Beam dynamics for cyclotrons

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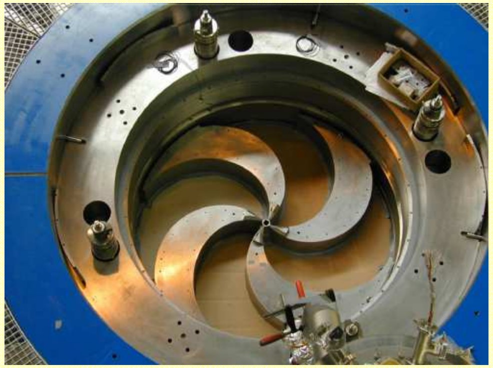
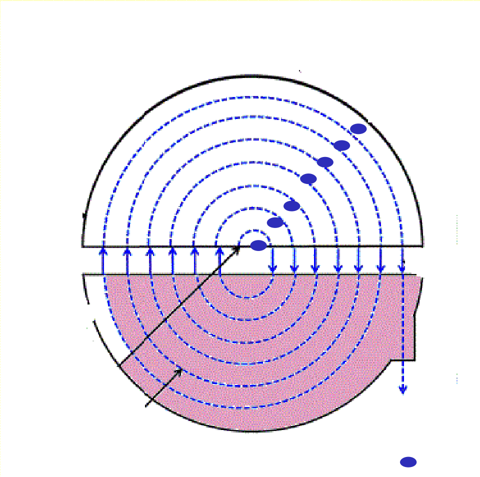
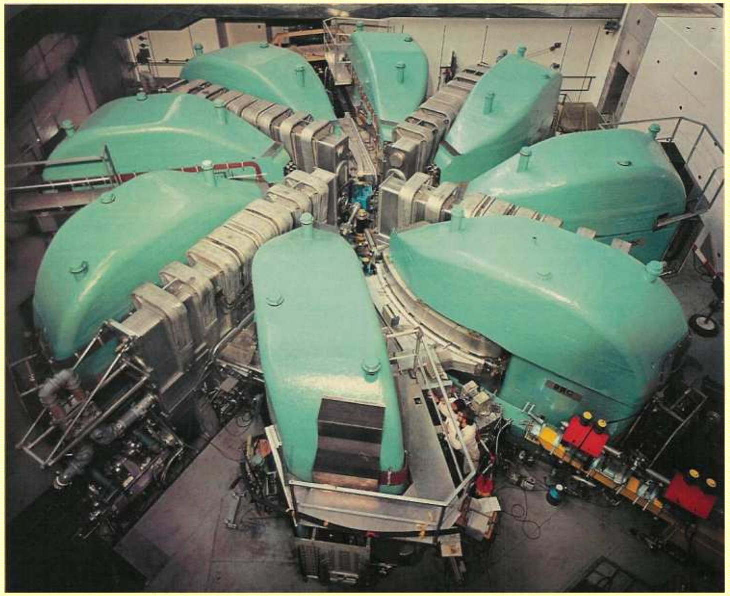






compact cyclotrons

Separated sectors (ring cyclotrons)

Synchrocyclotrons

Fixed energy

Variable energy

Superconducting

Normal conducting

OUTLINE

Chapter 1 : theory

- Principle
- Basic equation
- Longitudinal dynamics
- Transverse dynamics

Chapter 2 :specific problems

- Longitudinal dynamics
- Acceleration
- Injection
- Extraction

Chapter 3 : design

- Design strategy
- Tracking
- Simulations

Chapter 4 :

-Theory vs reality (cost, ,tunes,isochronism,...)

Exemples

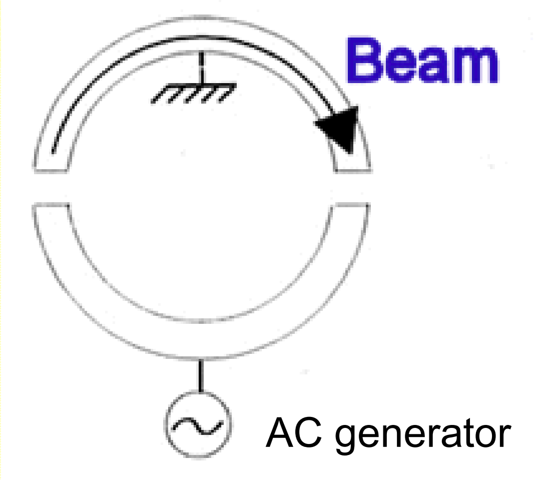
-Medical cyclotron

-Reseach facility

CYCLOTRON HISTORY

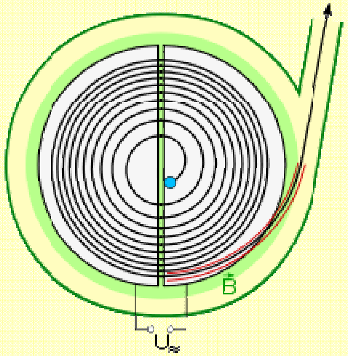
The Inventor, E. Lawrence, get the Nobel in Physics (1939) (first nuclear reactions without alpha source)

- brilliant idea (E. Lawrence, Berkeley, 1929) : RF accelerating field is technically complex and expensive.



So Let 's use only 1 RF cavity, but many times

A device is put into a magnetic field, curving the ion trajectories and only one electrode is used several times.

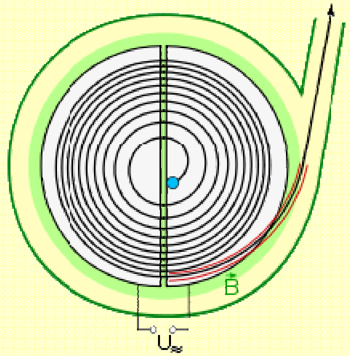


What is a CYCLOTRON ?

- RF accelerator for the ions :

  from proton A=1 to Uranium A=238

- Energy range for proton 1MeV -1GeV (γ~1-2)
- Circular machine : CW (and Weak focusing)
- Size Radius=30cm to R=6m
- RF Frequency : 10 MHz -60 MHz



APPLICATIONS : Nuclear physics

(from fundamental to applied research)

: Medical application

Radio Isotopes production (for PET scan,....)

