} g\, v \;, M_Z = \frac{1}{2} g_Z\, v \,, \Rightarrow \frac{M_Z}{M_W} = \frac{1}{cos\theta_W}$$

This relation must hold experimentally!!! (-up to higher correct.)

Gauge inv. $\Rightarrow$ relations between couplings, masses

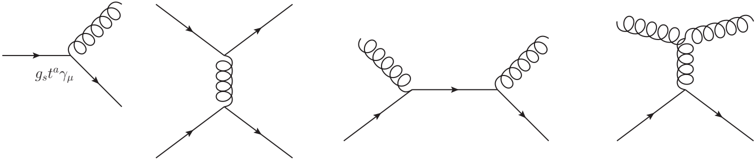
Quantum chromo dynamics (QCD)

$$\mathcal{L}_{QCD} = \overline{q}\,\gamma^{\mu}\left(i\partial_{\mu} - g_s t^a A^a_{\mu} - m_q\right) q - \frac{1}{4}F^{a,\mu\nu}F^a_{\mu\nu}$$

The field tensor an extra term

$$F^a_{\mu\nu} = (\partial_{\mu}A^a_{\nu} - \partial_{\nu}A^a_{\mu}) + g_s f^{abc}A^b_{\nu}A^c_{\nu}$$

- All the six quark fields q $(q = u, d, s, c, b, t)$ have 3 components (i.e 3 Dirac fields). t^a are generators (3×3-matrices) of the gauge group $SU(3)_c$)
- Important: Vector bosons interact among themselves! We obtain triple and quartic vertices. The triple gluon vertex has dramatic consequences for QCD compared to QED.
- Argument for (at least) three colors: Otherwise the paricle $\Omega^-(sss)$ cannot be explained! Remember the Pauli-principle !



Quark-quark and quark-gluon scattering

- QCD is significantly different from QED due to triplet coupling
- The gauge copling g_s is *universal*, the same for quark-gluon and triple gluon couplings
- Perturbative QCD breaks down for low energies (say 1-2 GeV). See later.

Higgs-production and decay

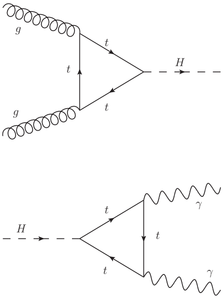


Figure: Feynman diagram for Higgs production through gluon fusion. And decay of Higgs to two photons

NB! : A proton at high energy contains gluons. Two protons collide, and the Higgs particle appear as a collision between a gluon from each proton

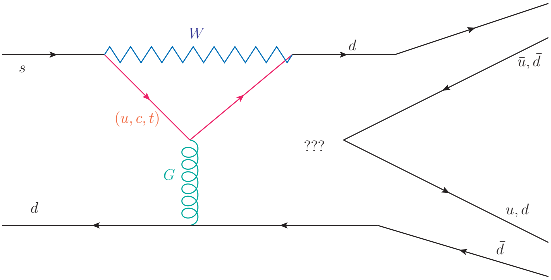


Figure: Example: “Penguin diagram”.Perturbative weak and strong interactions in play. AND Non-perturbative QCD

A diagram to illustrate $K_L \to \pi^+\pi^-$ -or $2\pi^0$. CP-violation is different for charged and neutral pions (The ϵ'-effect) The non-perturbative QCD part difficult...(Attacked by Lattice gauge theory).

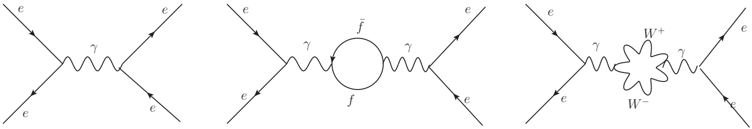


Figure: Diagrams explaining α_{em}, which grows with energy! $\alpha_{em} \simeq 1/129$ at energy = M_Z. Alternative: Modification of Coulomb potential

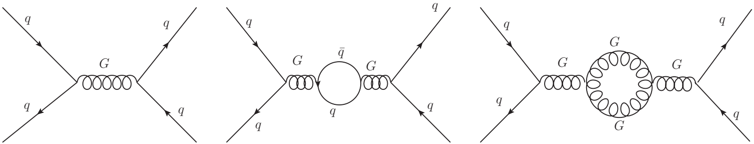


Figure: Diagrams explaining α_s, which become smaller with growing energy. The quark and the gluon loops have different signs. The gluon loop dominates.Dramatic consequenses.

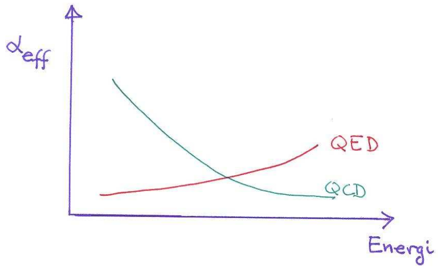


Figure: Qualitative behaviour of effective α_{em} and of effective α_s

*Perturbative QCD breaks down at small energies! Other methods needed. Quark models ?, Lattice gauge theory
Exp.: $\alpha_s(M_Z^2) \simeq 0.12 \; \Rightarrow \alpha_s((1GeV)^2) \simeq \; 0.5\,(\pm 10\%)$,
$\alpha_s((0.7GeV)^2) \simeq 0.6$ to 1.2

Breakdown of $SU(3)_L \times SU(3)_R$ - symmetry

* Chiral sym. L-R symm breaks down $\Rightarrow$ Goldsone bosons (π, K, η)
For light sector $q = (u, d, s)$ $(q = q_L + q_R)$

$$\mathcal{L}_{QCD} = \overline{q_L}\,(\gamma \cdot iD)\; q_L + \overline{q_R}\,(\gamma \cdot iD)\; q_R + m_q(\overline{q_L}\, q_R + \overline{q_R}\, q_L) + \mathcal{L}_G$$

is (for $m_q \to 0$) symmetric under the unitary transf.

