Chapter 1b

Cyclotrons Bz =f(R , θ)

Isochronism condition
(longitudinal)

Bz =B (R)

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}$$

Bz should increase with R (B~ R^{-n} with n<0)

Unstable Vertical oscillations (*Br* defocus in z plane)

Additive Vertical focusing is needed

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix}$$

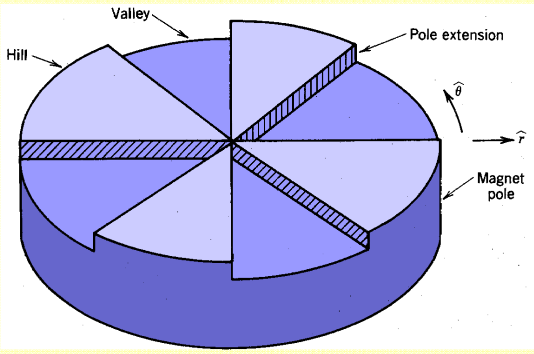
Bθ component needed (Fz =-q vr Bθ) : « AVF » Cyclo

Azimuthally varying Field (AVF) $B_z = f(R, \theta)$

an additive focusing vertical force $v_r \cdot B_\theta$

B_θ created by:

- Succession of high field & low field regions : $Bz = f(R, \theta)$
  - Valley : large gap, weak field
  - Hill : small gap, strong field



N=4 sectors

FLUTTER function (definition)

$$F_l = \frac{< (B - < B >)^2 >}{\langle B \rangle^2}$$

1 turn

$$F_l = \frac{\sigma_B^{\,2}}{\langle B \rangle^2}$$

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Azimuthally varying Field (AVF) cyclo

Vertical focusing $\langle F_z \rangle \propto v_r \cdot B_\theta$ →

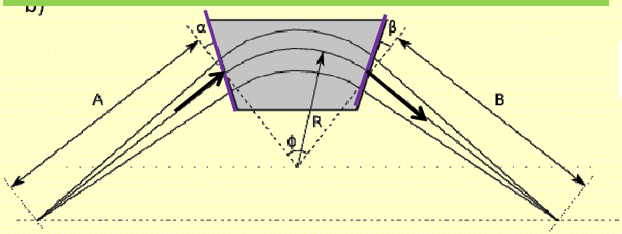
$V_r = dr/dt\, \vec{er}$ **created by :**

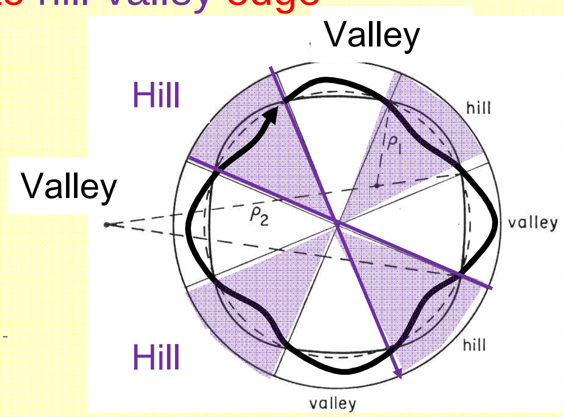
- Valley: weak field, large trajectory curvature
- **Hill : strong field**, small trajectory curvature

Trajectory is not a circle

- Orbit not perpendicular to hill-valley edge

Like edge focusing in Dipole Magnet





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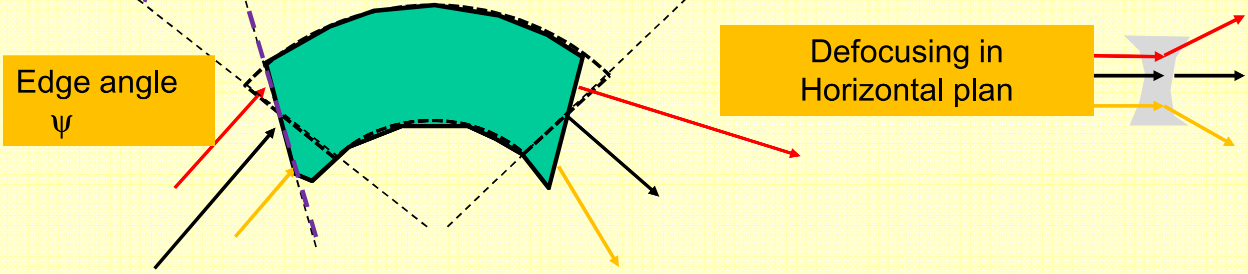
Edge focusing in dipole magnet recap

non perpendicular edges in dipole magnet
can generate horizontal defocusing + vertical focusing

