UiO : Department of Physics
University of Oslo

Introduction to Supersymmetry

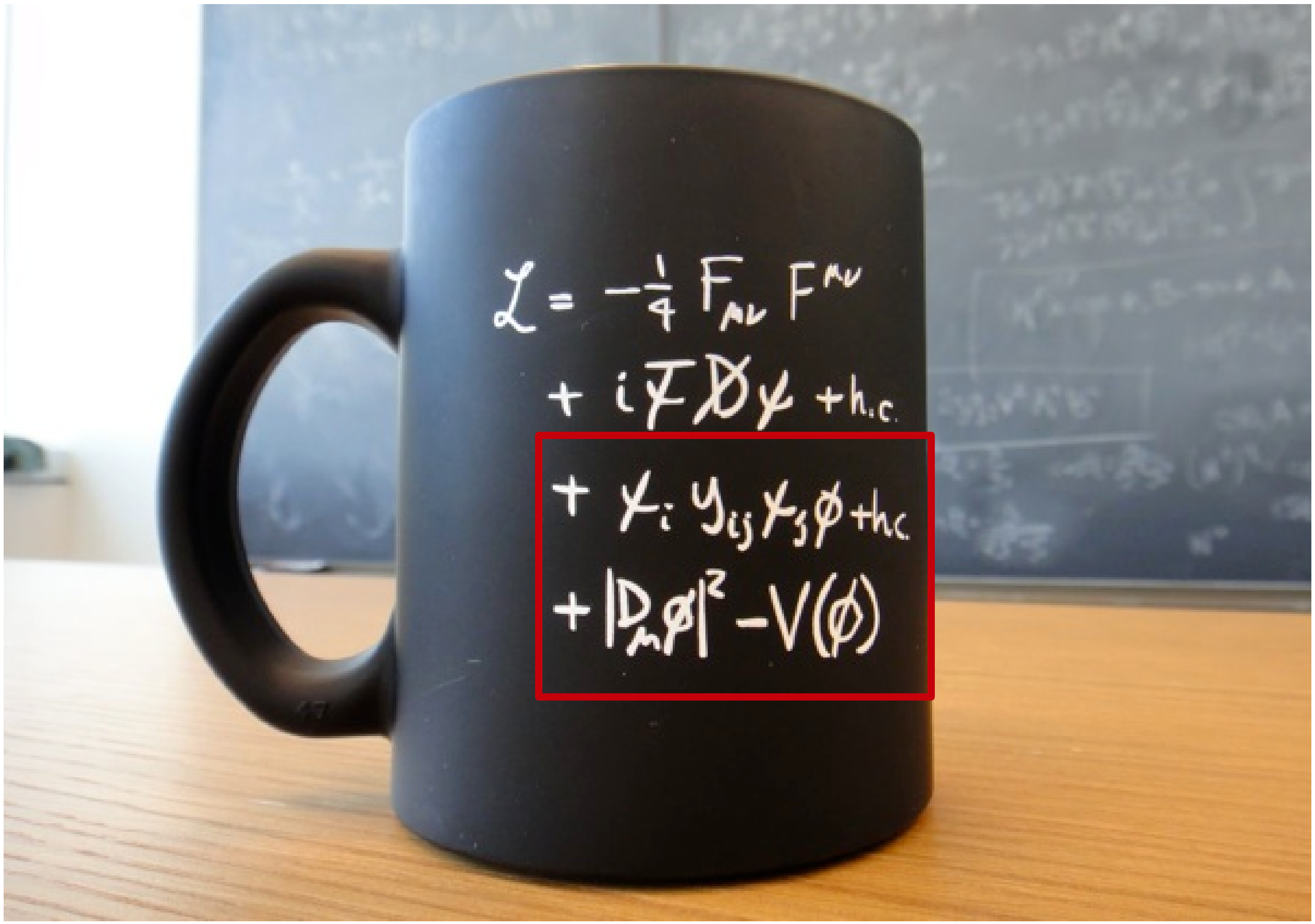
Are Raklev

Overview

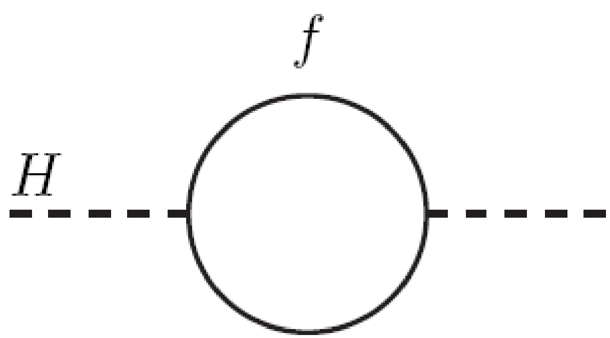
- Some motivation
- What is Supersymmetry?
- The Minimal Supersymmetric Standard Model
- Phenomenology

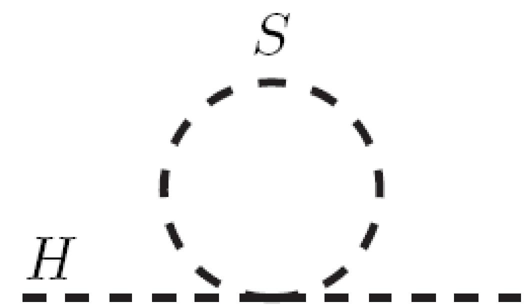
Motivation

Cup of coffee



The problem with the Higgs





$$\Delta m_H^2 = \frac{|\lambda_f|^2}{8\pi^2}\Lambda_{UV}^2 + \ldots \qquad \Delta m_H^2 = -\frac{\lambda_S}{16\pi^2}\Lambda_{UV}^2 + \ldots$$

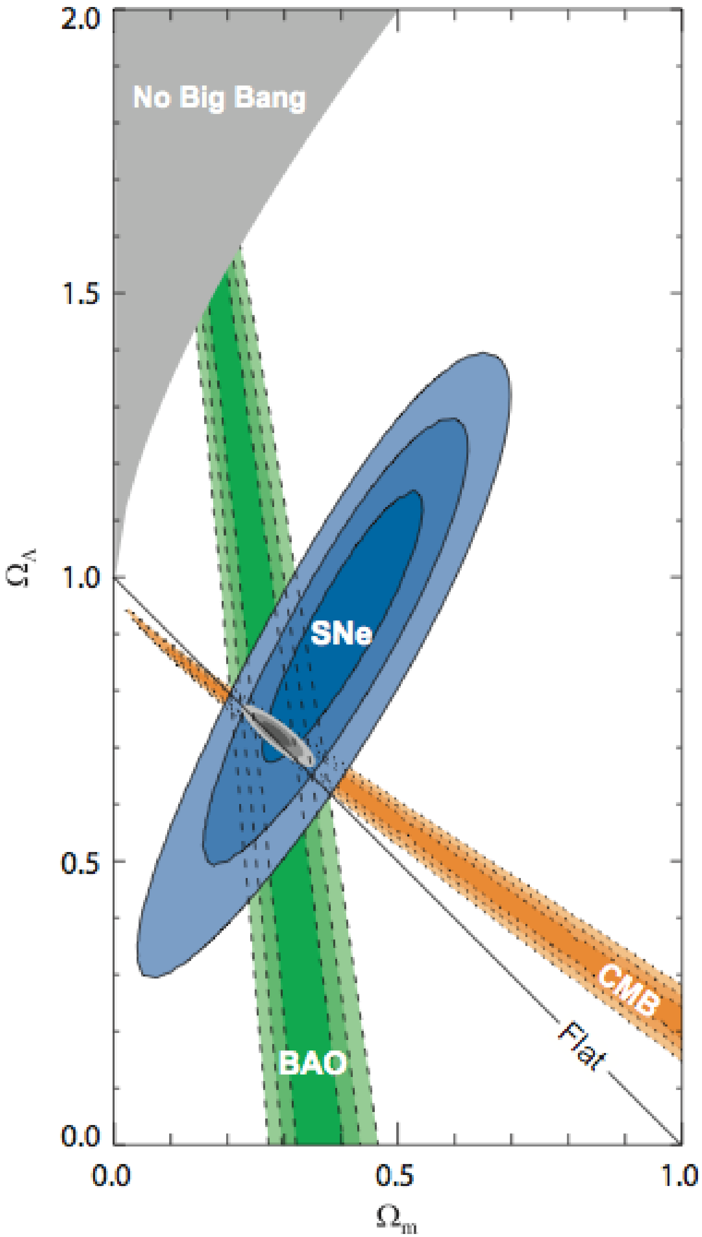
Planck scale cut-off: $\Lambda_{UV} = M_P \sim 10^{18}\ \text{GeV}$

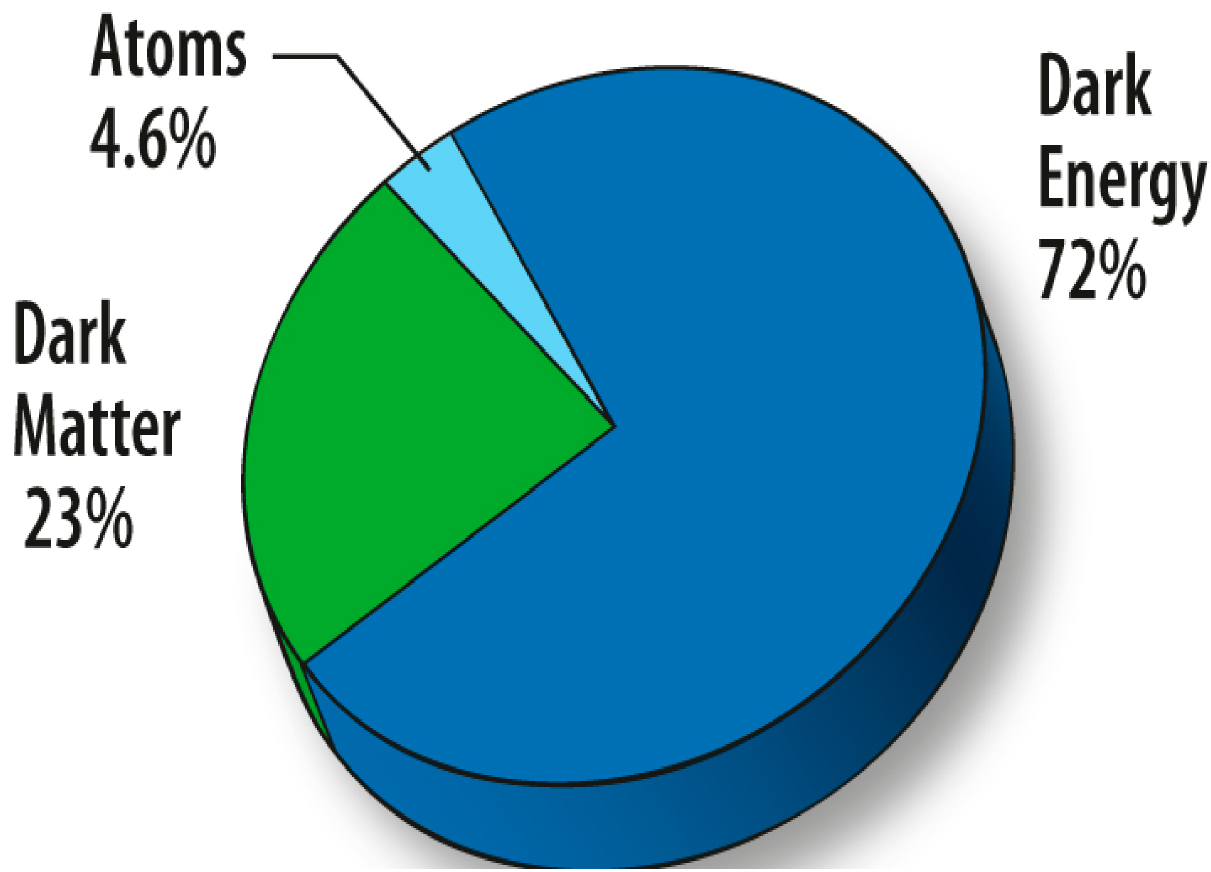
We need cancelations of the order of 10^{16}:
the Hierarchy problem

The problem with the Higgs

- So, what **are** the loopholes?
  1) There are no particles above the electroweak scale.
  2) The Higgs is not elementary.
  3) Some new physics ensures an exact cancellation.

