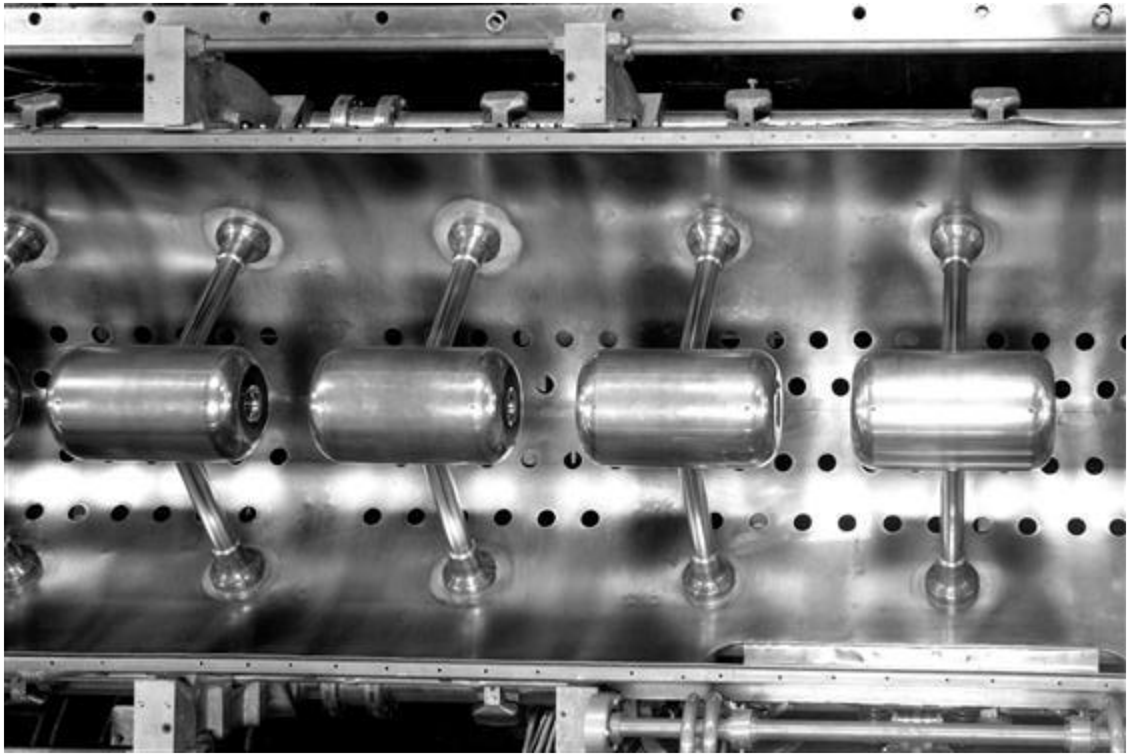
Part2: Cavities and Structures

- Modes in resonant cavity
- From a cavity to an accelerator
- Examples of structures



Wave equation

Maxwell equation for electromagnetics waves

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}-\frac{1}{c^2}\frac{d^2}{dt^2}\right)\vec{E}=0$$

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}-\frac{1}{c^2}\frac{d^2}{dt^2}\right)\vec{B}=0$$

- In free space the electromagnetic fieds are of the transverse electromagnetic, TEM type: Electric and magnetic field vectors are $\perp$ to each other and to the direction of propagation !

- In a bounded medium (cavity) the solution of the equation must satisfy the boundary conditions:

$$\vec{E}_{\parallel}=\vec{0}$$
$$\vec{B}_{\perp}=\vec{0}$$

TE and TM modes

- TE mode (transverse electric): The electric field is perpendicular to the direction of propagation in a cylindrical cavity.

$$TE_{mn}$$

m: azimuthal
n: radial

- TM mode (transverse magnetic): The magnetic field is perpendicular to the direction of propagation in a cylindrical cavity.

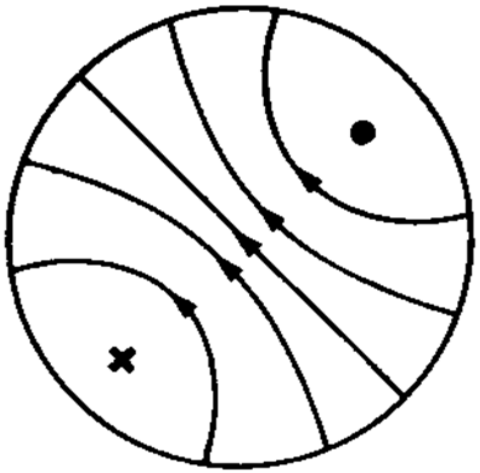
$$TM_{mn}$$

m: azimuthal
n: radial

TE modes

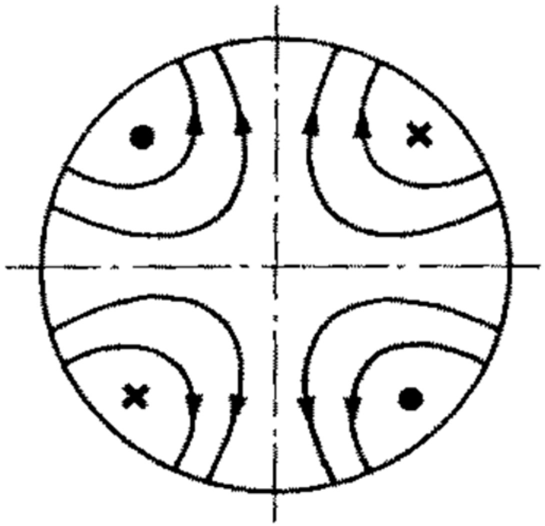
- The two Transverse Electric modes for accelerating structures are:

TE_{11}: **Dipole mode**



Empty cavity; mode TE_{11}

TE_{21}: **Quadrupole mode**

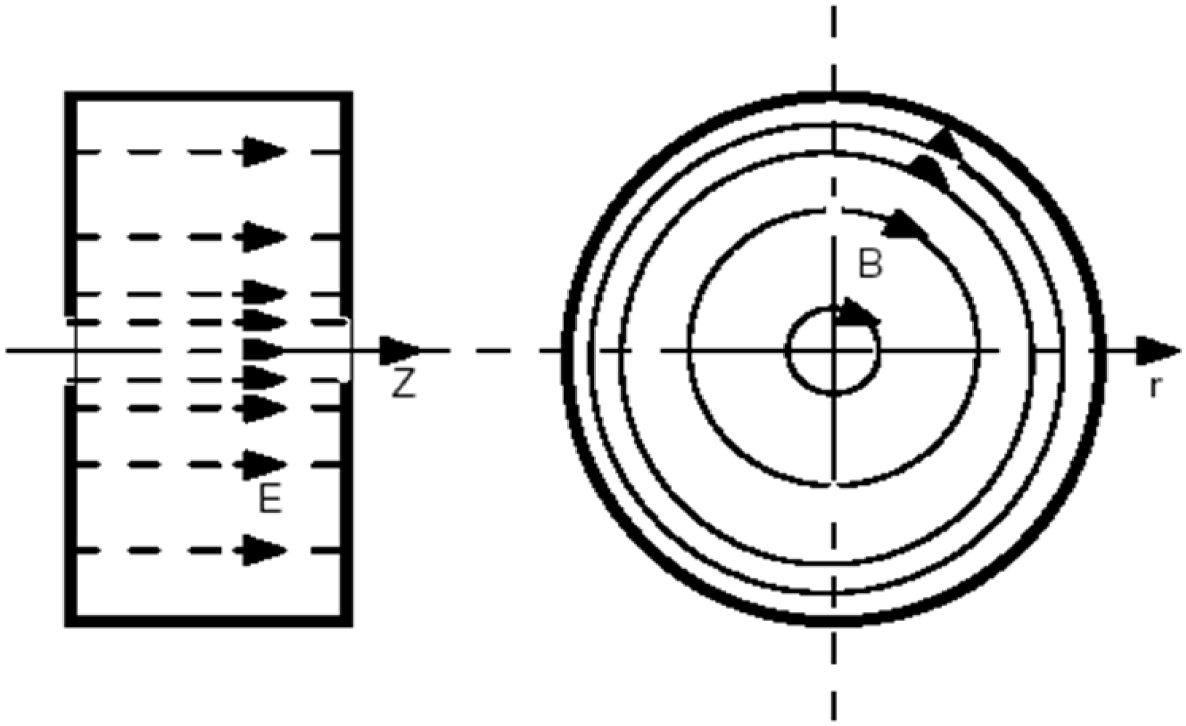


Empty cavity; mode TE_{21}

TM modes

- The most commonly used Transverse Magnetic mode for accelerating structures is:

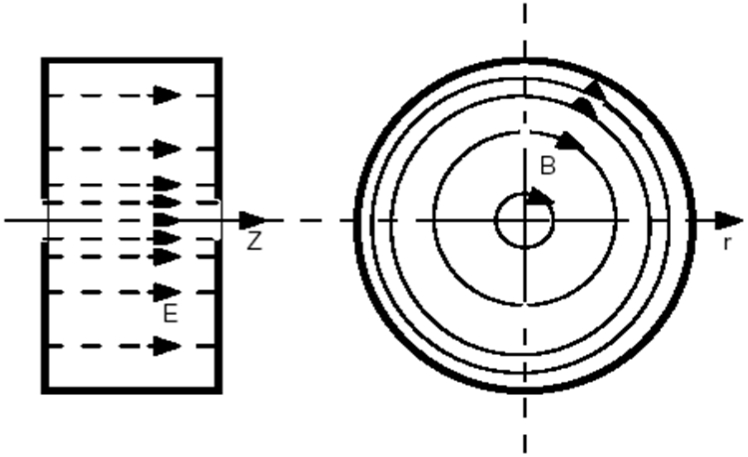
TM_{01}

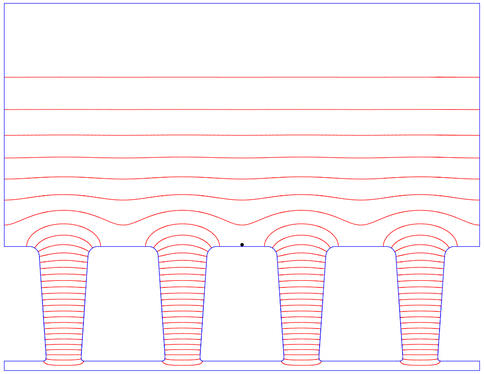


TM modes

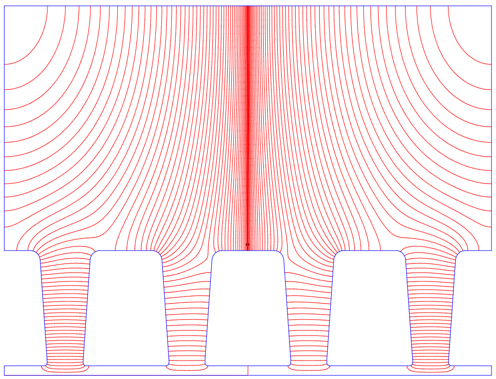
- The most commonly used for accelerating structures are the TM modes:

