~~HBT correlations~~
Correlation femtoscopy

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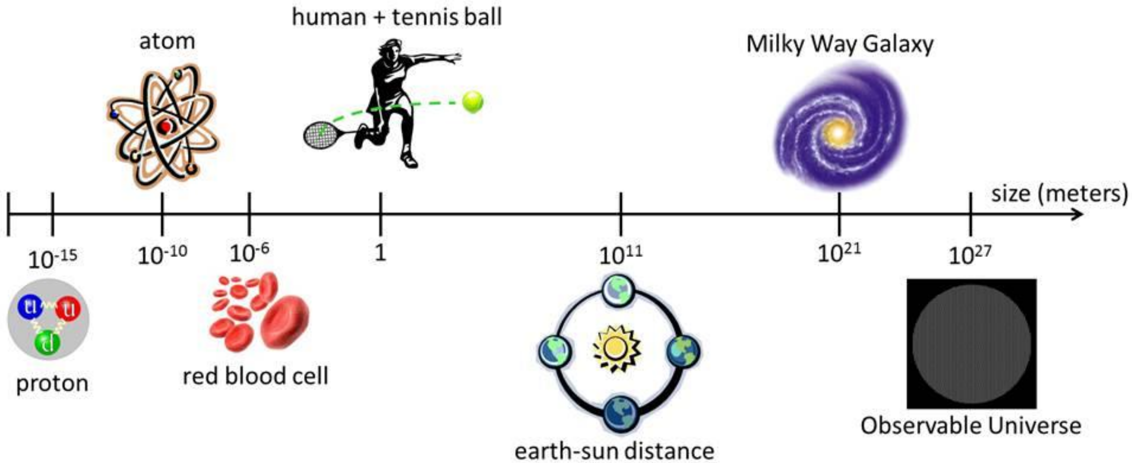
Outline

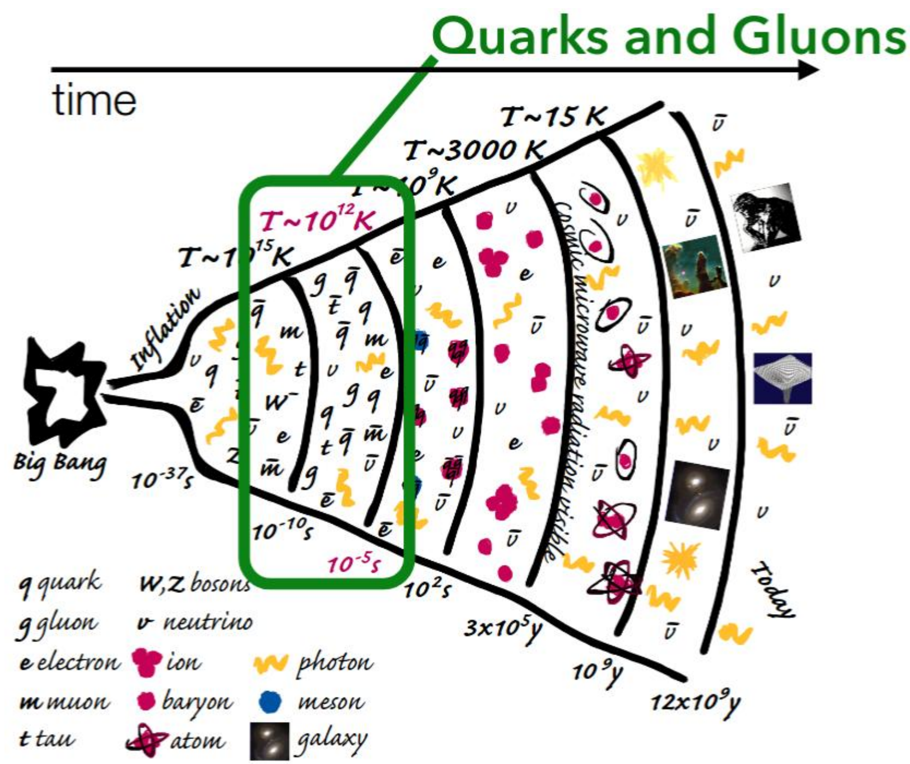
- Motivation
- Concepts of particle detection in HEP
- HBT interferometry
- Correlation femtoscopy: formalism

Sizes in Nature



In order to measure the object of the certain size, we need to measure how it interacts with something.

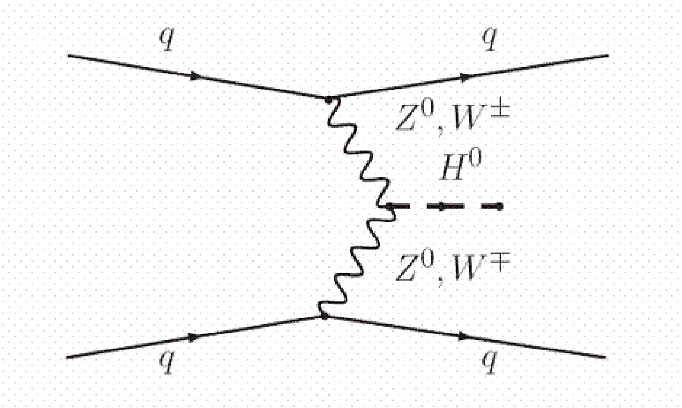
In high-energy particle and nuclear physics one need to measure the products of the interaction, i.e. measure the produced particles.



Concepts of particle detection in HEP

Theorist say:

We want to measure this process in order to study the Standard Model



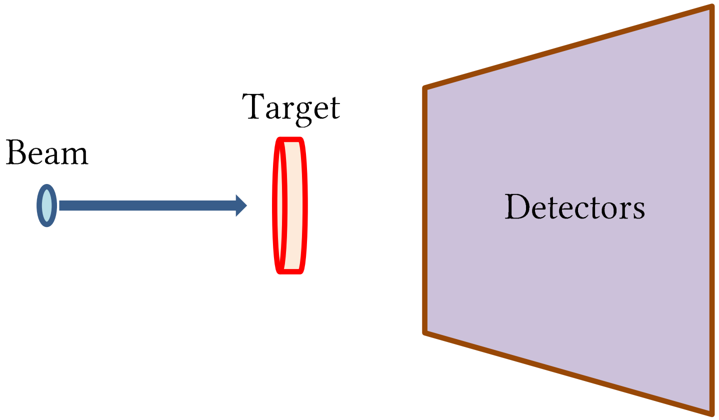
Experimentalist say:

We should make ion beams, accelerate them, collide with other ions, and detect the end products

- We must detect particles, understand what detectors are telling us in order to answer the theoretical questions
- Measurable properties: positions, momentum, charge, energy, particle type (mass)

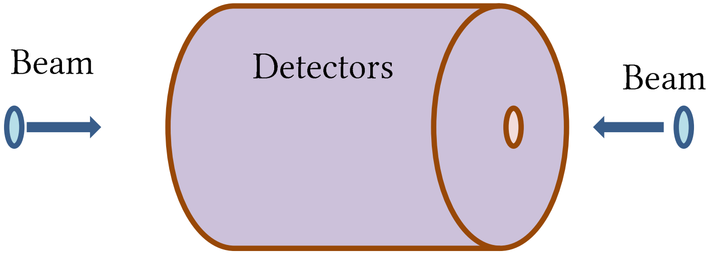
Fixed target and collider experiments

Fixed target (target stays at rest in the laboratory frame):



With a fixed-target experiment the particles produced generally fly in the forward direction, so detectors are cone shaped and are placed "downstream."