The Universe pie





What is SUSY?

Symmetries

- We know Einstein's (Poincaré's) symmetry well. Lengths of **external** four-vectors are invariant:

$$x^{\mu} \rightarrow x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + a^{\mu}, \quad (x' - y')^2 = (x - y)^2$$

- The SM also has **internal** gauge symmetries:

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

- Can the Poincaré symmetry be extended? Yes
- Can we unify internal and external symmetries? Yes, but not the way we would have liked...

Symmetries

- Continuous symmetries are described by a Lie group and its algebra (relations of generators).
- The Poincaré algebra:

$$
\begin{aligned}
[P_\mu, P_\nu] &= 0 \\
[M_{\mu\nu}, P_\rho] &= -i(g_{\mu\rho}P_\nu - g_{\nu\rho}P_\mu) \\
[M_{\mu\nu}, M_{\rho\sigma}] &= -i(g_{\mu\rho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\rho})
\end{aligned}
$$

- *No-go theorem*, Coleman & Mandula (1967).
- Haag, Lopuszanski and Sohnius (1975): allow for anti-commutators in algebra.

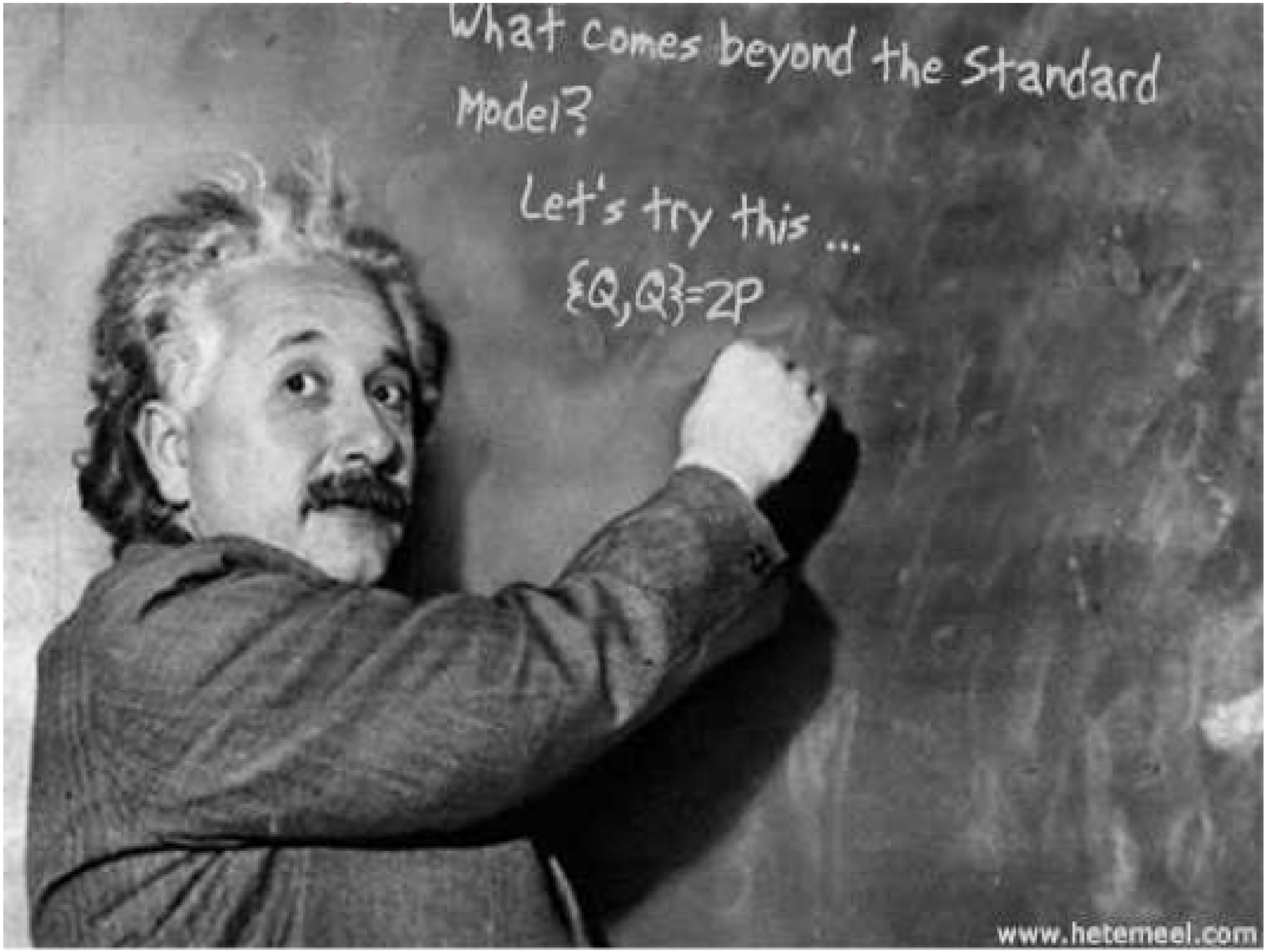
Supersymmtry

- Introduce new generator Q_a (a Majorana spinor) mapping fermion states to bosons and back:

$$Q|\mathrm{fermion}\rangle = |\mathrm{boson}\rangle, \;\; Q|\mathrm{boson}\rangle = |\mathrm{fermion}\rangle$$

- The (N=1) super-Poincaré algebra:

$$\begin{aligned}
[P_\mu, P_\nu] &= 0 \\
[M_{\mu\nu}, P_\rho] &= -i(g_{\mu\rho}P_\nu - g_{\nu\rho}P_\mu) \\
[M_{\mu\nu}, M_{\rho\sigma}] &= -i(g_{\mu\rho}M_{\nu\sigma} - g_{\mu\sigma}M_{\nu\rho} - g_{\nu\rho}M_{\mu\sigma} + g_{\nu\sigma}M_{\mu\rho}) \\
[Q_a, P_\mu] &= 0 \\
[Q_a, M_{\mu\nu}] &= (\sigma_{\mu\nu}Q)_a \\
\{Q_a, \bar{Q}_b\} &= 2\not{P}_{ab}
\end{aligned}$$


What comes beyond the Standard Model?
Let's try this ...
{Q,Q}=2P
www.hetemeel.com

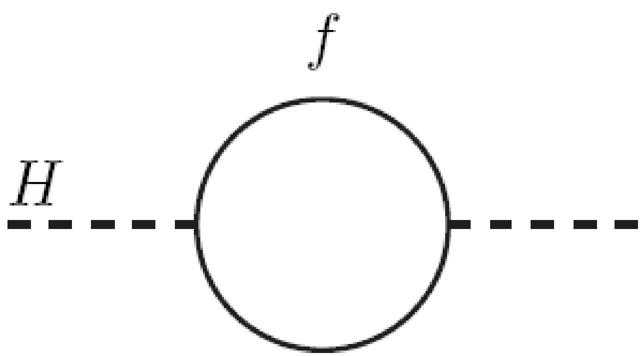
Supersymmtry

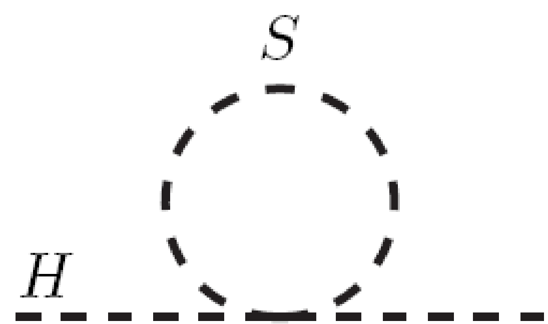
- Some immediate consequences:
  - Equal number of fermion and boson states (not particles!)
  - New partner states inherit couplings.
  - And have the same mass.
- These properties are vital to the solution of the Higgs hierarchy problem.
- But where have all the bosons gone?

Supersymmetry to the rescue

- Supersymmetry predicts that there are twice the number of S compared to f, and that

$$|\lambda_f|^2 = \lambda_S$$





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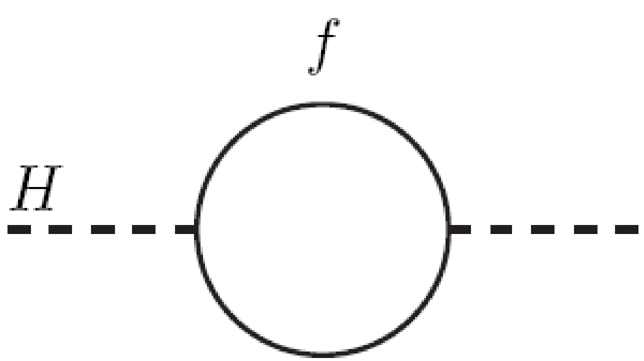
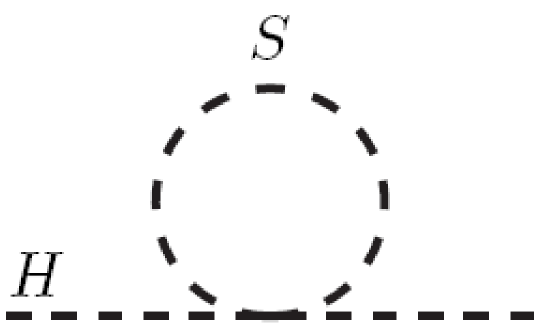




$$\Delta m_h^2 = 0 + \ldots$$

Spontaneous SUSY breaking

- Just as in the SM Higgs mechanism we can use the scalar potential of the theory to break SUSY.
- However, we are limited by the supertrace relation

$$\mathrm{STr}\ M^2 = \sum_s (-1)^{2s}(2s+1)\ \mathrm{Tr}\ M_s^2 = 0$$

- All new scalars can not be heavier than all the fermions!
- Solution: put breaking at a high scale with extra fermions.