TM_{01}

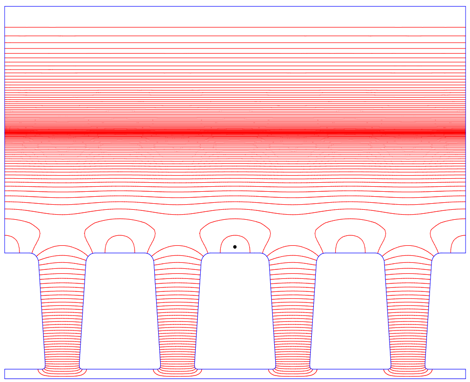




TM_{010} : f=352.2 MHz

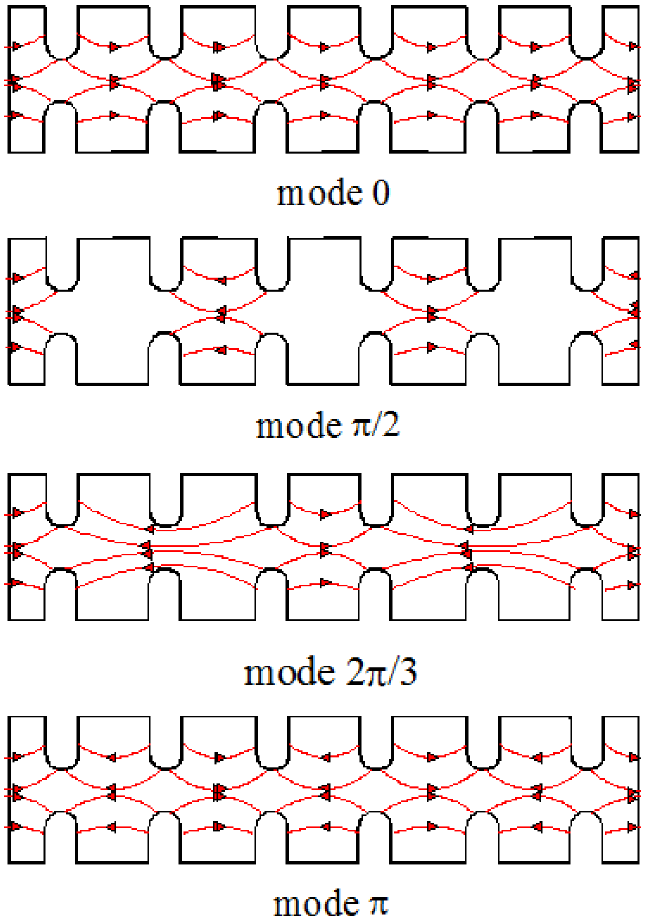


TM_{011} : f=548 MHz



TM_{020} : f=952 MHz

Standing Wave Modes



Named from the phase difference between adjacent cells.

Mode 0 also called mode 2π.

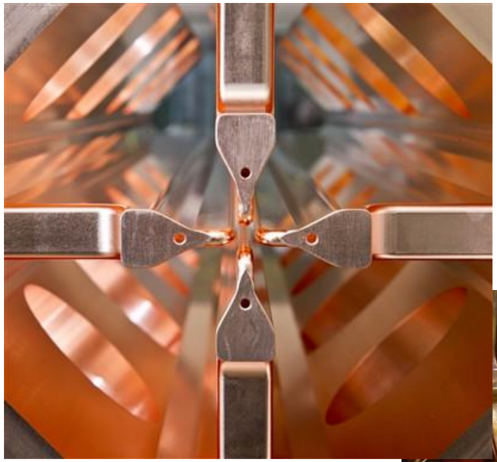
For synchronicity and acceleration, particles must be in phase with the E field on axis (will be discussed more in details in part.3).

During 1 RF period, the particles travel over a distance of $\beta\lambda$.

The cell L lentgh should be:

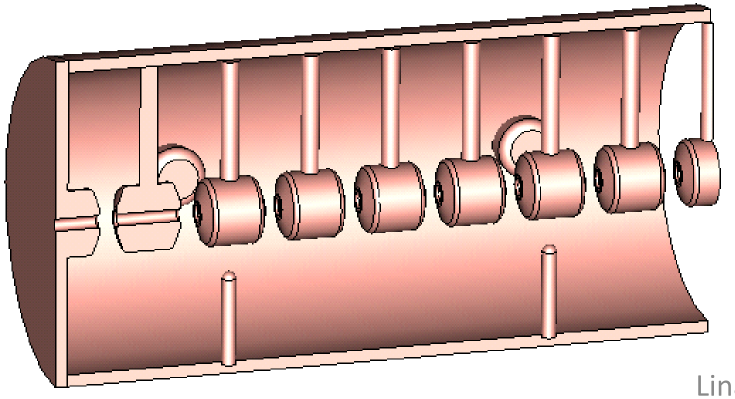
Mode	L
2π	$\beta\lambda$
$\pi/2$	$\beta\lambda/4$
$2\pi/3$	$\beta\lambda/3$
π	$\beta\lambda/2$

Basic accelerating structures

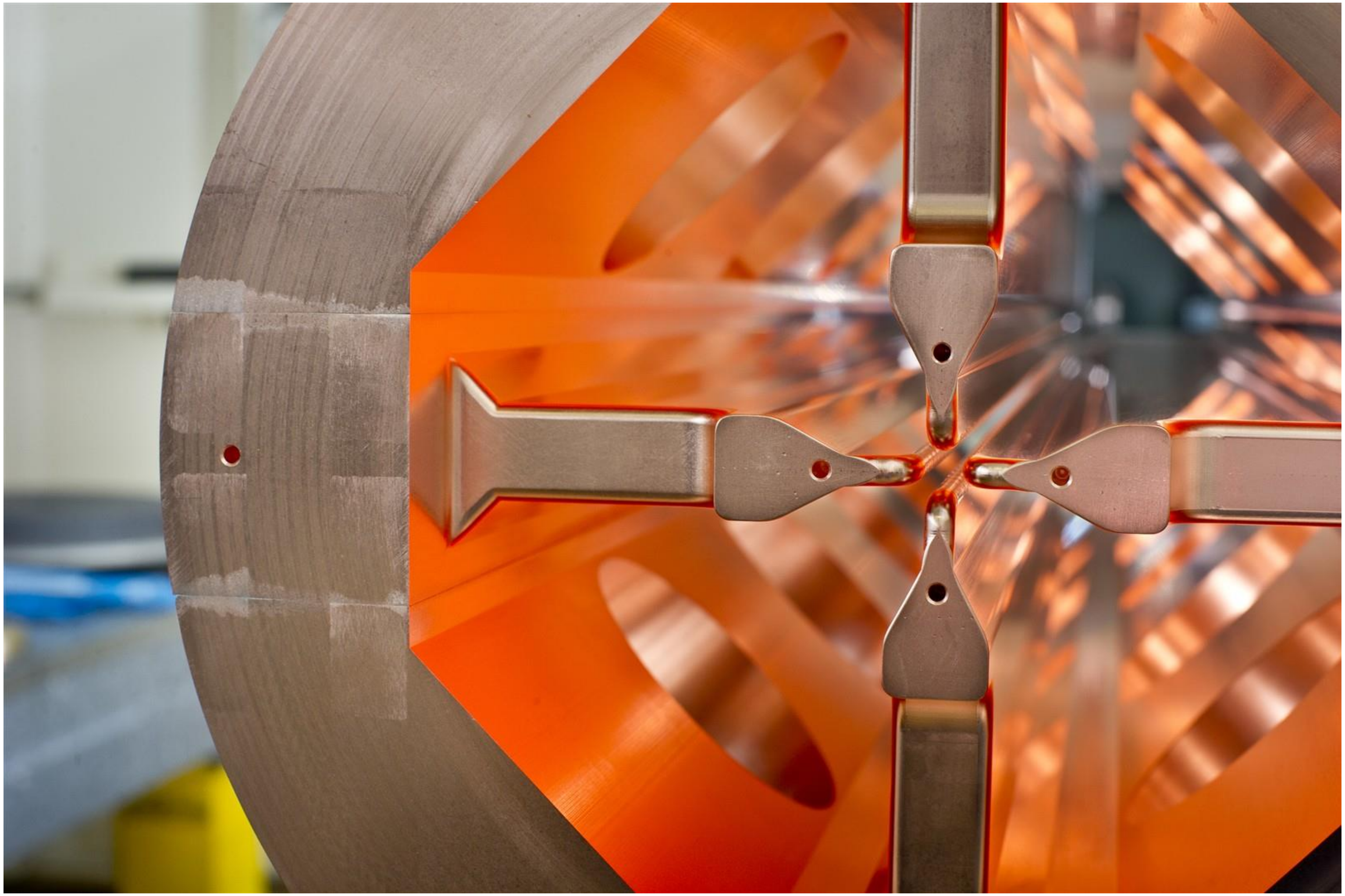


- TE mode:
  - Radio Frequency Quadrupole: RFQ
  - Interdigital-H structure: IH

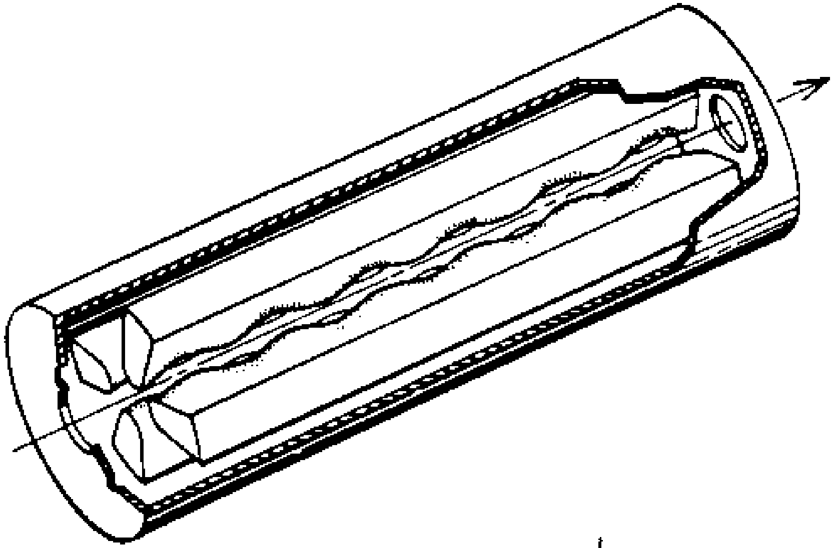
- TM mode:
  - Drift Tube Linac: DTL
  - Cavity Coupled DTL: CCDTL
  - PI Mode Structure: PIMS
  - Superconducting cavities