Colliding beams (center of mass frame):

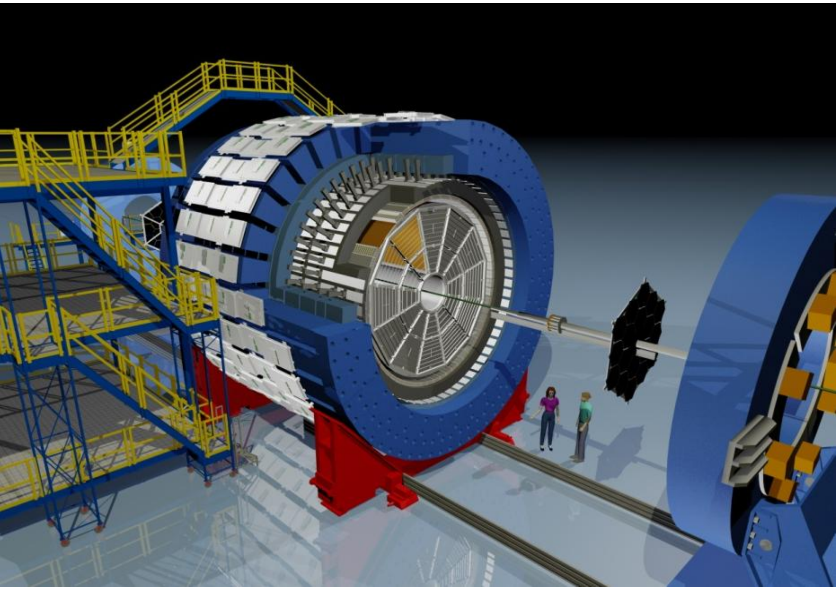


During a colliding-beam experiment, the particles radiate in all directions, so the detector is spherical or, more commonly, cylindrical.

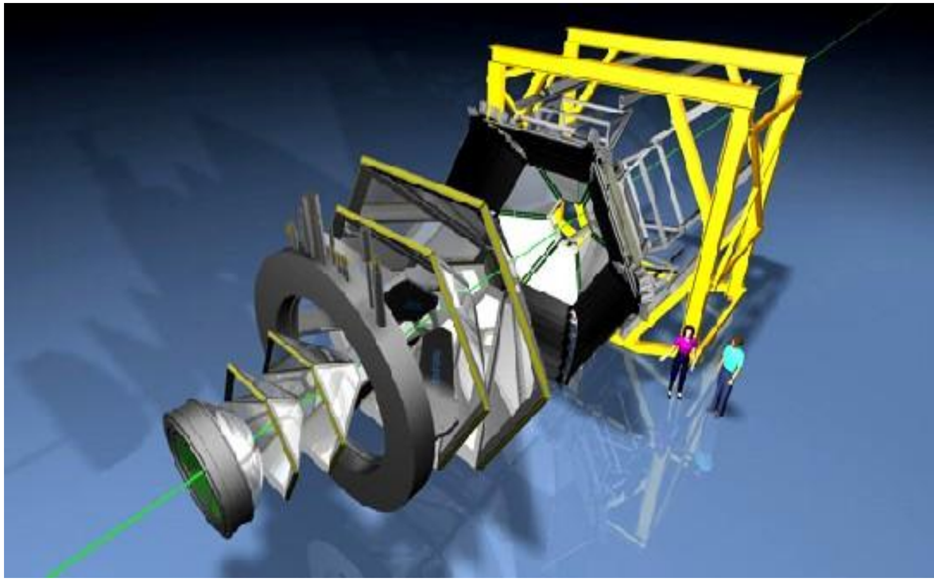
Different physics goals and kinematics impose the usage of different types of particle detectors and their geometries.

Fixed target and collider experiments

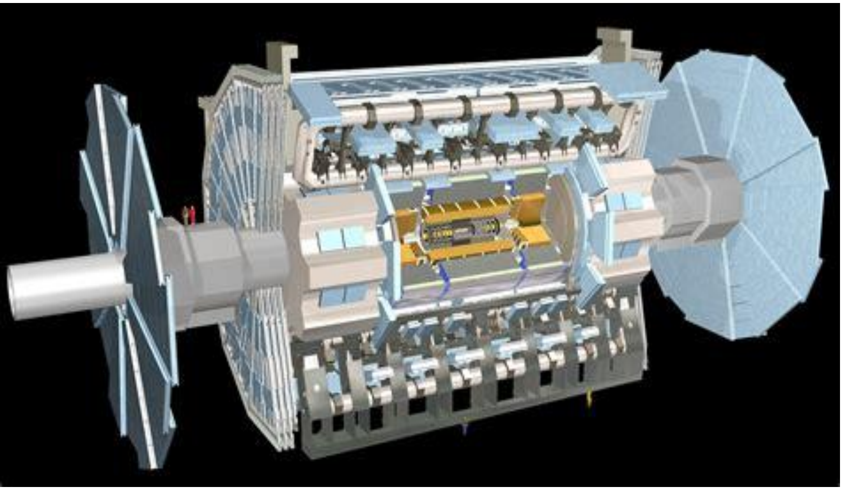
STAR @ BNL



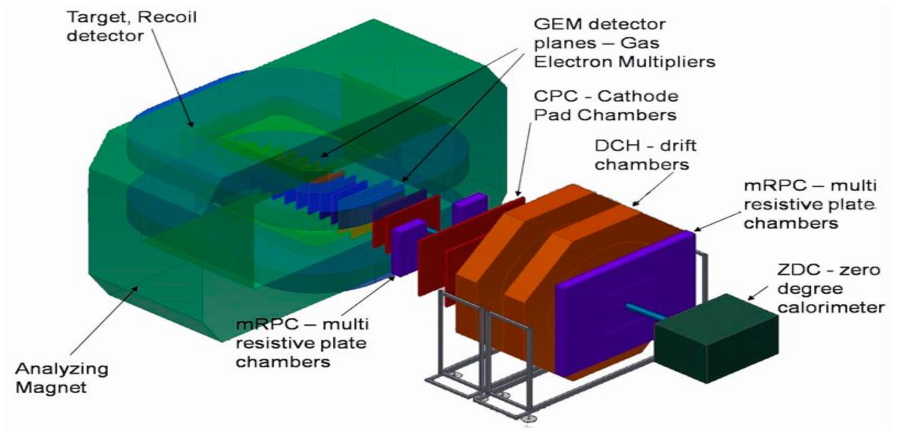
HADES @ GSI



ATLAS @ LHC



BM@N @ JINR



Particle interactions

There are 4 types of interactions in Nature but we use only 2 of them to detect particles:

- Strong interaction
  - Hadronic showers
- Electromagnetic interaction
  - Bremsstrahlung
  - Pair production
  - Ionization
  - Scintillation
  - Cherenkov radiation
  - Transition radiation

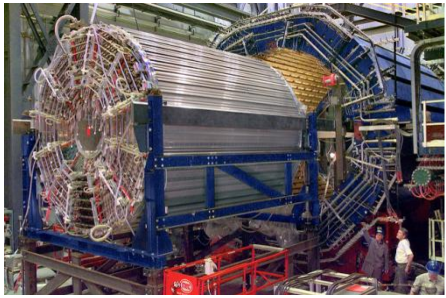
Calorimeters

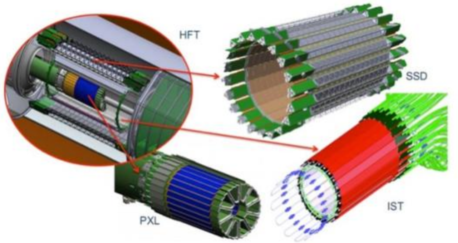
Tracking detectors

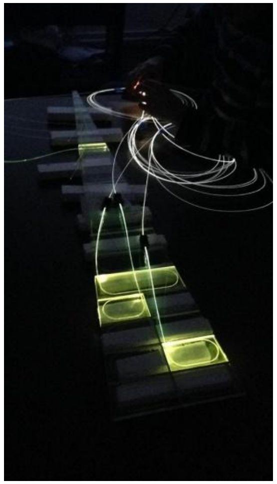
Cherenkov/TRD

Tracking detectors

- Purpose:
  - Measurement of momentum and charge determination
  - Tracking (production position)
- Material:
  - Gaseous detectors (drift chambers, straws)
  - Solid state (silicon detectors)
  - Scintillating (fiber trackers)
- Important concepts:
  - Energy loss
  - Resolution
  - Possibility to place a magnetic field