Spontaneous SUSY breaking

- We parametrize our ignorance of the exact mechanism by adding SUSY breaking terms by hand

$$\mathcal{L}_{\rm soft} = -\frac{1}{2}M\lambda^A\lambda_A - (\frac{1}{6}a_{ijk}A_iA_jA_k + \frac{1}{2}b_{ij}A_iA_j + t_iA_i + c.c.) - m_{ij}^2 A_i^* A_j$$

- These are the **soft breaking terms** (do not reintroduce the hierarchy problem).

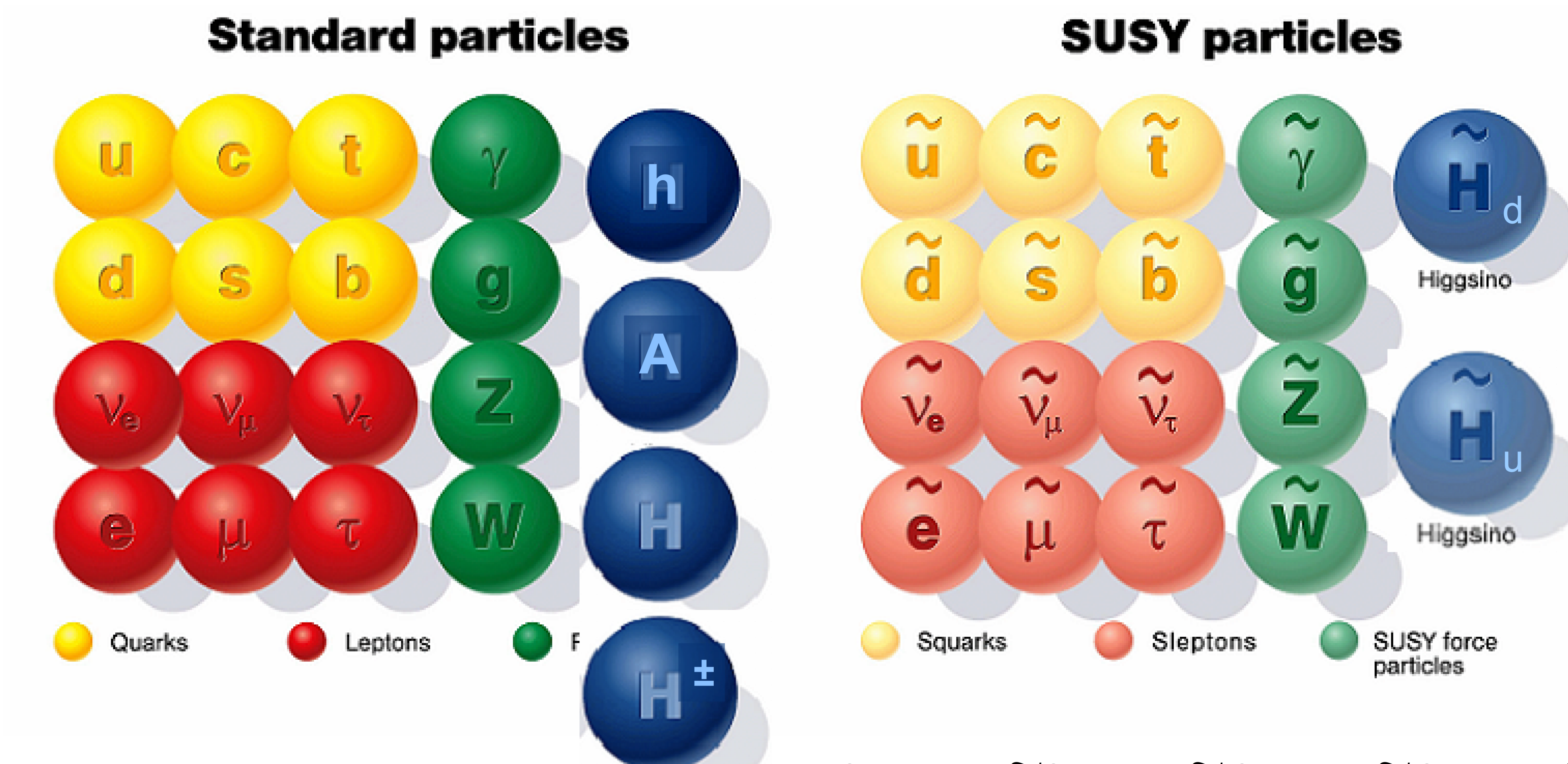
Softly broken SUSY: $\Delta m_h^2 = \frac{\lambda}{16\pi^2} m_{\rm SUSY}^2 \ln \frac{\Lambda_{UV}^2}{m_{\rm SUSY}^2} + \ldots$

Minimal Supersymmetric Standard Model

MSSM

- The Minimal Supersymmetric Standard Model (MSSM) is the smallest model in terms of fields that contain all SM particles.
- In addition to the partners of all SM particles it is necessary to introduce two Higgs doublets.
  - Anomaly cancellation.
  - Give mass to both up- and down-type quarks.

MSSM



Neutralinos $\tilde{\chi}_i^0 = N_{i1}\tilde{B}^0 + N_{i1}\tilde{W}^0 + N_{i3}\tilde{H}_u^0 + N_{i4}\tilde{H}_d^0$

Charginos $\tilde{\chi}_i^+ = C_{i1}\tilde{W}^+ + C_{i1}\tilde{H}_u^+$

R-parity

- To remove lepton & baryon number violating interactions we introduce a new multiplicative quantum number R-parity

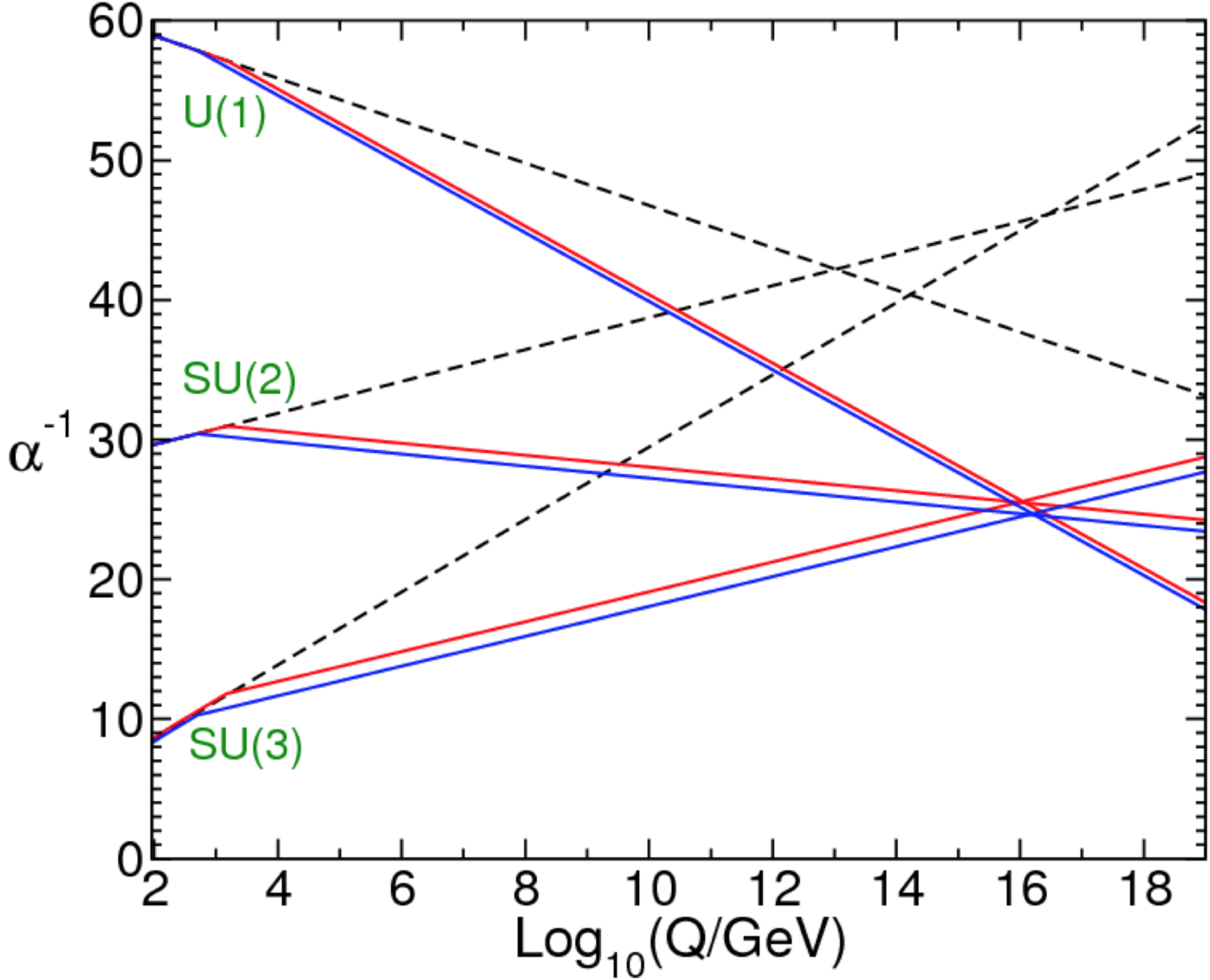
$$R=(-1)^{3B+L+2s}$$

  - All interactions have an even number of sparticles.
  - Sparticles can only be pair-produced.
  - The lightest sparticle (LSP) is absolutely stable. (Usually the lightest neutralino.)