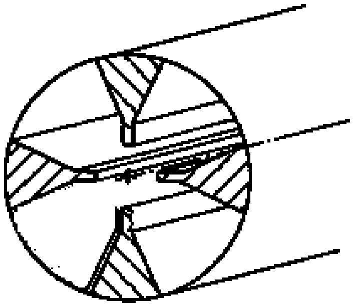


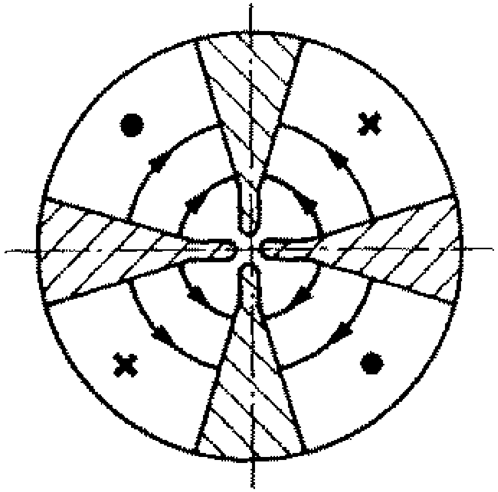
Radio Frequency Quadrupole



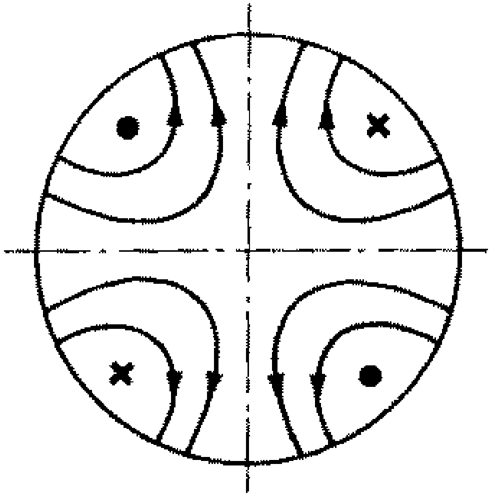
Radio Frequency Quadrupole





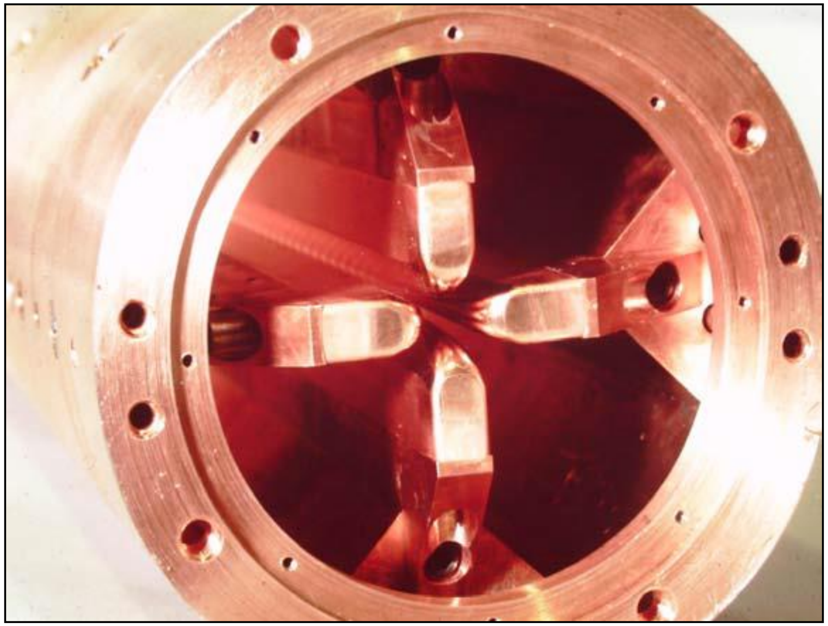
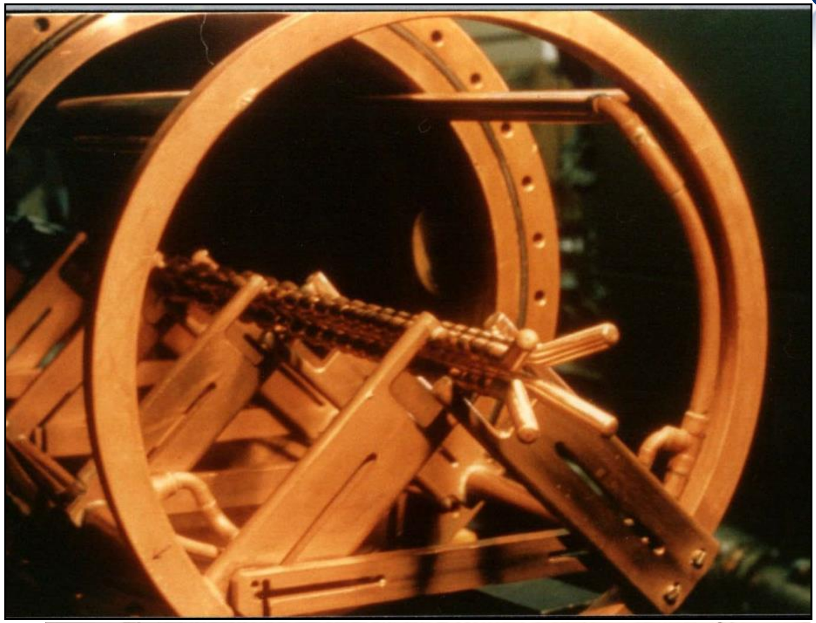
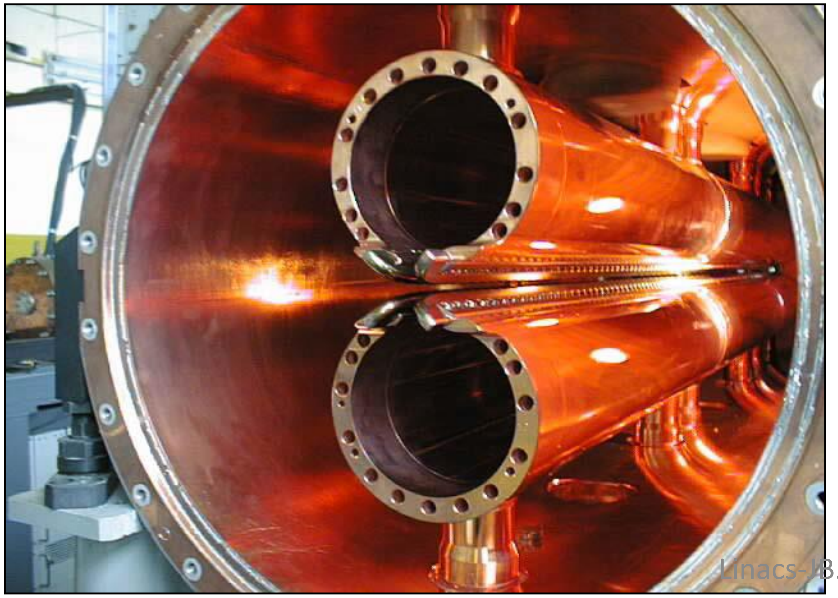
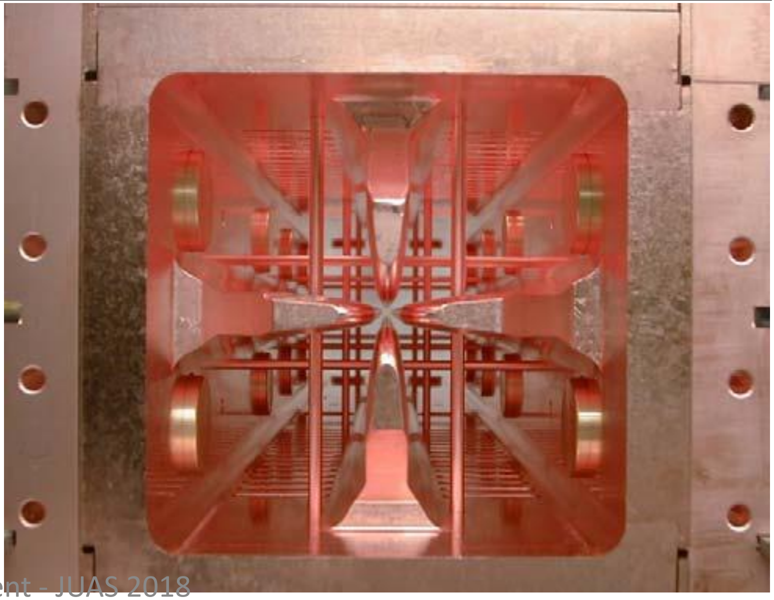


