

Cyclotrons

Chapter 3 Cyclotron Design

- Isochronism
- Maximal energy (B_{\max} , R , stability)
- **Simulation // tracking**
(numerical integration, realistic cyclo simulation)

Design Strategy for $K=10$ MeV cyclo

Design Strategy for $K=250$ MeV cyclo

Design Strategy for a research facility (E/A vs I)

Magnet design

How to adjust $B(R)$: $\langle B \rangle \sim R^{-n}$

- Pole Gap evolution $\langle B(R) \rangle$: $n(R) = 1 - \gamma^2$
- Correction coils (trim coils)

$$FLUTTER \approx \frac{(B_{hill} - B_{val})^2}{8 \langle B \rangle^2}$$

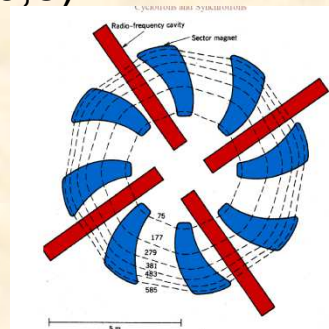
Sufficient FLUTTER F for axial stability ($v_z^2 > 0$)

- Valley // hill B field
- sector angle
- spiral angle ε

$$v_z^2 = n + \frac{N^2}{N^2 - 1} F (1 + 2 \tan^2 \varepsilon) > 0$$

Space for injection beam line and RF

- Azimuthally Varying Field or Separated Sectors : $B = B(R, \theta)$
- large Number of sectors N (3,4, 6,8)



Isochronism : Field $B=f(r)$

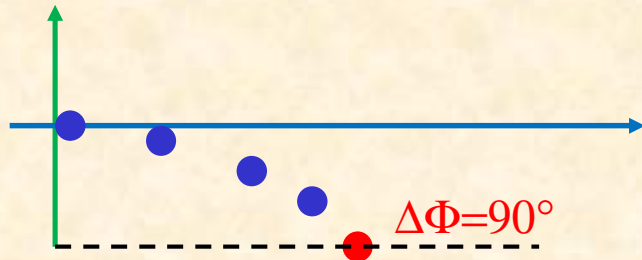
$B_z(R)$ adjusted to get $h \omega_{rev} = \omega_{rf}$

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R) m}$$

$$\gamma(R) = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - (R\omega_{rev})^2/c^2}}$$

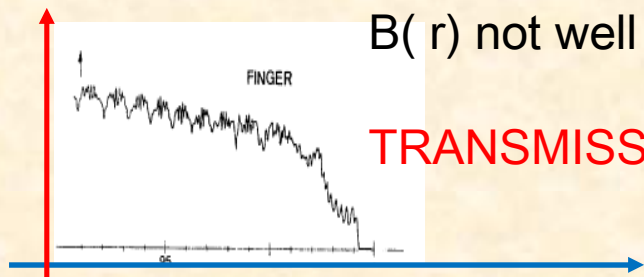
$$\langle B_z(R) \rangle = \langle B_{z0} \rangle / \sqrt{1 - (R\omega_{rev})^2/c^2}$$

$$\Delta\Phi = \Delta T / \omega_{rf}$$



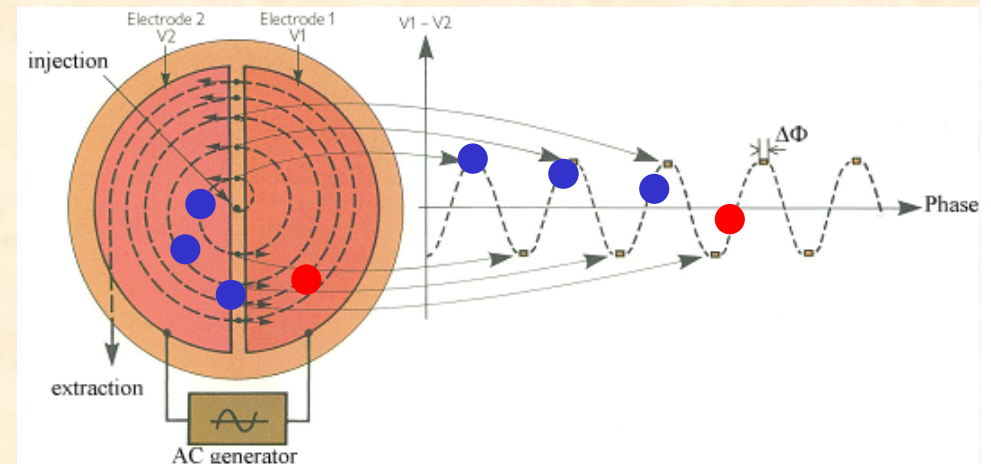
$B(\text{Radius})$ obtained -with correction coils
or -with pole shapes