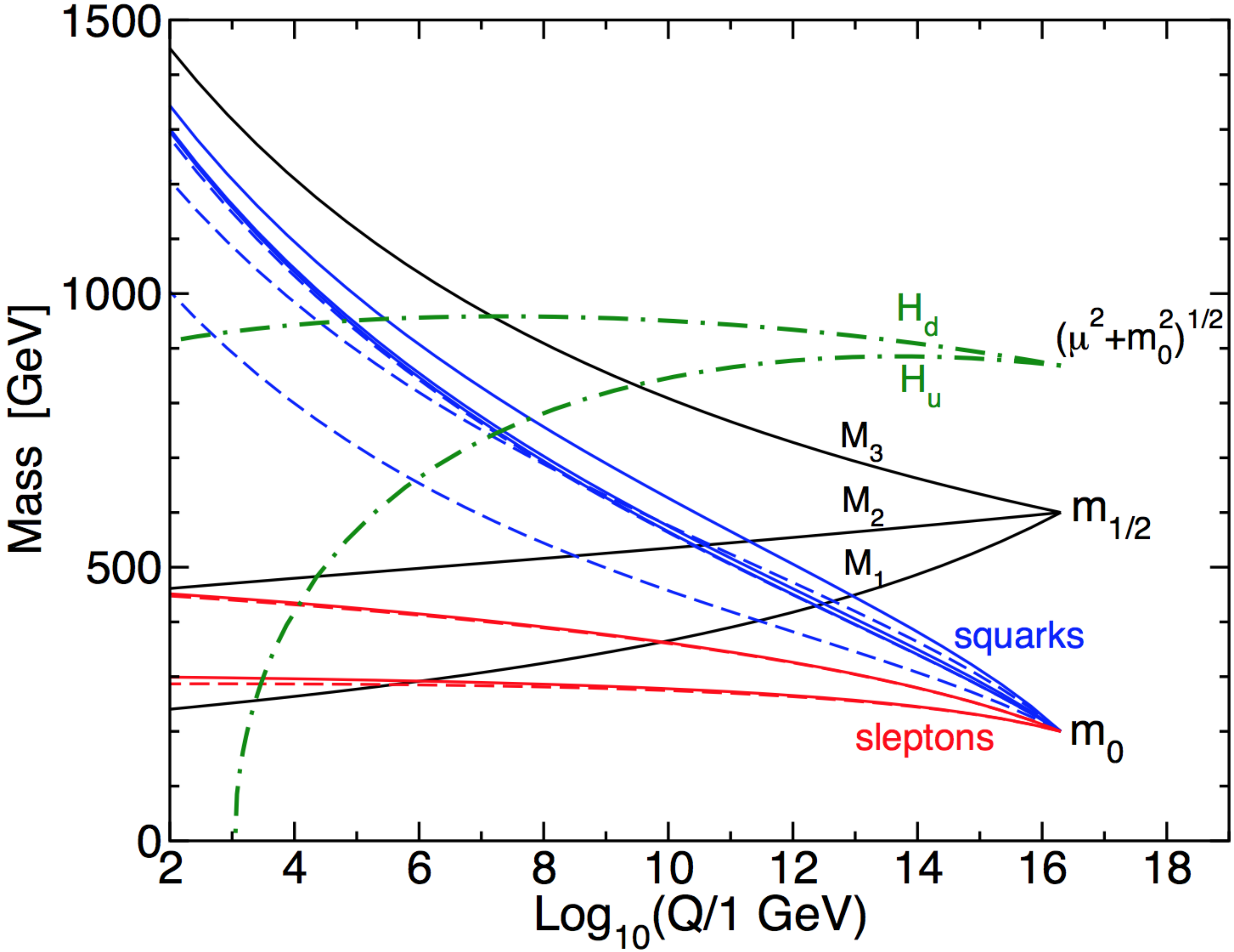
MSSM

- The extra Higgs doublet adds only one new parameter μ coupling the two Higgs doublets.
- For historical reasons no neutrino mass. (Or right handed neutrinos.)
- There are 104 new soft breaking parameters!

  (In addition to 19 free parameters in the SM)
- Does this ruin predictability?


60
50
40
30
20
10
0
2
4
6
8
10
12
14
16
18
α^{-1}
$Log_{10}(Q/GeV)$
U(1)
SU(2)
SU(3)