Cavity with vanes

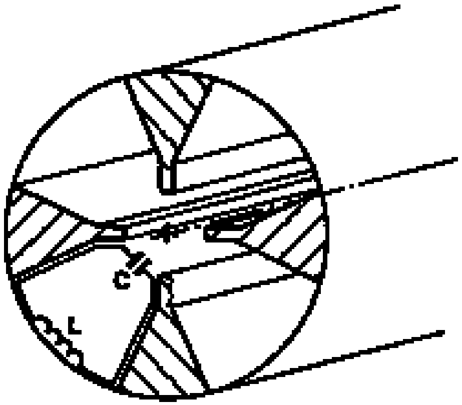


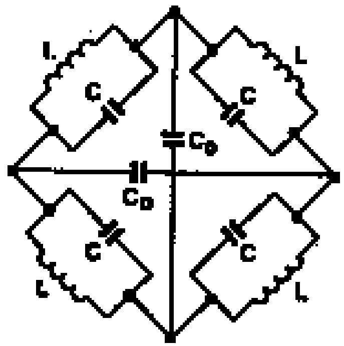
Empty cavity; mode TE_{21}

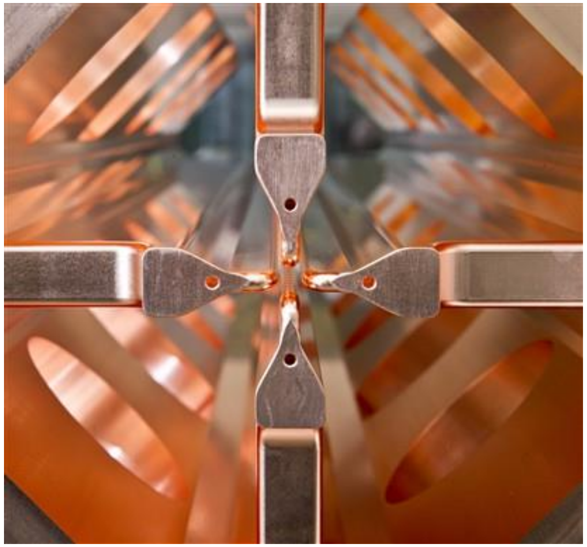
Radio Frequency Quadrupole

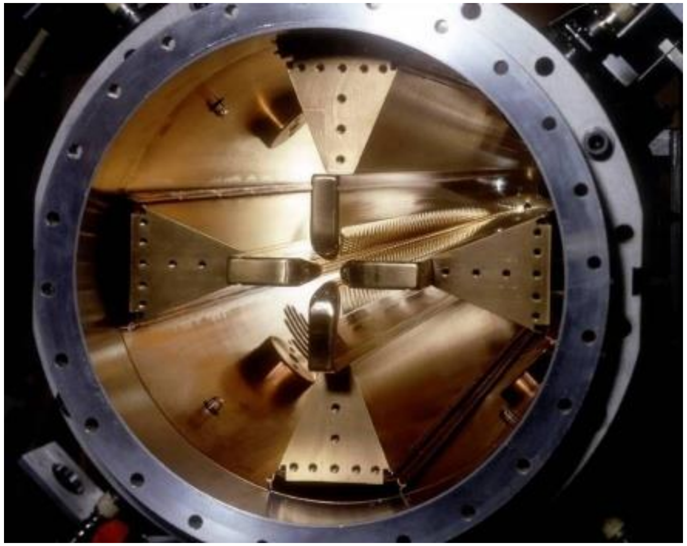





4 vane-structure



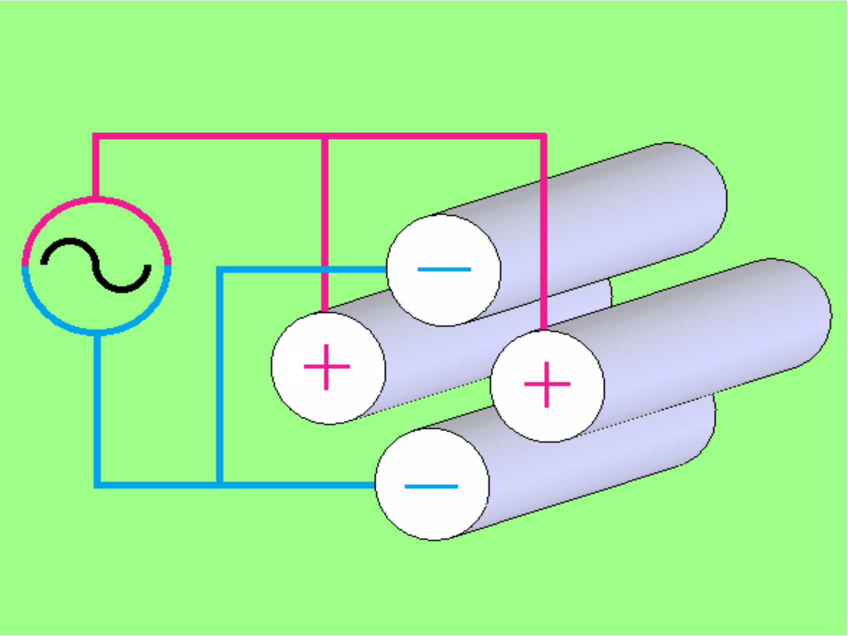


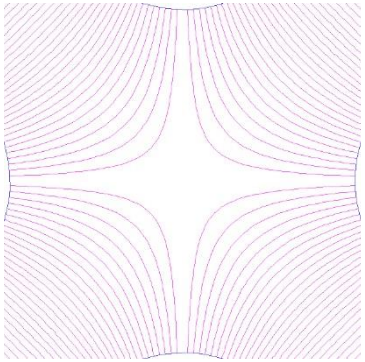


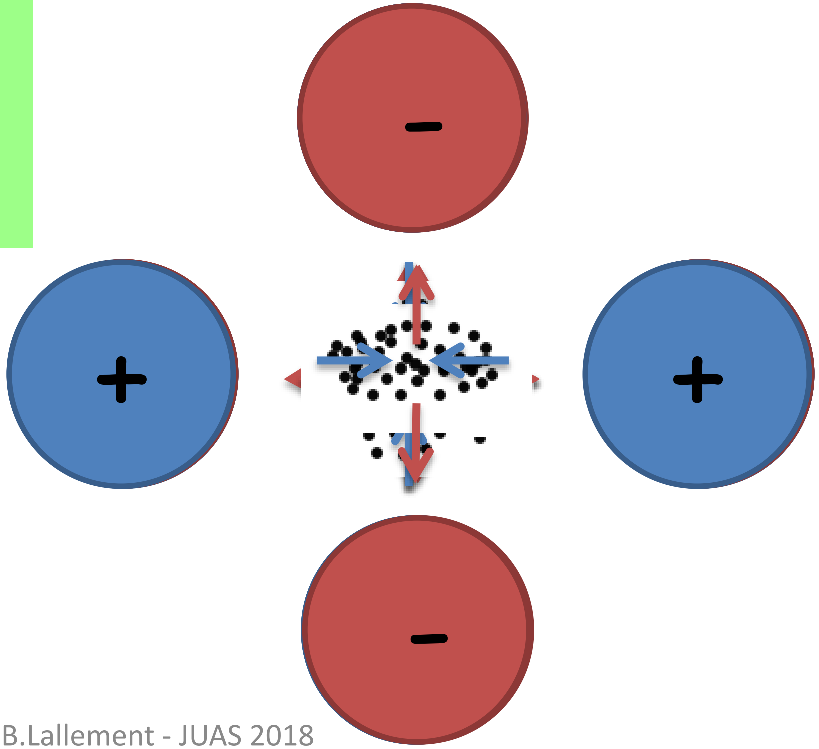


1. Capacitance between vanes, inductance in the intervane volume.
2. Each quadrant is a resonator
3. Frequency depends on cylinder dimensions.
4. Vane tip are machined by a computer controlled milling machine.
5. Need stabilization (problem of mixing with dipole mode TE11).

