

## Chapter 2

# Cyclotrons : specific techniques

- **Acceleration** and RF cavities
- Injection (axial or radial)
- Extraction  
(stripping, turn separation, precession...)



## Cyclotrons Tutorial 4

- An cyclotron is supposed to accelerate ions with **A nucleons** and a **charge state Q**.
- Demonstrate than the maximal kinetic energy **E/A** of a cyclotron is

$$E/A = Kb \cdot (Q/A)^2$$

**Nota :** Give the **Kb** factor in a non relativistic approximation using the extraction radius **R**, the maximal average magnetic field **B**.

The mass of the ions is **m= Am<sub>0</sub>** & the charge of the ions is **q= Qe<sub>0</sub>**

## Cyclotrons Tutorial 5

- A COMPACT CYCLOTRON have a  $K_b$  factor of 30 MeV  
( $E/A = K_b \cdot (Q/A)^2$ )

*What is the maximal energy*

*we could reach with such a cyclotron magnet*

- With a proton beam*
- With a carbon beam (with  $Q=6+$ )*

*The cyclotron magnet have  $\langle B \rangle = 1$  Tesla, what is the revolution frequency ? (  $F_{rev} = \omega / 2\pi$  )*

- of a proton beam*
- of a carbon beam (with  $Q=6+$ )*

*Can we work with the same RF cavity for the two beams ?*

*( $\omega f = h \quad \omega = h qB/m\gamma$ )*

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## Acceleration

- The final energy is independent of the accelerating potential  $V = V_0 \cos\phi$ .

If  $V_0$  varies, the **number** of turn varies. ( $B\rho_{final} = \langle B \rangle \cdot R_{extraction}$ )

- **The energy gain** per turn depends on the peak voltage  $V_0$ , but is constant, if the cyclotron is **isochronous** ( $\phi = \text{const}$ ):

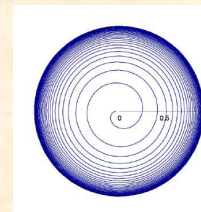
$$\delta E = N_g q V_0 \cos\phi$$

$N_g$  : number of gaps per turn

- The **radial separation**  $\delta r$  between two turns varies as  $1/r$  ( $\gamma \sim 1$ ):

$$\frac{\delta r}{r} = \frac{\delta B\rho}{B\rho} = \frac{\delta p}{p} = \frac{\gamma}{\gamma + 1} \frac{\delta E}{E} \approx \frac{qV_0 \cos\phi}{2E} \propto \frac{1}{r^2}$$

$$\delta r \propto \frac{1}{r}$$

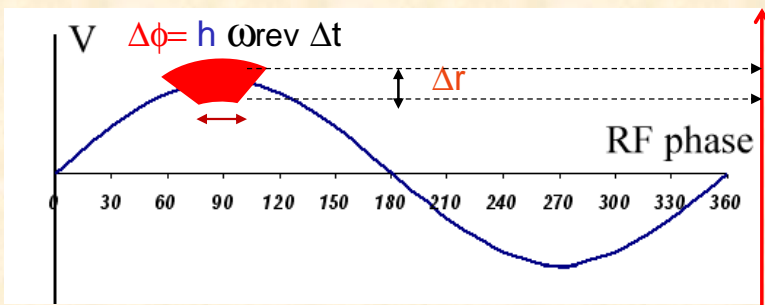


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## Acceleration & bunch length $\Delta t$

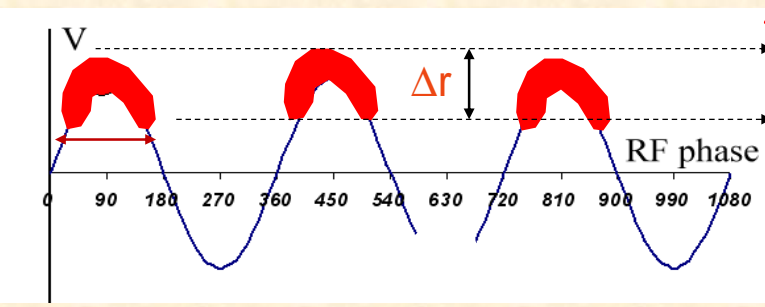
- The bunch length  $\Delta t$  induces radial dispersion  $\Delta r$  & energy dispersion

$$\frac{\Delta r}{r} = \frac{\Delta B\rho}{B\rho} = \frac{\gamma}{\gamma+1} \frac{\Delta E}{E} \approx \frac{1}{2} \frac{\Delta[qV_0 \cos(h\omega_{RF}t)]}{E}$$



harmonics=1

$$\Delta r \sim qV \omega_{rf} \Delta t$$



harmonics > 1

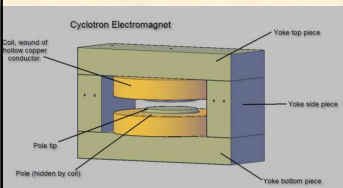
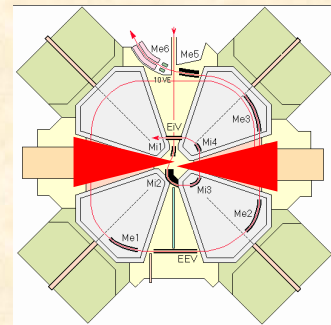
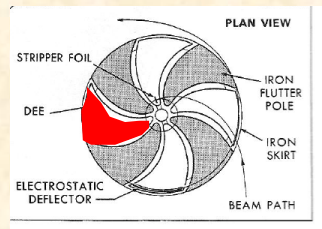
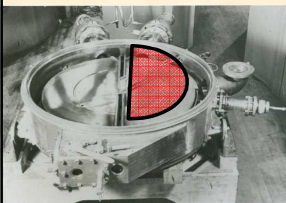
Worst beam quality

$$\Delta r \text{ larger} \sim qV H \omega_{rf} \Delta t$$

~ Energy dispersion larger

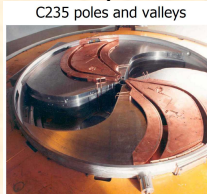
# Acceleration RF Technology

Magnetic structure => RF cavity's shape



The classical "D" shape

"Curved sector"  
For spiral AVF  
C235 poles and valleys



"Triangle" shape  
For separated sector cyclo

The choice of the pole shape and the number of sectors  $N$  have a great impact on the available space for RF systems. Dees have to fit into the gaps and/or valley sections

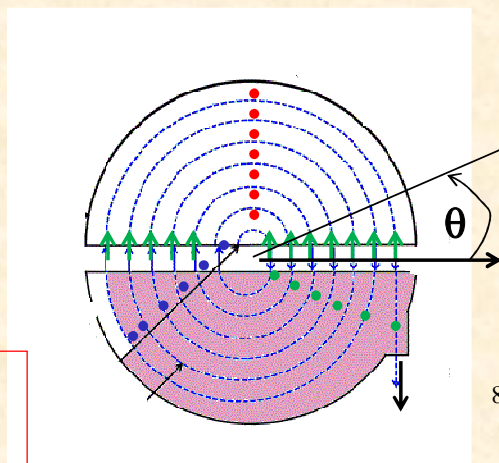
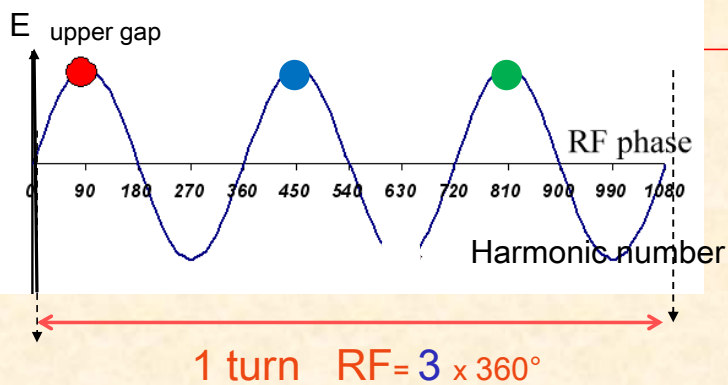
Harmonic number  $h = F_{RF} / F_{rev}$