GUT-motivated models



CMSSM

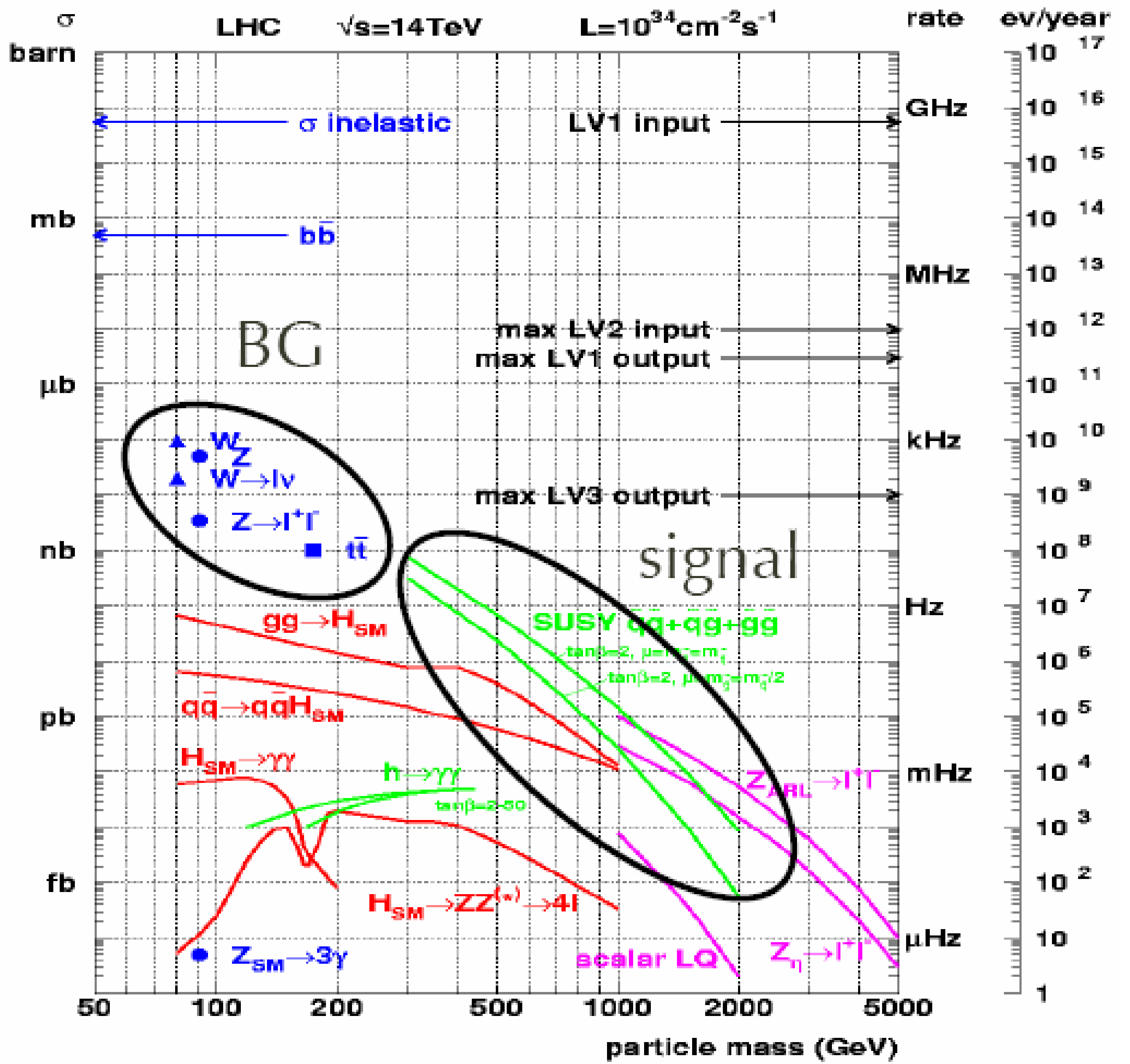
m_0 = 200 GeV

$m_{1/2}$ = 600 GeV

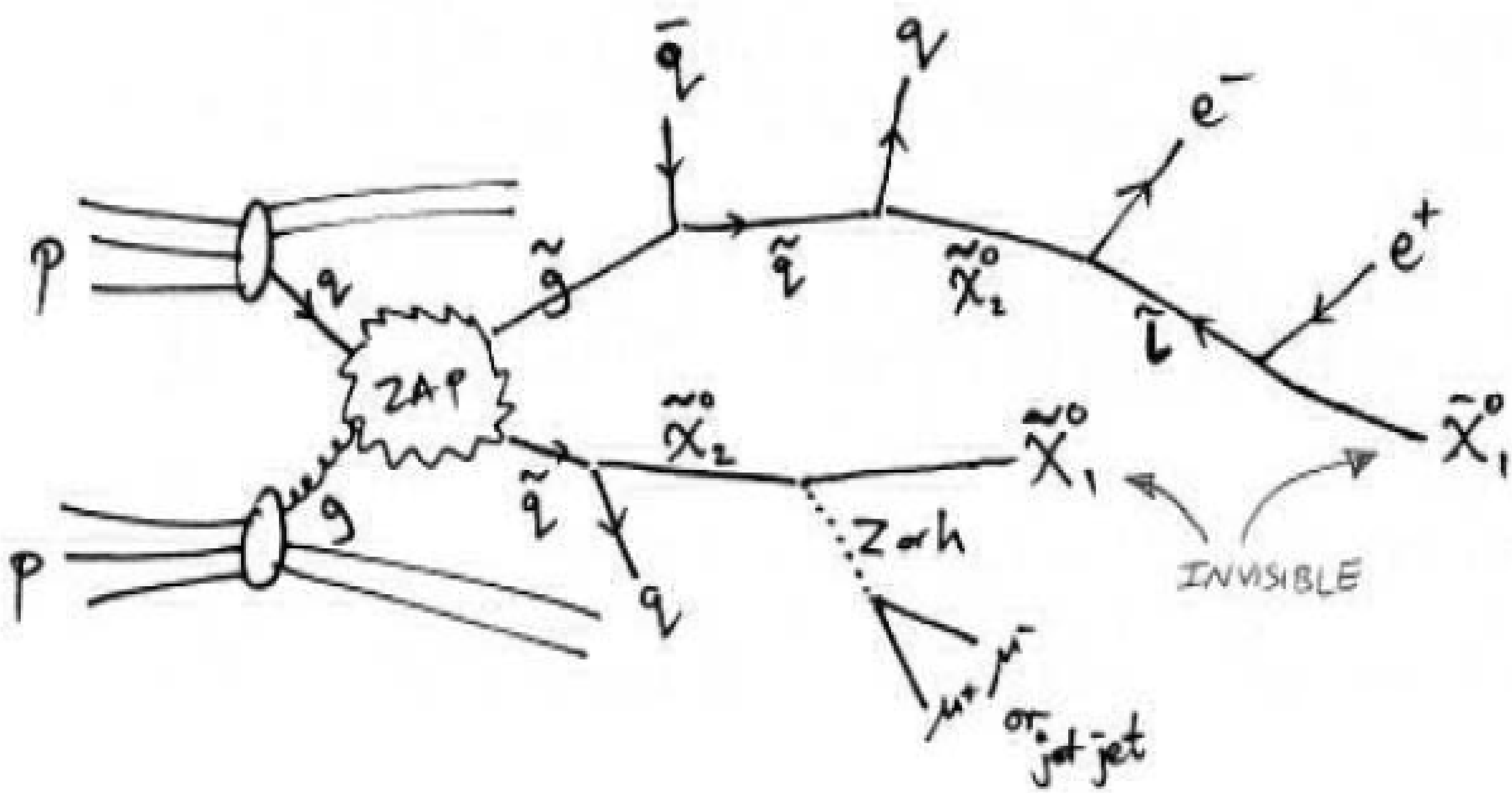
A_0 = 0 GeV

tan β = 10

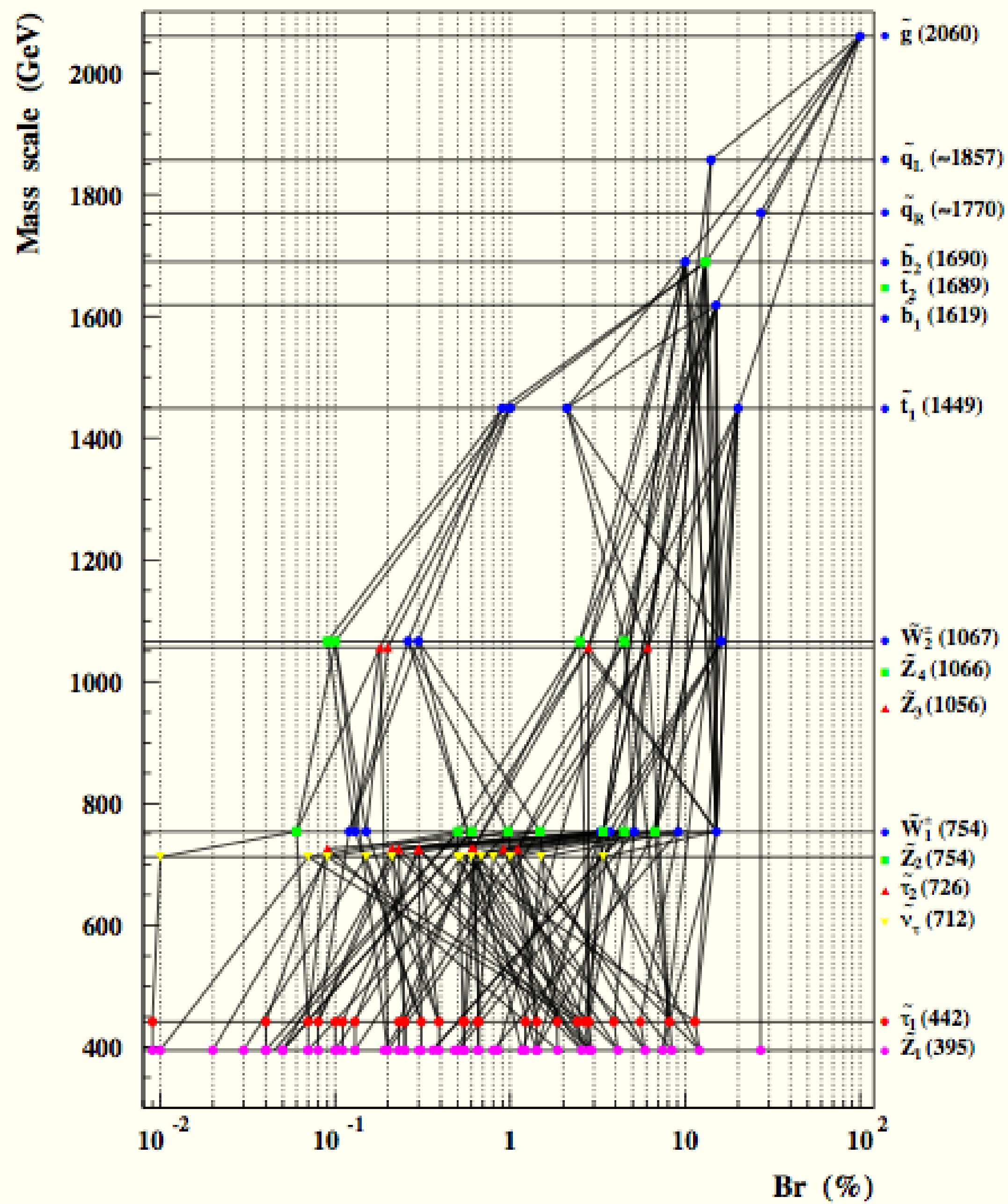
Phenomenology


σ
barn
LHC
√s=14TeV
L=10³⁴cm⁻²s⁻¹
rate
ev/year
σ inelastic
LV1 input
GHz
mb
bb̄
MHz
BG
max LV2 input
max LV1 output
μb
kHz
W
Z
W→lν
max LV3 output
Z→l⁺l⁻
nb
tt̄
signal
Hz
gg→H_SM
SUSY q̃q̃+q̃g̃+g̃g̃
qq̄→qq̄H_SM
pb
H_SM→γγ
h→γγ
mHz
fb
H_SM→ZZ(*)→4l
μHz
Z_SM→3γ
scalar LQ
Z_η→l⁺l⁻
Z_ARL→l⁺l⁻
50
100
200
500
1000
2000
5000
particle mass (GeV)

SUSY cascades

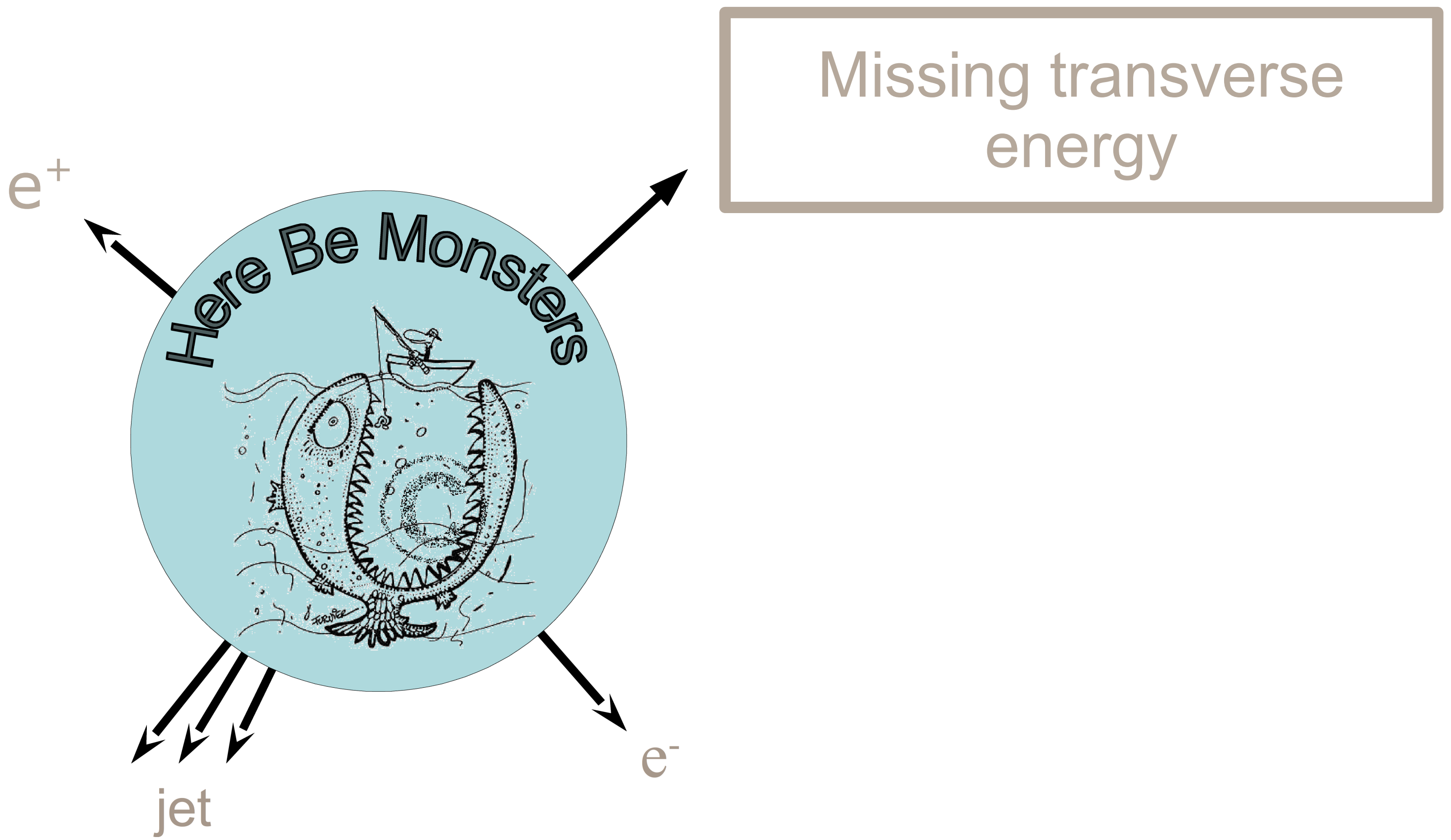


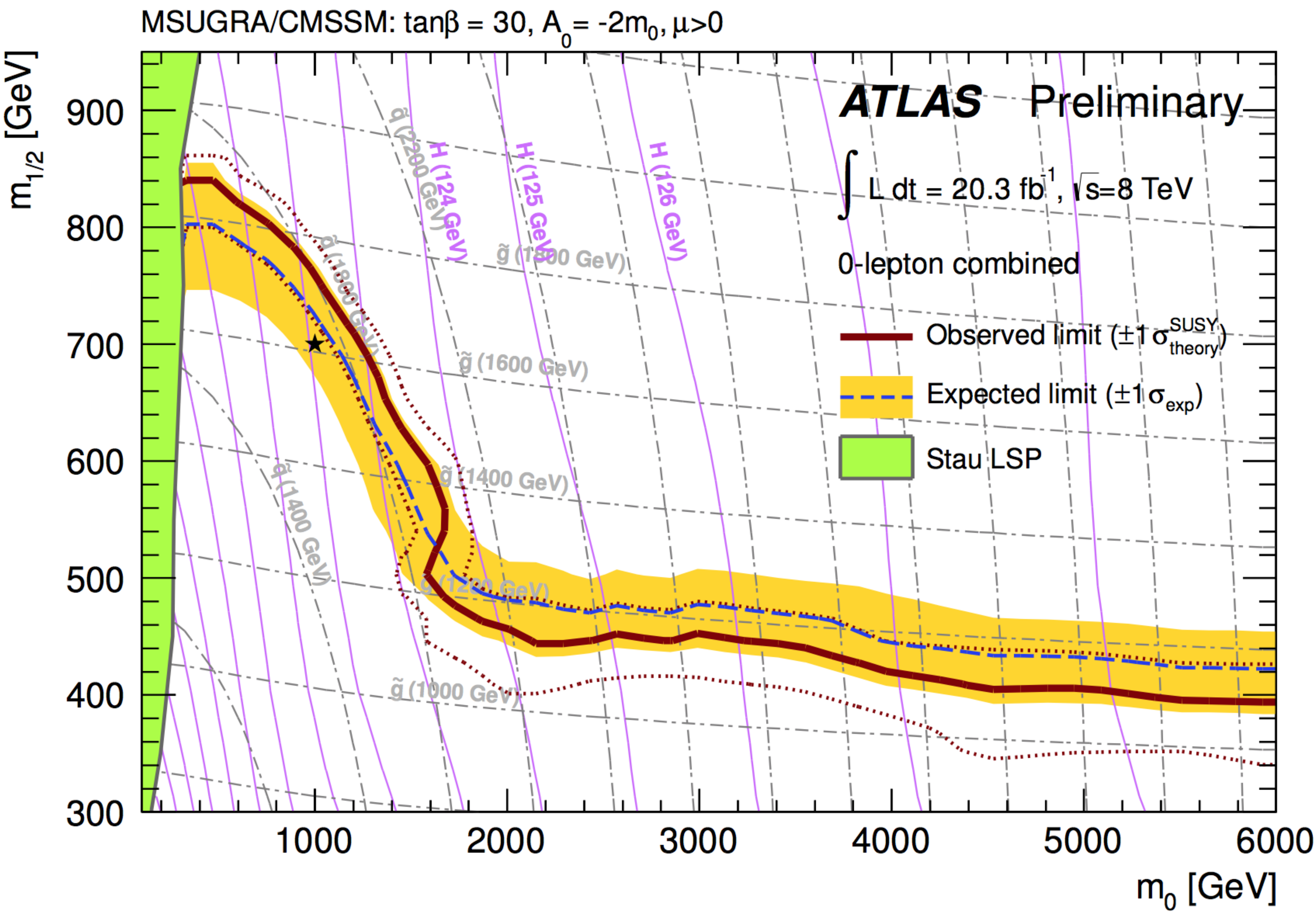
[Drawing by Chris Lester]