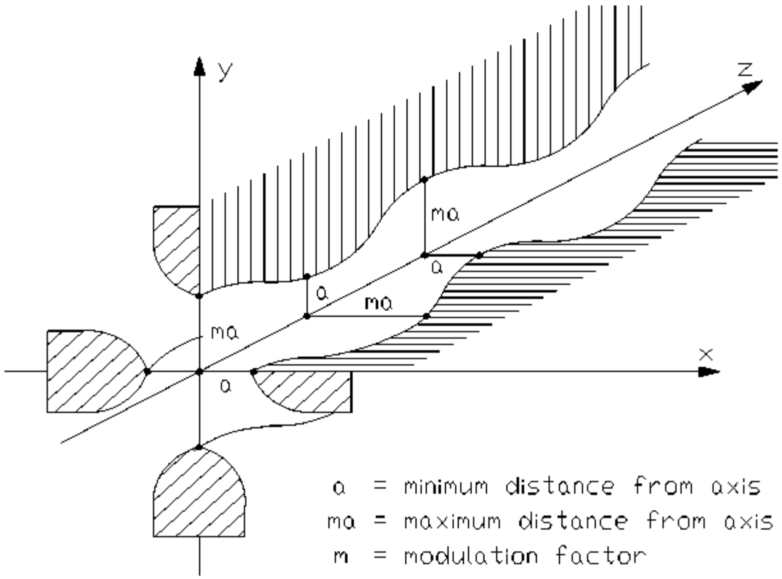
Radio Frequency Quadrupole

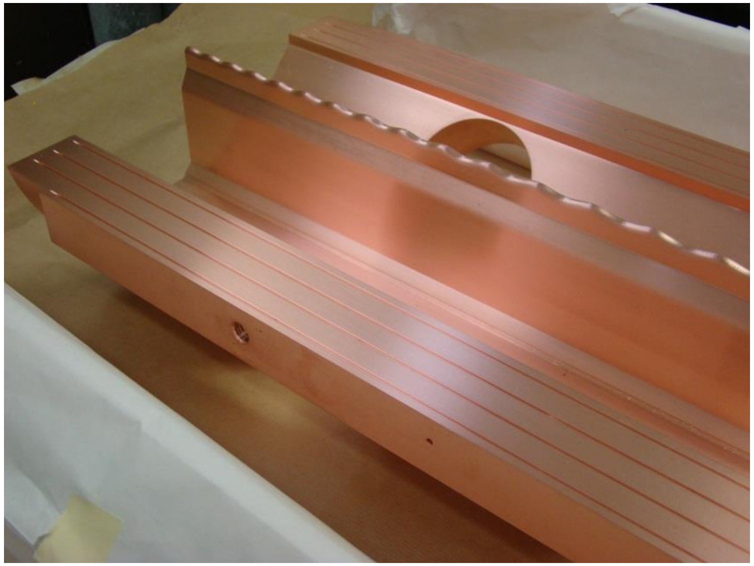




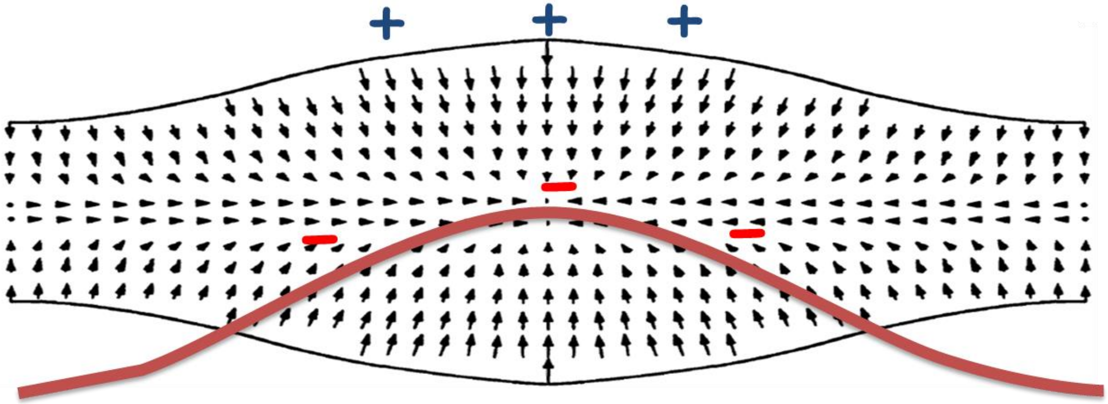


Acceleration in RFQ





longitudinal modulation on the electrodes creates a longitudinal component in the TE mode



RFQ parameters

$$B=\left(\frac{q}{m_0}\right)\left(\frac{V}{a}\right)\left(\frac{1}{f^2}\right)\frac{1}{a}\left(\frac{I_o(ka)+I_o(mka)}{m^2 I_o(ka)+I_o(mka)}\right)$$

Focusing term

$$E_0T=\frac{m^2-1}{m^2 I_o(ka)+I_o(mka)}\cdot V\frac{2}{\beta\cdot\lambda}\frac{\pi}{4}$$

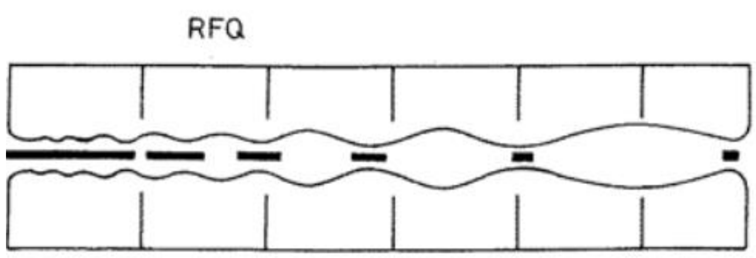
Acceleration term

a=bore radius
m=modulation
m_0=rest mass
β=reduced velocity
λ=wave length
f=frequency
k=wave number
V=vane voltage
$I_{0,1}$= zero and first order Bessel function

$$\left(\frac{I_o(ka)+I_o(mka)}{m^2 I_o(ka)+I_o(mka)}\right)+\frac{m^2-1}{m^2 I_o(ka)+I_o(mka)}\cdot I_0(ka)=1$$

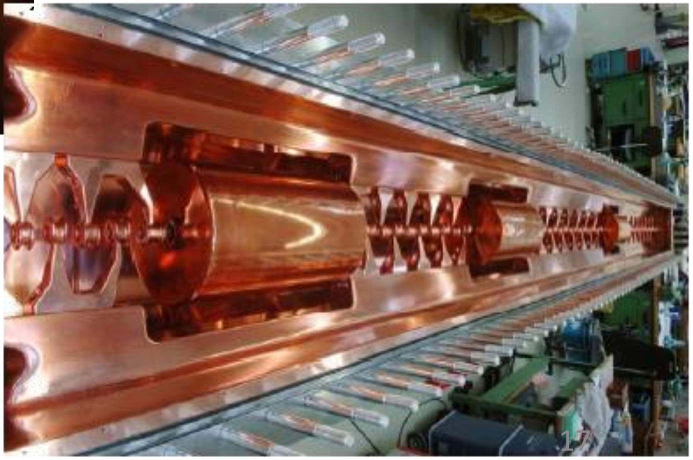
RFQ

- The resonating mode of the cavity is a focusing mode
- Alternating the voltage on the electrodes produces an alternating focusing channel
- A longitudinal modulation of the electrodes produces a field in the direction of propagation of the beam which bunches and accelerates the beam
- Both the focusing as well as the bunching and acceleration are performed by the RF field
- The RFQ is the only linear accelerator that can accept a low energy CONTINOUS beam of particles
- 1970 Kapchinskij and Teplyakov propose the idea of the radiofrequency quadrupole (I. M. Kapchinskii and V. A. Teplvakov, Prib.Tekh. Eksp. No. 2, 19 (1970))

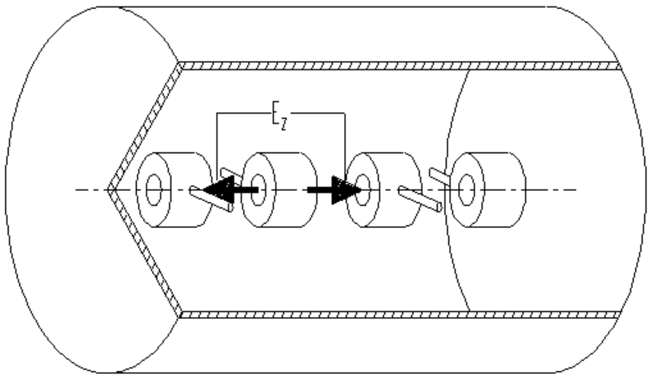


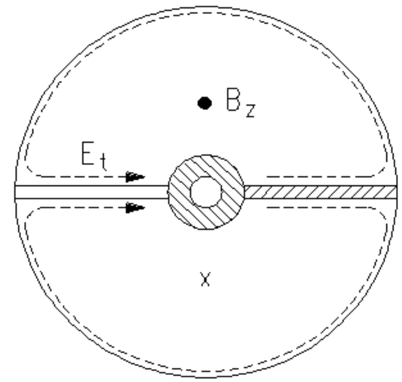
Acceleration
Bunching
Transverse focusing

Interdigital H structure

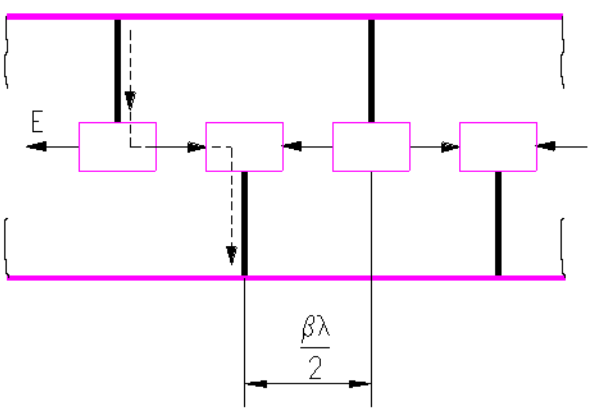


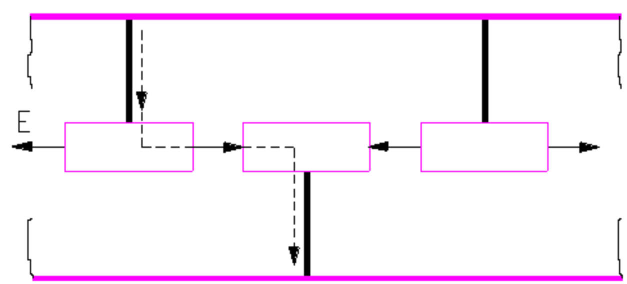
Interdigital H structure





TE$_{11}$ mode





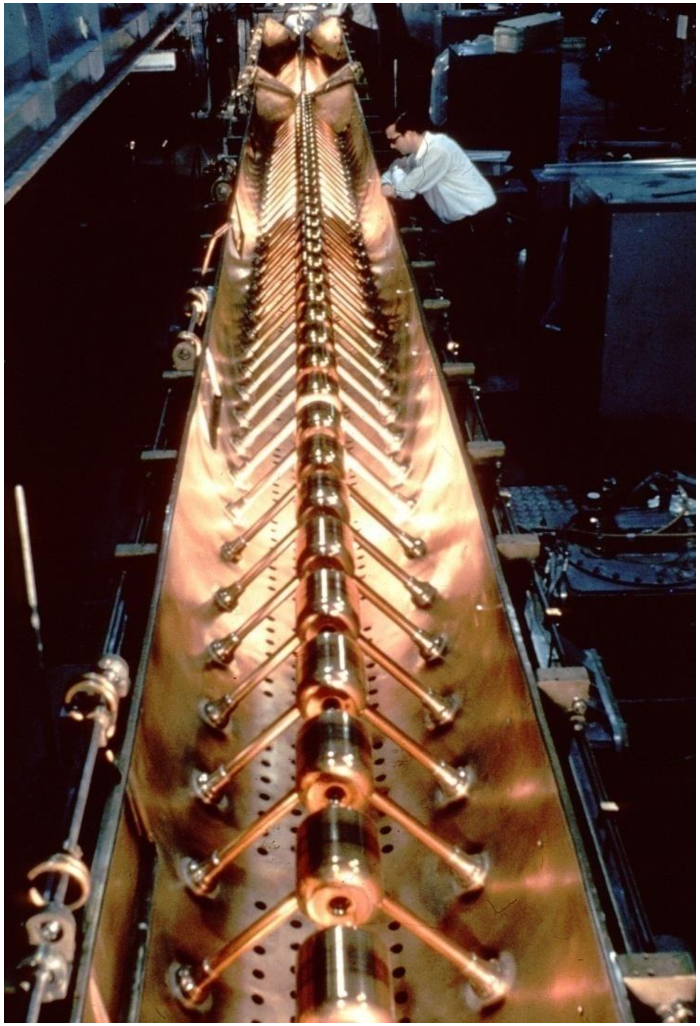
•Stem on alternating side of the drift tube force a longitudinal field between the drift tubes