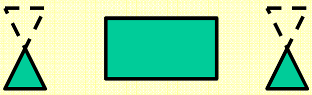


Non perpendicular edge in dipole magnet can provide

1) additive focusing in vertical + 2) defocusing in horizontal plane

The optical Transfer Matrix M is

Mdipole = M*edge*1. Mbody. M*edge2*



$$Mz_{edge}\begin{bmatrix} z \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\dfrac{\tan\psi}{R} & 1 \end{bmatrix}\begin{bmatrix} z_0 \\ z_0' \end{bmatrix}$$

Equivalent effect in AVF Cyclo

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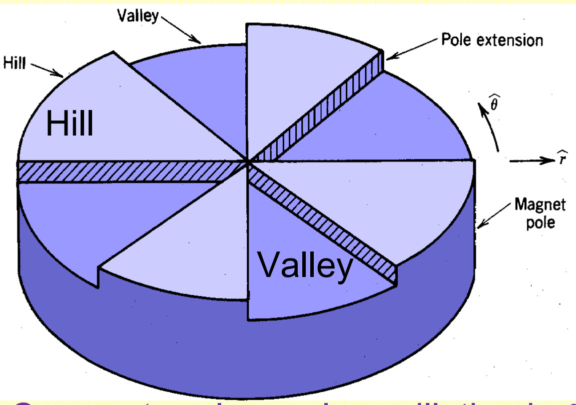
Vertical focusing with sectors



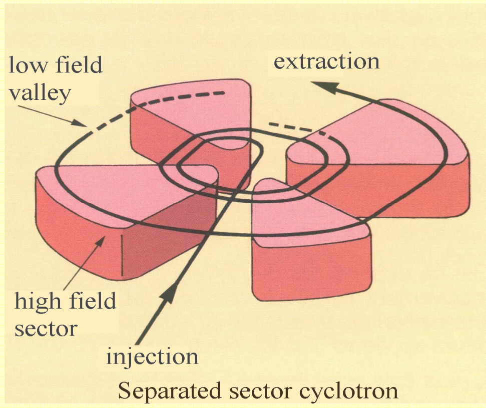
Compact cyclo : pole oscillation in θ

Bz= <B0> [1+ f. cos (N θ)]

f <1 f = 0.5. (Bhill- Bvalley)/<B0>

FLUTTER function (definition)

$$F_l = \frac{< (B - < B >)^2 >}{\langle B \rangle^2}$$ <1turn>



Separated magnets
generate field oscillation in θ

Bz= <B0> [1+ cos (N θ)]

Separated sector cyclotron

The FLUTTER is larger

Larger vertical focusing

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Tutorial 1 :

Give the **Lorentz force** in a **cyclotron**
and explain the focusing and defocusing effect in Vertical plane
of the **B**r (radial) and **B**θ (azimuthal) components

$$m\gamma \frac{d^2\mathbf{r}}{dt^2} = m\gamma \frac{d^2(r\mathbf{e_r} + z\mathbf{e_z})}{dt^2} = q(\mathbf{v} \times B) = ?$$

$$B_z = B_{0z} R^{-n}$$

$$m\gamma \frac{d^2 z}{dt^2} = F_z = ?$$

$$B_r = ?$$

« Curl B =0 »

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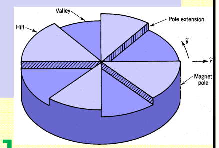
Tutorial 2 : Flutter F in AVF cyclo