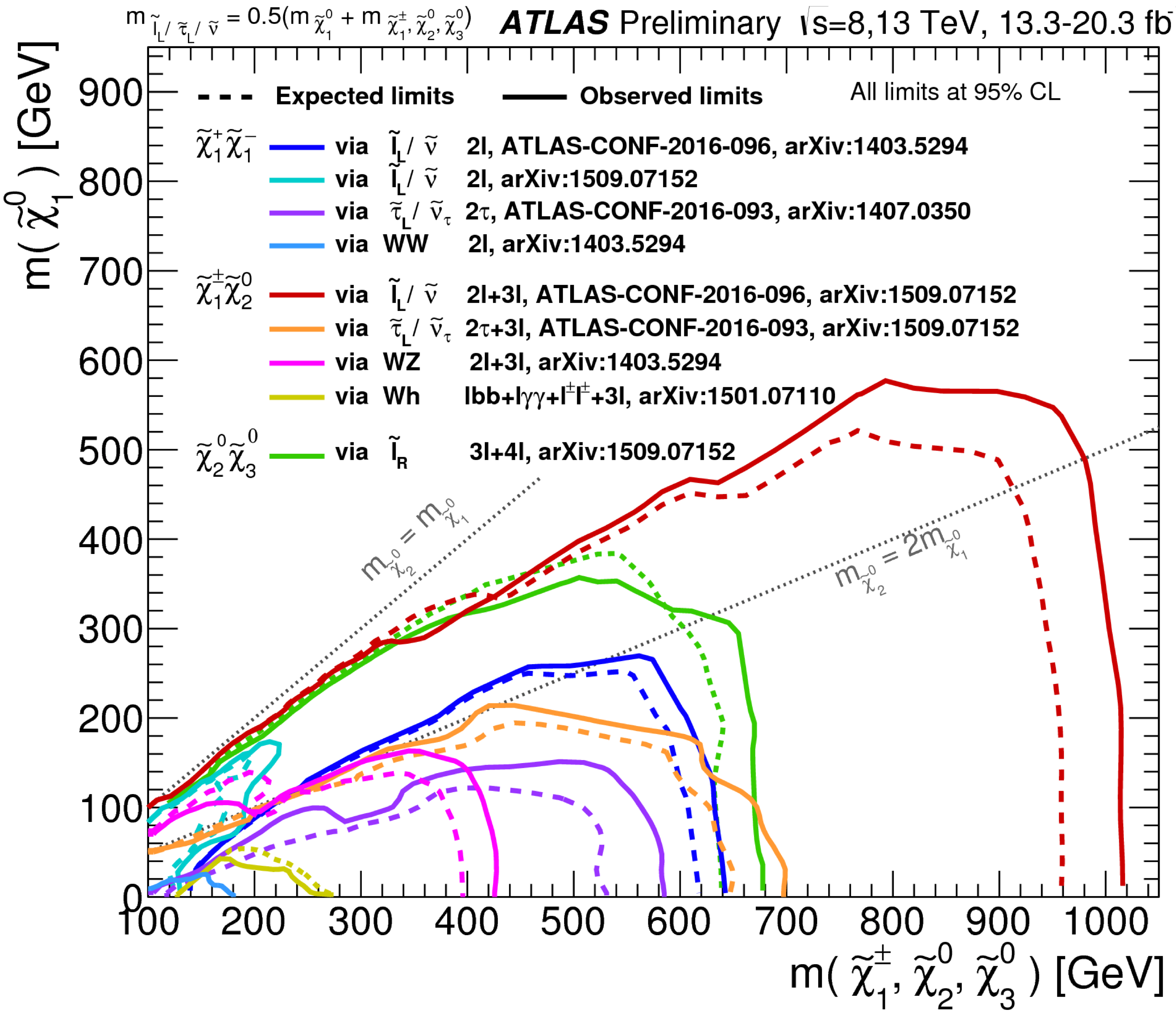


$\tilde{Z}_1$ qq	(27.0 %)	$\tilde{Z}_1$ τνWWWbb	(4.1 %)
$\tilde{Z}_1$ τνWbb	(12.1 %)	$\tilde{Z}_1$ ττbb	(2.9 %)
$\tilde{Z}_1$ ττWWbb	(8.4 %)	$\tilde{Z}_1$ ττqq	(2.9 %)
$\tilde{Z}_1$ WWbb	(7.4 %)	$\tilde{Z}_1$ τνZWbb	(2.8 %)
$\tilde{Z}_1$ τνqq	(5.9 %)	$\tilde{Z}_1$ τνhWbb	(2.6 %)

Realistic detector




MSUGRA/CMSSM: tanβ = 30, A0 = -2m0, μ>0
ATLAS Preliminary
∫ L dt = 20.3 fb-1, √s=8 TeV
0-lepton combined
Observed limit (±1 σ SUSY theory)
Expected limit (±1 σexp)
Stau LSP
m1/2 [GeV]
m0 [GeV]
900
800
700
600
500
400
300
1000
2000
3000
4000
5000
6000
g̃ (2200 GeV)
g̃ (1800 GeV)
g̃ (1600 GeV)
g̃ (1400 GeV)
g̃ (1200 GeV)
g̃ (1000 GeV)
H (124 GeV)
H (125 GeV)
H (126 GeV)



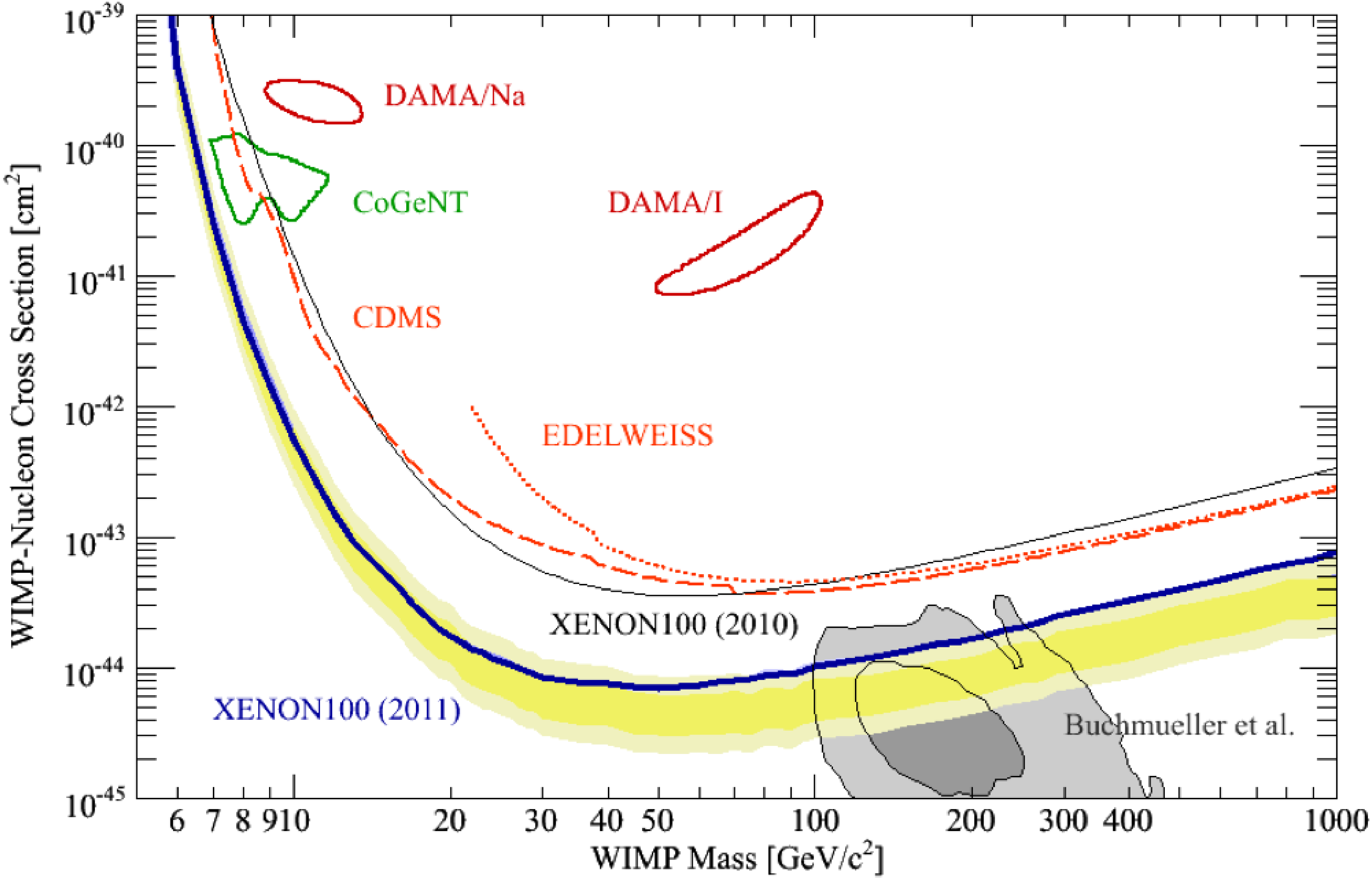
ATLAS Preliminary √s=8,13 TeV, 13.3-20.3 fb⁻¹
Expected limits
Observed limits
All limits at 95% CL
via l̃_L / ν̃ 2l, ATLAS-CONF-2016-096, arXiv:1403.5294
via l̃_L / ν̃ 2l, arXiv:1509.07152
via τ̃_L / ν̃_τ 2τ, ATLAS-CONF-2016-093, arXiv:1407.0350
via WW 2l, arXiv:1403.5294
via l̃_L / ν̃ 2l+3l, ATLAS-CONF-2016-096, arXiv:1509.07152
via τ̃_L / ν̃_τ 2τ+3l, ATLAS-CONF-2016-093, arXiv:1509.07152
via WZ 2l+3l, arXiv:1403.5294
via Wh lbb+lγγ+l±l±+3l, arXiv:1501.07110
via l̃_R 3l+4l, arXiv:1509.07152
[GeV]

