Large-matrix inversion with MillePede-II and application to track-based alignment.

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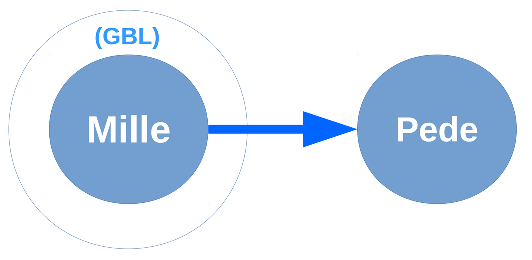
Introduction

Purpose
> Developed in context of tracker alignment.
> Intended for inversion of large, sparse matrix.
> Implement various tricks of linear algebra to perform matrix inversion (block matrix algebra & MINRES).

Context & History
> Development of Millepede in Fortran 77 started in 1996 by Volker Blobel (Uni Hamburg) [1].
> First implementation in 1999 for H1 experiment as `SuperFit` macro in H1 by Claus Kleinwort (DESY).
> Later, with CMS as main customer & for use at larger scale, the need for a better interfacing motivated a full re-implementation, namely MillePede-II.
> Official code in Fortran 90 (most performant solution), but re-implementations in C++ exist.
> Development still very active, mainly by Claus Kleinwort [2].
> Successfully used by many HEP experiments.



Mille
> To be implemented by the user according to actual problem (*e.g.* GBL in context of tracker alignment).
> Outputs residuals, derivatives & uncertainties for local and global parameters ($\mathbf{d}_k$, m_k & σ_k).

Pede
> Standalone application.
> Perform (approximate) inversion.
> Fortran 90 (best performance).
> Parallelisation with OpenMP (gcc4).

Motivation: Tracker Alignment

At mounting

Mechanical alignment is performed but performance is still limited:

$$\sigma_{\rm align} \gg \sigma_{\rm hit}$$

Goal

Compute a correction for each module in order to improve the tracking performance:

$$\sigma_{\rm align} \lesssim \sigma_{\rm hit}$$

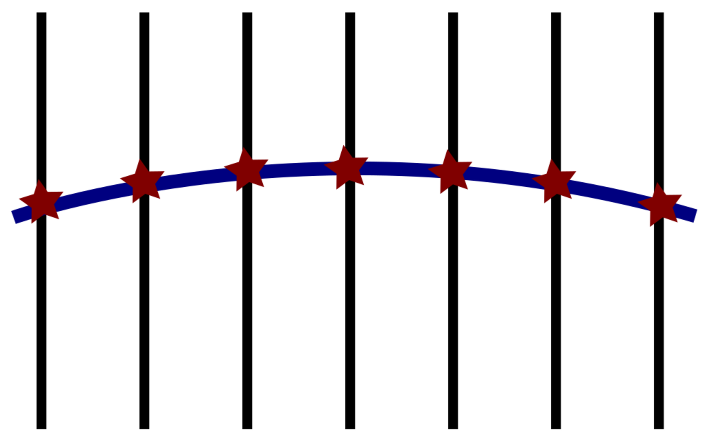
Track-based alignment

> module and track parameters $\mathbf{p}$ and $\mathbf{q}_j$
> measured and predicted position m_{ij} and f_{ij}
> measurement uncertainty σ_{ij}

$$\chi^2(\mathbf{p},\mathbf{q}) = \sum_j^{\rm tracks}\sum_i^{\rm hits}\left(\frac{m_{ij}-f_{ij}(\mathbf{p},\mathbf{q}_j)}{\sigma_{ij}}\right)^2$$

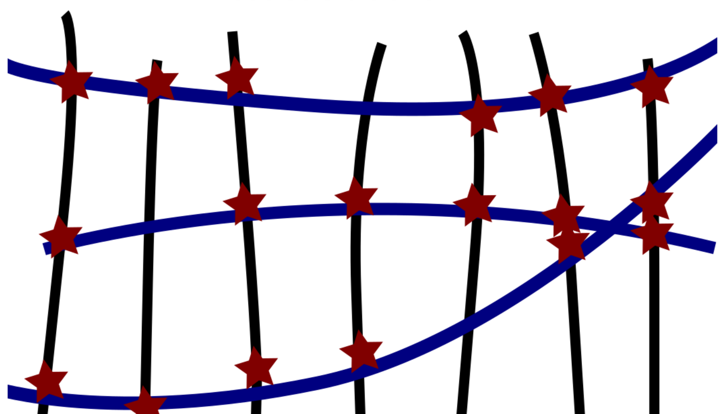
Linearise and reformulate in matrix-inversion problem