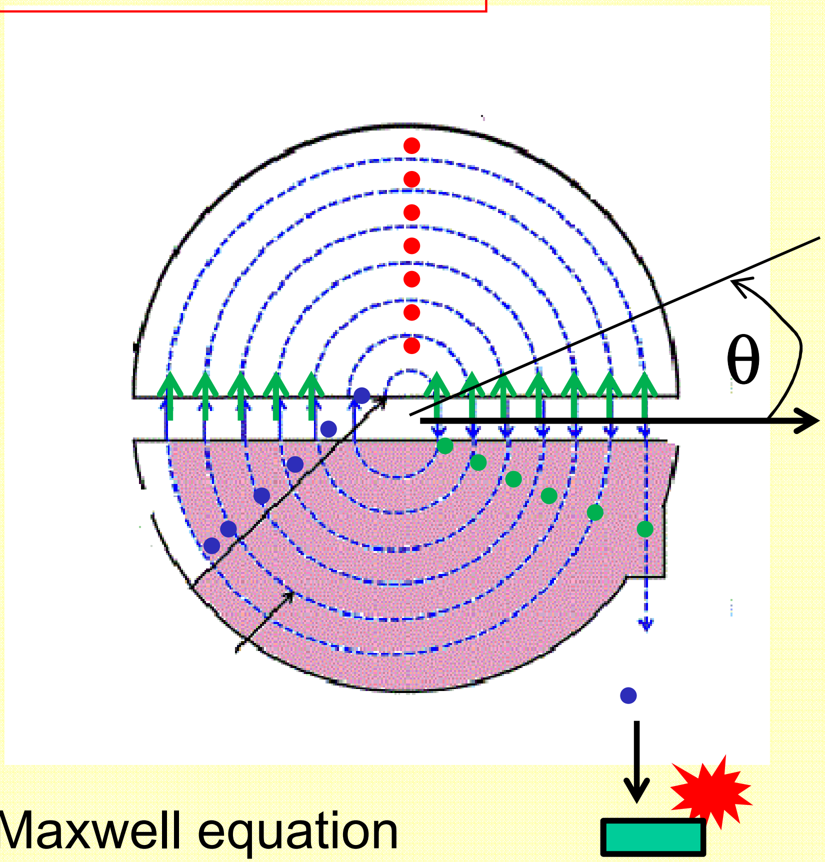
Cancer treatment

Quality : Compact and Cost effective

Useful concepts for the cyclotrons

$$B\rho = \frac{P}{q} = \frac{\gamma\, m.v}{q}$$

$$E_K = (\gamma - 1).mc^2$$



Maxwell equation

$\nabla \times B = 0$

Cyclotron coordinates

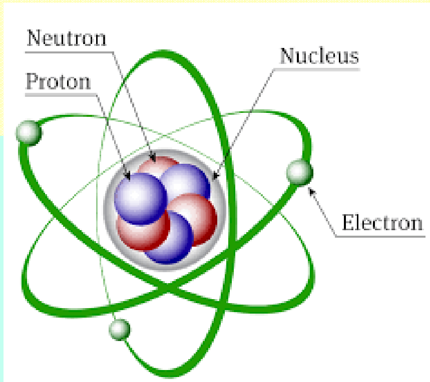
r Radial = horizontal

z Axial = vertical

θ « Azimuth » = cylindrical angle

MeV/A= kinetic energy unit in MeV per nucleon

Ions : $^{A}_{Z}X^{Q}$

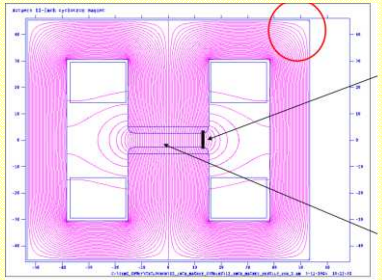


A : nucleons number

Z: protons number

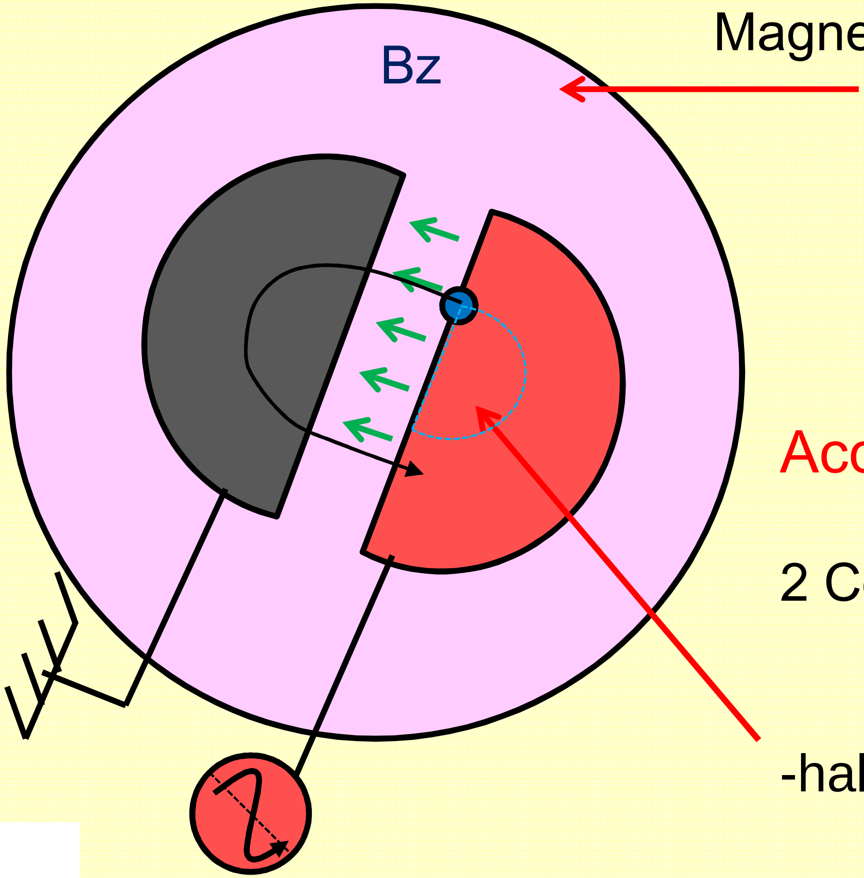
Q : charge state : 0+,1+,2+,.....