The field of a cyclotron is $B_z(r,\theta)= B_0(r)\ [1+ f.\ \cos(4\ \theta)\ ]$

A: How many hills and valleys have such a cyclotron

B: Compute B_θ With *Curl B=0*

$$\nabla\times\mathbf{B}=\left[\mathbf{e}_r\frac{\partial}{r\partial \mathrm{r}}+\mathbf{e}_z\frac{\partial}{\partial \mathrm{z}}+\mathbf{e}_\theta\frac{\partial}{r\partial\theta}\right]\times\left[B_r\mathbf{e}_r+B_z\mathbf{e}_z+B_\theta\mathbf{e}_\theta\right]$$

C: Compute the flutter function « *Fl* »

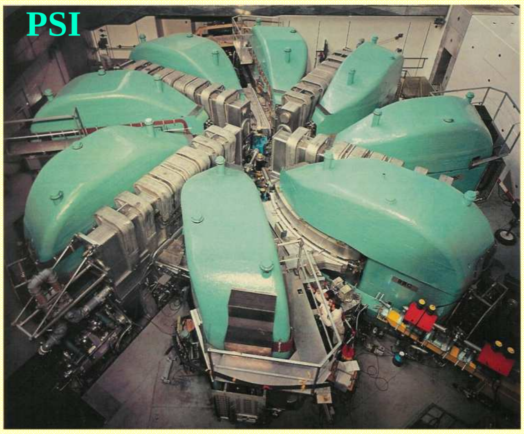
Separated Sectors(ring) Cyclotron

Focusing condition limit: (n<0)

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l + \ldots > 0$$

Increase the flutter F_l, using separated sectors where $B_{valley} = 0$

$$F_l = \frac{< (B - < B >)^2 >}{\langle B \rangle^2}$$



PSI= 590 MeV proton
γ=1.63

⇒ Separated sectors cyclotron needed at "High energies" ($n(R) = 1-\gamma^2 << 0$)

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Vertical focusing and isochronism

2 conditions to fulfill

- Increase the vertical focusing force strength: $\nu_z^2 = n + \frac{N^2}{N^2-1}F_l + \ldots > 0$
- Keep the isochronism condition true: n<0

$$n = -\frac{R}{B_{0z}}\frac{\partial B_z}{\partial R} = 1 - \gamma^2 < 0$$

So we should have:

$$\frac{N^2}{N^2-1}F_l > \gamma^2 - 1$$

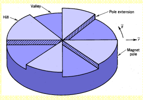
For High Energy cyclotron : 2 solutions for vertical stability

1) Larger Flutter (separated sectors) *Fl*

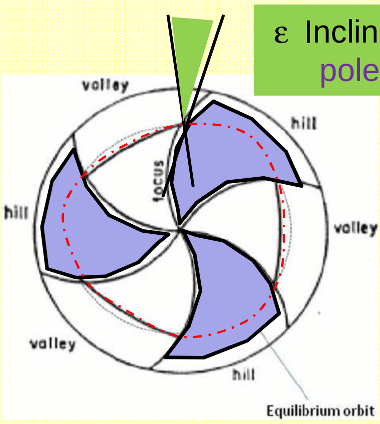
2) Other idea ??? Yes (spiralled sectors)

Increase *N* sectors doesn't help Z stability but it reduces resonnances 11

Better vertical focusing : Spiralled sectors



AVF with straight sectors



ε Inclination angle
pole edge AND Trajectory

AVF cyclo with spiralled sectors

Larger vertical focusing

Bz= <B0>(1+ f(r).g (r, θ))

Spiral eq. r = A.(θ+ 2π j /2N)

tan ε =2r/A

Additive vertical focusing : + FLUTTER .(1+ 2 $\tan^2 \varepsilon$)

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l \, (1 + 2 \tan^2 \varepsilon)$$

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Spiralled sectors

By tilting the edges (ε angle) :

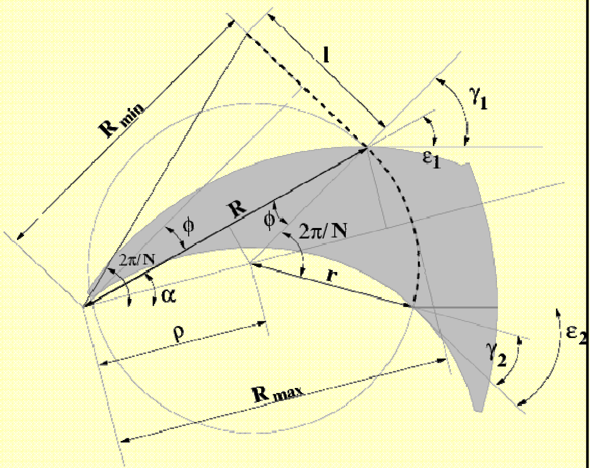
- The valley-hill transition became more focusing
- The hill-valley transition became less focusing