Ideal tracker



Modules Tracks Hits

Realistic tracker



H1 @ HERA (DESY)

H1 tracker system [3]
Central drift chambers with Rφ resolution ~ O(140 mm)

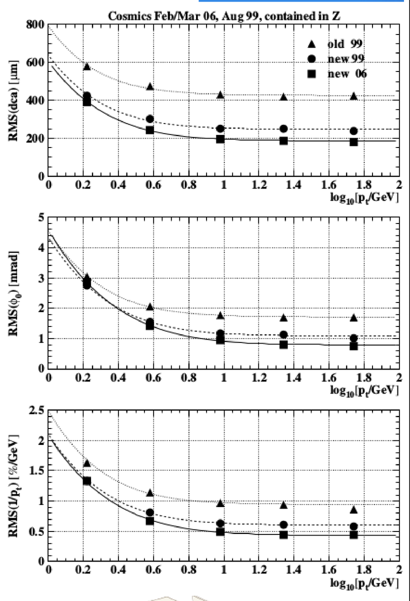
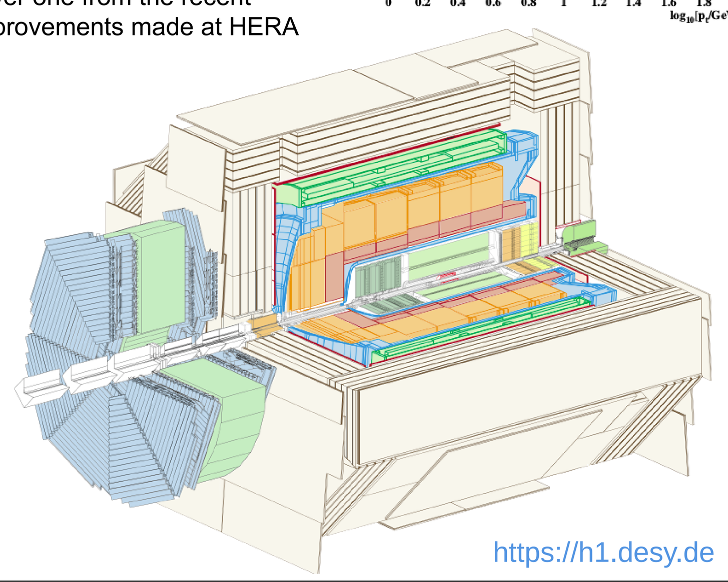


Figure [4]
"CJC track parameter resolution in *Rφ* deduced from the comparison of cosmic track halves. As a function of the transverse momentum (p_t) from 1 to 100 GeV, the resolution in distance to the interaction point (d_{ca}), the angle at the interaction point (φ_0), and the curvature ($1/p_t$) are shown. The upper curve represents results before alignment and calibration with Millepede, the middle one from afterwards, and the lower one from the recent improvements made at HERA II."



https://h1.desy.de

Mathematical principles [5,6,7]

General formulation

> m_k and σ_k describes independent measurements
> **A** represents all parameters to be minimised
> $\mathbf{d}_k$ represents the first derivative of the prediction

$$\chi^2(\mathbf{A}) = \sum_k\left(\frac{m_k - \mathbf{d}_k^{\mathsf{T}}\cdot\mathbf{A}}{\sigma_k}\right)^2$$

At minimum

$$\underbrace{\left(\sum_k \frac{1}{\sigma_k^2}\left(\mathbf{d}_k\cdot\mathbf{d}_k^{\mathsf{T}}\right)\right)}_{\equiv\mathbf{C}}\times\mathbf{A} = \underbrace{\sum_k\frac{m_k}{\sigma_k^2}\mathbf{d}_k}_{\equiv\mathbf{B}}$$

Local & global parameters

> Given *N* measurements:

$$\text{residuals} = \underbrace{\sum_{i=1}^{n} a_j\cdot d_j}_{\text{global parameters}} + \underbrace{\sum_{j=1}^{\nu}\alpha_j\cdot\delta_j}_{\text{local parameters}}$$