Principle :the hardware



H-type
electro-magnet

$$\vec{E} = [\,V_1(t) - V_0\,]/d$$
$$\sim \cos(\omega t)$$



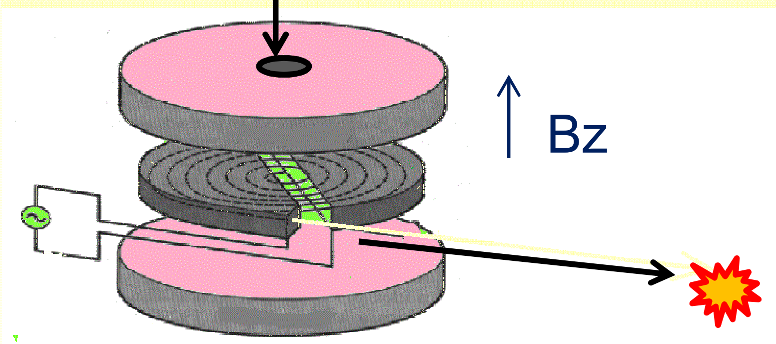
AC generator

Accelerating Dee’s

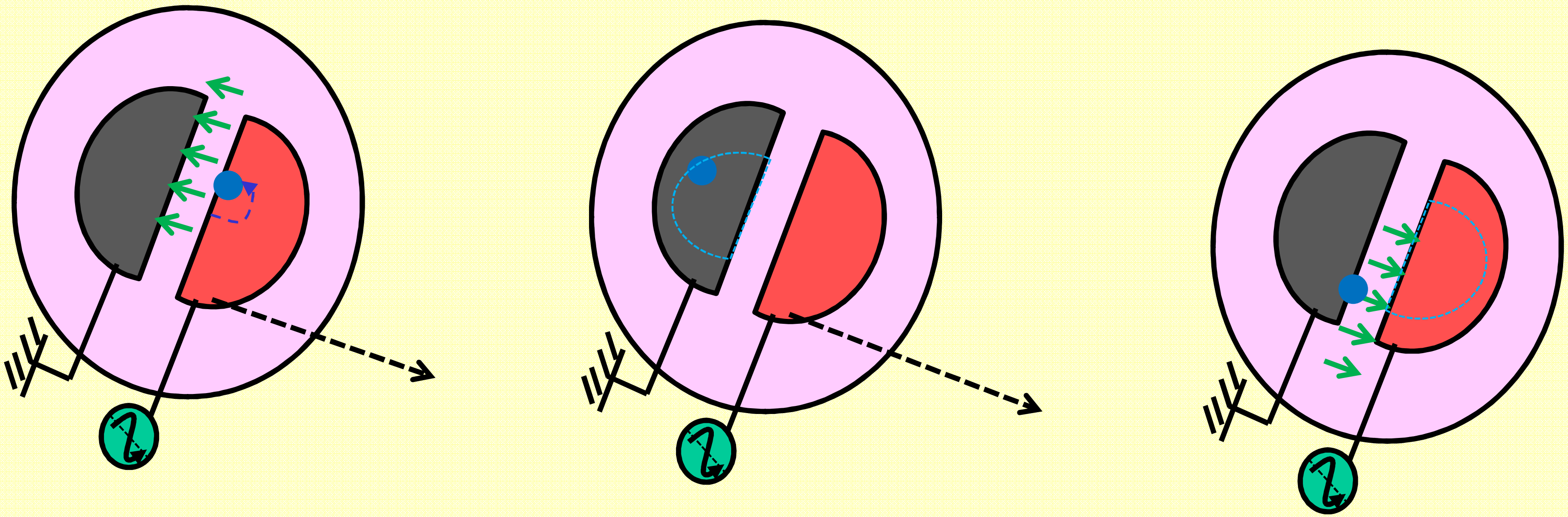
2 Copper boxes
≠ potential

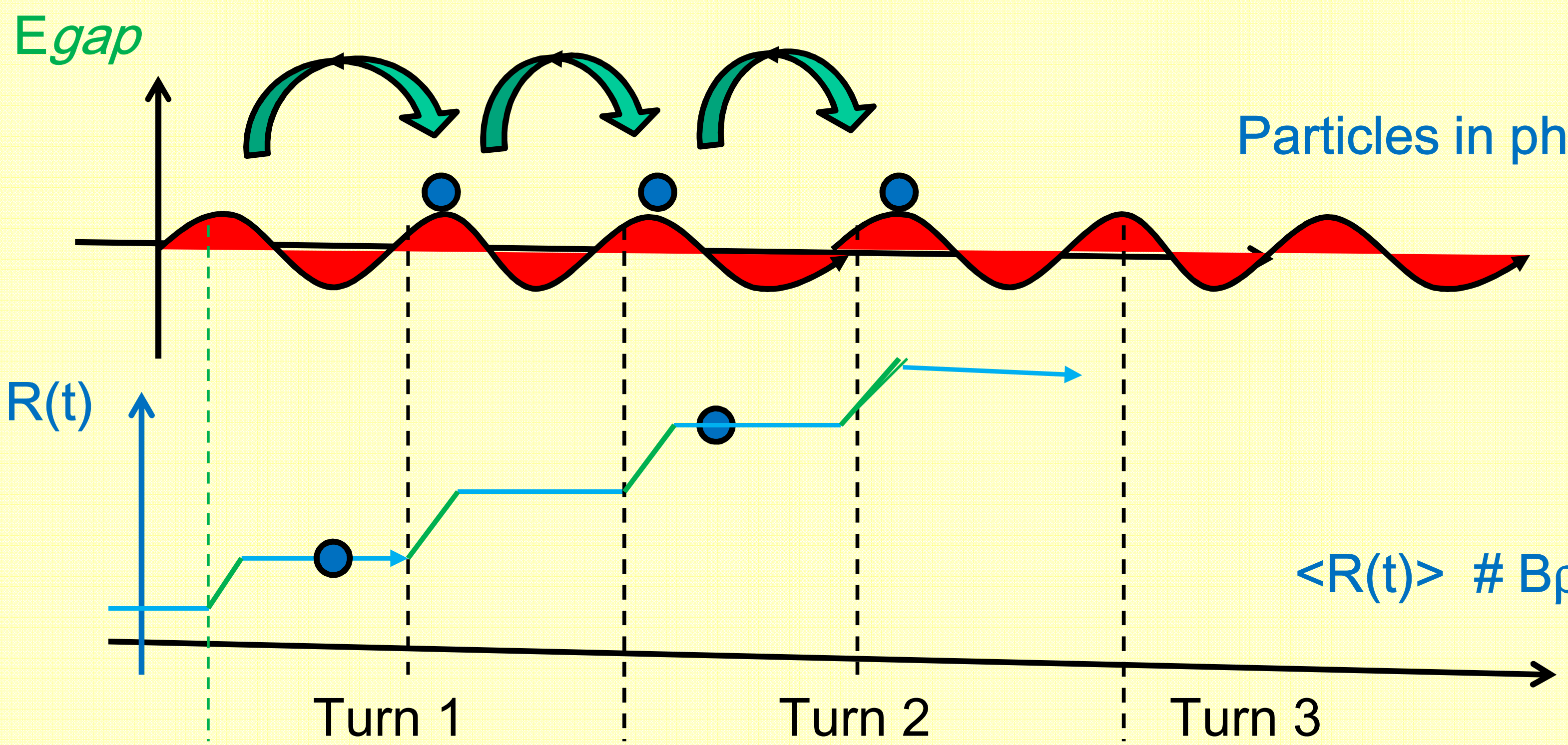
-half is $V1(t) \sim \cos(\omega t)$

- half is
at the ground potential

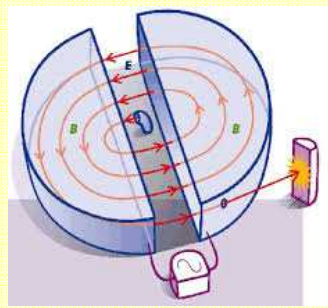


Principle B: the trajectories



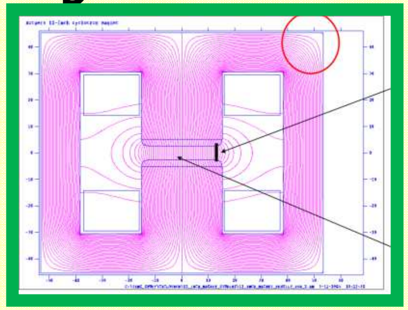


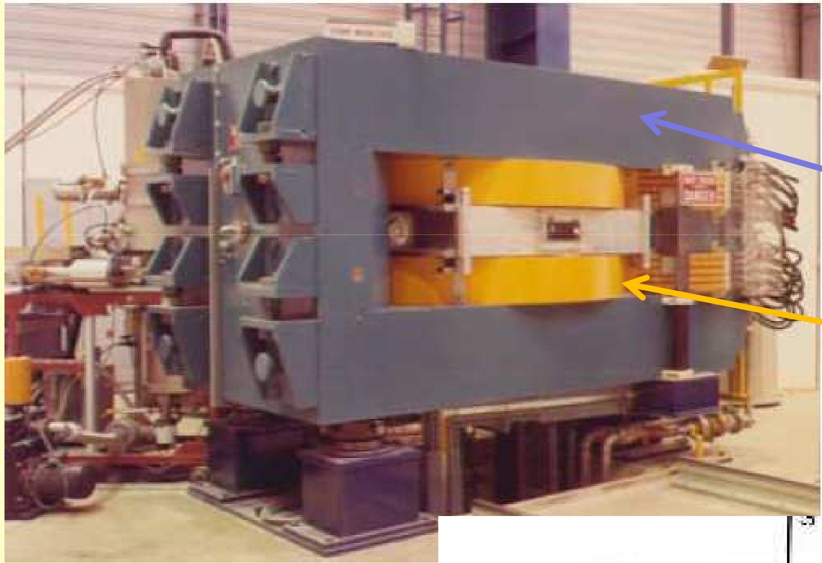
Particles in phase with *RF*

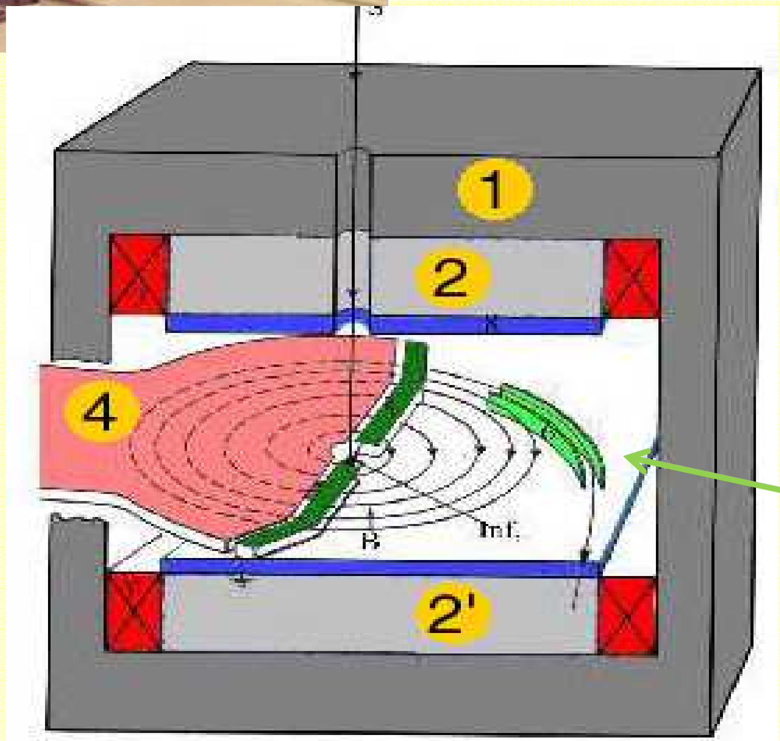


<R(t)> # Bρ / <B>

A compact cyclotron in reality







Magnet (Bz) :
1) Yoke
2) poles
3) coils

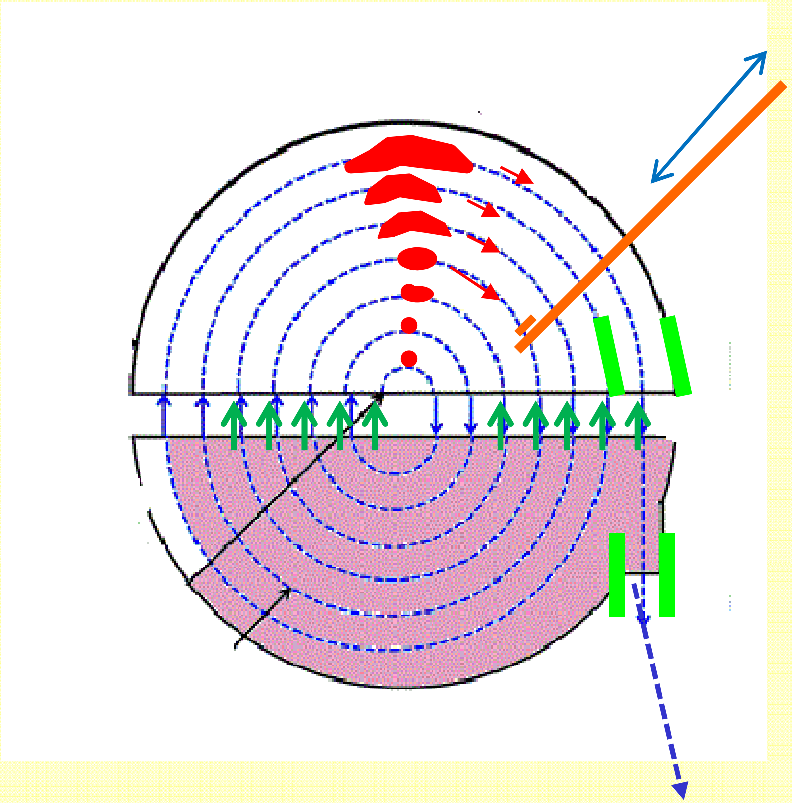
RF cavities
4) Dee

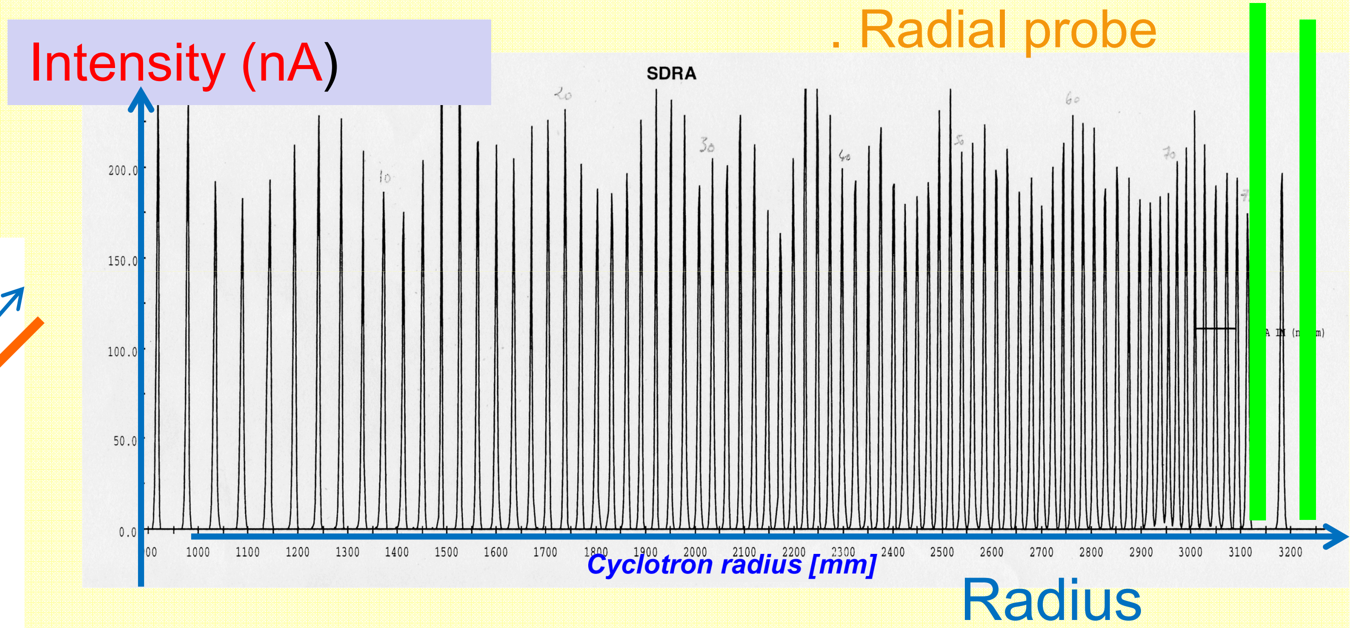
Electrostatic Deflector

Radial probes

useful tool to check the acceleration

Monitoring beam with a
Radial Probe





Radial probe : I = F(Radius)

Radius in the cyclo = f(Turn Number)

Turn separation : δr =R(turn N) - R(turn N-1)

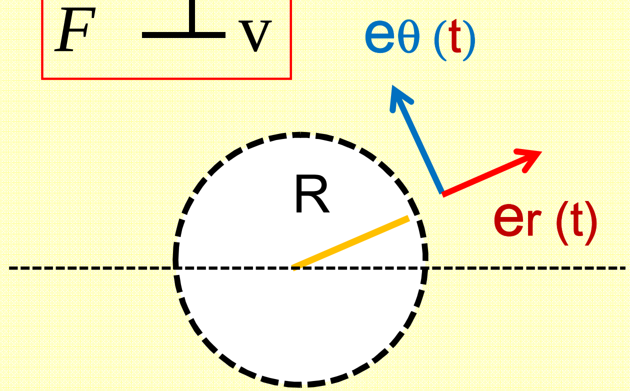
Trajectory in uniform B field

$$\frac{d(\gamma m\vec{v})}{dt} = \vec{F}$$

Let's consider an ion with a charge ***q*** and a mass ***m*** circulating at a speed $\boldsymbol{v_\theta}$ in a uniform induction field ***B***.=*(0,0,Bz)*