$h = integer$

$$\omega_{rf} = h \omega_{rev}$$

$h$  Rf oscillations

in a revolution time



$h = 1, 2, 3$  Number of bunch per turn

$h$  bunches by turn  $\omega_{rf} = h \omega_{rev}$

## RF Cavities in variable energy cyclotron

Often, in research facility Cyclotrons must provide ions at variable energy

How to adjust the final energy to the needs ?

$$\omega_{rev} = \frac{qB}{\gamma m} = \frac{\omega_{RF}}{h}$$

$$\omega_{rf} = \omega_{rev} / h$$

Adjust **Bz** which modify  **$\omega_{rev}$**  for a given ion (m,q)

**$\omega_{RF}$**  should be adjusted as well

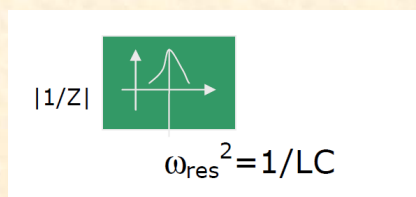
Variable frequency ACCELERATING CAVITY ARE needed

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## RF Cavities for “variable ions machine” Adjust the resonance of the cavity

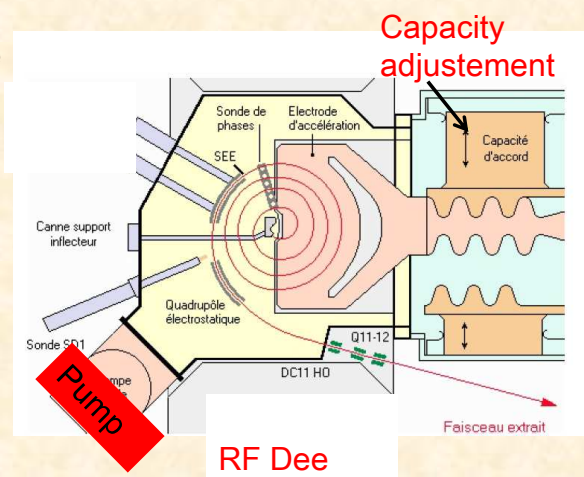
Resonance for a cavity  
= minimal impedance (Z) for maximal Voltage



Cyclotron : Variable Energy with  
B and Frf variable

$$1/Z = 1/R + j\omega(C - 1/L\omega^2)$$

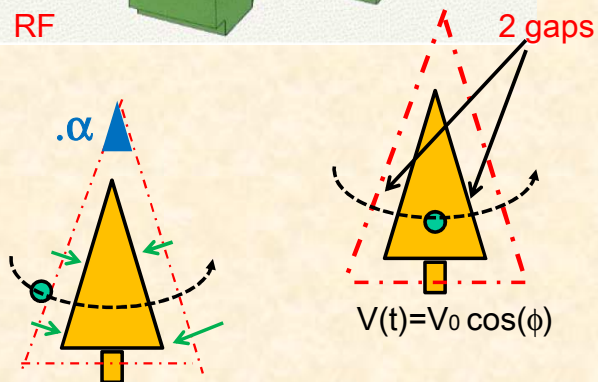
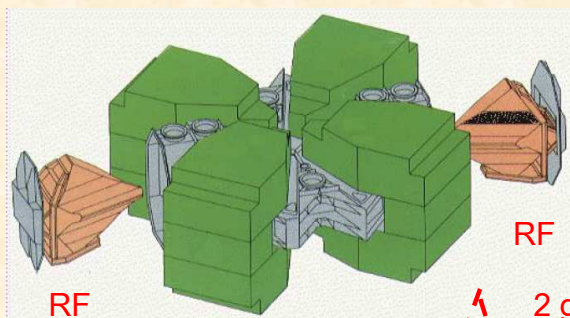
$$\omega_{rev} = \frac{qB}{\gamma m} = h\omega_{RF}$$



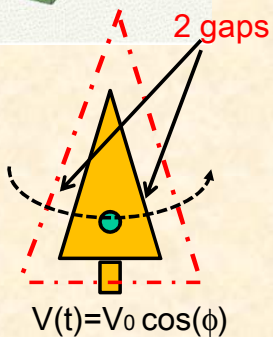
Variation of the Capacity C :  
to adjust  $\omega_{resonance}$

$$\omega_{resonance} = \omega_{rf} = \omega_{rev} / h$$

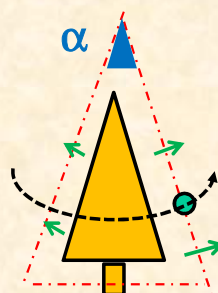
# RF Cavities : example 1 for Separated Sectors Cyclo



$$V(t) = V_0 \cos(\phi - h\alpha/2)$$



$$V(t) = V_0 \cos(\phi)$$



$$V(t) = V_0 \cos(\phi + h\alpha/2)$$

## Example 1: RF Cavities (not Dees)

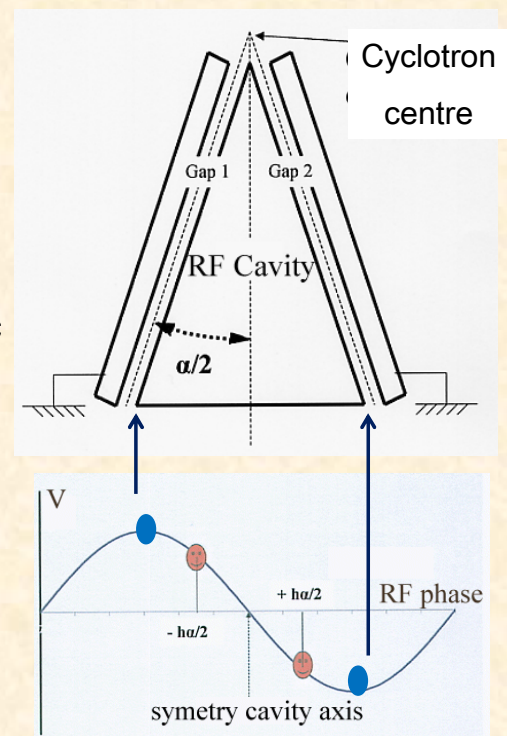
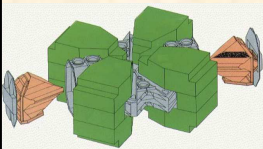
Energy gain in 1 gap :

$$\delta E = qV_0 \sin\left(\frac{h\alpha}{2}\right) \cos \varphi$$

- For a maximum energy gain ( $\cos\varphi = 1$ ) the particle passes the symmetry cavity axis
- Energy gain per gap for the various harmonic mode

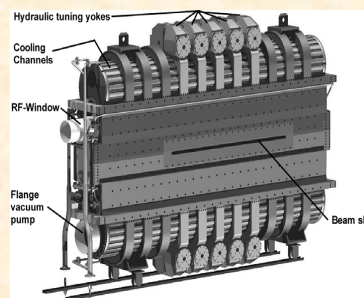
$$\delta E = qV_0 \sin\left(\frac{h\alpha}{2}\right)$$

$\delta E$  optimum is  
for  $h.\alpha/2 = 90$  degree



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example 2 :separated sector cyclotron:  
the PSI ring cyclotron (proton kb=590 MeV)