(*n+Nν*) equations with computing time proportional to (*n+Nν*)³

A bit of block-matrix algebra

$$\begin{pmatrix} \sum\mathbf{G}_i & \cdots & \mathbf{\Gamma}_i & \cdots \\ \vdots & \ddots & 0 & 0 \\ \mathbf{\Gamma}_i^{\mathsf{T}} & 0 & \mathbf{L}_i & 0 \\ \vdots & 0 & 0 & \ddots \end{pmatrix}\begin{pmatrix}\mathbf{a}\\ \vdots \\ \alpha_i \\ \vdots\end{pmatrix} = \begin{pmatrix}\sum\mathbf{b}_i \\ \vdots \\ \beta_i \\ \vdots\end{pmatrix}$$

> source term $\quad \mathbf{b}' \equiv \sum\left(\mathbf{b}_i - \mathbf{L}_i\alpha_i^*\right)$

> local solution $\quad \alpha_i^* \equiv \mathbf{L}_i^{-1}\mathbf{b}_i$

> (inverse of) Schur complement $\quad \mathbf{C}' \equiv \sum\left(\mathbf{G}_i - \mathbf{\Gamma}_i\mathbf{L}_i^{-1}\mathbf{\Gamma}_i^{\mathsf{T}}\right)$

$$\longrightarrow \boxed{\mathbf{C}'\mathbf{a} = \mathbf{b}'}$$

More local measurements will not significantly affect computing time

***n* equations with computing time proportional to *n*³**

Solution

Full inversion
> In principle best (since exact) solution, including correlated uncertainties.
> However very demanding in computing resources.
> Reasonable for matrix with up to *n* ~ *O(1000)*.

MINRES-QLP
> Iterative approach in Krylov space.
> At iteration *i*, the solution is given by a linear combination of *b'*, *C'b'*, *C'²b'*, *C'³b'*, etc.
> Preconditioning the matrix to help the convergence (typically with Cholesky decomposition).
> Stop iterative process once || *C'a* - *b'* || is "small" enough.
> Able to deal with possibly singular matrices.

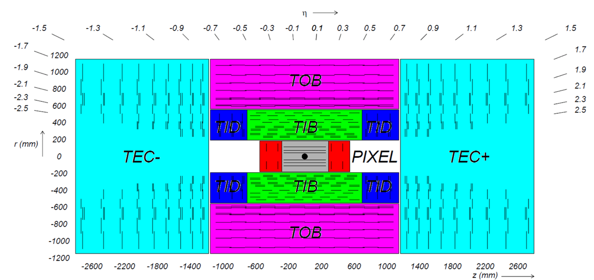
Invert large, symmetric matrices

***n* equations with computing time proportional to n_{it}×*n*²**

CMS @ LHC (CERN)

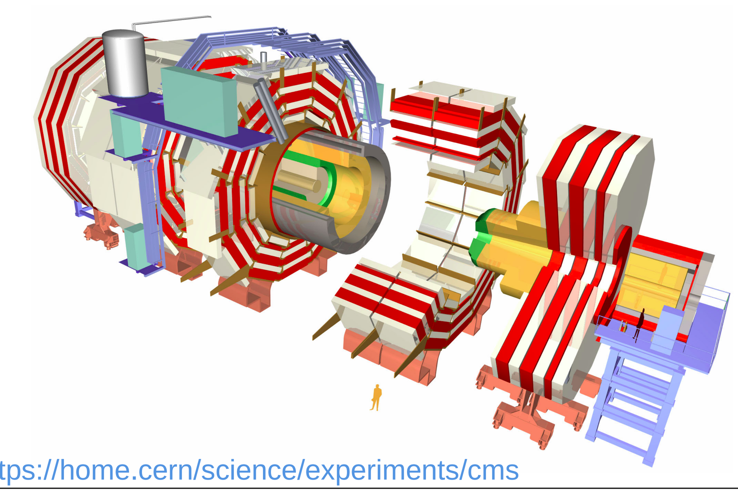
Current history
> *Phase-0*: Run-I & Run-II 2016 [8]
> *Phase-I*: Run-II 2017-2018 [9]

Central tracking system
> *Inner pixel detector*: PXF + PXB
> *Outer strip detector*: TIB + TID + TOB + TEC



> Each sensor has to be aligned [10,11].
> 3+3+3 parameters for **position**, **orientation** and **curvature**.
> Include **time dependence** with hierarchy approach.
> **Additional calibration** may be included (*e.g.* Lorentz drift in pixel sensors)