Ionization energy loss

The mean rate of energy loss by moderately relativistic charged heavy particles is well-described by the “Bethe equation”:

$$\left\langle -\frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

where

A, Z: atomic mass and atomic number of absorber

z: charge of incident particle

β, γ: relativistic velocity, relativistic factor of incident particle

δ(βγ): density correction due to relativistic compression of absorber

I: ionization potential

W_{max}: maximum energy loss (for a particle with mass M) in a single collision

$$W_{\max} = \frac{2m_e c^2\, \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

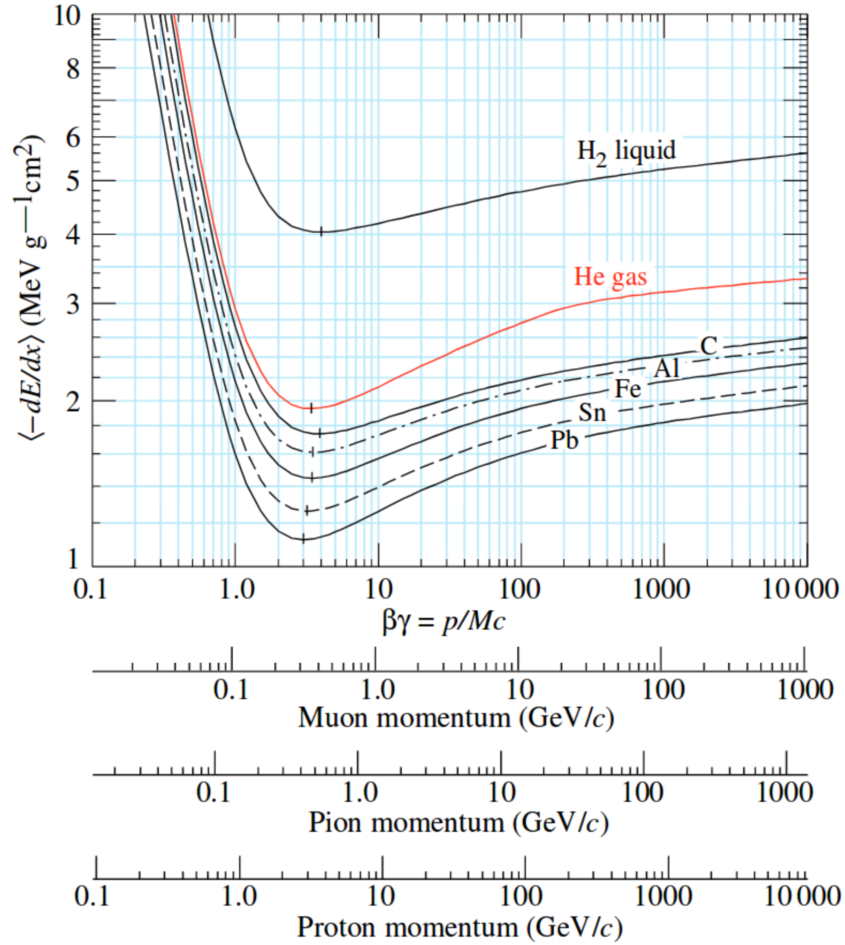
dE/dx has units of [MeV cm^2/g]

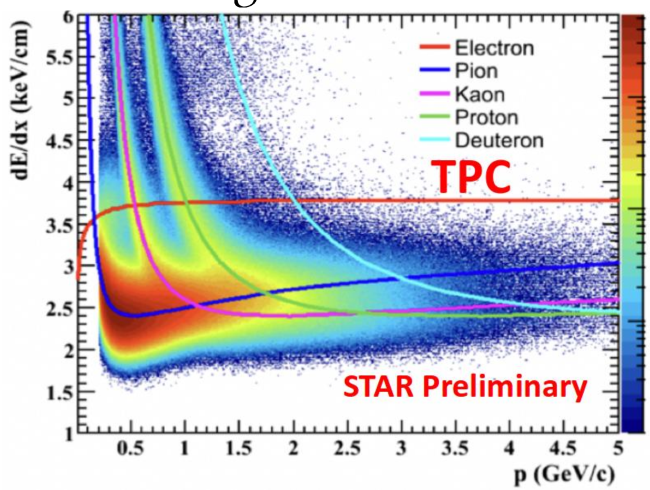
It describes the mean rate of energy loss in the region 0.1≤ βγ ≤ 1000 for intermediate-Z materials with an accuracy of a few percent.

The equation for electrons/positrons will be more complicated!

Ionization energy loss

Common features of the shown dependences are fast growth, as $1/\beta^2$, at low energy, a wide minimum in the range $3 \le \beta\gamma \le 4$, and slow increase at high energy. A particle having dE/dx near the minimum is often referred to as a minimum-ionizing particle or MIP. The MIP's ionization losses for all materials except hydrogen are in the range between 1 and 2 MeV/(g/cm^2) slightly increasing from low to large Z.





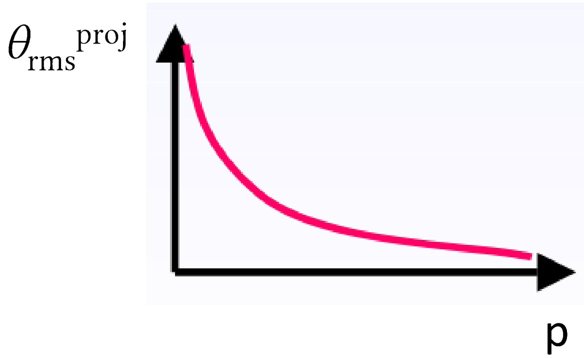
Multiple scattering of charged particles

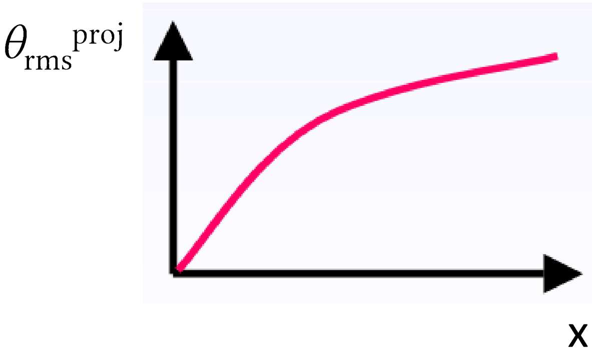
A particle passing through material undergoes multiple small-angle scattering, mostly due to large-impact-parameter interactions with nuclei. Then an initially parallel particle beam gets the angular spread after traveling through the layer of material.

For small scattering angles it is normally distributed around the average value θ=0. Larger scattering angles caused by rare collisions of charged particles with nuclei are, however, more probable than expected from a Gaussian distribution. The root mean square of the projected scattering-angle distribution is expressed as:

$$\theta_{\mathrm{rms}}^{\mathrm{proj}} = \sqrt{\langle \theta^2 \rangle} = \frac{13.6\,\mathrm{MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln(x/X_0)\right]$$

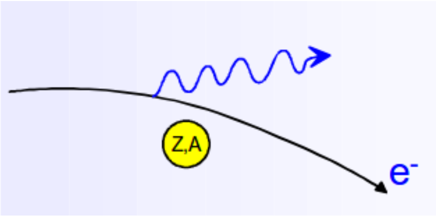
Where X_0 is a mean distance over which particle's energy decreases to 1/e of the initial value (radiation length)





Bremsstrahlung

Energy loss by Bremsstrahlung – charged particle radiate photons in the Coulomb field of the nuclei of the absorber medium:



$$-\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} z^2 (\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mc^2}) E \ln\frac{183}{Z^{1/3}} \propto \frac{E}{m^2}$$

QED process where electron is hit by plane electromagnetic wave (for large speed v).

Since energy loss is proportional to $1/m^2$, hence the effect is important for electrons/positrons and ultra-relativistic muons (p>1 TeV/c).