The motion equation can be derived from the **Newton's law** and the **Lorentz force *F*** ***in a cylindrical coordinate system*** (er,eθ,ez):

$$\vec{F} \perp \vec{v}$$



$$\frac{d(\vec{v})}{dt} = \vec{a} = \frac{d^2(R.\vec{er})}{dt^2} = -\left[\left\|\vec{v}\right\|^2 / R\right].\vec{e_r}$$

$$\frac{d\vec{p}}{dt} = \vec{F} = q(\vec{v}\times\vec{B}) = -q v_\theta B_z \vec{e_r}$$

$$\gamma\frac{m v^2}{R} = -q v_\theta B_z$$

$$v_\theta = R\dot{\theta}$$

$$R = \frac{\gamma m v}{q B_z}$$

Trajectory in uniform B field

$$R = \frac{B\rho}{B_z} = \frac{\gamma\, m\mathrm{v}}{qB_z}$$

$$Frevolution = \frac{\mathrm{v}}{2\pi\, R} = \frac{1}{2\pi}\frac{qB}{\gamma\, m}$$

$$\omega_{rev} = 2\pi\, F_{rev} = \dot{\theta} = \frac{d\theta}{dt} = \frac{\mathrm{v}_\theta}{R} = \frac{qB}{\gamma\, m}$$

$$\omega_{rev} = \frac{qB}{\gamma\, m}$$

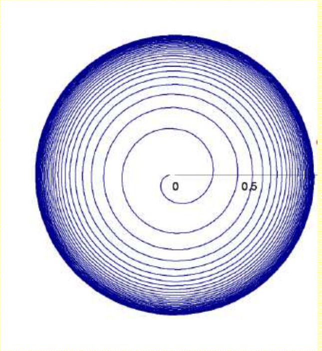
Centrifugal force = Magnetic force

$$\gamma\frac{m\mathrm{v}_\theta^2}{R} = q\mathrm{v}_\theta B_z$$

Let's accelerate ions, in a constant vertical field B*z*

The Radius evolves with P/q :

$$R(t) = \frac{\gamma\, m\mathrm{v}}{qB_z} = \frac{B\rho}{B_z}$$



For *non relativistic* ions (low energy) ⇨ $\gamma \sim 1$

In this domain, if B*z* =const ⇨ ω=const

$$\omega_{rev} = \frac{qB_z}{\gamma m} \approx \quad const$$

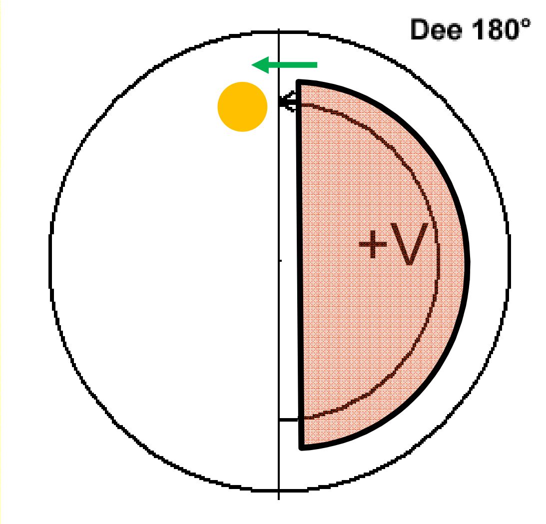
same ΔT for each Turn

So, it is easy to synchronize an Accelerating cavity (RF)