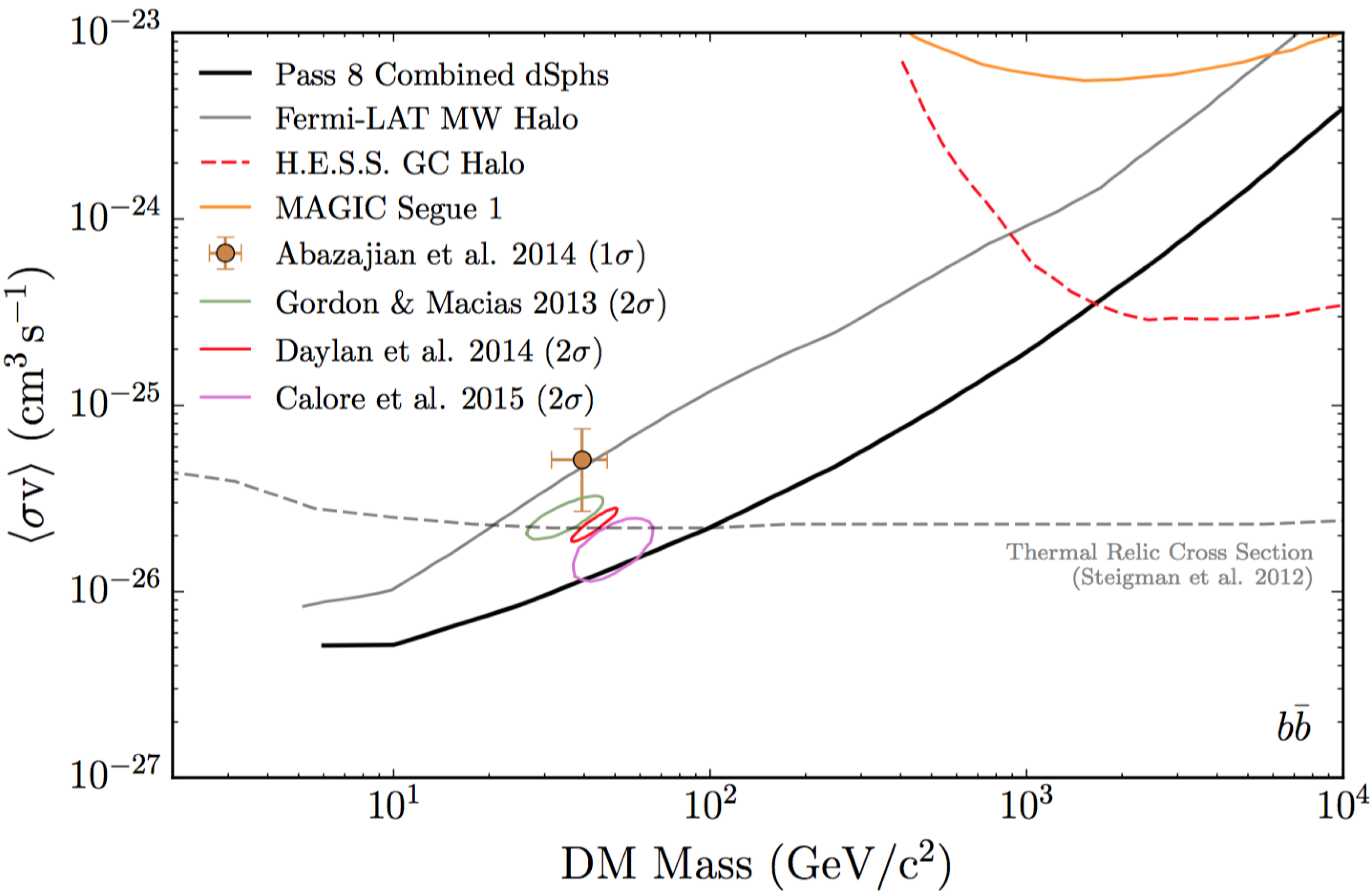
Neutralino Dark Matter

- A neutralino LSP is a good Dark Matter candidate:
  - It is stable (R-parity).
  - It has no charge (electric or strong).
  - It is weakly interacting $\Rightarrow$ WIMP candidate.

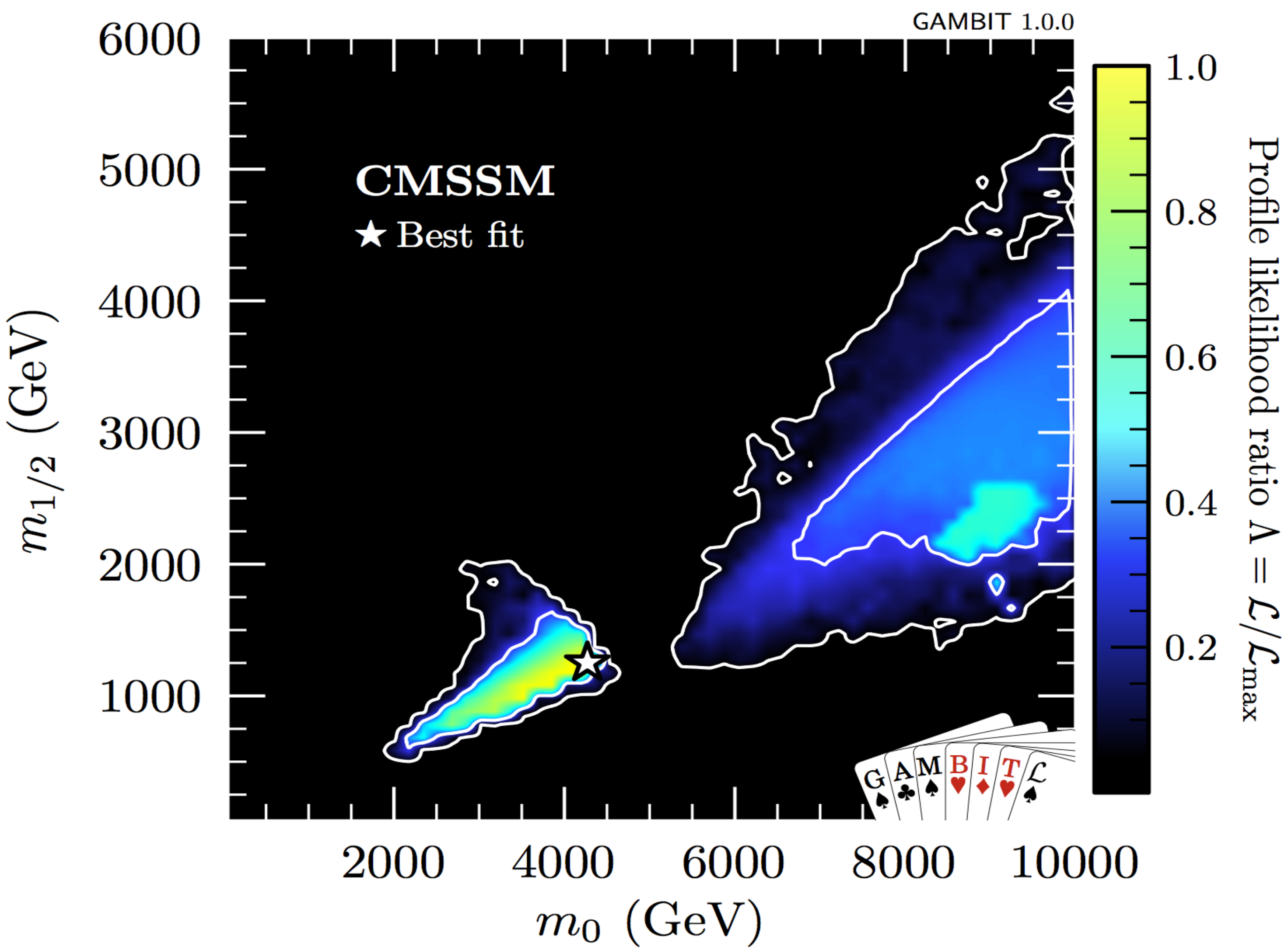
$$\Omega h^2 \simeq 0.1 \times \frac{3 \cdot 10^{-26}\,\mathrm{cm}^3\,\mathrm{s}^{-1}}{\langle \sigma v \rangle}$$

$$\langle \sigma v \rangle \approx \frac{\alpha^2_{\mathrm{weak}}}{m^2_{\mathrm{weak}}} \approx 10^{-25}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$$


DAMA/Na
CoGeNT
DAMA/I
CDMS
EDELWEISS
XENON100 (2010)
XENON100 (2011)
Buchmueller et al.
WIMP-Nucleon Cross Section [cm^2]
WIMP Mass [GeV/c^2]
10^{-39}
10^{-40}
10^{-41}
10^{-42}
10^{-43}
10^{-44}
10^{-45}
6 7 8 910 20 30 40 50 100 200 300 400 1000


Pass 8 Combined dSphs
Fermi-LAT MW Halo
H.E.S.S. GC Halo
MAGIC Segue 1
Abazajian et al. 2014 (1σ)
Gordon & Macias 2013 (2σ)
Daylan et al. 2014 (2σ)
Calore et al. 2015 (2σ)
Thermal Relic Cross Section
(Steigman et al. 2012)
$b\bar{b}$
$\langle\sigma v\rangle$ (cm^3 s^{-1})
10^{-23}
10^{-24}
10^{-25}
10^{-26}
10^{-27}
10^1
10^2
10^3
10^4
DM Mass (GeV/c^2)

Final slide (almost)



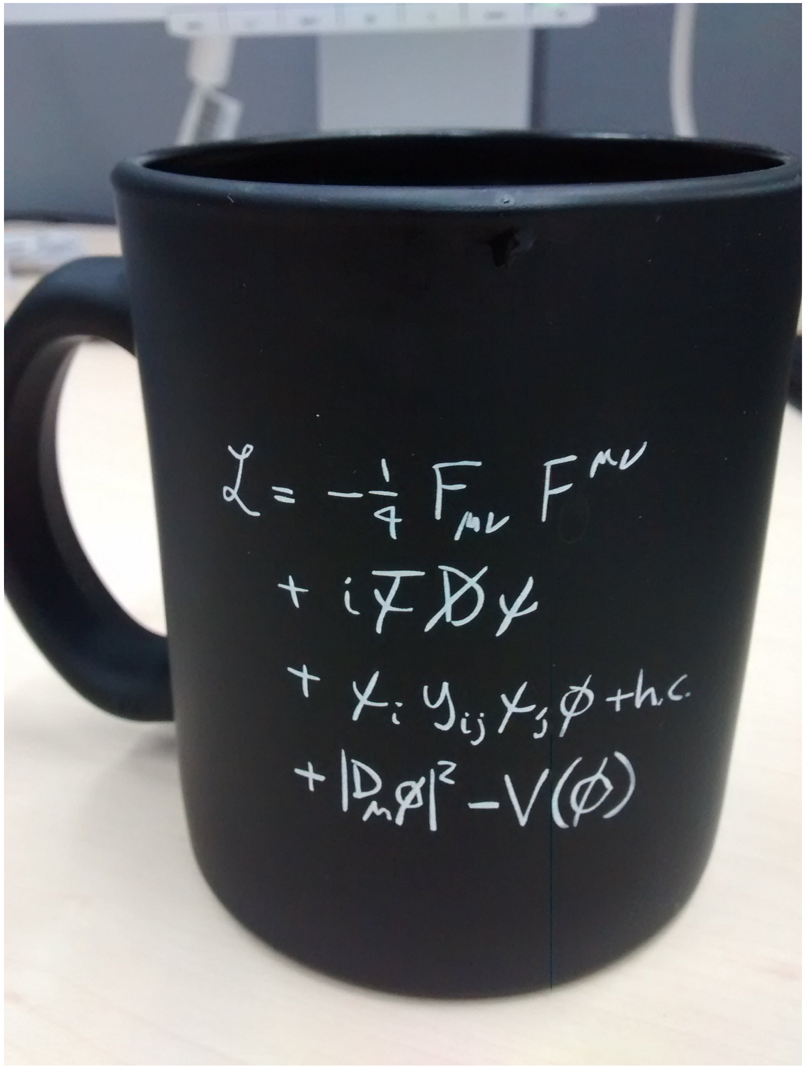
[The GAMBIT Collaboration, EPJC 77 (2017) 824]

Final slide (really!)