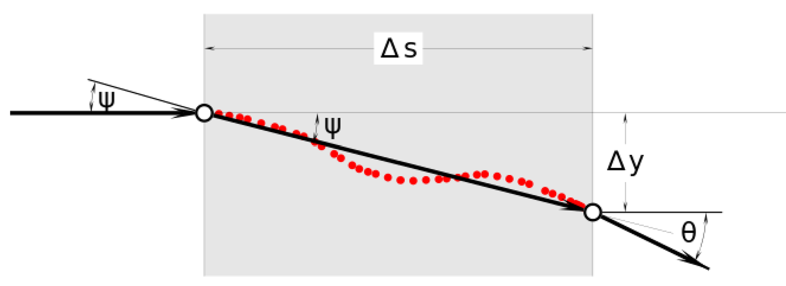
→ O(100k) parameters are determined in O(10h)!



https://home.cern/science/experiments/cms

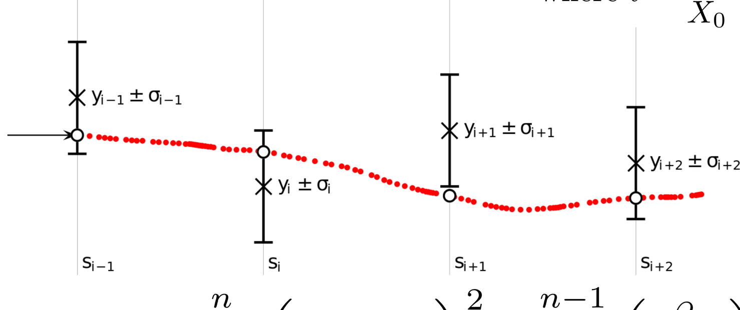
General Broken Lines [13,14,15]

Fast track model necessary for tracker alignment

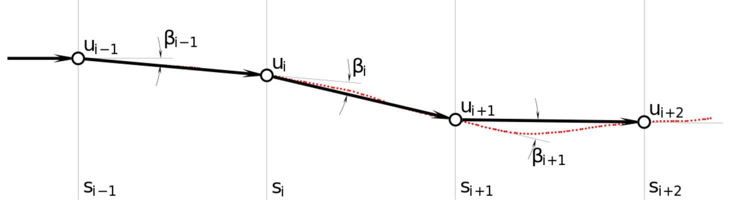


$$V\begin{bmatrix}\theta\\ \psi\end{bmatrix} = \begin{bmatrix}1 & 1/2\\ 1/2 & 1/3\end{bmatrix}\cdot\left(\frac{13.6MeV}{\beta pc}\right)^2\cdot t\cdot(1+0.038\ln t)^2$$

$$\text{where } t = \frac{\Delta s}{X_0}$$



$$\chi^2(\mathbf{u}) = \sum_{i=1}^{n}\left(\frac{y_i-u_i}{\sigma_i}\right)^2 + \sum_{i=2}^{n-1}\left(\frac{\beta_i}{\sigma_{\beta,i}}\right)^2$$



Introduction
> Mathematically equivalent to Kálmán filter for track fitting (standard in tracking in HEP).
> Computationally different**,** with bordered band matrix.
> Implementation available for Fortran, C++ & Python, including Mille step in context of tracker alignment.

GBL in a nutshell
> Main deviations from perfect helix (5 parameters) are due to multiple scattering in modules.
> *Broken lines* means that the trajectory is described in terms of scatterers and kink angles.
> *General broken lines* include arbitrary magnetic field for arbitrary measurements and material distribution.

Purpose
> Describes trajectory in term of scatterers and kink angles.
> Five parameters per scatterer: curvature *q/p*, offset **u**, slope d**u**/d*w*.
> Re-write the problem in term of bordered band matrix.

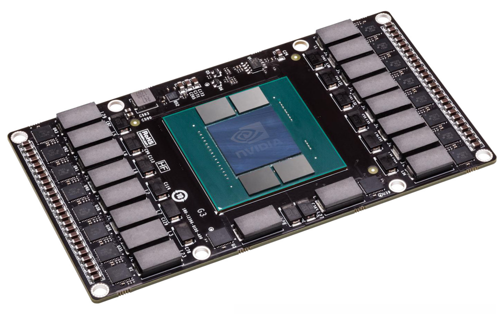
$$\mathbf{p}_{\rm local} = \left(\frac{q}{p}, \mathbf{u}, \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}w}\right)$$

$$\text{where } \mathbf{u} = (u_1, u_2)$$

Root-free Cholesky decomposition
> Fast due to bordered band matrix structure
> For *n* measurements, time for track fit (track fit + covariance matrix) ~ *n* (*n*²), while inversion ~ *n*³