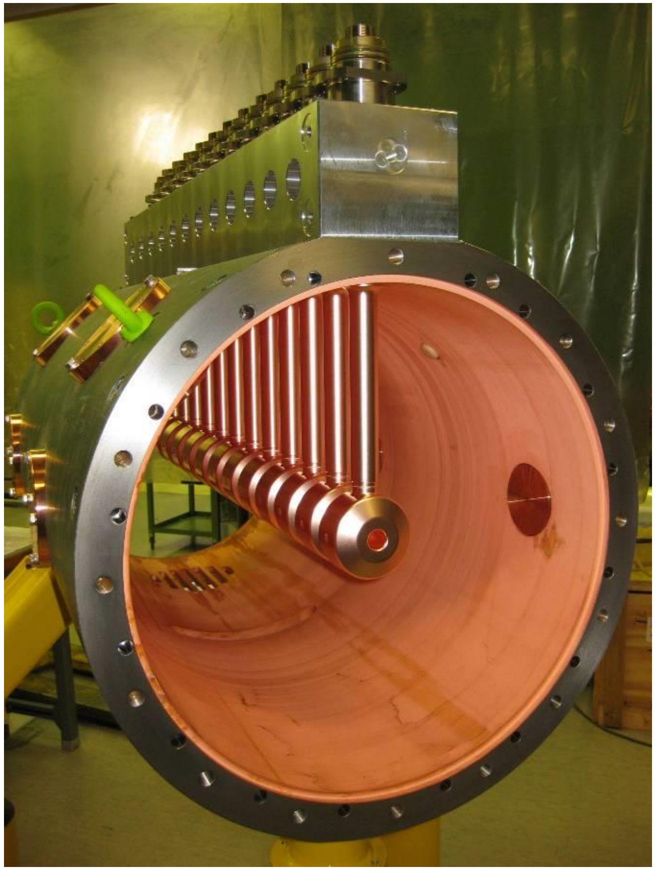
•Focalisation is provided by quadrupole triplets places OUTSIDE the drift tubes or OUTSIDE the tank

Interdigital H structure

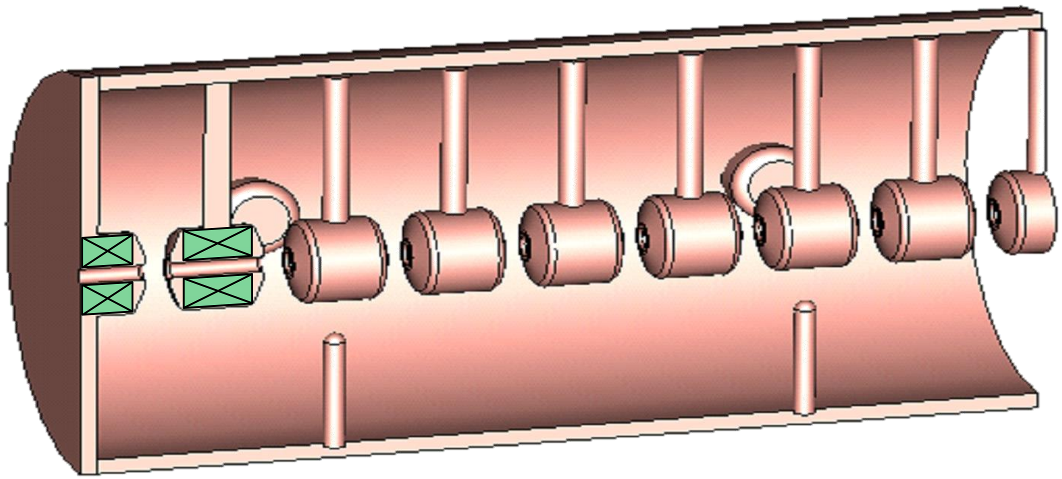
- very good shunt impedance in the low beta region (($\beta \cong 0.02$ to 0.08) and low frequency (up to 200MHz)
- not for high intensity beam due to long focusing period
- ideal for low beta heavy ion acceleration

Drift Tube Linac

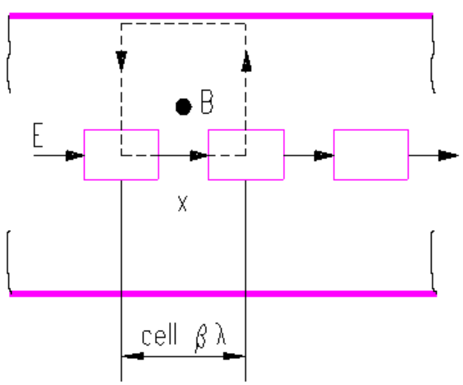


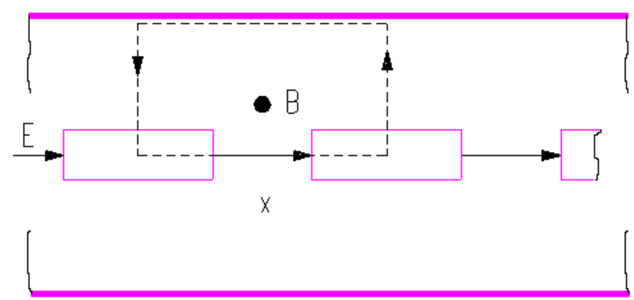


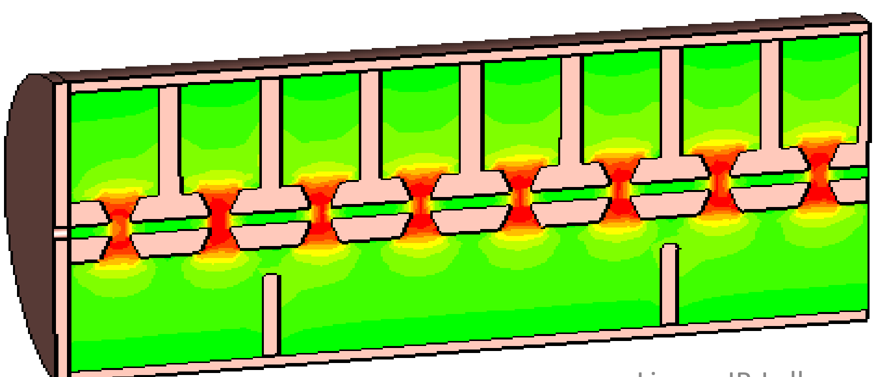
Drift Tube Linac

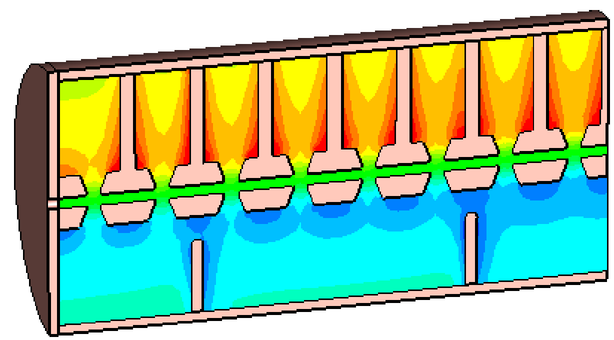


Tutorial !

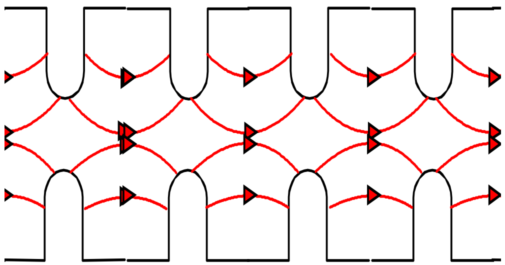




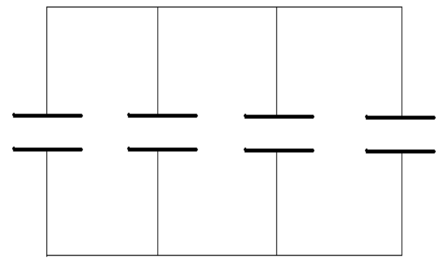




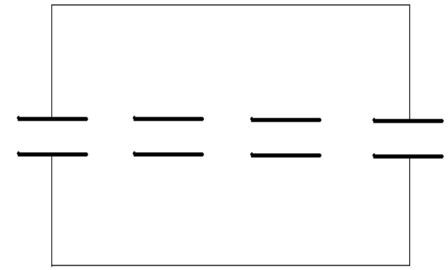
Drift Tube Linac



Disc loaded structure
Operation in 2π mode



Add tubes for high shunt impedance

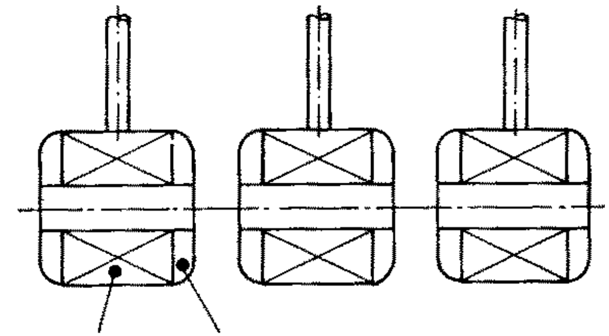


Remove the walls to increase coupling between cells

Particles are inside the tubes when the electric field is decelerating.
Quadrupole can fit inside the drift tubes.
β=0.04-0.5 (750 keV – 150 MeV)