But by the strong focusing principle (larger betatron amplitude in focusing, smaller in defocusing), the net effect is focusing (cf F+D quadrupole).

$$\nu_z^2 = n + \frac{N^2}{N^2-1} F_l \left(1 + 2\tan^2 \varepsilon\right)$$

n<0

Fl > 0



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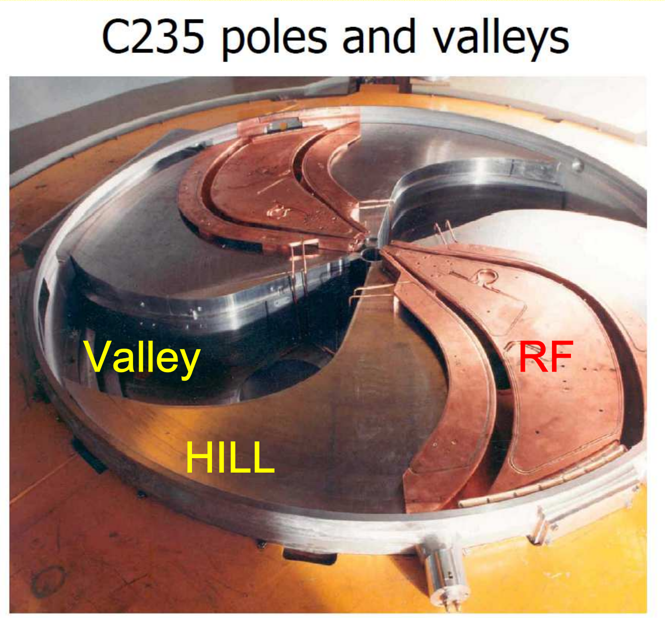
Exemple : 235 MeV compact proton cyclo
4 spiralled sectors



-2 RF cavities (Dees)
Inserted in the valleys

4 Spiralled sectors:

Higher energy=
Higher axial focusing
required

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Exemple : 235 MeV compact proton cyclo

C235 lower part

Without RF

4 Spiralled sectors:

Valley gap =60cm
Hill gap =10cm-1cm

BHill~ 2-3T
Bval~1T

<B> ~ 1.7 T in center
<B> ~ 2.1T extraction 15

Beam dynamics in the ISOCHRONOUS cyclotrons

B=Constant ≠Isochronism condition

A STRONG LIMITIATION in energy γ=1

to get the ions synchrone With RF

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}$$

$B_z = B_0 . g(R)$

B_z increase with R (field index n < 0)

Unstable Vertical oscillations

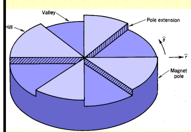
strong limitation in transmission

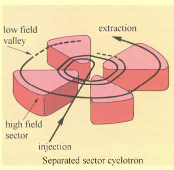
Additive Vertical focusing is needed : -N sectors (Hills//valleys)

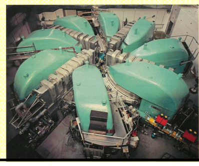
-separated straight sectors

-spiralled sectors

-separated spiralled sectors







$B_z = B_0 . g(R,\theta)$

4 techniques

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One other possibility

SYNCHRO CYCLOTRON (NOT ISOCHRONOUS)

Acceleration condition with Bz decreasing (n>0)

ω_{rev} =not constant

Not isochronous !!

But no vertical instabilities!!