having a "D" shape, with accelerated ions

$$V = V_0 \cos(\omega_{RF}\, t)$$

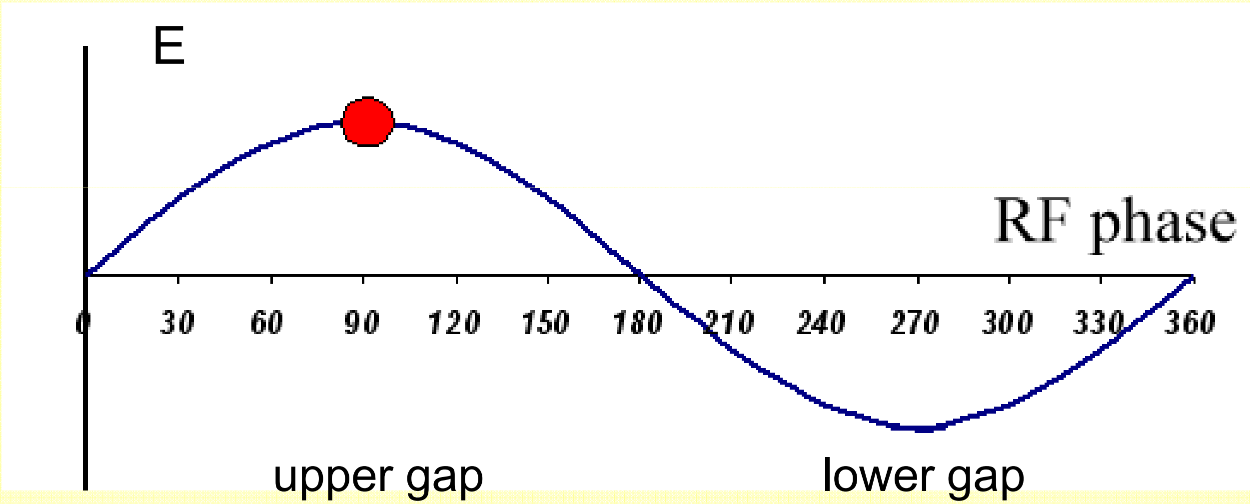
$$\omega_{RF} = h \;\; \omega_{rev}$$

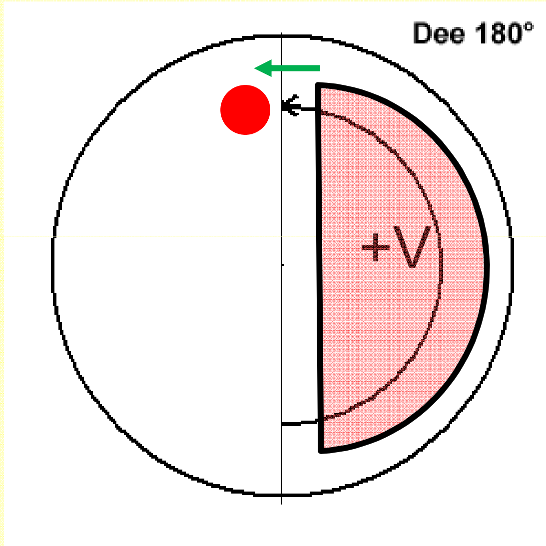


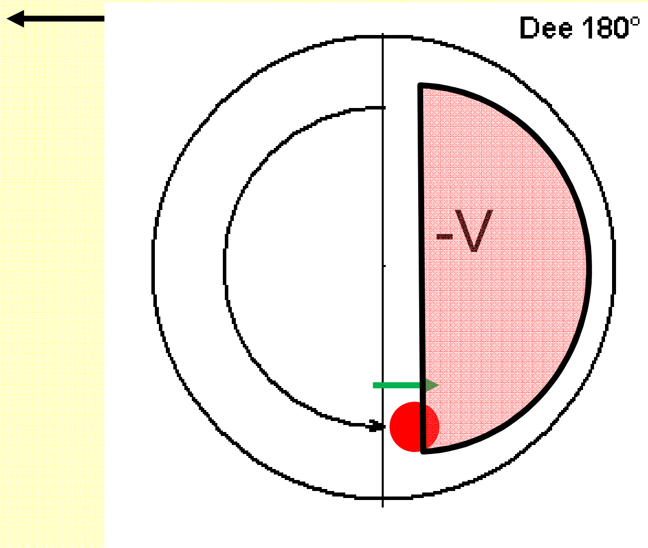
h = 1, 2, 3, … called the RF harmonic number (integer)

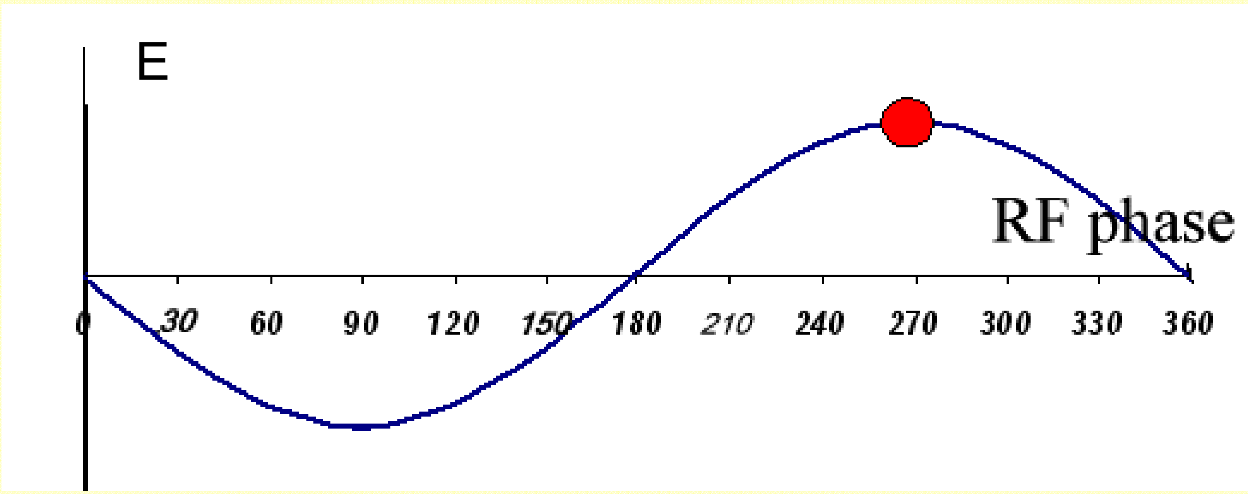
Harmonic number h=FRF/Frevolution

h = 1 *1 bunch by turn* $\omega_{rf} = \mathbf{h}\ \omega_{rev}$





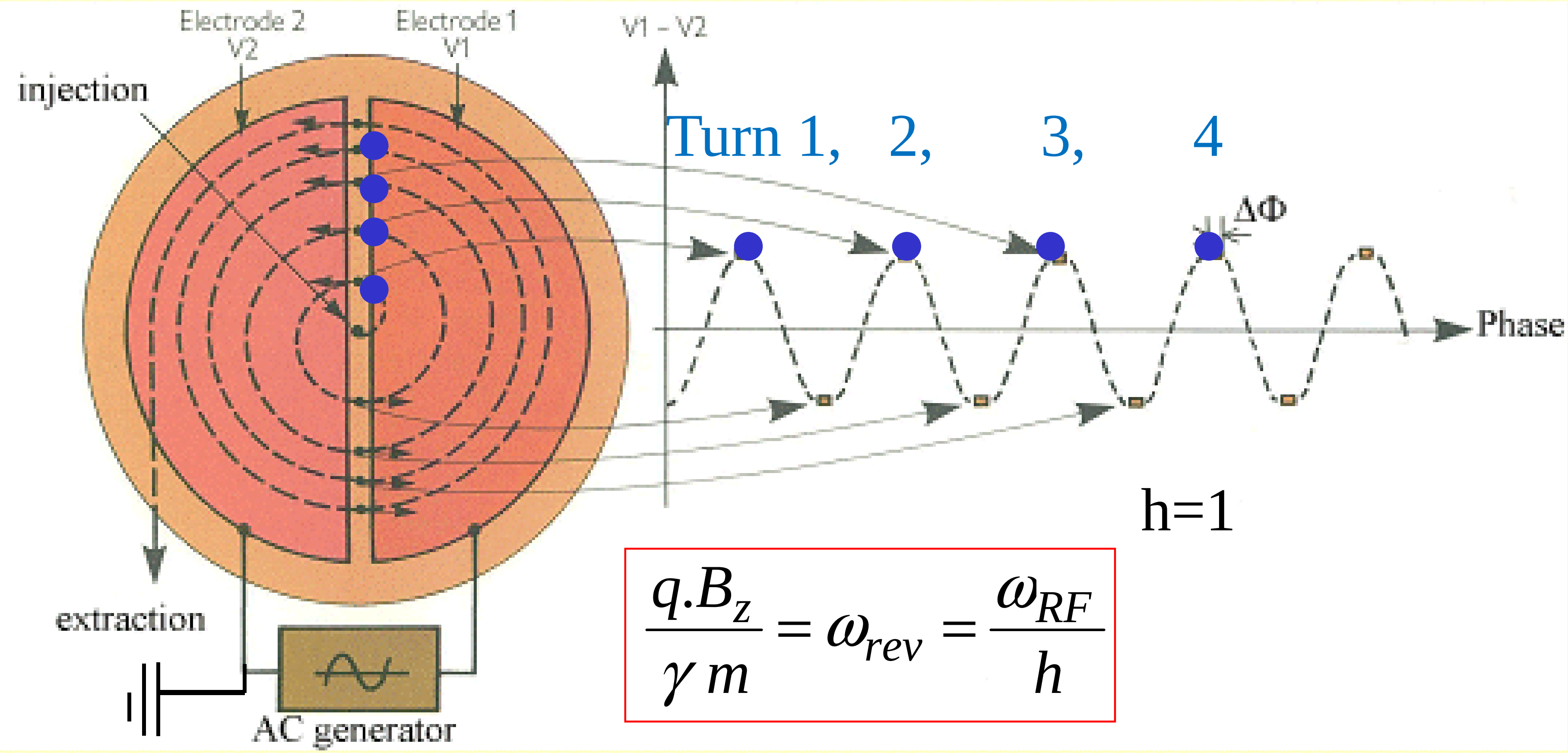




Isochronism condition: The particle takes the same amount of time to travel one turn : (constant revolution frequency ω_{rev} =const)

and with $\omega_{rf} = h\ \omega_{rev}$, the particle is synchronous with the RF wave.