$R_{\text{extraction}} = 4.5 \text{ m}$

$K_b = 590 \text{ MeV}$

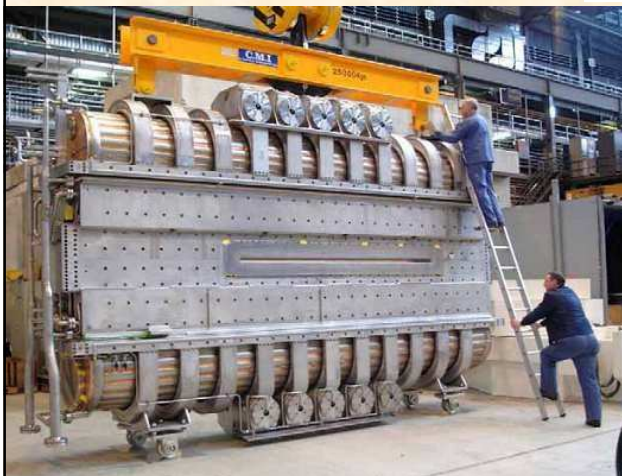
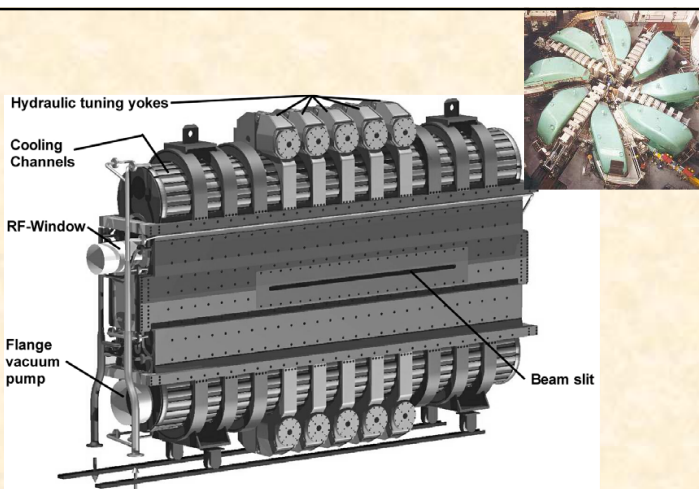
**4 RF Cavities**

Typical 'Separated Sector Cyclotron' (SSC). the PSI 590 MeV (p) ring cyclotron,  
with 8 sector magnets and 4 accelerating cavities

PSI Proton Beam ~1 Mwatt

The Challenge :  
Single turn extraction

Turn separation  $\delta r$  large  
But size  $\Delta r$  small



4 cavities : 50 MHz, CW  
Voltage: 0.9-1 MVolt

Harmonics h=6

Proton Beam ~1 Mwatt  
(I=2 mA)

if  $\delta r$  ( $\sim N_{gap} \cdot V_{rf}$ ) Large  
No beam losses  
T= 99.99%

# Beam injection

-THE ION SOURCES (internal and external )

Low energy :

AXIAL INJECTION FOR COMPACT CYCLOTRON

- Infector (spiral, hyperboloid;...)

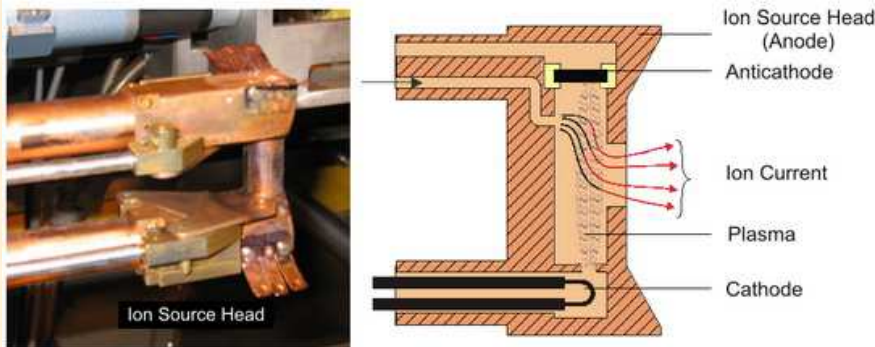
Higher energy :

RADIAL INJECTION FOR SEPARATED SECTOR  
CYCLOTRON



# Cold Cathode PIG Ion Source

Penning or Philips Ionization Gauge (*PIG*) ion source

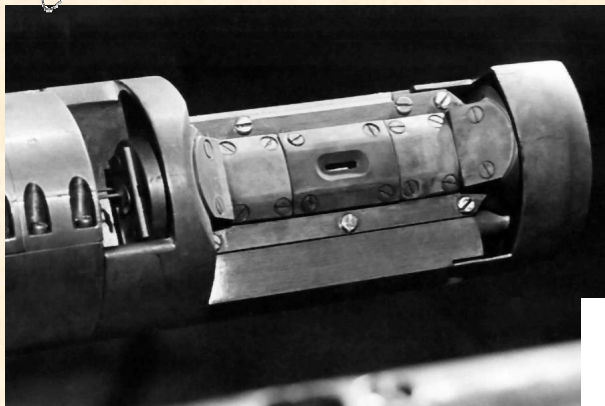


- Electron emission due to electrical potential on the cathodes
- Electron confinement due to the magnetic field along the anode axis
- Electrons produced by thermionic emission and ionic bombardment
  - Start-up: 3 kV to strike an arc
  - At the operating point : 100 V
- Cathodes heated by the plasma (100 V is enough to pull an outer e- off the gas atoms)

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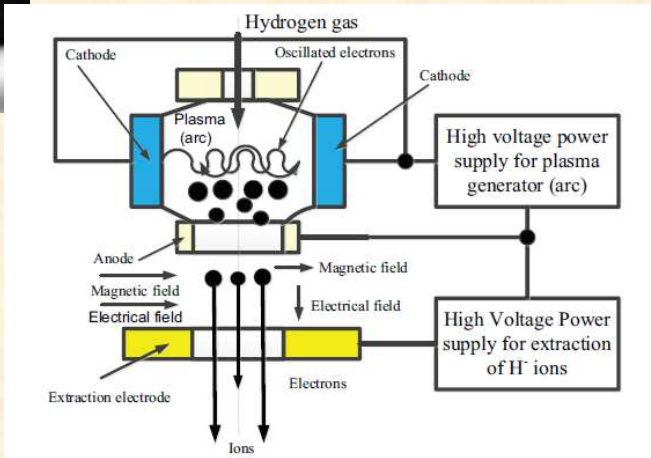
# Example of PIG source



FLNR, PIG test-bed, 1992.  
The head of MC400 cyclotron vertical ion source

Small size

Inserted in the cyclotron gap

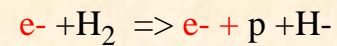
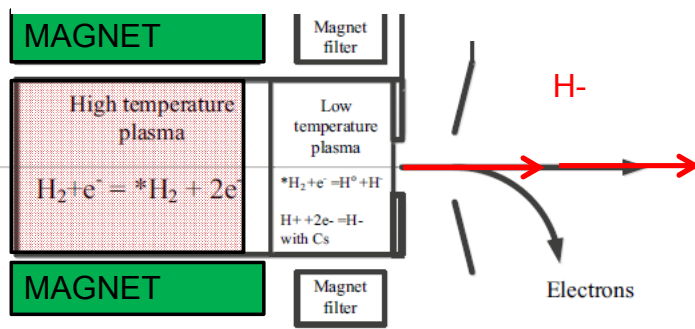




## Multi-CUSP source

negativ ions : H-//D- with high current

Confinement + filtering + extraction



- Larger Than the PIG source (**Magnets**)
- Better emittance
- Larger current (Magnet confinement+ Filter)

Larger Size  $\Rightarrow$  External Source

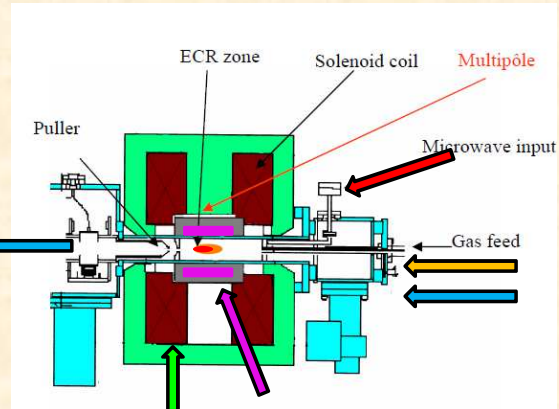
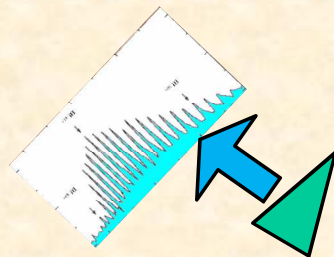
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# ECR ion source : positiv Heavy ions