Synchronism condition for 2π mode : $$L = \frac{\beta c}{f} = \beta\lambda$$

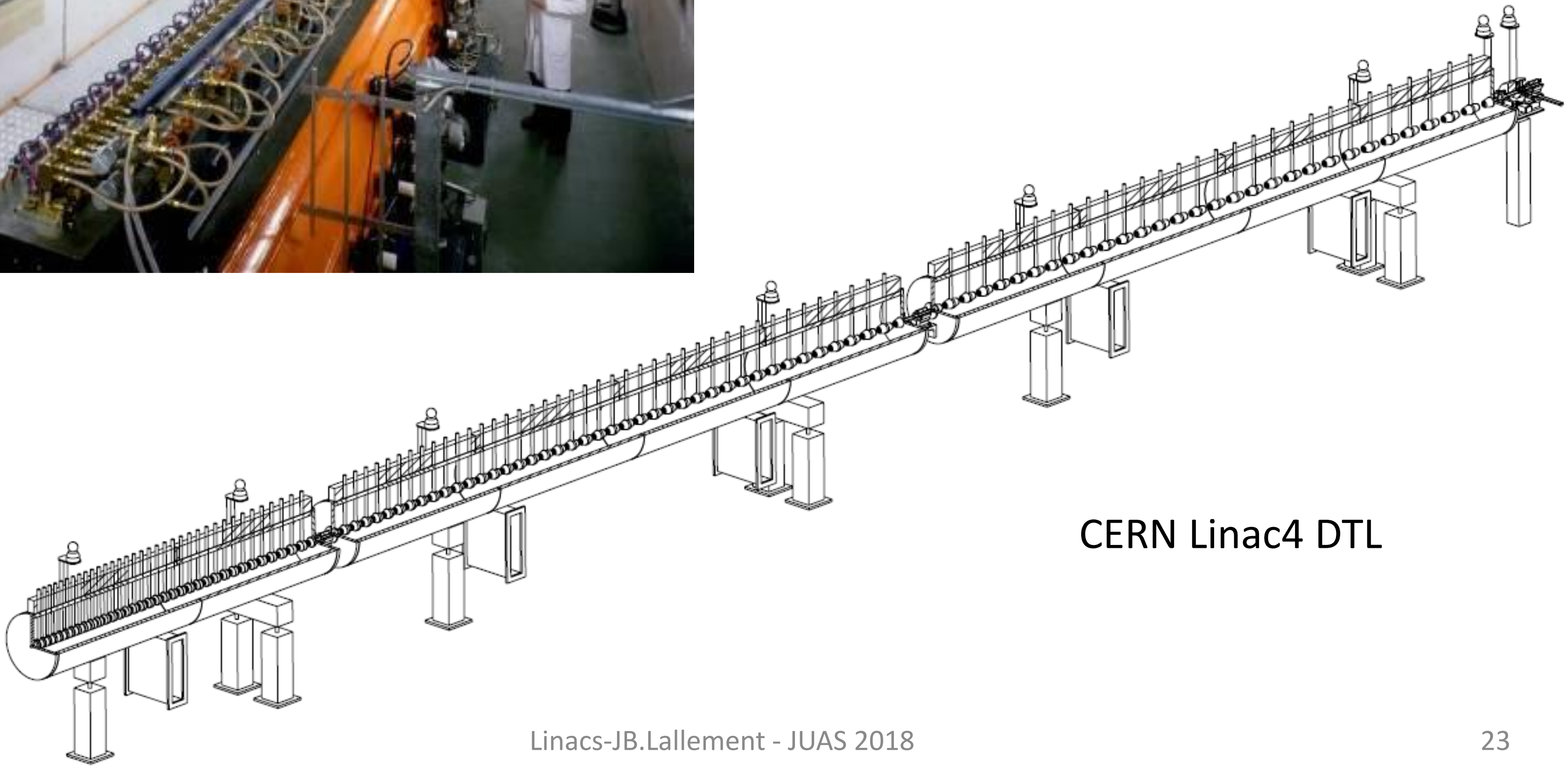
Cell length should increase to account for the beta increase