- Supersymmetry is well motivated:
  - It solves the Higgs hierarchy problem.
  - It has (multiple) good Dark Matter candidates.
  - It can provide GUT-models.
- Searches have turned up nothing:
  - Performed on very simplified models.
  - Collider searches blind to mass degeneracies.
  - The Higgs mass tells us that SUSY should be heavy.

Bonus material

The canonical coffee



The full superalgebra

$$
\begin{aligned}
\{Q_a^\alpha, Q_b^\beta\} &= \{\bar{Q}_a^\alpha, \bar{Q}_b^\beta\} = 0 \\
\{Q_a^\alpha, \bar{Q}_b^\beta\} &= 2\delta^{\alpha\beta}\gamma_{ab}^\mu P_\mu \\
[Q_a^\alpha, P_\mu] &= [\bar{Q}_a^\alpha, P_\mu] = 0 \\
[Q_a^\alpha, M^{\mu\nu}] &= \sigma_{ab}^{\mu\nu} Q_b^\alpha \\
[Q_a^\alpha, B_l] &= i S_l^{\alpha\beta} Q_a^\beta \\
[B_k, B_l] &= i c_{klm} B_m \\
\{Q_a^\alpha, Q_b^\beta\} &= \epsilon_{ab} Z^{\alpha\beta} \\
Z^{\alpha\beta} &= -Z^{\beta\alpha} \\
[Z^{\alpha\beta}, B_l] &= 0
\end{aligned}
$$

[Haag, Lopuszanski and Sohnius, '75]

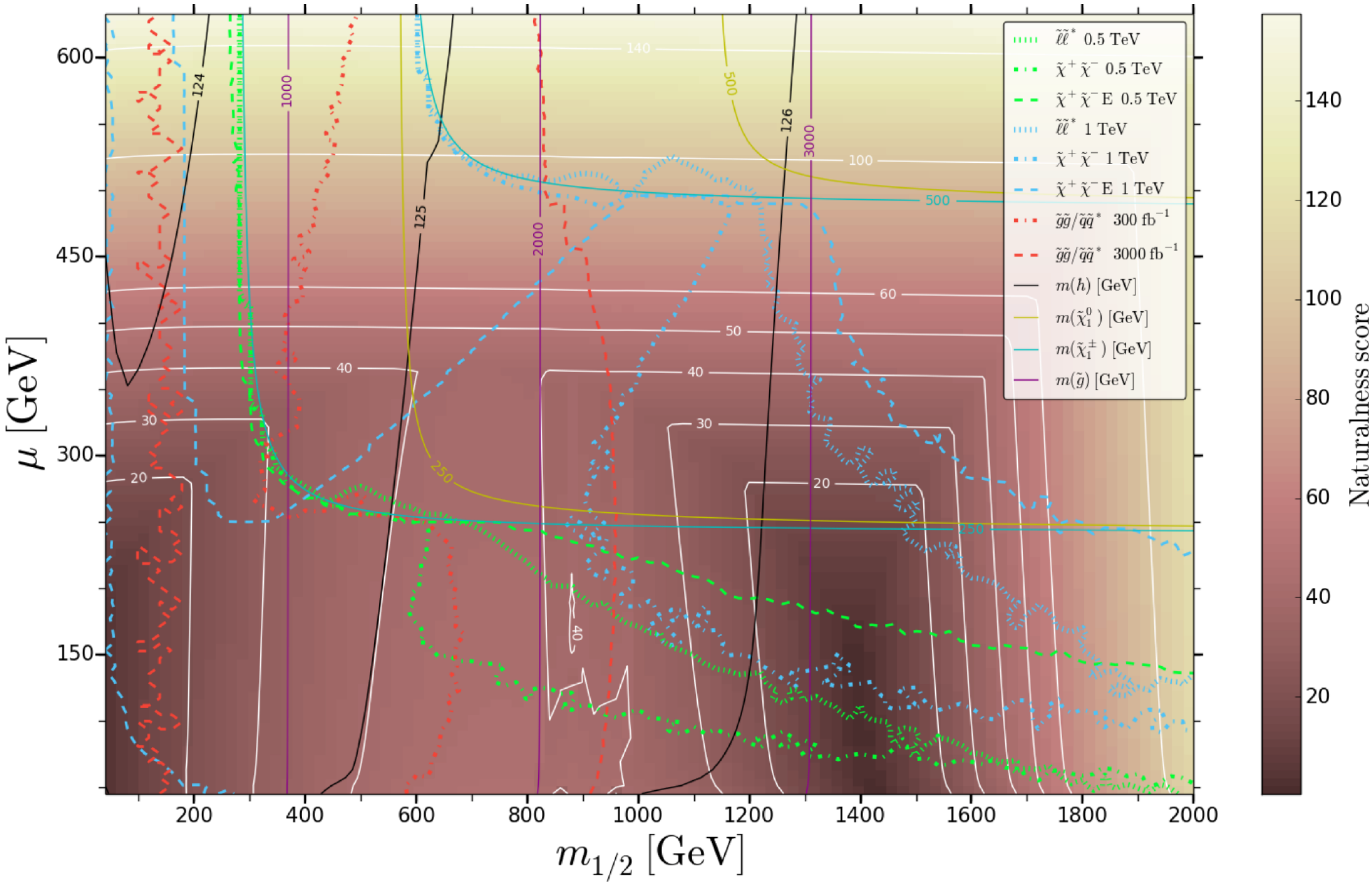
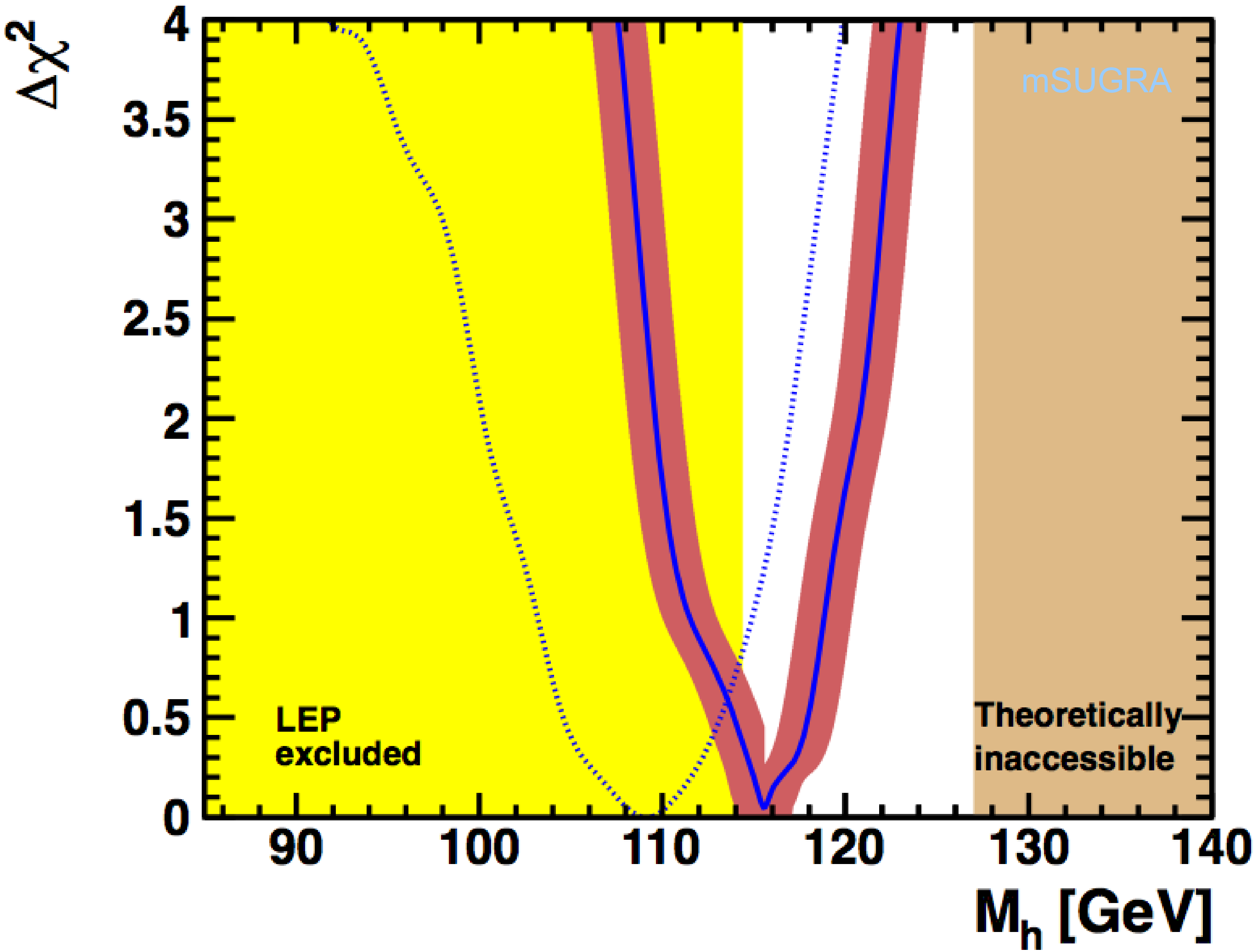


Figure 6.6: The NUHM2 scenario with $m_0 = 4$ TeV, $\tan\beta = 15$, $A_0 = -1.6m_0$, $m_A = 1$ TeV. The white lines are contours for the naturalness score.

[E. Rye, MSc. thesis]


Δχ²
4
3.5
3
2.5
2
1.5
1
0.5
0
90
100
110
120
130
140
M_h [GeV]
mSUGRA
LEP
excluded
Theoretically
inaccessible