Pantechnic  
Nanogan®

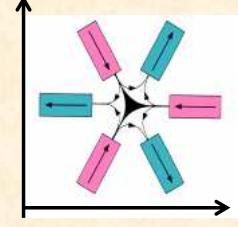
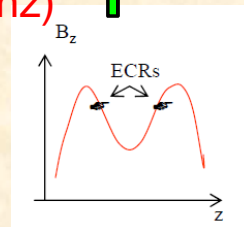
Charge state distribution



Gas (He,O,..) + RF(microWave 10-18Ghz)  
Plasma (ions + electrons) :  
+ ATOMS

electrons + ions impacts

Ionize any injected heavy atoms  
( He,Li,.....U)

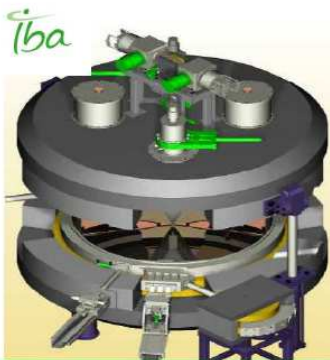


Bz + Br  
3D plasma confinement

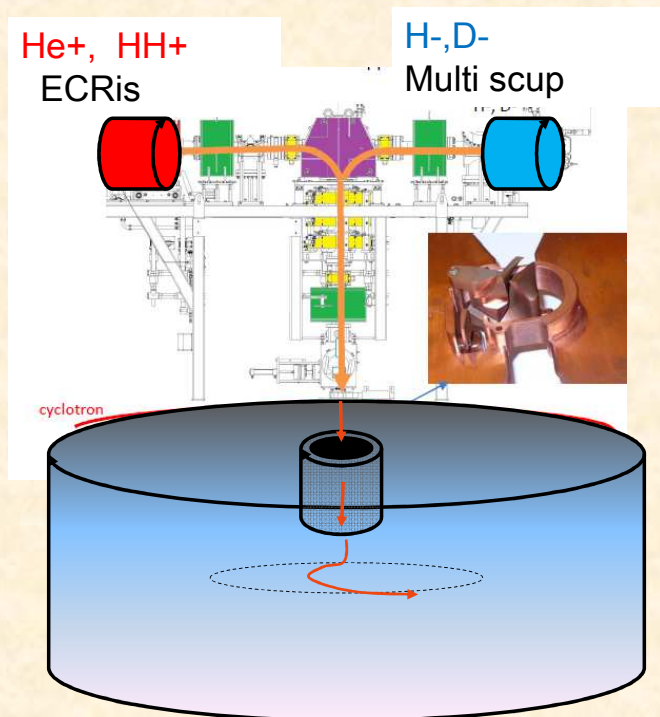
Exemple ARRONAX (Nantes, France)  
2 external sources in a  $K_b=70$  MeV cyclotron

$K_b=70$  MeV

- Radiolotopes production
- Radio-Biology studies
- Irradiation



Cyclotron ARRONAX

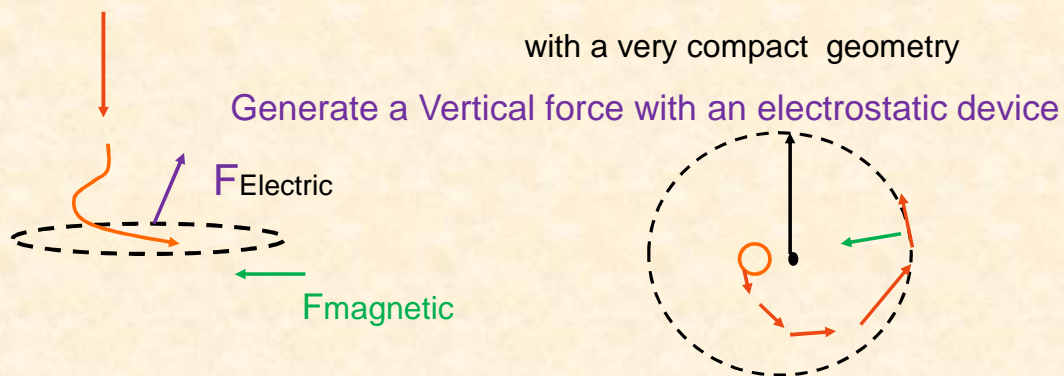


Injection from the Top (AXIAL injection)

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## Axial injection with inflector

- Goal : Put **the beam** on the « **good orbit** » at the good phase



Outside cyclotron

axial motion (vertical)

Inside cyclotron ( Magnetic force is radial)

radial motion (horizontal )

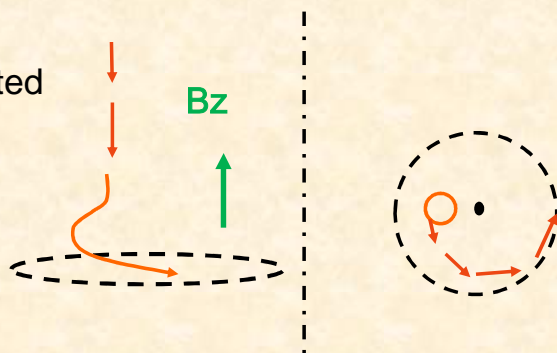
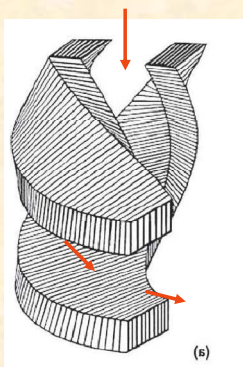
$$R_m = B_\rho / B_{center}$$