$\nu_z^2 = n > 0$ stable oscillation

Revolution frequency evolves $F_{rev}(t) = F_{rev}(Radius)$

beam has to be synchrone With RF :

$$\omega_{rev}(R)/h = \omega_{RF}(R)$$

Revolution frequency is evolving $F_{RF}(R)$ (like a synchrotron)

Pulsed Machine $\omega_{RF}(t)$: SYNCHRO CYCLOTRON

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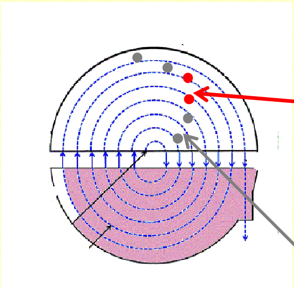
Synchro-cyclotrons : RF cycled, but stable in z

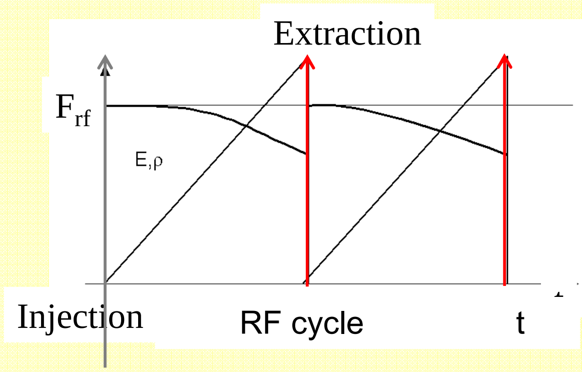


Pulsed beam

Accelerated bunches

$F_{RF} = h\ F_{rev}$

Not accelerated



Less intensity (pulsed) avalaible (not cw)

Exemple : medical aplication
Superconducting synchrocyclo.

ProteusOne® (IBA) : 250 MeV proton

Bz = 5.7 - 5.0 Tesla (very compact)

Rextraction =0.6 m / harmonics=1

F_{RF}= 93 MHz -63 MHz (Rextraction)

Beam pulse : Every 1 ms

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CYCLOTRONS
The Family

$$\omega_{rev} = \frac{qB_z}{\gamma.m}$$

1

Conventional cyclotrons

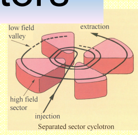
B_z = uniform
F_{rev} = evolves with γ !!!
F_{RF} = constant

NOT USED anymore
Limited in energy :EK<1MeV

2: isochronous

Compact cyclotrons
1 magnet
with modulation

Separated sectors



B_z = NOT uniform =f(R)
$F_{revolution}$ = Constant
F_{RF} = constant

Vertical focusing with
B_z = f(R,θ)

Isochronous

$$\omega_{rev} / h = \omega_{RF}$$

3 : non isochronous

Synchro-cyclotrons

F_{rev} = NOT Constant
F_{RF} = NOT Constant = beam pulsed

Not Isochronous

Less intensity (pulsed) ≠ cw

$$\omega_{rev}(R) / h = \omega_{RF}(t)$$

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Chapter 1 :Part b

End Chapter 1 :

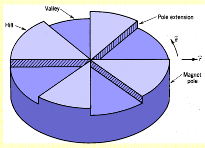
important facts to remenber

1) isochronous = constant revolution time

2) $\omega_{RF} = \mathbf{h}\ \omega_{rev}$

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m}$$

3) Axial stability can be obtained with azimutal field variations $Bz = f(R, \theta)$



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Few other slides
for questions

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Tutorial 3 : What is the field index n(R) ?

n(R) : it gives the radial evolution of *Bz*

$$n = -\frac{R}{B_0}\frac{\partial B_z}{\partial R}$$

⟺

Equivalent definition

$Bz(R) = R^{-n} B_0$

The field index is not constant in a cyclotron n = n (R)

Isochronous cyclotron n(R) <0 : $<Bz(R,\theta)>_{turn}$ increases with R

$Bz(R) = \gamma(R)\ B_0$
$= R^{-n}\ B_0$

$$< B_z(R) >= \frac{B_0}{\sqrt{1-[R\ \omega/c]^2}} = B_0.R^{-n(R)}$$

$$\gamma = \frac{1.}{\sqrt{1-[v/c]^2}} = \frac{1.}{\sqrt{1-[R\omega/c]^2}}$$

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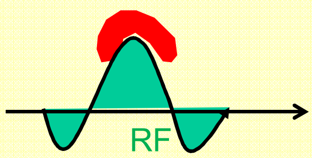
Dynamics in cyclotron

summary

$Qe_0 \hat{V} \cos\phi .\, N_{gap}$ — ***Energy gain per turn***

$\phi_0 \approx 0°$ — ***Central RF phase , Ion bunches are centered at 0°***



$\omega_{RF} = h\omega_{rev} = const$ — ***RF synchronism = Isochronism (h - harmonic number)***