In other words, the particle arrives always at the same RF phase in the middle of the accelerating gap.



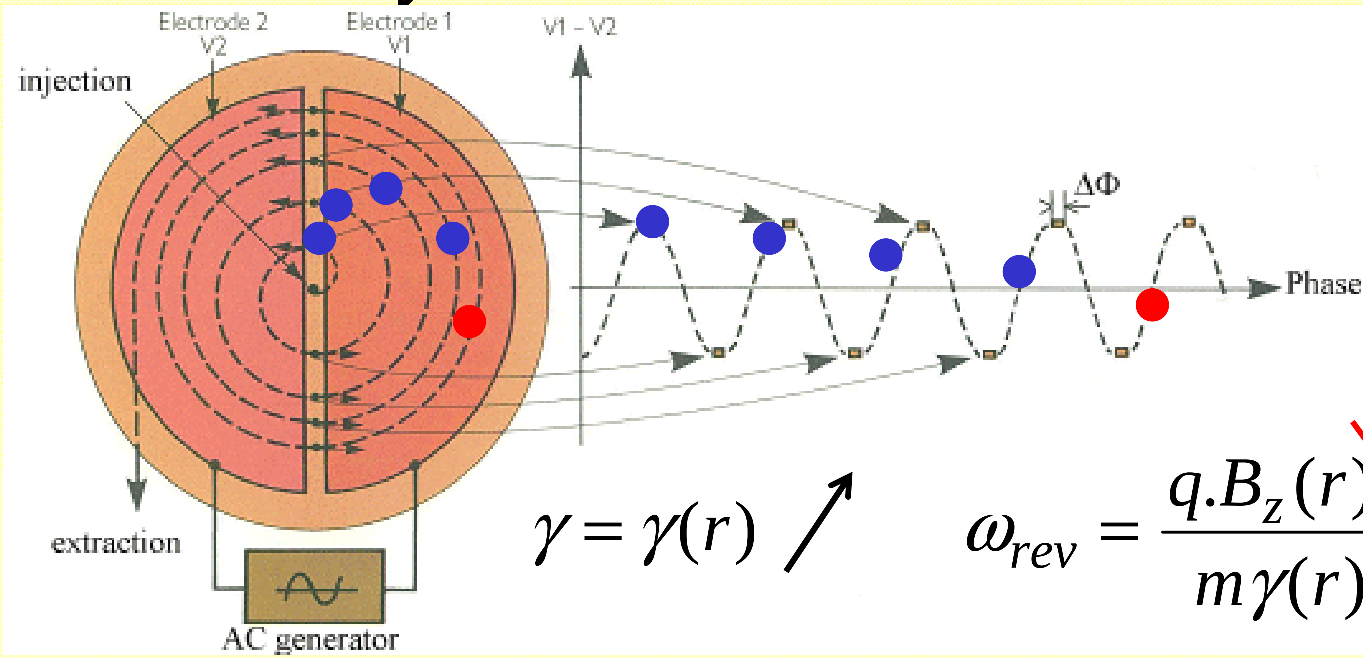
$$\frac{q.B_z}{\gamma\, m} = \omega_{rev} = \frac{\omega_{RF}}{h}$$

Longitudinals with relativistic particles

With Bz = constant , relativistic γ increases AND ω_{rev} decreases

$$\omega_{rev} = \frac{qB_0}{\gamma\, m}$$

→ Isochronism condition not fullfilled



$$\gamma = \gamma(r) \nearrow \qquad \omega_{rev} = \frac{q.B_z(r)}{m\gamma(r)} \searrow$$

$$\omega_{rev} = \frac{q.B_z(r)}{\gamma(r)\, m}$$

→ Isochronism condition fullfilled if

Bz(r)/ γ(r) =CONSTANT

Transverse dynamics in the cyclotrons

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m} = const$$

Isochronism condition
(longitudinal)

Bz =Bz(R) ~ γ

We will show that that isochronism have a bad consequence on vertical oscillations

Cyclotrons Tutorials 1

•An **isochronous cyclotron** uses a **RF cavity** at **60 MHz** at the RF harmonic **h=3**

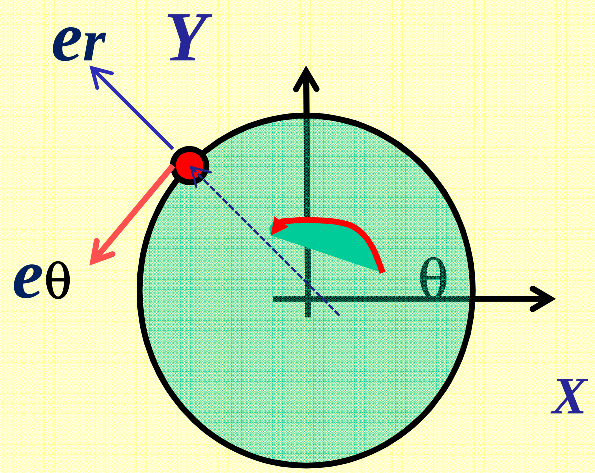
a. Compute **the time needed to perform one turn T_{rev} for the accelerated ions.**

b. Compute **the average field B_z needed to accelerate a proton beam (in a non relativistic approximation)**

Cyclotrons Tutorials 2

•**Demonstrate than in a** **uniform circular motion** **, the radial acceleration is**

$$a_r = |V^2 / R| .$$



Nota : ***You can use parametric equations*** **:**

$$X(t) = R\cos(\omega t)$$
$$Y(t) = R\sin(\omega t)$$

Then compute the velocity and the acceleration.

Demonstrate that the acceleration is radial

Nota : $\omega t = \theta$ $\omega = d\theta/dt$

ion trajectory in cyclotrons

Steenbeck 1935, Kerst and Serber 1941

We will use cylindrical coordinates (er, eθ, ez)

We will show that

In Radial plane (horizontal)

$$\text{radius}(t) = R(t) + x_0 \cos(\nu_r \, \omega_{rev} \, t)$$

Radial tune ν_r

In the Vertical (axial) plane

$$z(t) = z_0 \cos(\nu_z \, \omega_{rev} \, t)$$

axial tune ν_z

3 slides to compute ν_z & ν_r

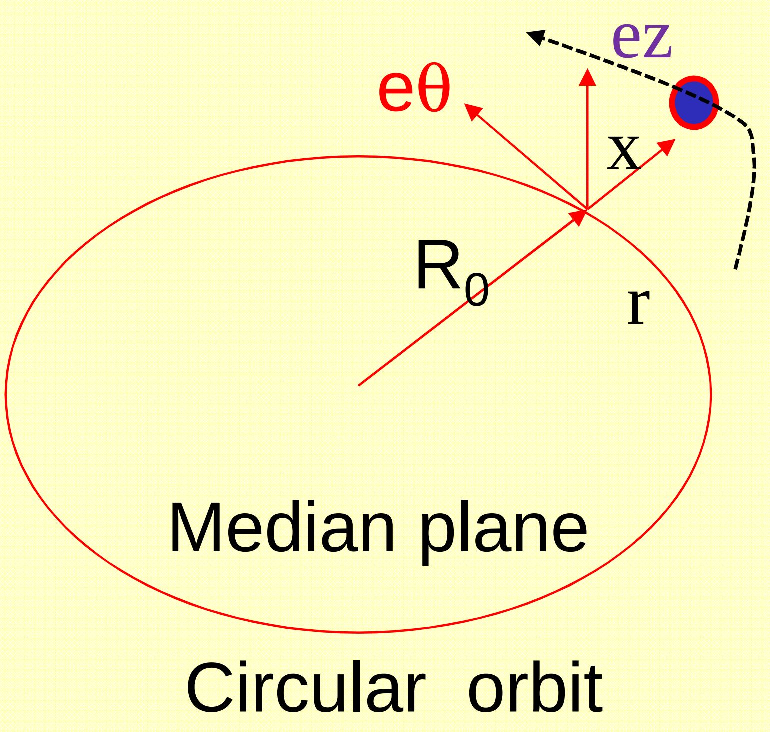
Transverse dynamics with Bz(R)

cylindrical coordinates (**er**, **e**θ, **ez**)

and

define **x** a small orbit deviation with Bz=Bz(r) (not constant)

$$\vec{\mathbf{r}} = [R + x(t)].\overrightarrow{er} + z(t).\overrightarrow{ez}$$



$Bz(R) = \gamma(R)\ B_0$ Isochron field

$= R^{-n}\ B_0$

Uniform Circular motion x=0

Motion Eq. With x≠ 0 ?????

$$m\frac{d(\overrightarrow{\mathbf{v}})}{dt} = m\frac{d^2(\mathbf{r})}{dt^2} = ?$$