$$\left(\begin{array}{c|ccccccccc} d & b & b & b & b & b & b & b & b & b\\ \hline b & d & m & m & 0 & 0 & 0 & 0 & 0 & 0\\ b & m & d & m & m & 0 & 0 & 0 & 0 & 0\\ b & m & m & d & m & m & 0 & 0 & 0 & 0\\ b & 0 & m & m & d & m & m & 0 & 0 & 0\\ b & 0 & 0 & m & m & d & m & m & 0 & 0\\ b & 0 & 0 & 0 & m & m & d & m & m & 0\\ b & 0 & 0 & 0 & 0 & m & m & d & m & m\\ b & 0 & 0 & 0 & 0 & 0 & m & m & d & m\\ b & 0 & 0 & 0 & 0 & 0 & 0 & m & m & d \end{array}\right)$$

Future



Implementation
> Massive parallelisation
> Probably on GPUs using OpenCL

OpenCL

Applications
> CMS Phase-II upgrade [16]
> Apply to other detectors & calibrations
> Investigate application in machine learning

What about *your* project?

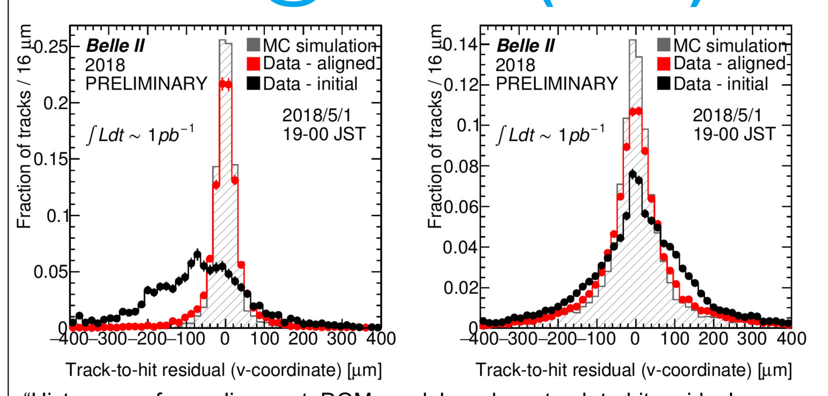
Acknowledgements

Millepede (I/II) has been developed by Volker Blobel (U. Hamburg) since 1997 and the support and further development has been handed over to the German Terascale Alliance in 2009.

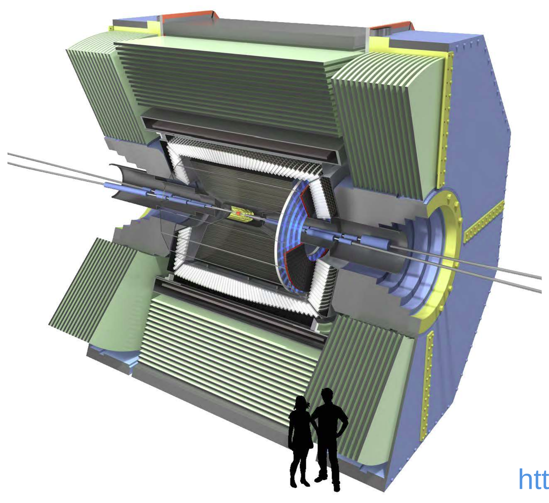
A new fast track-fit algorithm based on broken lines has been developed by V. Blobel since 2005 (for H1) and extended to GBL (for CMS) in 2011 by C.K.

We also express our gratitude to the organisers of the school for giving us the opportunity to participate and present this work.

Belle-II @ KEKB (KEK)



"Histograms from alignment DQM modules show track-to-hit residuals from all PXD (left) and SVD (right) sensors after standard reconstruction and track fitting is performed. Distributions are shown for the residuals of the local v-coordinate, parallel to z/beam axis. The data from some of the first collisions in Phase 2 (limited vertex detector) are used to illustrate the improvement of the position resolution **before** and **after** the first preliminary alignment is computed and used in reconstruction. All 108 parameters (18 sensors x 6 rigid body parameters, CDC fixed as reference) are determined simultaneously. A sample of 20k charged particles originating near IP (electrons, positrons, protons) is generated to check the correspondence of the data and MC simulation (grey). All compared histograms are normalized." [12]



https://www.belle2.org

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