# Transverse Beam Dynamics

JUAS 2018 - tutorial 4

## 1 Exercise: Double-Bend Achromat (DBA) lattice

A Double-Bend Achromat (DBA) can be made from two dipoles with a horizontally focusing quadrupole between them. The transfer matrix through the achromat is:

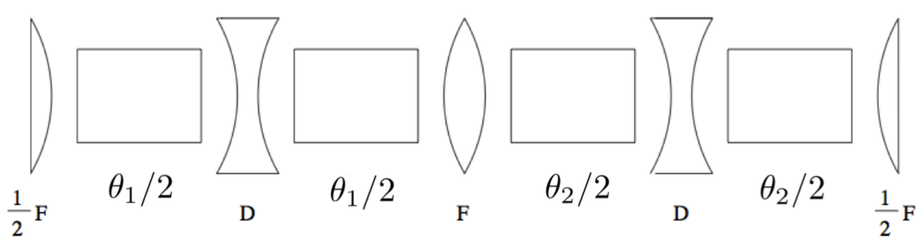
$$M_{\mathrm{DBA}} = M_{\mathrm{bend}} M_{\mathrm{drift}} M_{1/2\mathrm{F}} M_{1/2\mathrm{F}} M_{\mathrm{drift}} M_{\mathrm{bend}}$$

Note that this magnet configuration does not produce vertical focusing, therefore it will not be enough to create a stable lattice. A full DBA typically comprises additional quadrupole doublets before and after the bending section. For sake of simplicity, we will neglect them.

1. Use the thin-lens approximation for quadrupoles and small-angle approximation for bends to find the dispersion in the middle of the quadrupole. Write the focal length in terms of the drift and bend parameters.
2. Show that the dispersion vanishes after the bend.
3. Compute the parameters $L$, $f$ for a 10-meter long DBA which bends the beam by an angle of 1 radians. What is the dispersion in the centre? Given a particle with 1% energy deviation, compute the displacement at the centre of the cell.

## 2 Exercise: Dispersion suppressor

In several straight sections of the accelerator, like the ones hosting RF cavities, extraction systems or other devices such as detectors, it is preferable to have no dispersion $\eta(s) = \eta'(s) = 0$. For example, in big colliders, such as the LHC, where small spot sizes are required at the interaction points, the dispersion is reduced to zero at the detector positions. The most common dispersion suppressors consists of two FODO cells of equal length $L$ and equal quadrupole strengths. Bending magnets are placed in the space between the quadrupoles with a different bending field in each FODO. Figure below shows a typical dispersion suppressor.




1. Considering two FODO cells with different total bend angles, $\theta_1 \neq \theta_2$, calculate the relation between the angles $\theta_1$ and $\theta_2$ which must be satisfied to cancel the dispersion at the end of the lattice.

   **Hint:**

   For each FODO cell, $M_{\mathrm{FODO}} = M_{1/2\mathrm{F}} \cdot M_{\mathrm{dipole}} \cdot M_{\mathrm{D}} \cdot M_{\mathrm{dipole}} \cdot M_{1/2\mathrm{F}}$, in thin-lens approximation we have the following $3 \times 3$ matrix:

$$M_{\text{FODO } j} = \begin{pmatrix} 1-\frac{L^2}{8f^2} & L\left(1+\frac{L}{4f}\right) & \frac{L}{2}\left(1+\frac{L}{8f}\right)\theta_j \\ -\frac{L}{4f^2}\left(1-\frac{L}{4f}\right) & 1-\frac{L^2}{8f^2} & \left(1-\frac{L}{8f}-\frac{L^2}{32f^2}\right)\theta_j \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} \cos\mu & \beta\sin\mu & \frac{L}{2}\left(1+\frac{L}{8f}\right)\theta_j \\ -\frac{\sin\mu}{\beta} & \cos\mu & \left(1-\frac{L}{8f}-\frac{L^2}{32f^2}\right)\theta_j \\ 0 & 0 & 1 \end{pmatrix}$$

where $j = 1, 2$ (1=first cell, 2=second cell).

The following condition must be satisfied:

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = M_{\text{suppressor}} \begin{pmatrix} \eta_0 \\ 0 \\ 1 \end{pmatrix} \tag{1}$$

where $\eta_0$ is the initial dispersion (at the middle of the first focusing quadrupole). It can be demonstrated that for a FODO lattice the dispersion has its maximum at the middle of the focusing quadrupole:

$$\eta_0 = \frac{4f^2}{L}\left(1+\frac{L}{8f}\right)\theta \tag{2}$$

with $\theta = \theta_1 + \theta_2$ the total bend in the suppressor.

2. Obtain the relation between the angles for the cases of phase-advance per cell $\mu = \pi/3$ and $\pi/2$.