Radial dynamics with Bz(R) *(No RF)*

- Taylor expansion of the field B_z around the median plane:
- *definition of n(R)* $Bz = B_0 R^{-n}$ n =field index (Bz is never uniform)

so

$$B_z = B_{0z} + \frac{\partial B_z}{\partial x} x + \ldots \approx B_0 (1 - n \frac{x}{R})$$

$$with \quad n = -\frac{R}{B_0} \frac{\partial B_z}{\partial R}$$

- How evolves an ion, in this non uniform Bz : *r(t)= R+x(t)*

$$m\gamma \frac{d^2 \vec{r}}{dt^2} = -q \, \mathbf{v} \times \mathbf{B}$$

$$r = R(1 + x / R)$$

$$\frac{d^2 (r \cdot \overrightarrow{er})}{dt^2} = (\ddot{x} - \frac{v_\theta^2}{r})\overrightarrow{er} + 2\dot{x}\dot{e}_r$$

$$= \left[\ddot{x} - \frac{v_\theta^2}{R}(1 - \frac{x}{R}) \right] \overrightarrow{\mathbf{er}} + 2\dot{x}.\overrightarrow{\mathbf{e_\theta}}$$

$$\frac{1}{r} = \frac{1}{R \; (1 + \frac{x}{R})} \approx \frac{1}{R}(1 - \frac{x}{R})$$

*radial motion (**e**r)*

$$m\gamma \left(\ddot{x} - \frac{v_\theta^2}{R}(1 - \frac{x}{R}) \right) = -q \, \mathbf{v}_\theta B_{0z} (1 - n \frac{x}{R})$$

$$m\gamma\left(\ddot{x}-\frac{v_\theta^2}{R}(1-\frac{x}{R})\right)=-q\,B_{0z}(1-n\frac{x}{R}).v_\theta$$

and $\omega_{rev}=\dfrac{qB_{0z}}{\gamma m}=\omega_0\approx\dfrac{v_\theta}{R}$

After simplification :

$$\ddot{x}+\omega_0^2\cdot(1-n)x=0\Rightarrow$$

$$\ddot{x}+[\nu_r\omega_0]^2x=0 \qquad \nu_r^2=(1-n)$$

Harmonic oscillator with the frequency $\omega_r=\sqrt{1-n}\;\omega_0$

Horizontal stability condition (νr real) : **n < 1**

n <1 : Bz could decrease//or increase with the radius R

Horizontal stability is generally easy to obtain

Horizontal stability condition (ν_r real) :

Harmonic oscillator with the frequency

ν_r Radial tune

$$\ddot{x} + [\nu_r \omega_0]^2 x = 0$$

$$\nu_r = \sqrt{1-n}$$

$$x(t) = x_0 \cos(\nu_r \omega_0 t)$$

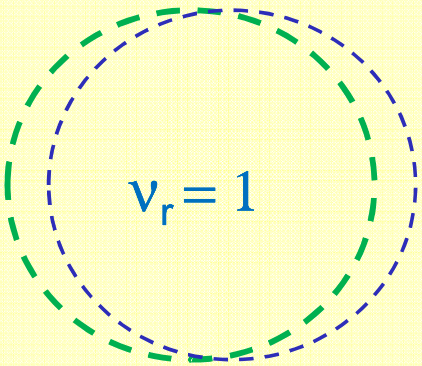
Horizontal stability if n <1

$$\nu_r^2 = 1-n > 0$$

n<0 : isochronism condition Bz **should increase**

n<1 : stability condition ($\nu_r^2 > 0$)

$$r(t) = R_0(t) + x_0 \cos(\nu_r \omega_0 t)$$



Vertical dynamics with B (r)

Vertical motion in the non uniform Bz(r)

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix}$$

$$m\gamma\frac{d^2z}{dt^2} = F_z = q(\mathrm{v}\times B)_z = -q(\dot{r}\,\cancel{B_\theta} - r\,\dot{\theta}\,B_r)$$

$$Bz = B_0\, r^{-n}$$

Because $\nabla\times B = 0$ $\quad \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} = 0$

$$B_r = -n\frac{B_{oz}}{r}z$$

Motion equation $\quad \ddot{z} + [\nu_z\omega_0]^2 z = 0$

Harmonic oscillator with the frequency $\quad \nu_z = \sqrt{n}$

Vertical stability condition : n >0 (ν_z real)

$$\nu_z^2 = n > 0$$

NOT COMPATIBLE WITH ISOCHRONISM

Watch the vertical oscillations !!

Isochronism condition :

n < 0 : $B_z(R) \sim R^{-n}$:increase with R

Vertical tune

$$\nu_z = \sqrt{n}$$

n < 0

$$\ddot{z} + [\nu_z \omega_0]^2 z = 0$$

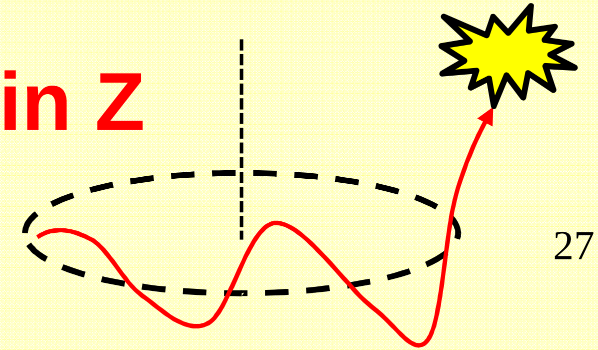
$$\nu_z = i\sqrt{|n|}$$

Isochronism condition will induce Unstable oscillations

$$z(t) \sim z_0 \exp(-i\ \nu_z\ \omega_{rev}\ t) = z_0 \exp(+|\nu_z|\ \omega_{rev}\ t)$$

Unstable oscillations in Z

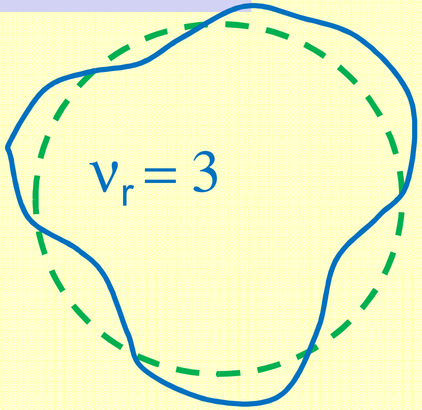
= exponential growth =beam losses



Tunes : ν_r & ν_z

oscillations around reference trajectory

$$r(t) = R_0(t) + x_0 \cos(\nu_r \, \omega_{rev} \, t)$$

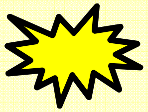


ν_r :Number of radial oscillations per cyclotron turn in horizontal (radial) plan

$\nu_r^2 = 1-n$ stable oscillations

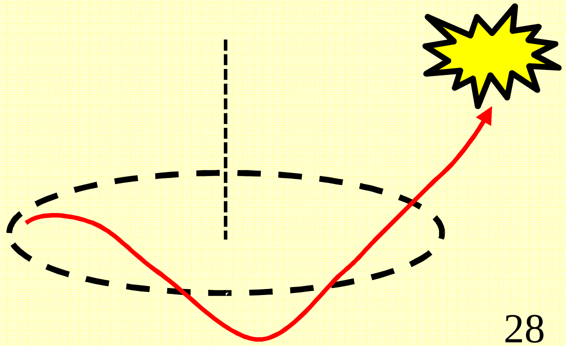
$$z(t) = z_0 \cos(\nu_z \, \omega_{rev} \, t) = z_0 \cos(\nu_z \, \theta)$$

$\nu_z^2 = n < 0$ unstable oscillations



$(\nu_z = i \, | \nu_z |)$

$$z(t) \sim z_0 \exp(\pm | \nu_z | \, \omega_{rev} \, t)$$



Vertical stability ≠ Isochronism

Isochronism condition
(longitudinal)

B=Bz(R)

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}$$

Bz should increase with R ($Bz = B_0 R^{-n}$ n<0)

Unstable Vertical oscillations (*Br* defocus in z plane)