$$R_E = mV^2/Q / E_{inflector}$$

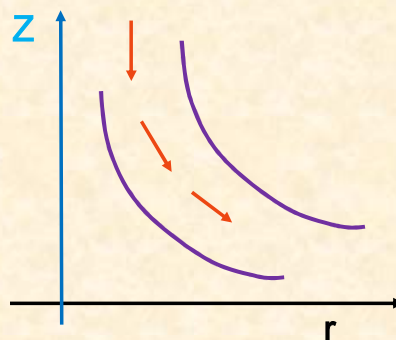
# Axial injection : Spiral inflector

## 1. Spiral inflector (or helical channel)

principle: 90° electrostatic deflector twisted

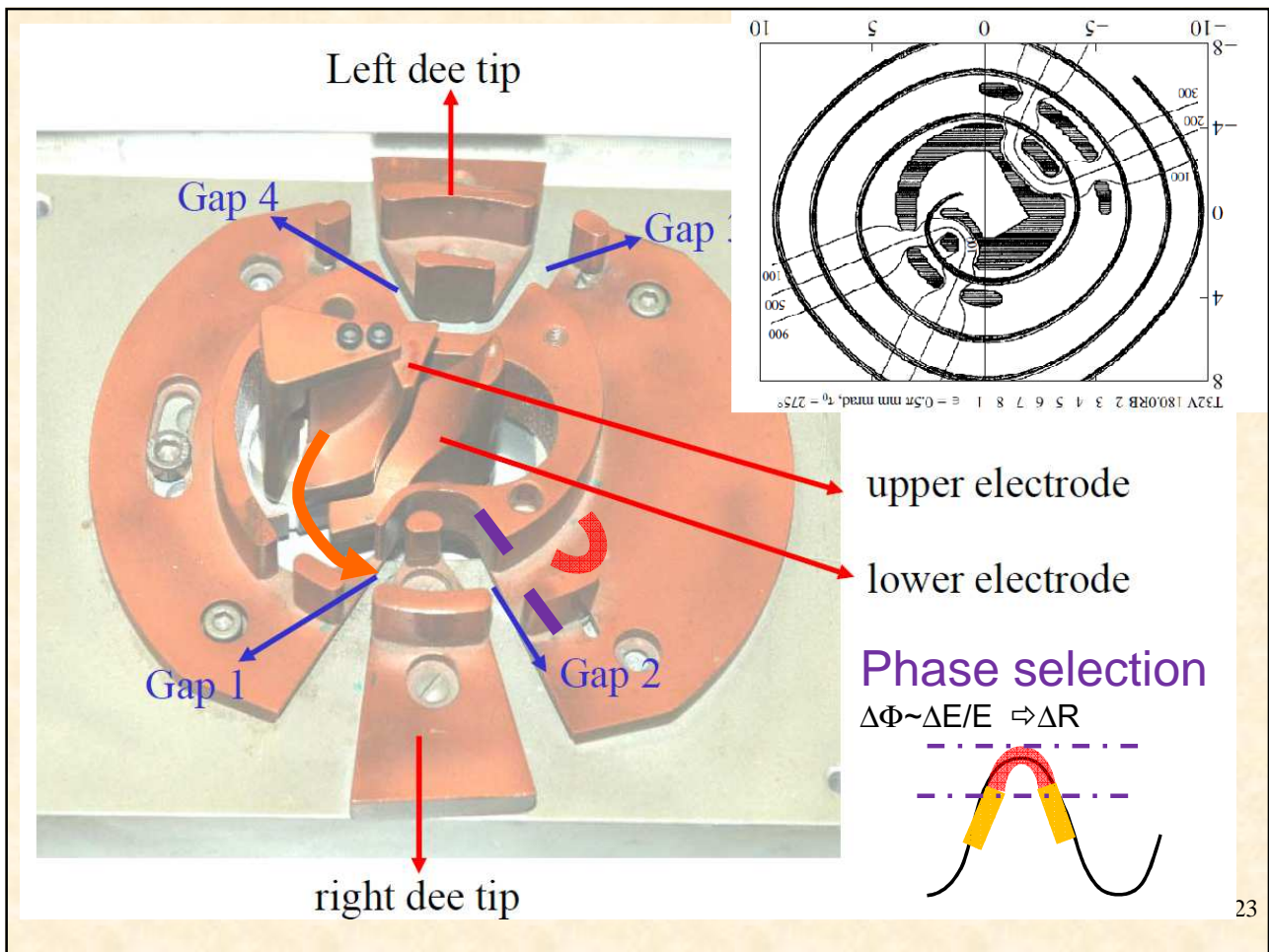


$E$  always perpendicular to  $v$   
 $B=B_z$  constant (cyclo center)



Complex geometry , very compact

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## Axial injection 1: Spiral inflector

$$m\ddot{x} = qE_x - qv_y B_0,$$

$$m\ddot{y} = qE_y + qv_x B_0,$$

$$m\ddot{z} = qE_z.$$

Trajectory Equations are very funny :

Parametric equation of the trajectory  $\theta = [0, \pi/2]$

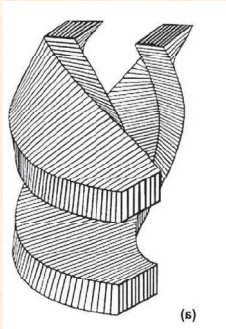
$$x_c = \lambda(1 - \sin k\theta \sin \theta - \cos k\theta \cos \theta)$$

$$y_c = \lambda(\sin k\theta \cos \theta - \cos k\theta \sin \theta)$$

$$z_c = A(\sin \theta - 1)$$

$$k = A/R_m + k'$$

$$\lambda = A/(k^2 - 1)$$



Two parameters : A the inflector Height  
k' the tilt

2 forces bend the beam

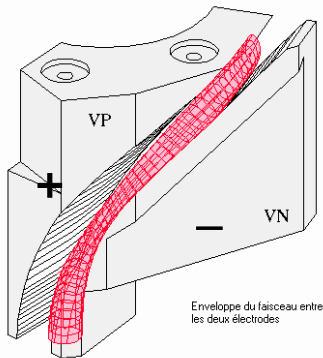
Electric radius  $A = RE = mV^2/Q / E_0$

Magnetic radius  $R_m = B\rho / B_0$



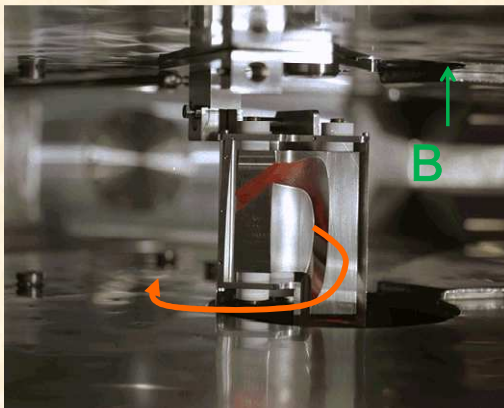
## Axial injection 1: Spiral inflector

Inflecteur CIME



- Consists of 2 cylindrical capacitors which have been twisted to take into account the spiraling of the ion trajectory from magnet field.

- $\vec{v}_{beam} \perp \vec{E}$  : central trajectory lies on an equipotential surface. Allows lower voltage than with mirrors.



- 2 free parameters (spiral size in z and xy ) giving flexibility for central region design
- 100 % transmission



## Axial injection 2: **hyperboloid inflector**

Spiral electrodes are complex :

**hyperboloid inflector** have simpler electrode

two electrodes equation :  $r^2 - 2z^2 = r_1$     $r^2 - 2z^2 = r_2$

$$V = -Kz^2/2 + Kr^2/4 + c$$

**Vertical field**  $E_z = -Kz$

$$x = \frac{r_0}{2} \{-b \cos(akt) + a \cos(bkt)\},$$

$$y = \frac{r_0}{2} \{-b \sin(akt) + a \sin(bkt)\},$$

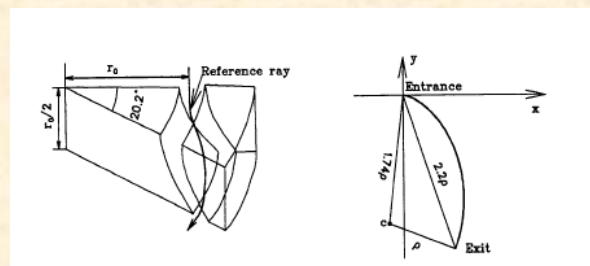
$$z = \frac{r_0}{2} \sin(kt),$$

$$k^2 = \frac{qK}{m},$$

$$r_0 = (2\sqrt{6})\rho.$$

$$k^2 = -qv^2/2$$

$$r_0 = 2 \cdot 6^{1/2} R_m$$



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$$\left. \begin{aligned} \dot{r}_1 &= 1 + A_1 v_1 \\ \dot{r}_2 &= -1 + A_2 v_2 \\ \dot{z}_1 &= -A_1 v_1 / \sqrt{2} \\ \dot{z}_2 &= -A_2 v_2 / \sqrt{2} \end{aligned} \right\} (14)$$

$$\left. \begin{aligned} \dot{r}_1 &= A_1 v_1 + A_2 v_2 \\ \dot{z}_1 &= A_1 v_1 + A_2 v_2 \end{aligned} \right\} (15)$$

By transforming eq. (13) we get

$$\left( \frac{dr}{dt} \right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} r \\ z \\ v_1 \\ v_2 \end{pmatrix} \quad (16)$$

$$r = 2z \sin \omega t \cos \omega t + 2z \cos^2 \omega t + \frac{1}{2} (v_1^2 + v_2^2) \sin^2 \omega t \quad (17)$$

Simpler geometry than spiral inflector

But No free parameter ( Rinjection=Rm it fixes all parameters)

## Radial injection

**Radial Injection** for pre-accelerated beam :

- Compact inflector not possible (axial inj. not possible) :
- Higher rigidity (electrostatic field have “low efficiency”)  
**need space to bend the beam with large magnet !!**

1. Injection into separated sector cyclotron (most common)

- More room for injection pieces and excellent transmission

2. Other Specific examples ( not described here)

- Injection with Charge exchange (internal stripper foil)  
in a compact superconducting cyclotron NSCL
- 3 ....