For electrons: $$-\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 E \ln\frac{183}{Z^{1/3}}$$

Considering also interaction with electrons in atom:

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = 4\alpha N_A \frac{Z(Z+1)}{A} r_e^2 E \ln\frac{287}{Z^{\frac{1}{2}}} = \frac{E}{X_0}$$

Thus: $\mathbf{E=E_0 \exp(-x/X_0)}$

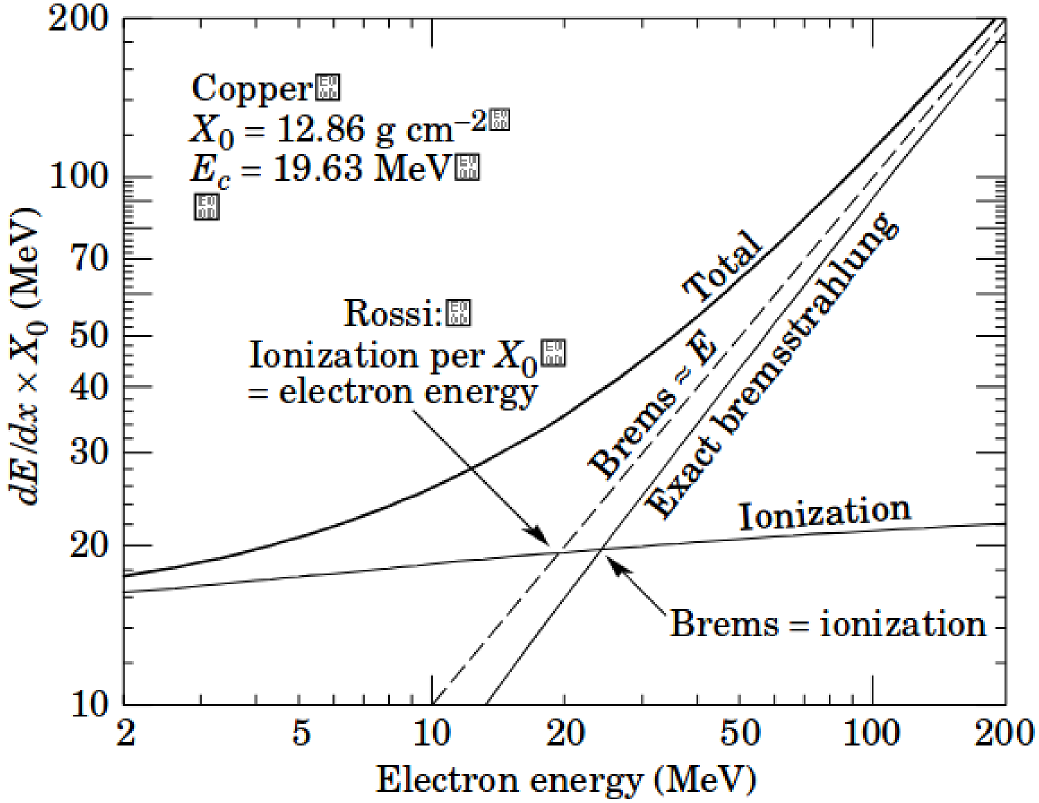
Bremsstrahlung

$$-\frac{dE}{dx} \text{ by ionization } \propto \ln E$$

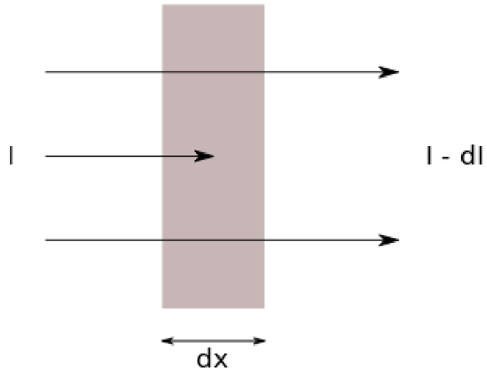
$$-\frac{dE}{dx} \text{ by bremsstrahlung } \propto E$$

The existence of crossing point beyond which bremsstrahlung dominates is called critical energy E_c:

$$\left(\frac{dE}{dx}\right)_{\text{ion}} = \left(\frac{dE}{dx}\right)_{\text{brems}}$$



Interaction of photons with matter



characteristic for photons: in a single interaction a photon can be removed out of beam with intensity I

$$dI = -I\mu\,dx$$

$$\mu(E, Z, \rho) \rightarrow \text{absorption coefficient}$$

Lambert-Beer law of attenuation:

$$I = I_0 \exp(-\mu x)$$

- mean free path of photon in matter: $\lambda = 1/n\sigma = 1/\mu$

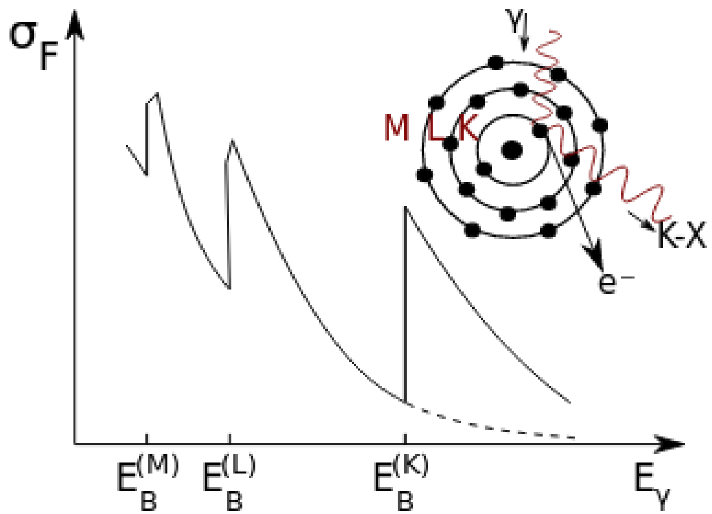
to become independent of state (gaseous, liquid) and reduce variations → introduce

mass absorption coefficient $\tau = \frac{\mu}{\rho} = N_A \frac{\sigma}{A}$

Photoelectric effect (photoeffect)

Photoelectric effect implies an absorption of a photon by the electron bound in atom and transfer of the photon energy to this electron. The photoeffect cross section for the photon of energy $E_\gamma > E_K$, where E_K is the K-shell energy, is particularly large for the K-shell electrons. The total cross section is:

$$\sigma_{\mathrm{ph}}^{\mathrm{K}} = \sqrt{\frac{32}{\zeta^7}} \alpha^4 Z^5 \sigma_{\mathrm{Th}} \; [\mathrm{cm}^2/\mathrm{atom}]$$



Where:

$$\zeta = E_\gamma / m_e c^2$$

And:

$$\sigma_{\mathrm{Th}} = \frac{8}{3} \pi r_e^2 = 665 \text{ mb}$$

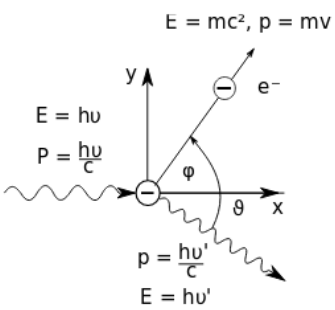
is the cross section of Thompson scattering.

The photoelectric cross section has sharp discontinuities when E_γ is equal to the binding energy of the atomic shells. After a photoelectric effect in the K shell, the atomic electrons are rearranged and characteristic X rays (like Kα) or Auger electrons are emitted.

