

OSLO WINTER SCHOOL

2 – 12 January 2018



Polarization radiation from bunches of correlated charged particles

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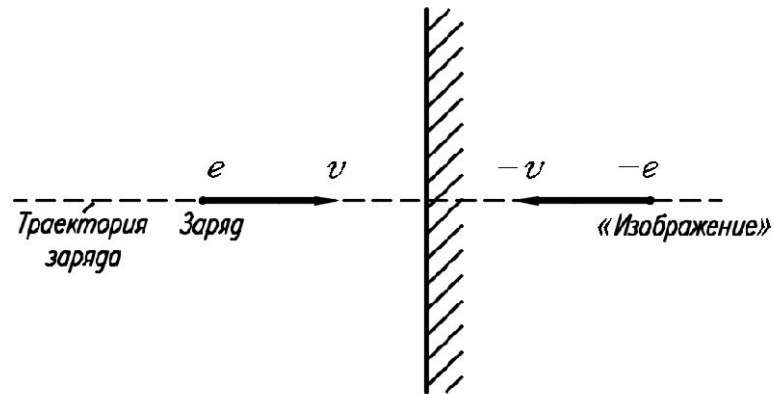
A.A. Tishchenko

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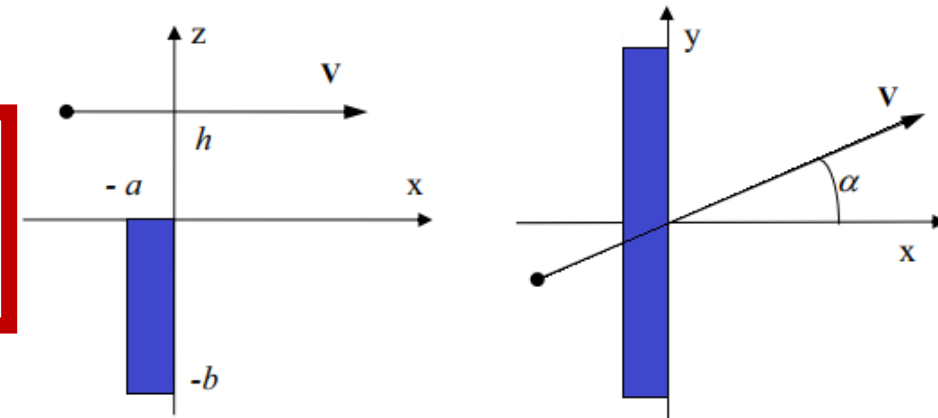
Introduction

Transition radiation



Goal: to calculate the effects of correlations in ultrashort bunches

Diffraction radiation



$$W_b^\pm(\omega, \mathcal{G}) = \frac{1}{2\pi} W^\pm(\omega, \mathcal{G}) F(\omega, \mathcal{G})$$

$$W_b^\pm(\omega, \mathcal{G}) = \frac{1}{2\pi} W^\pm(\omega, \mathcal{G}) (N + N(N-1)G_k(\omega, \mathcal{G}))$$

$$F(\omega, \mathcal{G}) = N + N(N-1) \iint d^3r d^3r' f_2(\mathbf{r}, \mathbf{r}') \exp \left\{ -i \left[\boldsymbol{\zeta}(\boldsymbol{\rho}_m - \boldsymbol{\rho}_n) + \frac{\boldsymbol{\omega}}{v} (z_m - z_n) \right] \right\}$$

$$f_2(\mathbf{r}, \mathbf{r}') = f_1(\mathbf{r}) f_1(\mathbf{r}') + f_0(\mathbf{r} - \mathbf{r}') \quad \longrightarrow \quad G_k(\omega, \mathcal{G}) = G_1(\omega, \mathcal{G}) + G_{cor}(\omega, \mathcal{G})$$

Form-factor and coherency factor for non-correlated bunch

$$G_k(\omega, \vartheta) = G_1(\omega, \vartheta) + G_{cor}(\omega, \vartheta)$$

$$f_2(\mathbf{r}, \mathbf{r}') \approx f_1(\mathbf{r})f_1(\mathbf{r}'),$$

$$f_1(\mathbf{r}) = f_\rho(\rho)f_z(z)$$

$$G_1(\omega, \vartheta) = \left| \int_{-\infty}^{\infty} dz f_z(z) e^{-i\frac{\omega}{v}z} \right|^2 \left| 2\pi \int_0^{\infty} d\rho f_\rho(\rho) J_0\left(\rho \frac{\omega}{c} \sin \vartheta\right) \rho \right|^2$$

Uniform distribution

Gaussian distribution

$$G_1(\omega, \vartheta) = \left| \frac{\sin\left(\frac{\omega a}{2v}\right)}{\frac{\omega a}{2v}} \right|^2 \left| \frac{2J_1\left(\frac{\omega}{c} b \sin \vartheta\right)}{\frac{\omega}{c} b \sin \vartheta} \right|^2$$

$$G_1(\omega, \vartheta) = \exp\left\{-\frac{\omega^2}{2}\left(\frac{a^2}{4v^2} + \frac{b^2}{c^2} \sin^2 \vartheta\right)\right\}$$

- G.M. Garibyan, Yang Shi, X-Ray Transition radiation, 1983.
- M. Ferrario, *Space Charge Effects*. Zeuthen: CAS-CERN, 2003.