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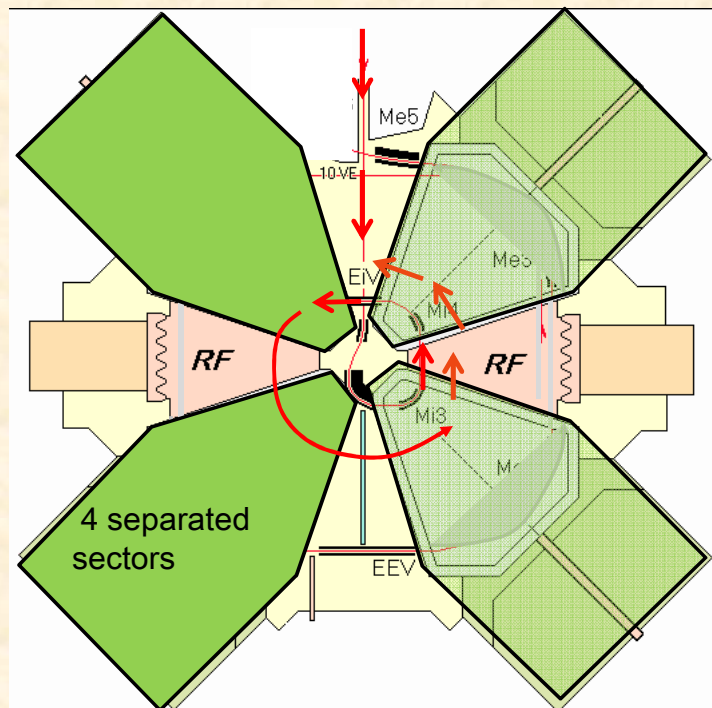
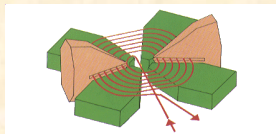
## Example: Radial injection in a ring cyclotron

- More room to insert bending elements.

Beam injected between sector magnets

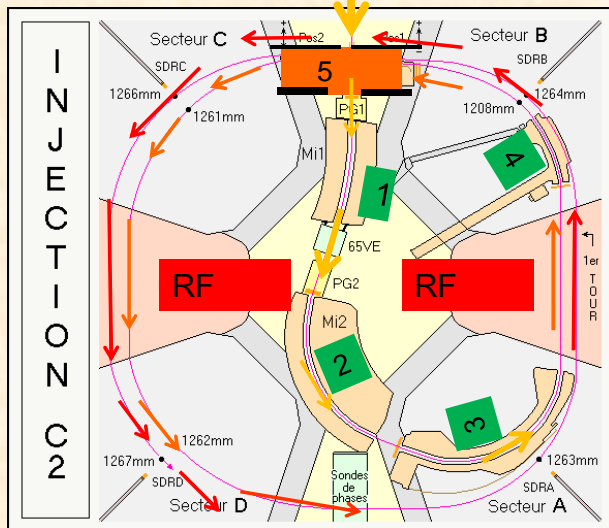
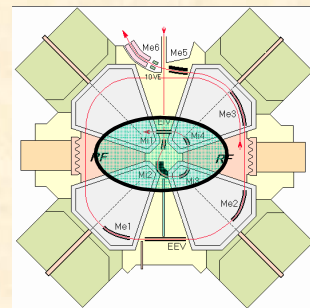
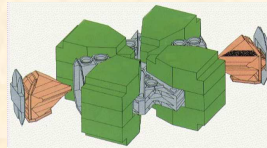
- The beam coming from the pre-injector enters the SSC horizontally.

- It is guided by 4 magnetic dipoles to the "good trajectory", then an electrostatic inflector deflect the beam behind the dipole yokes.



# Example: Radial injection

Beam coming from an other accelerator



1-4 : magnetic dipoles

5 : electrostatic inflector

- Very carefull centering
- RF Phase ajustement with bunches

### ELECTROSTATIC INFLECTOR SSC1 (GANIL)

#### Bending of the first turn

Septum: 0 Volt

**Electrostatic inflector**  
100 kVolt gap~ 1cm

**Injection of accelerated beam**  
(rigidity =difficult to bend)  
**Magnet requires more space**  
than electrostatic devices

# Cyclotron Extraction

## 1. Extraction by stripping negative ions

simpler and low cost , but restricted to Hydrogene isotopes

100% efficiency

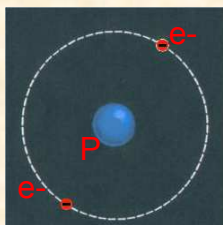
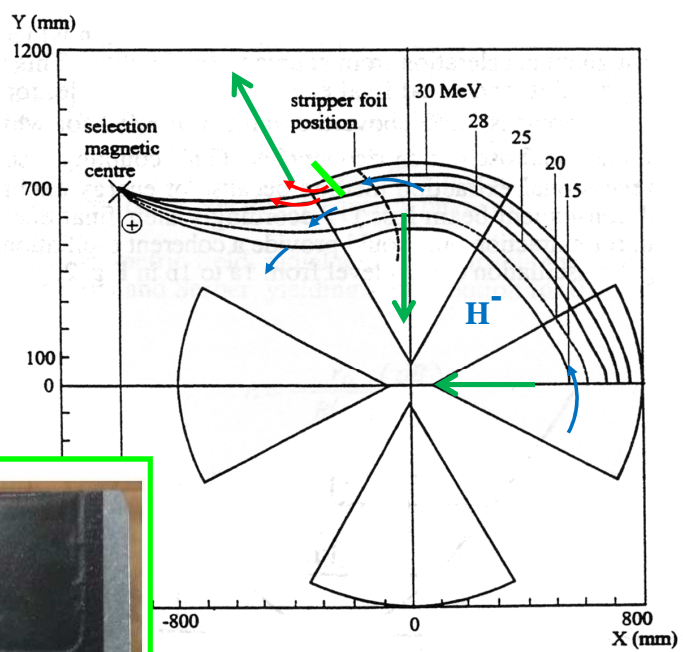
## 2. Extraction using the radial separation

between turn  $n^{\circ}N$  &  $n^{\circ}N+1$

...

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Extraction by stripping **negativ ions**  
 easy and efficient with **H<sup>-</sup>** ( 1 Proton+2 orbiting electrons)



The magnetic force is inverted

$F_r \sim - v \cdot Bz \Rightarrow + v \cdot Bz$

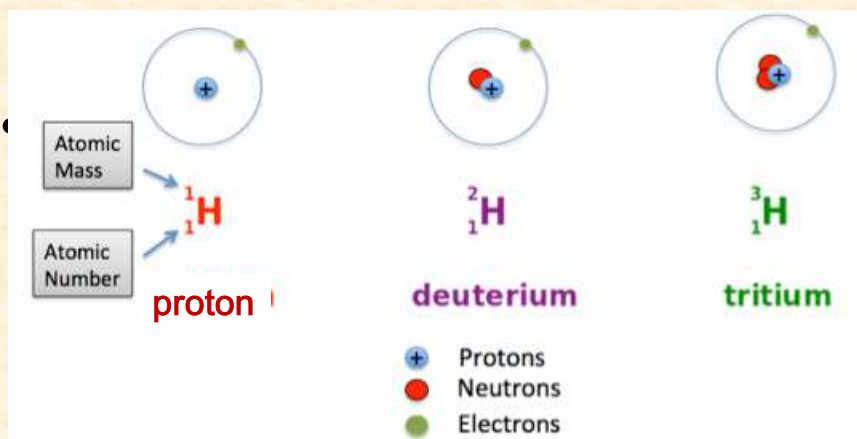
$Q = -1 \Rightarrow Q = +1$



Carbon foil Stripper

Extraction orbits in the IBA Cyclone 30

## Negative Hydrogene isotopes for **proton** & **deuteron** beam



PIG sources or multiscup sources for negative ions of H,D

H- (proton+ 2 e- )

D- (deuteron+2 e-)

D= good for several nuclear reactions  
(radio isotopes production)