Compton effect

The Compton effect is inelastic scattering of photons by quasi-free atomic electrons. After this scattering the photon energy, E'_γ, and the scattering angle, θ_γ, are related by the formula:

$$\frac{E'_\gamma}{E_\gamma} = \frac{1}{1 + \zeta(1 - \cos\theta_\gamma)}$$



Where: $\zeta = E_\gamma / m_e c^2$

The total cross section of Compton scattering derived by the integration of the Klein–Nishina formula is:

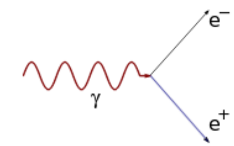
$$\sigma_C = \frac{\pi r_e^2}{\zeta}\left[\left(1 - \frac{2}{\zeta} - \frac{2}{\zeta^2}\right)\ln(1 + 2\zeta) + \frac{1}{2} + \frac{4}{\zeta} - \frac{1}{2(1 + 2\zeta)^2}\right]$$

This formula provides the cross section per one electron. In the ultrarelativistic case, when $\zeta \gg 1$, the formula for the Compton cross section reduces to:

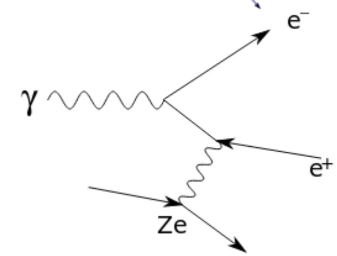
$$\sigma_C = \frac{\pi r_e^2}{\zeta}\left(\ln 2\zeta + \frac{1}{2}\right)$$

Electron-positron pair production

The production of an electron–positron pair by the photon becomes possible when the photon energy, E_γ, exceeds the threshold:



$$E_\gamma \geq 2m_e c^2 + \frac{2m_e^2 c^2}{M_{\text{nucleus}}} \approx 2m_e c^2$$



Not possible in free space but in Coulomb field of atomic nucleus to absorb recoil!

Cross section: for low energies impact parameters small, photon sees naked nucleus with increasing E_γ, range of impact parameter b is growing up to

$$\sigma_{\text{pair}} = 4\alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} - \frac{1}{54} \right) \; [\text{cm}^2/\text{atom}]$$

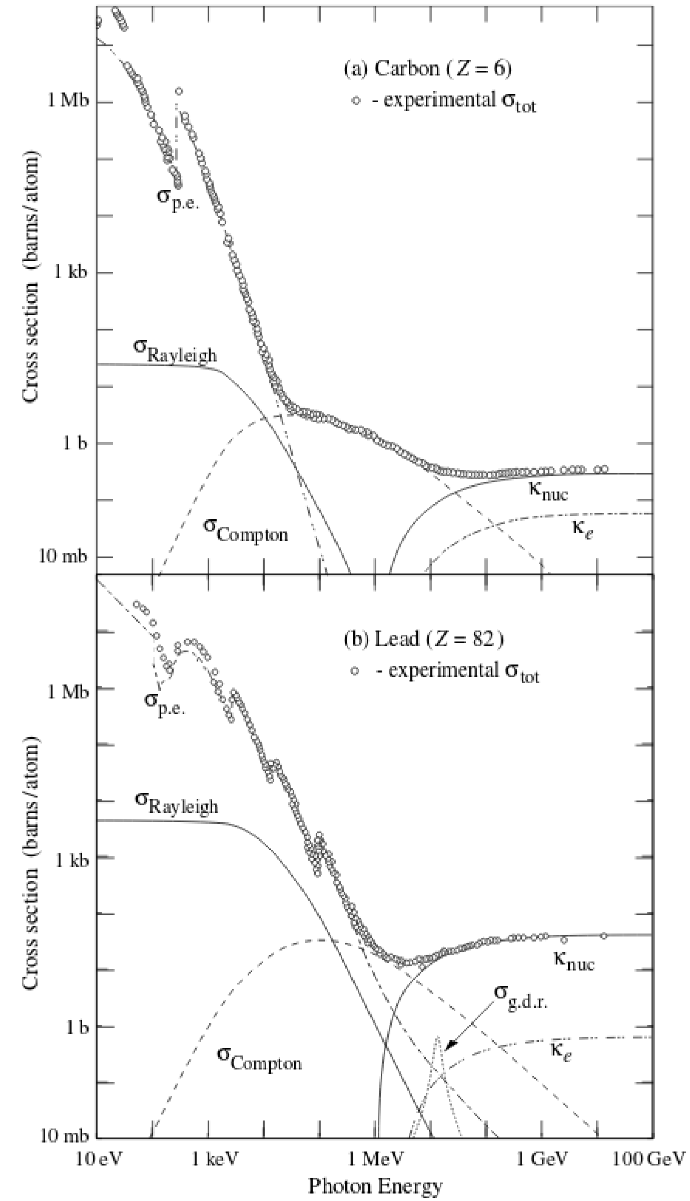
Total absorption coefficient

$$\sigma_{tot} = \sigma_{Ph} + \sigma_c + \sigma_p$$

$$\mu = \mu_{Ph} + \mu_c + \mu_p$$

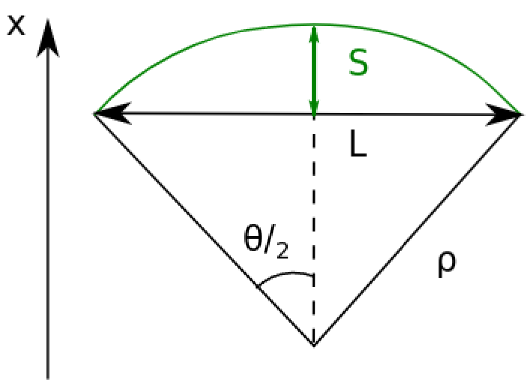
$$\mu_i = n\sigma_i = \frac{N_A \rho}{A}\sigma_i$$

Photon total cross sections as a function of energy in carbon and lead



Momentum measurement

The determination of the momentum of charged particles can be performed by measuring the bending of a particle trajectory (track) in a magnetic field



Sagitta $S = \rho - \rho\cos\frac{\theta}{2} = \rho\left(1 - \cos\frac{\theta}{2}\right)$

$= 2\rho\sin^2\frac{\theta}{4}$

for small θ $S \simeq \frac{\rho\theta^2}{8}$

with $\theta = \frac{qBL}{p_\perp}$ and $\sin\theta/2 \simeq \theta/2 = \frac{L/2}{\rho}$

$$S = \frac{qL^2B}{8p_\perp}$$

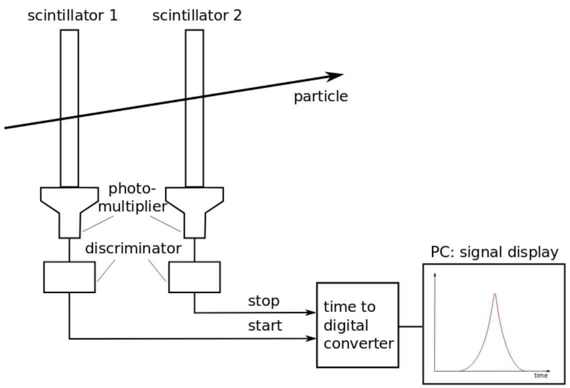
B in T, L in m, $p_\perp$ in GeV/c, q in e

$$S(\text{m}) = \frac{0.3\,qL^2B}{8p_\perp}$$

Time-of-Flight method

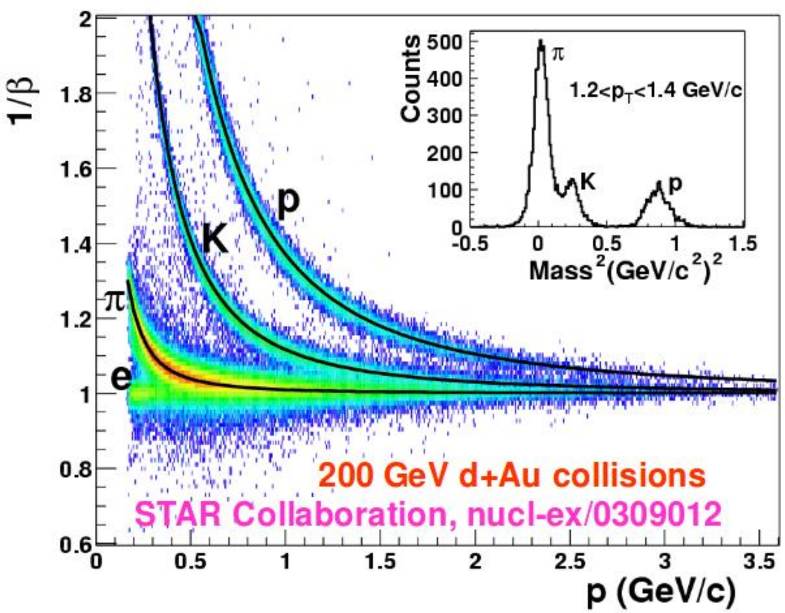
Time difference between two detectors with good time resolution: "start" and "stop"-counter:

- typically scintillator or resistive plate chamber, also calorimeter (neutrons)
- coincidence set-up or put all stop-signals into TDC (time-to-digital converter) with common start or stop from "beam" or "interaction"



For known distance L between start and stop counter, time-of-flight difference of two particles with masses $m_{1,2}$ and energies and $E_{1,2}$:

$$\Delta t = \tau_1 - \tau_2 = \frac{L}{c}\left(\frac{1}{\beta_1} - \frac{1}{\beta_2}\right)$$



$$\Delta t = \frac{L}{c}\left(\sqrt{\frac{1}{1-(m_1c^2/E_1)^2}} - \sqrt{\frac{1}{1-(m_2c^2/E_2)^2}}\right)$$

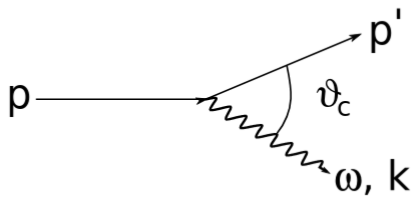
In the relativistic limit $E \approx pc \gg m_0c^2$:

$$\Delta t = \frac{Lc}{2p^2}(m_1^2 - m_2^2)$$

Cherenkov effect

Charged particles with a velocity |v| larger than the speed of light in a medium with refractive index n will emit Cherenkov radiation under an angle θ_c , given by Cherenkov:

$$\cos\theta_c = \frac{1}{\beta n}$$



With β=v/c, c being the speed of light in vacuum. The threshold velocity v_{thres} is given by: $\beta_{\text{thres}} = \frac{v_{\text{thres}}}{c} \geq \frac{1}{n}$, or $\gamma_{\text{thres}} = \frac{n}{\sqrt{n^2-1}}$,

with a maximum angle of θ^{max}_c=arccos(1/n).

The number of photons N emitted per energy interval dE and path length dl is given by Frank and Tamm: $\frac{d^2N}{dE\,dl} = \frac{\alpha}{\hbar c}\left(1 - \frac{1}{(\beta n)^2}\right) = \frac{\alpha}{\hbar c}\sin^2\theta$

or, expressed for a wavelength interval dλ:

$$\frac{d^2N}{d\lambda\,dl} = \frac{2\pi\alpha}{\lambda^2}\sin^2\theta$$

Ring Imaging Cherenkov detectors (RICH)

By measuring the Cherenkov angle θ_c one can in principle determine the velocity of the particle, which will, together with the momentum p obtained via a magnetic spectrometer, lead to the determination of the mass and therefore to the identification of the particle.

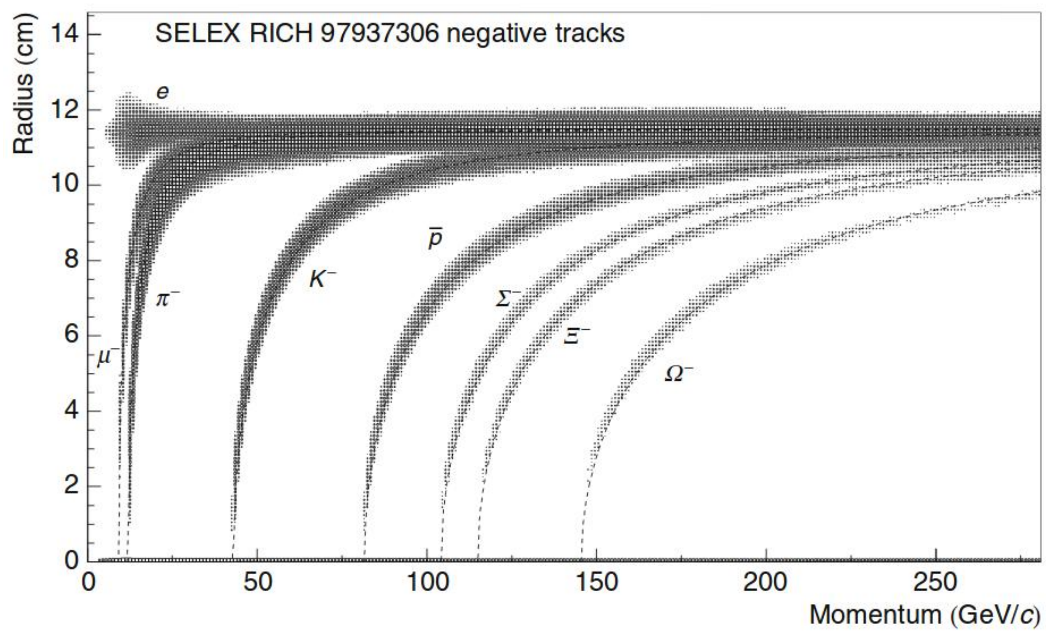
All the Cherenkov light (in one plane) is parallel, and can therefore be focused (for small θ_c) with a spherical mirror (radius R) onto a point. Since the emission is symmetrical in the azimuthal angle around the particle trajectory, this leads to a ring of radius r in the focus, which is itself a sphere with radius R/2.

The radius r is given by:

$$r = \frac{R}{2}\tan\theta_c \approx \frac{R}{2}\sqrt{2 - \frac{2}{n}\sqrt{1 + \frac{m^2c^2}{p^2}}}$$

The angular separation between two particles of masses m1 and m2 (in the small angle, relativistic, and small (n−1) approximation) is given by:

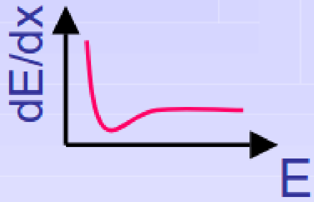
$$\theta_c\,\Delta\theta_c = \frac{m_1^2 - m_2^2}{2p^2}$$



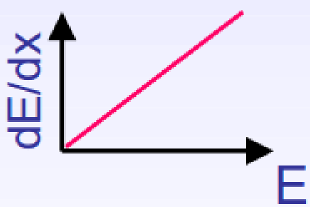
Basic electromagnetic interactions (reminder)