$$I=f(R)$$



$B(r)$ not well tuned

TRANSMISSION=0%



Max Energy for Cyclotrons

Heavy Ion

A= nucleon number

Q= charge number

Max Kinetic Energy

$$(\gamma-1)mc^2 \approx \frac{1}{2} m v^2$$

$$= \frac{1}{2} m (\text{Rextraction. } \omega_{rev})^2$$

$$\omega_{rev} = \frac{qB}{\gamma m} \approx \frac{qB}{m}$$

For ions: $m = A m_0 = A \cdot [1.6 \cdot 10^{-27} \text{ kg}]$ $q = Q e_0$

ex : $^{12}\text{C}^{4+}$ $A=12$ $Q=4$

$$[E / A]_{\max} (\text{MeV / nucleon}) = K_b \left\{ \frac{Q}{A} \right\}^2$$

with $K_b \approx 48.2 \left(\langle B \rangle \cdot R_{ext} \right)^2$

$\langle B \rangle$ limitation and size limitation (: Rextraction) for E_{max}

Cyclotron Design

1) Particle kind : proton, heavy ion ? Q/A

2) Max Kinetic Energy of reference ions E_{max}

3) Magnet (B_{max} , size, sectors, hill/valley gap) : Compute a field Map

4) Number of cavities (N_{gap}) : energy gain per turn

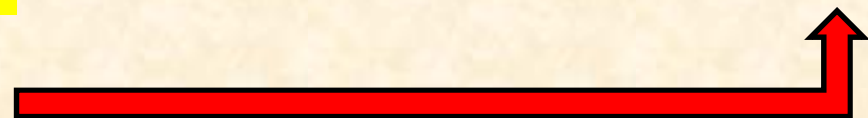
Then, let's start the SIMULATIONS

Multi_particlecode in « realistic » magnetic field

- compute reference orbit
- simulate injection
- simulate extraction

Analytical B Field + RF Kick
Computed field map B + RF Kick
Computed field map B + E

IS IT OK ?, IF NOT restart at 1)

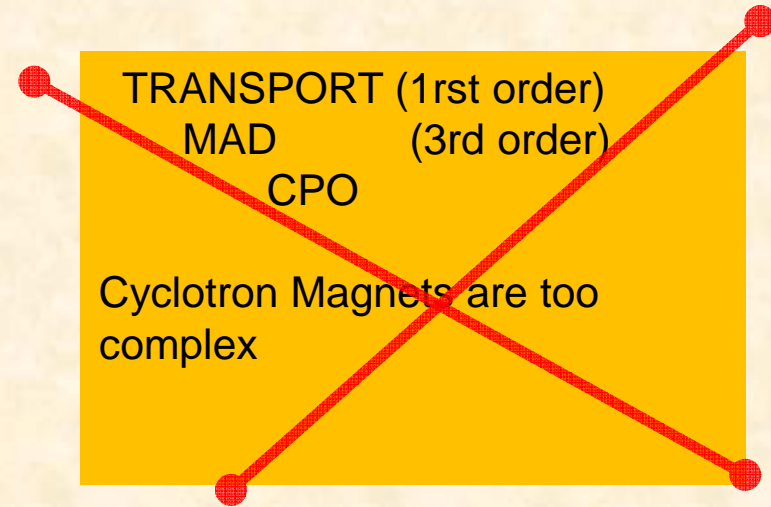


Cyclotron simulation : Particle Tracking with a computer code

SIMULATION : tracking ions (M,Q,v₀)

Multi-particle-code
in « realistic » magnetic field

In cylindrical coordinates



$$\mathbf{r} = r \cdot \mathbf{e}_r + z \cdot \mathbf{e}_z$$

Comoving Frame : $\mathbf{e}_r = f(t)$

Velocity : $\dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt} ?$

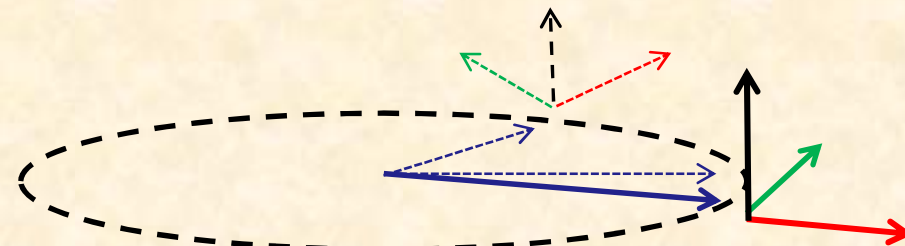
$$d\mathbf{e}_r = \mathbf{e}_\theta \cdot d\theta$$

$$d\mathbf{e}_z = 0$$

$$d\mathbf{e}_\theta = -\mathbf{e}_r \cdot d\theta$$

$$\dot{\mathbf{r}} = \dot{r} \cdot \mathbf{e}_r + \dot{z} \cdot \mathbf{e}_z + r \cdot \dot{\mathbf{e}}_r + z \cdot \dot{\mathbf{e}}_z$$

$$\frac{d}{dt} \left[m\gamma \dot{\mathbf{r}} \right] = q \cdot (\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B})$$



Unit vectors **are evolving** in time !!!