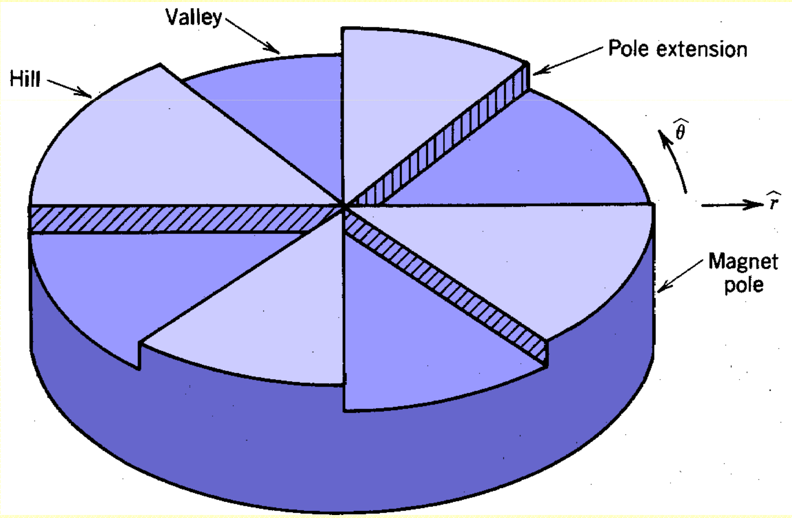
Additive Vertical focusing is needed

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix}$$

Bθ component needed (Fz =-q vr Bθ) : « AVF » Cyclo

Azimuthally Varying Field ("AVF")
Vertical weak focusing : $B_z = f(R,\theta)$

• $F_z \sim \langle q\, v_r\, B_\theta \rangle$: Vertical focusing



$B_z = f(R,\theta)$ $\quad \nabla \times B = 0$

$B_\theta = g(R,\theta)$

Like edge focusing in dipole magnet :
Bz variation
can produce vertical forces

Isochronism n<0 : $B_z(R)$ increase with R

Vertical stability : $B_z(R)$ Defocus + $B\theta$ *Focus*

Bz should oscillate with θ to compensate the instability

• Vertical force Fz , with component **B**θ

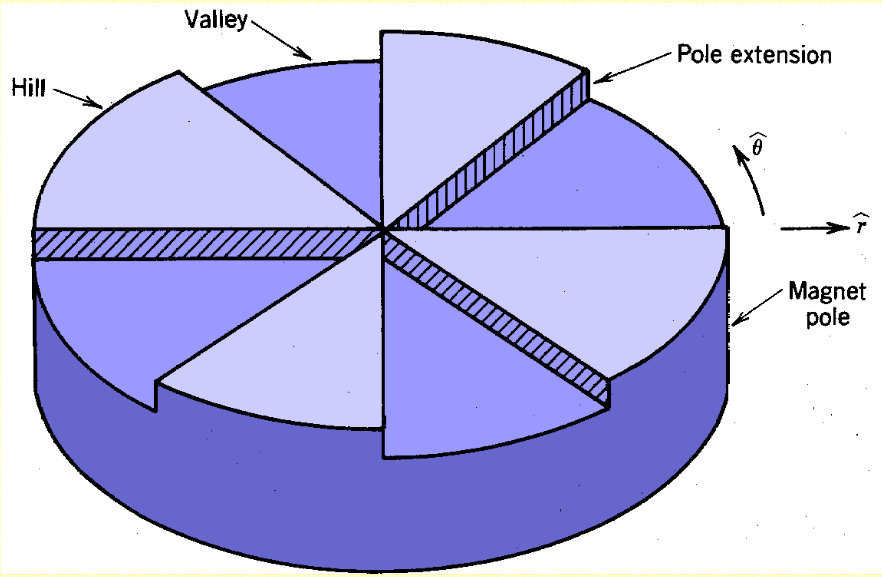
Azimuthally varying Field (AVF)

an additive focusing vertical force $\langle F_z \rangle = q \langle v_r \cdot B_\theta \rangle$

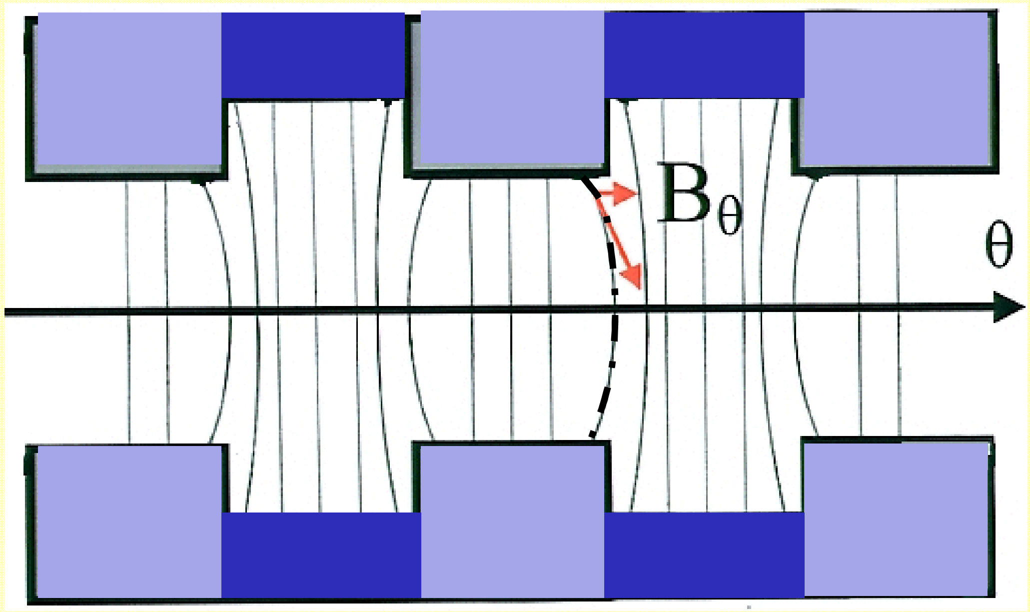
B_θ created by:

Succession of high field and low field regions : $B_z = f(R, \theta)$

- B_θ appears around the median plane
  - valley : large gap, weak field
  - Hill : small gap, strong field



N=4 sectors

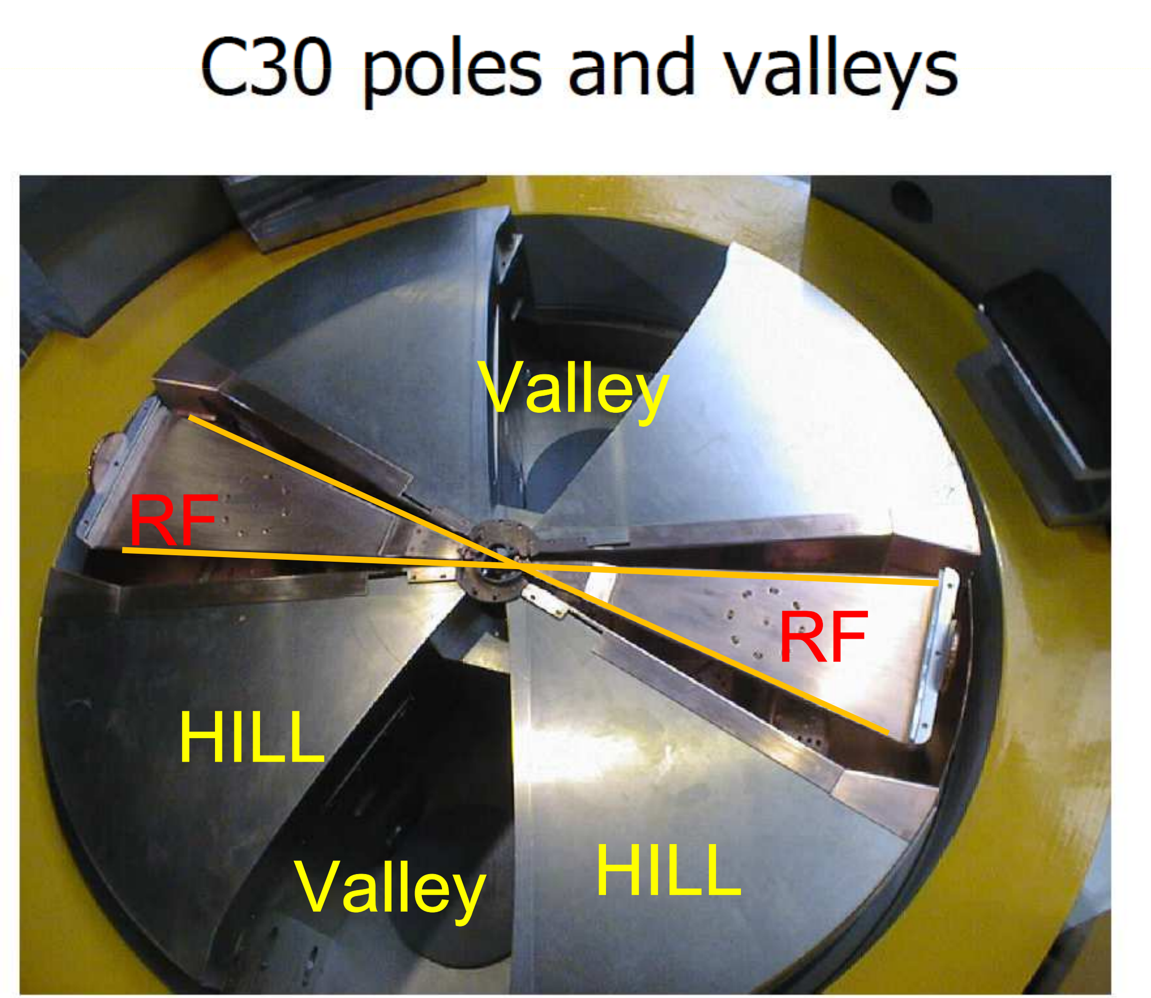


Hill valley Hill valley

Azimuthally varying Field (AVF)

Exemple : 30 MeV compact proton cyclo.

4 straight sectors



-2 RF cavities

Inserted in the valleys

= 4 accelerating gaps

4 Hills + 4 Valleys