e^+ / e^-

- Ionisation

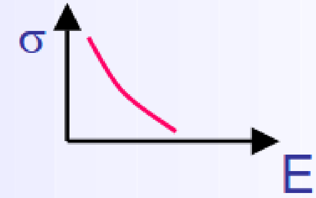


- Bremsstrahlung

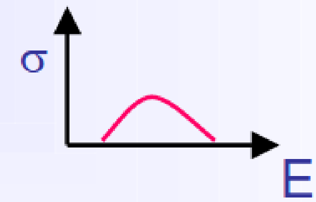


γ

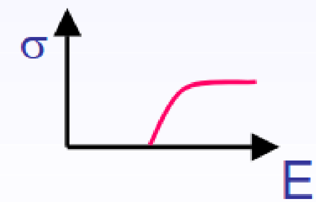
- Photoelectric effect



- Compton effect



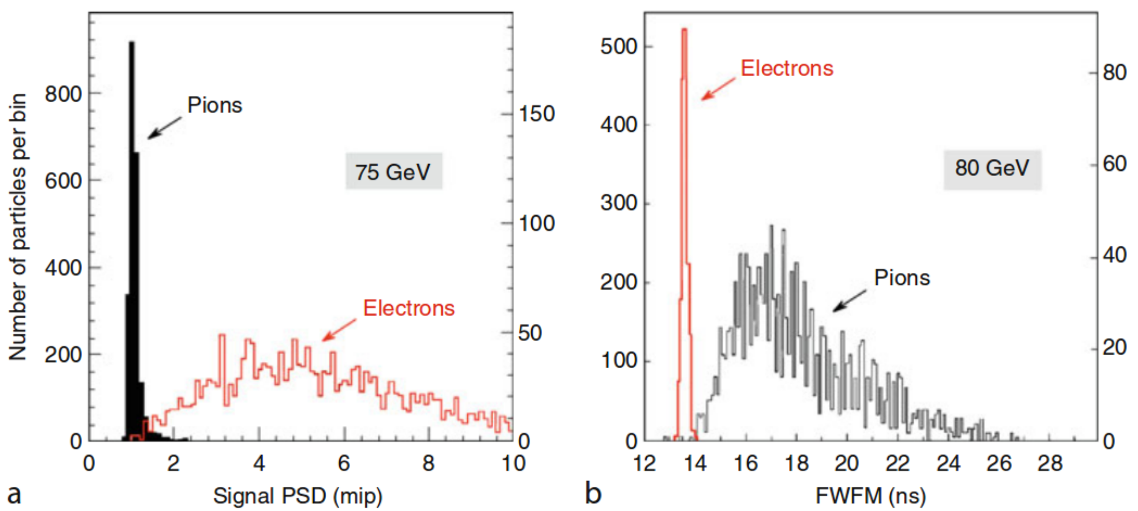
- Pair production



Calorimetry

In nuclear and particle physics, calorimetry refers to the detection of particles, and measurement of their properties, through total absorption in an instrument called a calorimeter. The signals from a properly instrumented absorber may be used to measure the entire four-vector of the particles. By analyzing the energy deposit pattern, the direction of the particle can be measured. The mass of the showering particle can be determined in a variety of ways:

- The E/p method, in which the energy measured in the calorimeter is compared with the momentum measured with a tracker in a magnetic field. This method only works for charged particles and relatively low energies.
- By analyzing the energy deposit profile. This method is frequently used to identify electrons. Especially in calorimeters with high-Z absorber material, em showers are much more shallow and concentrated around the shower axis than hadronic showers.
- By measuring the time structure of the calorimeter signals



Comparison of different PID method

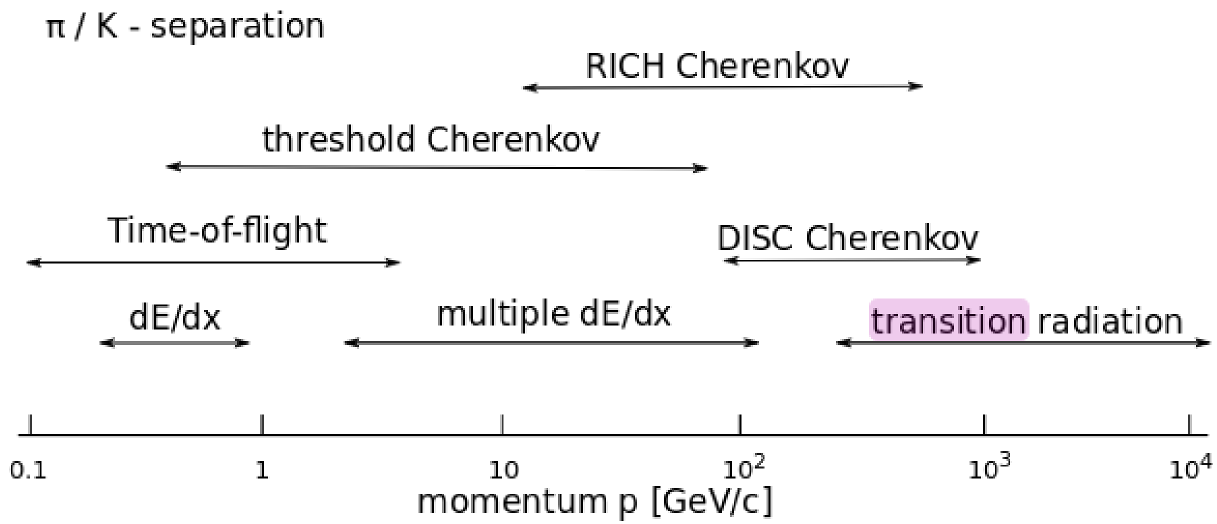
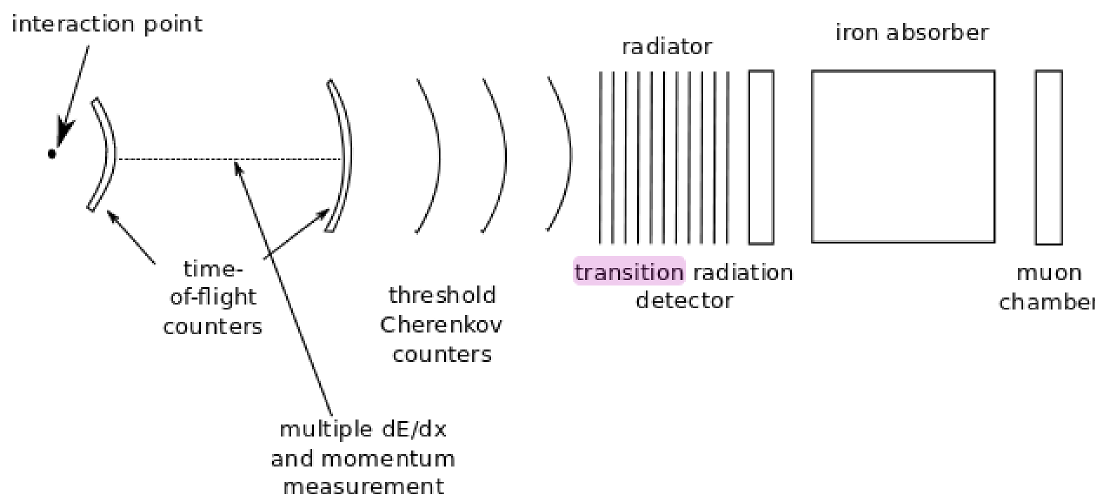
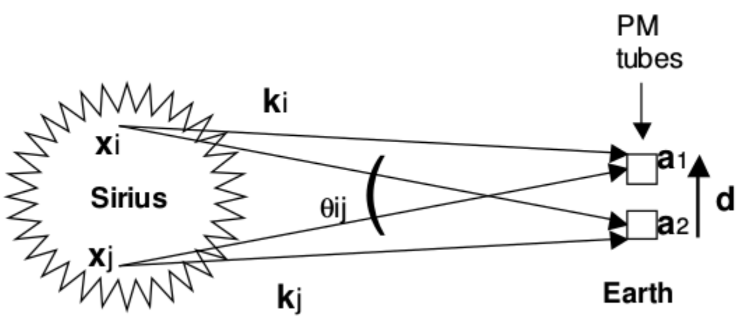


Illustration of various particle identification methods for K/π separation along with characteristic momentum ranges



HBT intensity interferometry

About fifty years ago, Robert Hanbury-Brown and Richard Twiss first suggested, and then proved in a table-top experiment, that photon pairs exhibit an interference effect if detected simultaneously in two detectors. They applied this technique to the measurement of the angular diameter of stars using pairs of photomultiplier tubes to detect optical photons and pairs of radiotelescope dishes to detect longer wavelength photons. [R. Hanbury-Brown and R.Q. Twiss, Phil. Mag. 45 (1954) 663]



Their measurement of the optical angular diameter of the star Sirius located in the constellation Canis Major. Sirius is shown emitting two photons with wavevectors k_i and k_j from points x_i and x_j, respectively, which are detected in two photomultiplier tubes located at positions a_1 and a_2 on the Earth.

HBT intensity interferometry

Assuming that photons are emitted incoherently from the star at each position and the photons are emitted as plane waves and taking a time-independent picture, the wavefunction to detect in coincidence the two photons in the two detectors on Earth, $\Psi(k_i, k_j; x_i, x_j)$, is:

$$\Psi(\mathbf{k}_i, \mathbf{k}_j; \mathbf{x}_i, \mathbf{x}_j) = b[\exp(i\mathbf{k}_i \cdot (\mathbf{a}_1 - \mathbf{x}_i))\exp(i\mathbf{k}_j \cdot (\mathbf{a}_2 - \mathbf{x}_j)) + \exp(i\mathbf{k}_i \cdot (\mathbf{a}_2 - \mathbf{x}_i))\exp(i\mathbf{k}_j \cdot (\mathbf{a}_1 - \mathbf{x}_j))]$$

where the second term arises due to the path ambiguity of detecting bosons, and b is a normalization constant. The probability to detect the two photons, P_{ij}, is just the square of Ψ, i.e. $\Psi * \Psi$, and thus given by:

$$P_{ij}(\Delta\mathbf{k}, \mathbf{d}) = b^2[1 + \cos(\Delta\mathbf{k} \cdot \mathbf{d})]$$

where $\Delta k = k_i - k_j$ and $d = a_1 - a_2$. The separation between the two detectors, d, is called the baseline. It is seen that (a) the *cos* term in P_{ij} is a result of the path ambiguity term in the first equation, and (b) P_{ij} does not depend on the photon emission positions at the star but only on the differences between the wavevectors and the detector positions.