## H<sup>-</sup> & D<sup>-</sup> commercial cyclotrons with two extracted beams

Low cost extraction beam line(s) :  
less complex than **electrostatic deflectors**

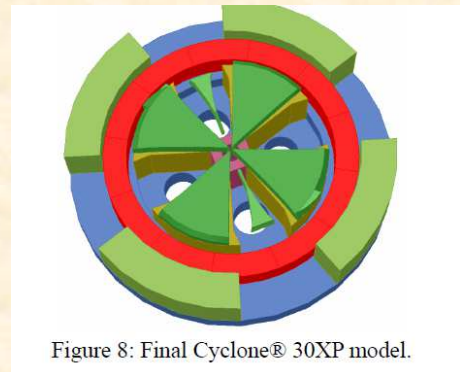
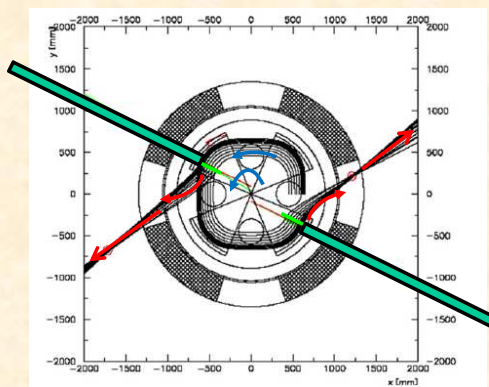
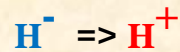


Figure 8: Final Cyclone® 30XP model.

H<sup>-</sup> production or D<sup>-</sup> production with an internal source (PIG)

2 strippers at extraction radius :



good beam quality, easy maintenance

## Extraction by turn separation

### 1. Extraction by acceleration (and fringe field)

The orbit radial  $\delta r$  separation between 2 turns is :

$$\delta r = r \times \frac{\delta E}{E} \times \frac{\gamma}{\gamma + 1} \times \frac{1}{v_r^2}$$

- $\delta E$  : Energy gain per turn as high as possible (RF)
- $v_r$  : Accelerate the beam to fringing field (Bz decrease,  $n > 0$ ,  $v_r \downarrow$ )

$$\frac{\delta B}{B} = -n \frac{\delta r}{r}$$

Demonstration :

$$\begin{aligned} \frac{\delta r}{r} &= \frac{\delta B \rho}{B \rho} = \frac{\delta \langle B \rangle R}{\langle B \rangle R} = \frac{\delta R}{R} \Big|_{acc} - \frac{\delta B}{B} \\ &= \frac{\delta P_{acc}}{P} + n \frac{\delta r}{r} = \frac{\delta P_{acc}}{P} \frac{1}{(1-n)} \approx \frac{\delta P}{P} \frac{1}{v_r^2} \approx \frac{1}{2} \frac{\delta E_{acc}}{E} \frac{1}{v_r^2} \end{aligned}$$

$$v_r = \sqrt{1-n}$$

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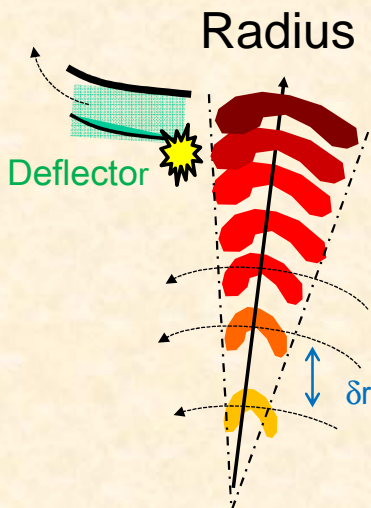
## Extraction using turn separation $\delta r$

if  $\Delta r < \delta r$  Each turn separated

Extraction with an electrostatic deflector

100% efficiency

SINGLE TURN EXTRACTION



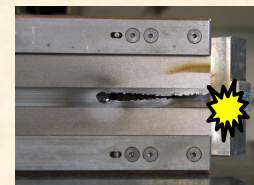
$\Delta r > \delta r$  (size > turn separation)

last turns are not separated

Beam losses in the extraction channel

Multi TURN EXTRACTION

Deflector sparking or damaged



$$\delta r \propto \frac{1}{\text{Radius}}$$

## Extraction : 3 mechanisms possible

Goal : High extraction efficiency with well separated orbit

$$\delta r = \text{Acceleration} + \text{Precession} + \text{increase oscillation by a field bump (resonance extraction)}$$

1. Extraction by acceleration (and fringe field + deflector)
  - Energy gain per turn as high as possible...
2. Precession extraction : radial oscillations help to separate orbits

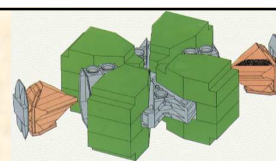
$$r(N) = r_0(N) + x_0 \sin(v_r \cdot \omega_0 t)$$

3. Resonant extraction : increase the precession by a field bump

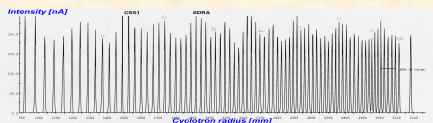
If turn separation not enough then magnetic perturbations are used. Particles are forced to oscillate around their equilibrium orbit with a magnetic bump

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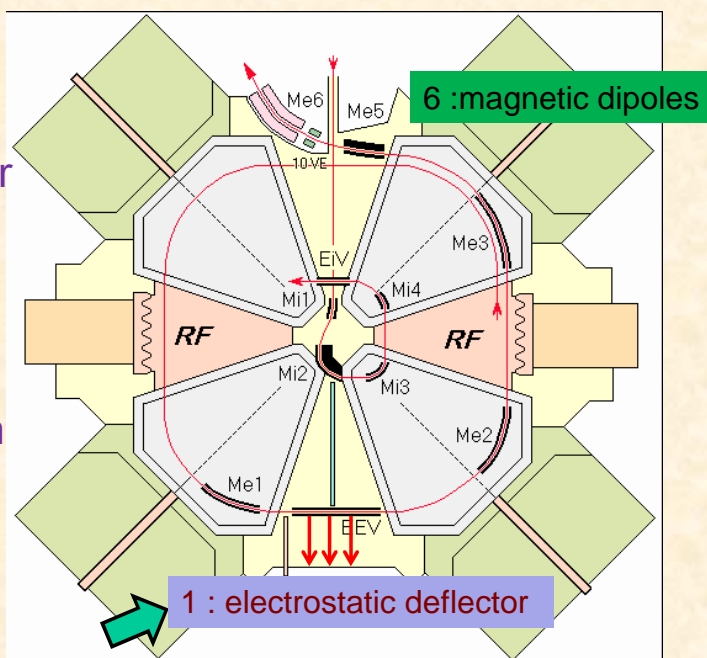
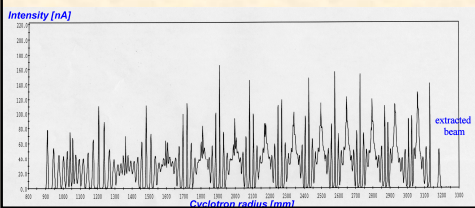
# Example: Ejection SSC



- 1) Acceleration (2 RF cavities)
- + Radial kick on the extracted turn With an electrostatic deflector