$$q_L \to V_L\, q_L \quad ;\; q_R \to V_R\, q_R \quad ;\; V_L^\dagger\, V_L = V_R^\dagger\, V_R = 1\,;\; V_L^\dagger V_R \neq 1\,.$$

SSB in QCD (NB. : No parity doublets!) $\Rightarrow$ quark condensate

$$\langle \bar{q}q \rangle \equiv \langle\, 0|\, \bar{q}q\, |\, 0 \rangle \simeq (-240\,\mathrm{MeV})^3 \neq 0$$

QCD has a non-trivial vacuum! $\Rightarrow SU_L(3) \times SU_R(3)$ symmetry of QCD (for $m_q \to 0$) *breaks down*.
Gluon condensate textcolorred $\langle \frac{\alpha_s}{\pi} G^2 \rangle \sim (300 \text{ to } 400\text{ MeV})^4 \neq 0$

Chiral perturbation theory

The meson octet (π, K, η) are Pseudo-Goldstone bosons

$$\mathcal{L}_{\chi PT} = \frac{f_\pi^2}{4} Tr[(\partial_\mu U)(\partial^\mu U^\dagger)] + ... \qquad (1)$$

where f_π is the decay constant from the decay $\pi \to \mu\nu_\mu$

$$U \equiv exp(i\lambda^a \pi^a) \qquad (2)$$

Sum runs over octet (π, K, η), λ^a are generators for $SU(3)_F$

- One has: $m_\pi^2 \sim -\langle \overline{q}q \rangle m_{u,d}/f_\pi^2$,
- $SU(3)_F$-symmetric Chiral perturbation theory (χPT)for mesons and baryons contains a lot of terms , and is *Non-renormalizable*.
- Chiral quark model (χQM) can be used to determine some coefficients of terms.(χQM connects quarks to mesons)

The SM is robust

Valid as far as we can measure

BUT: pert. QCD breaks down at low energies.
Lattice gauge theory may/should solve the problem

Gauge-symmetric theories chosen because:

Gauge theories are *renormalizable*-and have a minimum of parameters.

Various speculations beyond the SM

Thank you !

After the $SU(2)_L \times U(1)_Y$ transf. U_ξ has been applied. Electroweak interactions between fermion and gauge fields should be:

$$\left(\mathcal{L}_{fEW}^{int}\right)' = -g_\gamma \, j_\mu^{em} \, A^\mu - g_Z \, j_\mu^Z \, Z^\mu$$
$$- g_W \left(j_\mu^{(+)} \, W^{(-)\mu} + j_\mu^{(-)} \, W^{(+)\mu} \right)$$

Physical weak and electromagnetic currents are known (for first generation):

$$j_\mu^{em} = \sum_f Q_f \, \overline{f} \gamma_\mu f = -\overline{e} \, \gamma_\mu \, e + \frac{2}{3} \overline{u} \, \gamma_\mu \, u - \frac{1}{3} \overline{d} \, \gamma_\mu \, d$$

$$j_\mu^{(+)} = \overline{\nu_e} \, \gamma_\mu \, L \, e + \overline{u} \, \gamma_\mu \, L \, d$$
$$j_\mu^{(-)} = \overline{e} \, \gamma_\mu \, L \, \nu_e + \overline{d} \, \gamma_\mu \, L \, u \quad ;$$

where $L = (1 - \gamma_5)/2$, and the fermion fields are understood to be the weak eigenstates $f = f_W = f'_L + f'_R$.

$W^{\pm}$ are linear combinations of U_ξ transf. W^{a}:

$$W_\mu^{(\pm)} = \frac{1}{\sqrt{2}} \left(W_\mu^1 \pm i\, W_\mu^2\right)'$$

while $g_W = g/\sqrt{2}$, and the weak currents are

$$j_\mu^{(\pm)} = \left(j_\mu^1 \pm i\, j_\mu^2\right)'$$

The photon A_μ and Z-boson field Z_μ are linear combination of the transformed W_μ^3 and B_μ bosons:

$$A_\mu = c_W\, B_\mu' + s_W\, (W_\mu^3)' \; ;$$
$$Z_\mu = -\, s_W\, B_\mu' + c_W\, (W_\mu^3)'$$

$s_W \equiv sin\theta_W$ and $c_W \equiv cos\theta_W$, and θ_W = weak mixing angle
Requirement to obtain the correct electromagnetic current:

$$tg\theta_W = \frac{g'}{g} \; ; \; Q_f = (I_W^3)_f + \frac{1}{2}\, Y_f$$

consistent with

$$j_\mu^{em} = \left(j_\mu^3 + \frac{1}{2} j_\mu^Y\right)'$$

The elementary electric charge (= photon coupling) will be

$$g_\gamma = g \, sin\theta_W$$

The neutral current j_μ^Z *times* the coupling g_Z is now *completely determined*! One chooses

$$g_Z = g/cos\theta_W$$

$$j_\mu^Z = (j_\mu^3)' - sin^2\theta_W \, j_\mu^{em} \; ,$$

$$(j_\mu^3)' = \frac{1}{2}\left(\overline{\nu_e}\,\gamma_\mu \, L \, \nu_e - \overline{e}\,\gamma_\mu \, L \, e\right) + \frac{1}{2}\left(\overline{u}\,\gamma_\mu \, L \, u - \overline{d}\,\gamma_\mu \, L \, d\right) \; ,$$

where the fermion fields are understood to be the U_ξ transformed ones, $f = f_W = f_L' + f_R'$. Experimentally $sin^2\theta_W \simeq 0.23$