Since one can assume that: $k_i \cong k_j \equiv k$, $\Delta k \cdot d \cong |\Delta k|$, $|\Delta k| \cong \theta_{ij} k = 2\pi\theta_{ij}/\lambda$, taking b = 1, and defining the correlator, $C_{ij}(d) \equiv P_{ij} - 1$, resulting in:

$$C_{ij}(d) = \cos(2\pi\theta_{ij} d/\lambda) \,,$$

Where θ_{ij} is the angular diameter seen on Earth between the points on the star i and j and λ is the wavelength of the photons.

HBT intensity interferometry

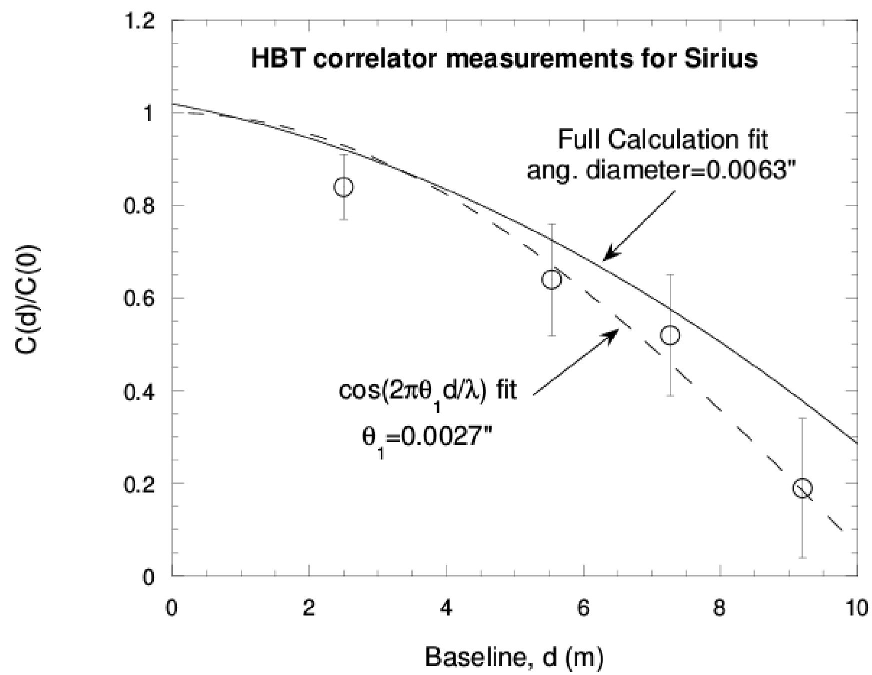
$C_{ij}(d)$ is proportional to the coincidence signal produced in the photomultipliers for the photons coming from these two points on the star, but in practice, the signal measured in the electronics is the sum over all of the photon emission point pairs from the star. If N is defined as the number of photon source points making up a star of brightness (intensity) I and radius r, to get the detected signal from the entire star, C(d), C_{ij} is summed over all of the unique pairs of photon emission points:

$$C(d) = \epsilon \sum_{i>j}^{N} C_{ij}(d) = \epsilon \sum_{i>j}^{N} \cos(2\pi\theta_{ij} d/\lambda)$$

where ϵ is a conversion factor to get the detector signal. If one looks at cases where the arguments of the *cos* terms are small such that the cos terms are not far from unity, e.g. for small baselines, then can approximately be expressed as:

$$C(d) \simeq \epsilon \frac{N^2}{2} \cos(2\pi\theta_1 d/\lambda)$$

where θ_1 is the average of the θ_{ij}.



GGLP effect

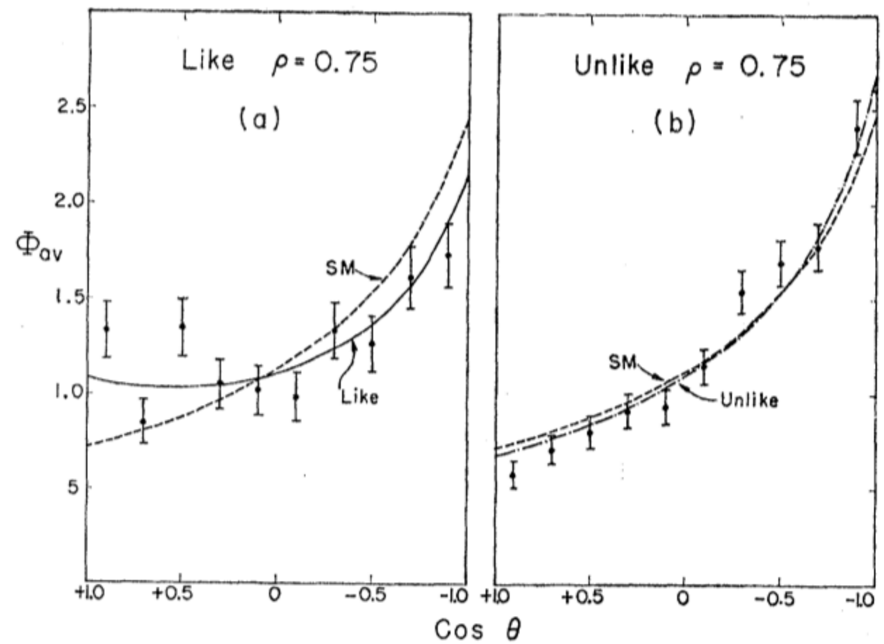
In 1960, a study has been made in a propane bubble chamber of "hydrogenlike“ annihilations of antiprotons of 1. 05-Bev/c laboratory-system momentum, corresponding to an energy release of 2.1 BeV in the center-of-mass system.

It was found that there is a clear difference between the angular distribution for pion pairs of like charge and that for pairs of unlike charge.

$$\bar{p}+p \longrightarrow 2\pi^{+}+2\pi^{-}+2\pi^{0}$$

[G. Goldhaber, S. Goldhaber, W.Y. Lee, A. Pais, Phys. Rev. 120 (1960) 300]

The experiment indicated that the distribution of the angle between pairs of pions deviates from the prediction of the conventional statistical model (SM).



It was interpreted within SM as Bose-Einstein enhancement depending on fireball Gaussian radius r_0.

SM multiplicity required radius r_0 by a factor of 3 larger.

Femtoscopy

In 70s, a similar (to GGLP effect) idea has been put forward in elementary-particle physics by Kopylov and Podgoretsky , namely, it has been suggested that it may be possible to use the interference between pairs of identical particles to determine the size, shape, and lifetime of an excited region in which elementary particles are produced.

[G.I. Kopylov, M.I. Podgoretsky, Sov. J. Nucl. Phys. 15 (1972) 219]

In astronomy, one measures the time difference between the arrival of the identical particles (photons) and the difference in the position of the detectors. This then may be looked upon as a space-time formulation of the experiment. In elementary-particle physics, on the other hand, one must measure the energy and momentum differences between the recorded identical particles (for example, positive pions).

In order to study the properties of the emitting source, they proposed:

1) to use two-particle correlation function $CF(q)=A^{corr}(q)/B^{uncorr}(q)$, use event-mixing technique to construct $B^{uncorr}(q)$ and two-body approximation to calculate theoretical CF
2) showed that sufficiently smooth momentum spectrum allows one to neglect space-time coherence at small q
3) clarified role of space-time production characteristics: shape & time source picture from various q-projections