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m} = const$$

$R = R(t) = R(N°turn\quad)$ — ***Orbit evolving***

$B\rho(t) = P/q \Rightarrow \langle B \rangle = B\rho / R$ — ***Average Magnetic field***

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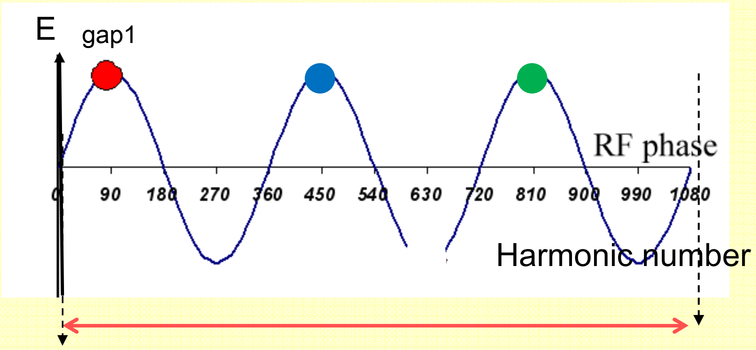
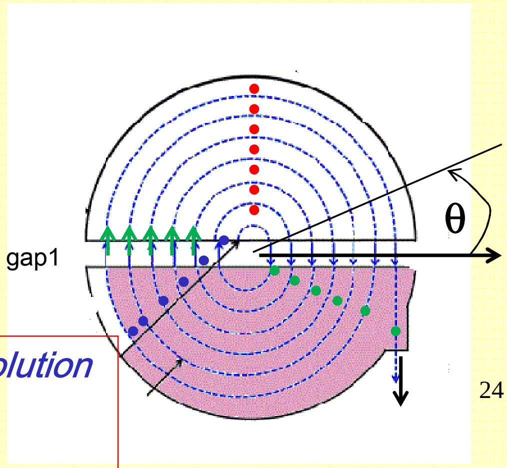
Harmonic number h=FRF/ Frev

h=integer $\omega_{RF} = \mathbf{h} \ \omega_{rev}$



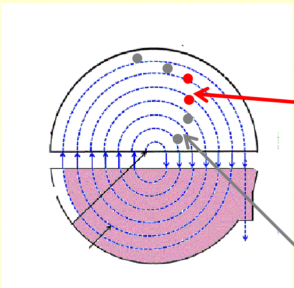
1 turn RF= h x 360°



h =1,2,3 *Number RF oscillations per revolution*

⇨ h *bunches by turn* $\omega_{rf} = \mathbf{h} \ \omega_{rev}$

Synchro-cyclotrons =not isochronous (pulsed machine)

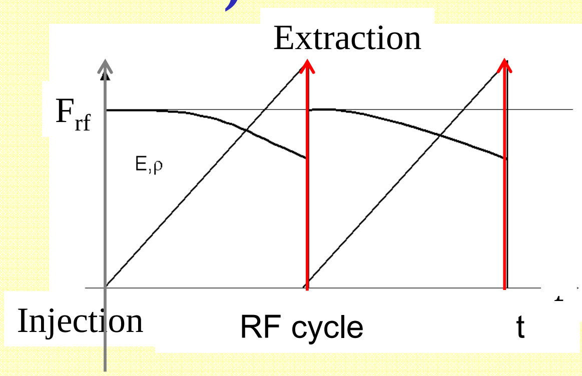


Pulsed beam

Accelerated bunches

$F_{RF} = h \; F_{rev}$

Not accelerated



Less intensity (pulsed) avalaible (since not cw)

$$\omega_{rev} = \frac{\omega_{RF}}{h} = \frac{qB_z}{\gamma(R)\, m}$$

$$\omega RF\,(injection) = h\,\frac{qB}{\gamma\, m} \approx h\,\frac{qB}{m}$$

$$\omega RF\,(extraction) = h\,\frac{qB_0}{\gamma(R).m}$$

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