Ideal for low β – low W - high current

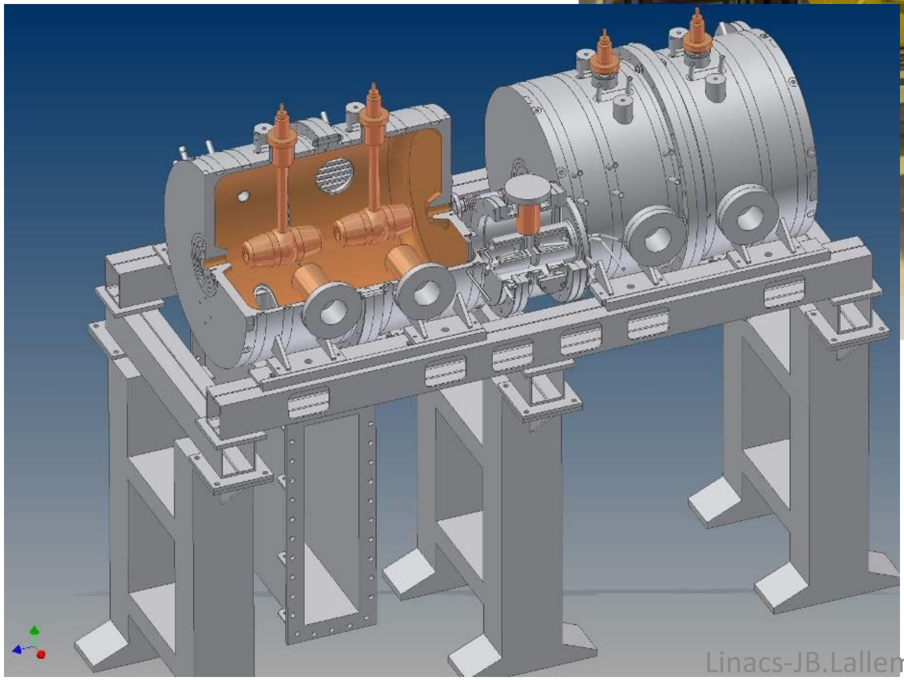
Drift Tube Linac

CERN Linac2 DTL

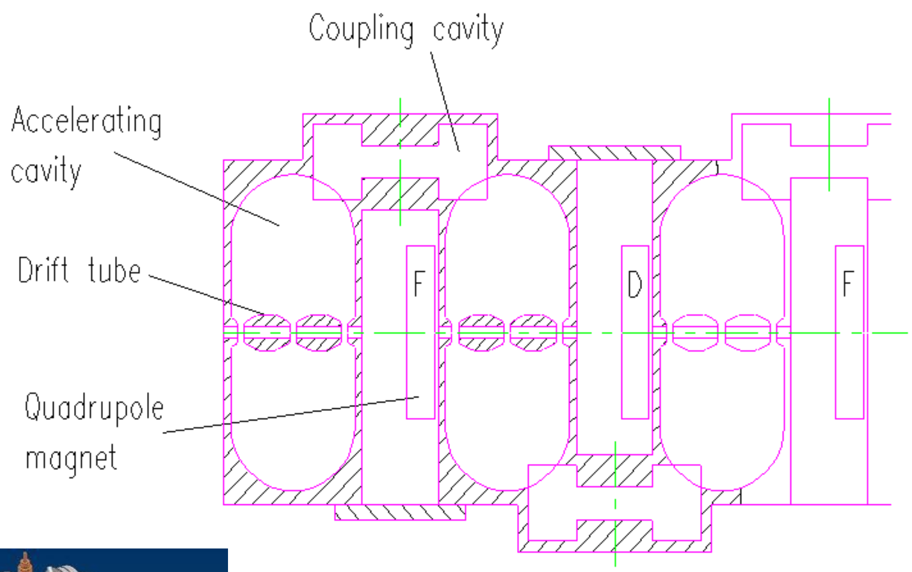


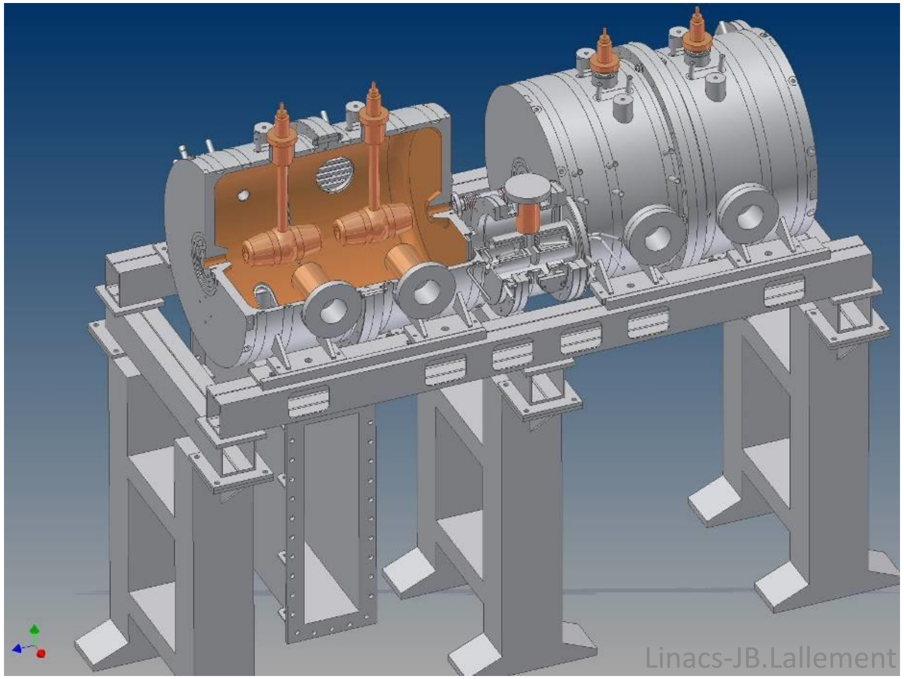
CERN Linac4 DTL

Coupled Cavity DTL

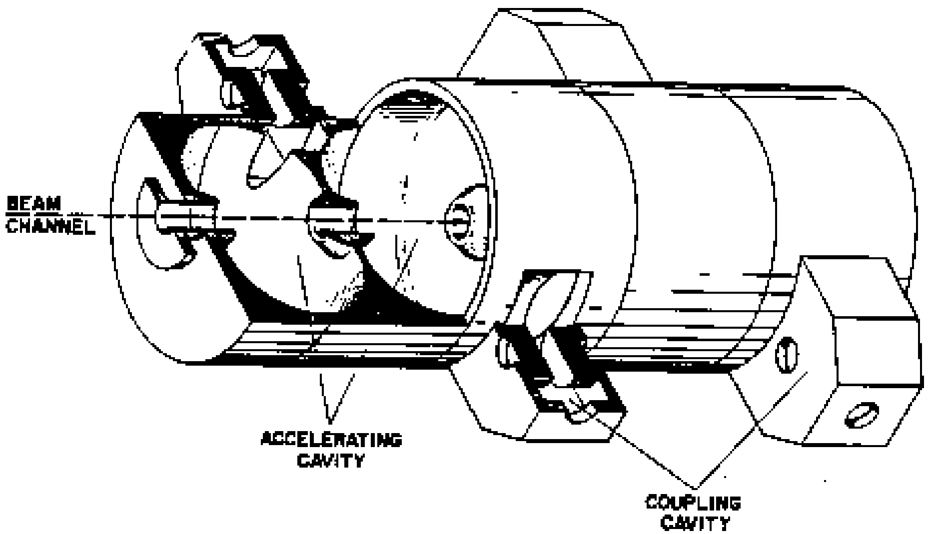


Coupled Cavity DTL





Side Coupled Cavity



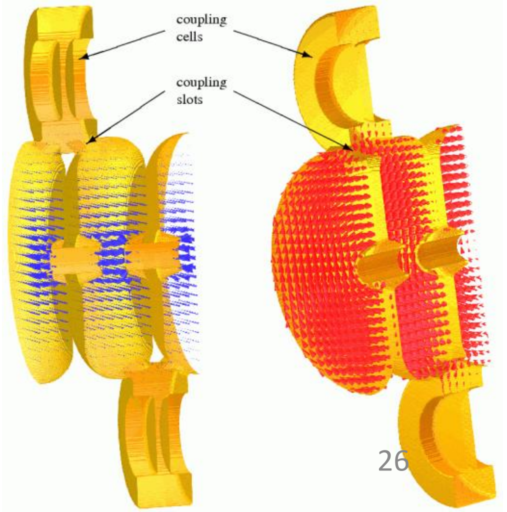
multi-cell Standing Wave structure in **p/2 mode**
frequency 800 - 3000 MHz
for protons (b=0.5 - 1)

Rationale: high beta ⇒ cells are longer ⇒ advantage for high frequencies

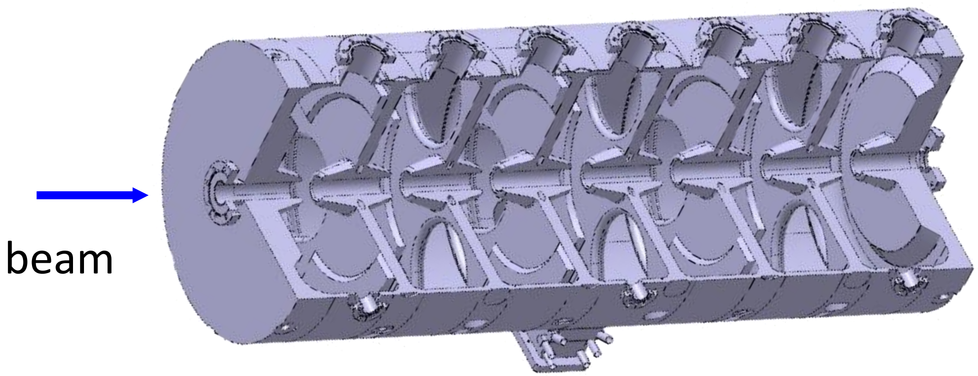
- at high *f*, high power (> 1 MW) klystrons available ⇒ long chains (many cells)
- long chains ⇒ high sensitivity to perturbations ⇒ operation in p/2 mode

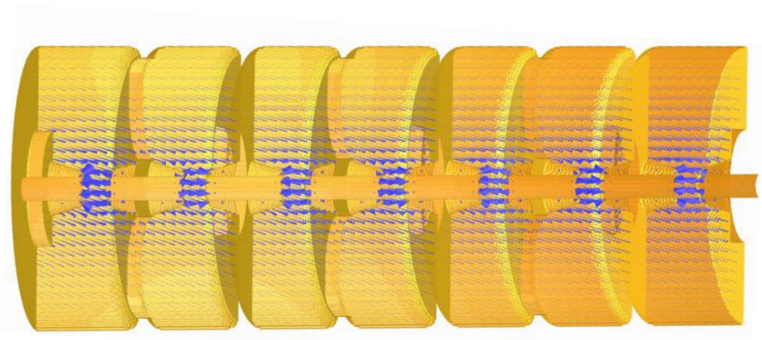
Side Coupled Structure:

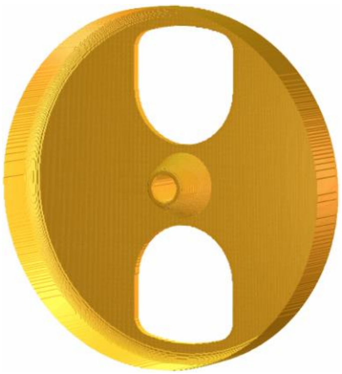
- from the wave point of view, p/2 mode
- from the beam point of view, p mode

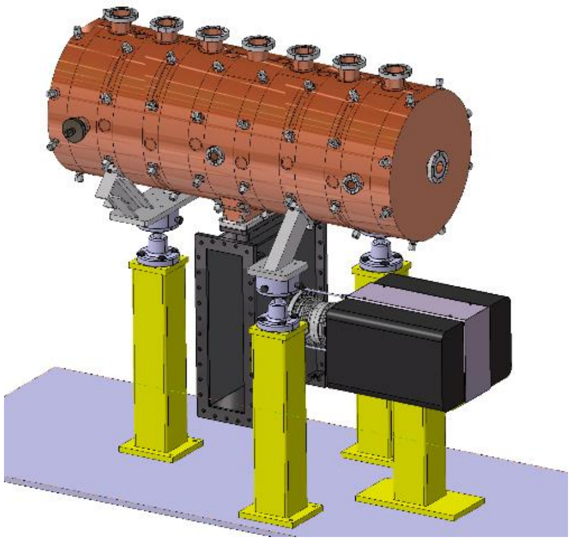


Pi Mode Structure









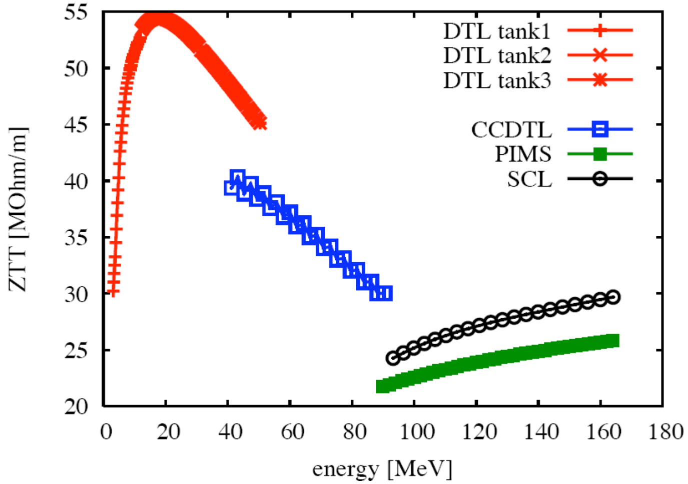
Superconducting cavities
Some examples

Multi gap cavities (elliptical)
Operate in π mode
β>0.5-0.7
350-700 MHz (protons)
0.35-3 GHz (electrons)

Other SC cavities (spoke, HWR, QWR)
β>0.1
From 1 to 4 gaps.
Can be individually phased.
Space for transverse focusing in between
Ideal for low β - CW proton linacs.

The choice of the structures

- Particle type : mass and charge
- Beam current
- Duty factor (pulsed, CW)
- Frequency
- Energy
- Operational constraints

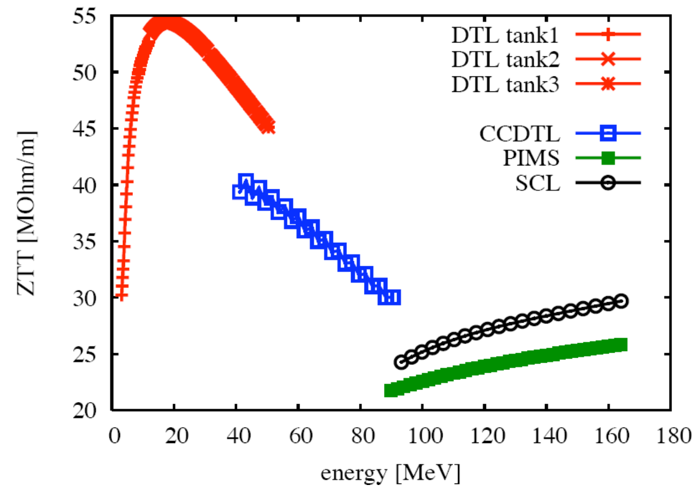


Cavity Type	Beta Range	Frequency	Particles
RFQ	Low! – 0.1	40-500 MHz	Protons, Ions
IH	0.02 – 0.08	40-100 MHz	Ions (Protons)
DTL	0.05 – 0.5	100-400 MHz	Protons, Ions
SCL	0.5 – 1 (ideal is 1)	600-3000 MHz	Protons, Electrons
HWR-QWR-Spokes	0.02-0.5	100-400 MHz	Protons, Ions
Elliptical	> 0.5-0.7	350 – 3000 MHz	Protons, Electrons

Not exhaustive list – To take with caution !!!

The choice of the structures

- Particle type : mass and charge
- Beam current
- Duty factor (pulsed, CW)
- Frequency
- Energy
- Operational constraints



To
Take with
CAUTION !!!

Cavity Type	Beta R[illegible]	Frequency	Particles
RFQ	Lo[illegible]	[illegible]500 MHz	Protons, Ions
IH	0[illegible]	[illegible]0 MHz	Ions (Protons)
DTL	[illegible]	[illegible] MHz	Protons, Ions
SCL	[illegible]	[illegible]MHz	Protons, Electrons
HWR-QWR-Sp[illegible]	[illegible]	[illegible]z	Protons, Ions
Elliptical	> 0.5-0.7	350 – 3000 MHz	Protons, Electrons

Not exhaustive list – To take with caution !!!