Correlation function and the correlation part of the coherency factor

Uniform distribution

$$G_{cor}^{Uniform}(\omega, \vartheta) = \left(\frac{2\pi n_{e0} e^2}{T} \right)^2 \left| \frac{\sin \left(\frac{a}{2} \sqrt{\kappa^2 + \alpha^2} - i \frac{\omega a}{2\nu} \right)}{\left(\sqrt{\kappa^2 + \alpha^2} \right) \left(\sqrt{\kappa^2 + \alpha^2} - i \frac{\omega}{\nu} \right)} \right|^2 e^{-a\sqrt{\kappa^2 + \alpha^2}}$$

$$\frac{\pi a}{\beta} \ll \lambda, \quad \pi a \sin \vartheta \ll \lambda$$

и $a \leq 2r_D$

Gaussian distribution

$$G_{cor}^{Gauss}(\omega, \vartheta) = \left(\frac{4e^2}{ab^2 \pi^{1/2} T} \right)^2 \left| \operatorname{Erfc} \left(\frac{b}{2r_D} \right) J_0 \left(\alpha \frac{b^2}{2r_D} \right) e^{b^2/4r_D^2} \frac{\pi b^2 a / 2}{\sqrt{a^2 / 4 + b^2}} \right|^2$$

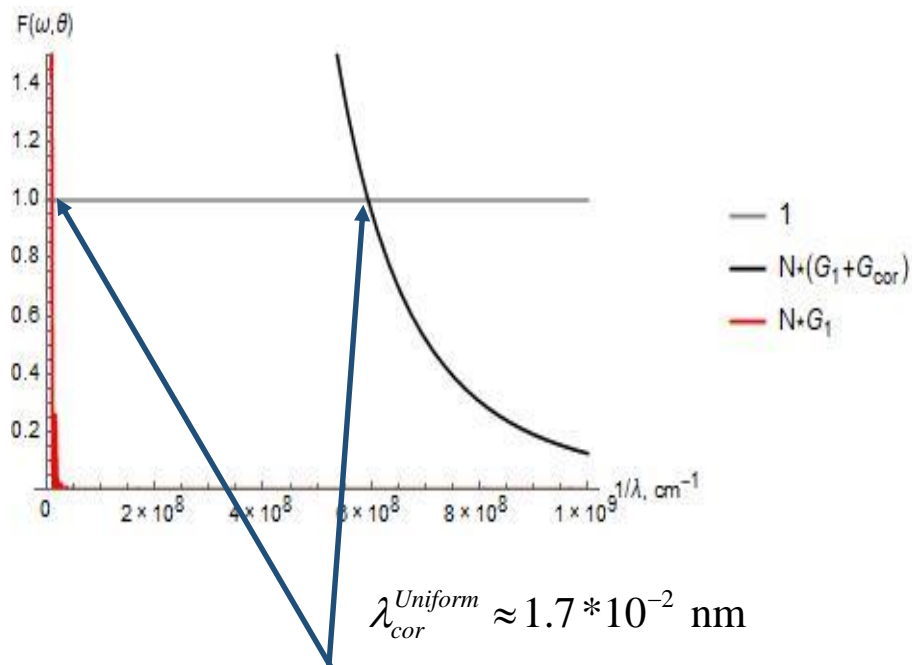
$$\frac{\pi b^2 \sin \vartheta}{r_D} \ll \lambda, \quad \frac{b}{2r_D} \leq 1$$

$$\alpha = \frac{\omega}{c} \sin \vartheta$$

Numerical estimates

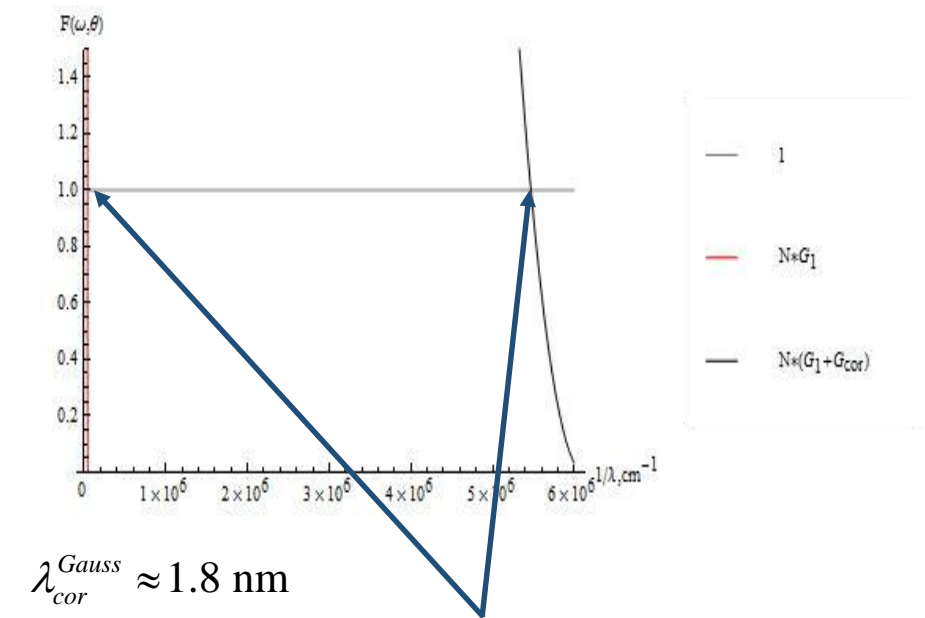
The value of the wavelength where the coherent radiation starts to exceed the incoherent one (λ_{coh}) with and without correlations in the bunch

Uniform distribution



difference $\approx 10^5$

Gaussian distribution



difference $\approx 10^3$

**Thank you for your
attention!**