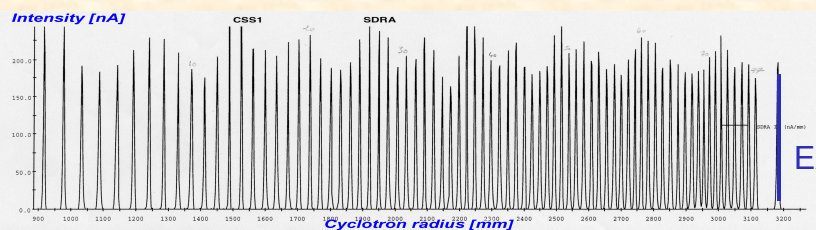


Precession : excited from injection



# Extraction with precession

Well centered beam orbits **Separated Sector Cyclo N°1 GANIL**



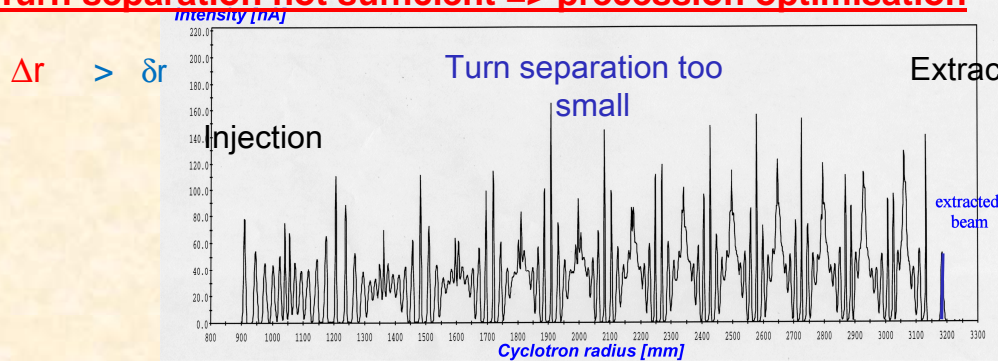
Turn separation sufficient

Extracted turn

$$\Delta r < \delta r$$

Precession for optimized extraction **Separated Sector Cyclo N°2 GANIL**

**Turn separation not sufficient => precession optimisation**



$$\Delta r > \delta r$$

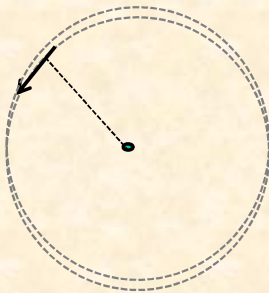
Turn separation too small

Extraction

transmission 95%

## Resonant extraction : with $Q_x = \nu_r \sim 1$

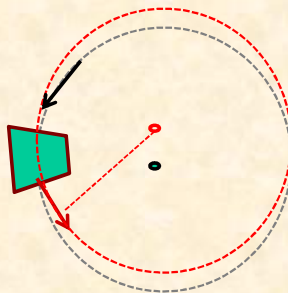
**Step 1** : circular motion  
+ small oscillations



$$\ddot{x} + \nu_r^2 \omega_0^2 \cdot x = 0$$

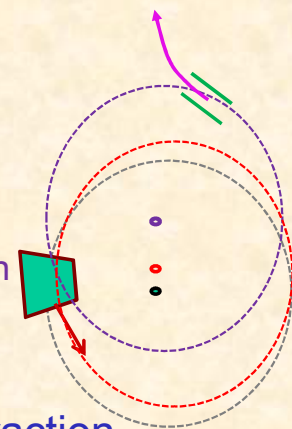
$\nu_r \sim 1$  : 1 oscillation per turn

**Step 2** : A magnetic bump  
shift of the orbit center  
larger deviation



**Step 3** : Several turns  
produce  
Large amplitude oscillation

Larger & Larger & Larger



Large  $\delta r$  = easy extraction

$$\delta r \approx \left[ \frac{1}{2} \frac{\delta E_{RF}}{E} \right] + \Delta x_0 \sin(\nu_r \cdot \omega t)$$



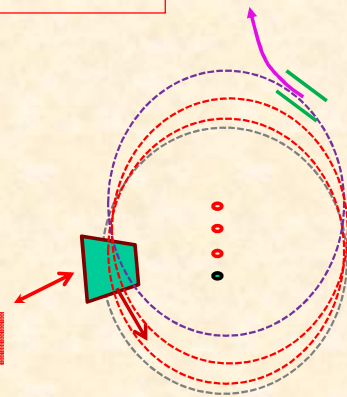
## Resonant extraction shown by equations

Radial Equation without Perturbation

$$\ddot{x} + v_r^2 \omega_0^2 \cdot x = 0$$

Equation with Perturbation  $\delta B_z \sim b_M(r) \cos(M\theta)$

$$\ddot{x} + [v_r \omega_0]^2 x = \omega_0^2 \frac{r}{B} \frac{db_M}{dr} \cos(M\omega_0 t)$$



Driven oscillator excited at the « frequency »  $M$

if the excitation is at the resonance frequency  $M = \nu_r$   
you get Large amplitude oscillations  $\delta r$  (easy extraction)

One field Bump correspond to harmonic  $M=1$





End Chapter 2 : important facts to remember

$$1) \quad (E/A)_{\max} = Kb \cdot (Q/A)^2$$



2) *“Compact cyclotron” have an axial compact inflector*  
*“Separated sectors” have radial injection*

3) *Turn separation for extraction  $\delta r$  ( $> \Delta r$ )*

$$\delta r = \text{Acceleration (RF)} \\
+ \text{Eventually Precession} \\
+ \text{Eventually resonance excitation}$$

*some Additive slides for questions...*

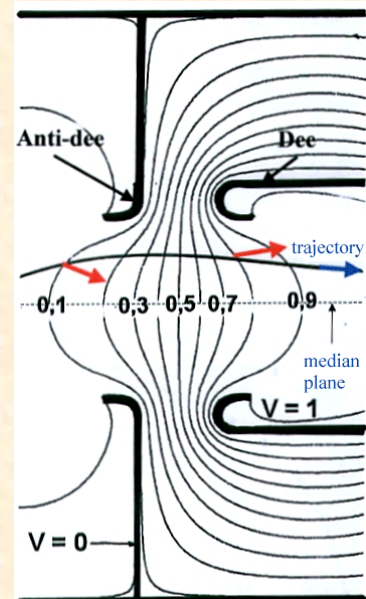
## Accelerating gap & Transit Time

The formula  $\delta E = QV_0 \cos \varphi$  corresponds to small accelerating gaps  
 Because of the gap geometry, the efficiency of the acceleration through  
 the gap ( $g$ ) is **modulated** by the **transit time factor**  $\tau$ :

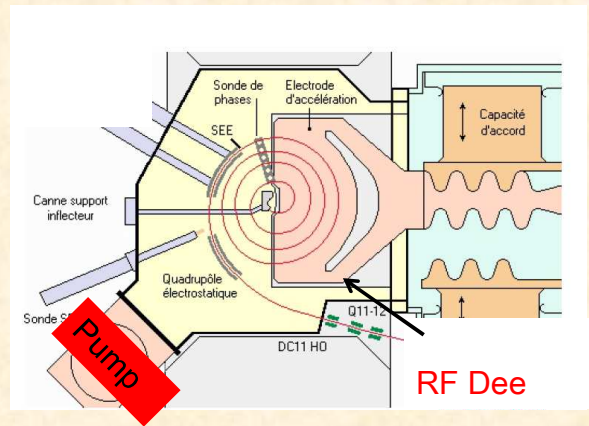
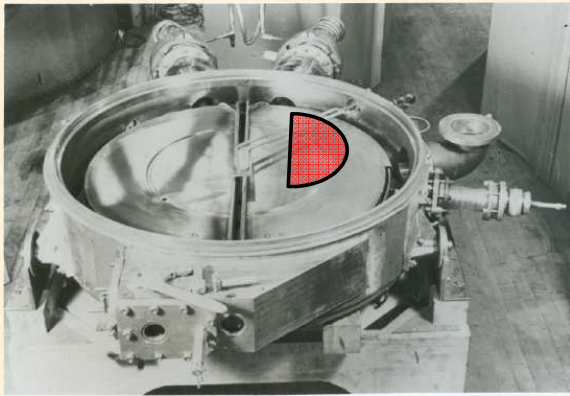
$$\delta E = QV_0 \tau \cos \varphi$$

$$\tau = \frac{\sin \left\{ \frac{hg}{2r} \right\}}{\frac{hg}{2r}} < 1$$

Finite size of gap **decreases** the efficiency of  
 accelerating cavity



## RF Cavities : with the 180° Dees



With the specific “180° Dees “ :

$h=1,3,5$  only odd number allowed

$h=2,4$  even number **forbidden**

Dee should change its voltage every half turn for a bunch

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