Cyclotrons simulation: cylindrical equation

$$\frac{d\mathbf{p}}{dt} = q \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{v} \times \mathbf{B} = \begin{vmatrix} \mathbf{e}_r & \mathbf{e}_z & \mathbf{e}_\theta \\ \dot{r} & \dot{z} & r\dot{\theta} \\ B_r & B_z & B_\theta \end{vmatrix} =$$

$$= (\dot{z} \cdot B_\theta - r \dot{\theta} \cdot B_z) \cdot \mathbf{e}_r + (r \dot{\theta} \cdot B_x - \dot{r} \cdot B_\theta) \cdot \mathbf{e}_z + (\dot{r} \cdot B_z - \dot{z} \cdot B_r) \cdot \mathbf{e}_\theta$$

Evolution in time t is not convenient, evolution in θ is better !!!

$$\frac{d}{dt} = \frac{d\theta}{dt} \frac{d}{d\theta} = \dot{\theta} \frac{d}{d\theta}$$

$$\frac{d\mathbf{p}}{dt} = \dot{\theta} \frac{d\mathbf{p}}{d\theta} = q \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\frac{d}{d\theta} [m\gamma \dot{r}] = \frac{d}{d\theta} [p_r] = m\gamma r \dot{\theta} + \frac{q}{\dot{\theta}} (\dot{z} \cdot B_\theta - r \dot{\theta} \cdot B_z)$$

$$\frac{d}{d\theta} [m\gamma \dot{z}] = \frac{d}{d\theta} [p_z] = \frac{q}{\dot{\theta}} (r \dot{\theta} \cdot B_x - \dot{r} \cdot B_\theta)$$

$$\frac{d}{d\theta} [m\gamma r \dot{\theta}] = \frac{d}{d\theta} [p_\theta] = \frac{q}{\dot{\theta}} \dots$$

$$\frac{dr}{rd\theta} = \frac{\dot{r}}{r\dot{\theta}} = \frac{p_r}{p_\theta}$$

$$\frac{dz}{rd\theta} = \frac{\dot{z}}{r\dot{\theta}} = \frac{p_z}{p_\theta}$$

Cyclotrons simulation: trajectory in $(r,z)=f(\theta)$

The integration of particle's equation can be obtained with numerical methods

The equation to be solved is a set of Ordinary Differential Equations (ODE).

START at $\theta=0$ ($r_0, Z_0, P_r, P_z, P_\theta$) : what are (r, Z) at $\theta=0+\Delta\theta$?

At first order , we can compute (r,z) and (p_r, p_z)

$$p_r(\theta_0 + d\theta) = p_r(\theta_0) + \frac{dp_r(\theta_0)}{d\theta} d\theta + O(d\theta^2) + \dots$$

$$\frac{d}{d\theta} [p_r] = m\gamma r \dot{\theta} + \frac{q}{\dot{\theta}} (\dot{z} \cdot B_\theta - r \dot{\theta} \cdot B_z)$$

$$r(\theta = \theta_0 + d\theta) = r_0 + \frac{dr}{d\theta} d\theta + \dots$$

$$\frac{dr}{r d\theta} = \frac{dr/dt}{r d\theta/dt} = \frac{v_r}{v_\theta} = \frac{p_r}{p_\theta}$$

$$\frac{dr}{d\theta} = r \cdot \frac{p_r}{p_\theta}$$

$$t = t_0 + \frac{dt}{d\theta} d\theta + \dots$$

$$\frac{dt}{d\theta} = \frac{1}{d\theta/dt} = \frac{m\gamma r}{P_\theta}$$

This the EULER method ! = 1st order expansion

Cyclotrons simulation : the algorithm

Loop $j=1, N_{\text{particles}}$

INITIAL position and momentum : $\theta = 0$ r, z, p_r, p_z, p_θ

Loop $i=1, N_{\text{step}}$ // step in $\Delta\theta$

$$B_r = BR(r, z, \theta) \quad B_z = BZ(r, z, \theta) \quad B_\theta = B\theta(r, z, \theta)$$

$$\theta = \theta_0 + \Delta\theta$$

$$r(\theta = \theta + d\theta) = r + \frac{dr}{d\theta} \Delta\theta$$

$$z(\theta = \theta + d\theta) = z + \frac{dz}{d\theta} \Delta\theta$$

$$p_r(\theta) = p_{r0} + \frac{dp_r}{d\theta} \Delta\theta$$

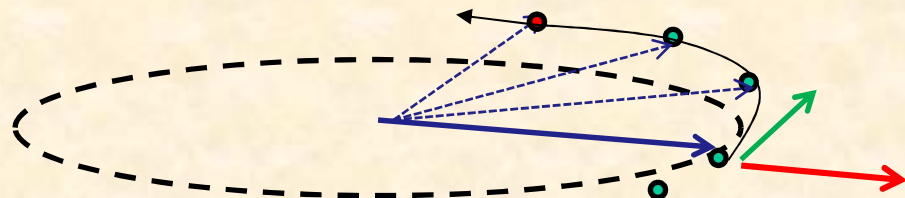
$$p_z(\theta) = \dots\dots$$

FIELD MAP

$$\frac{d}{d\theta} [p_r] = m\gamma r \dot{\theta} + \frac{q}{\dot{\theta}} (z \cdot \dot{B}_\theta - r \cdot \dot{B}_z)$$

Endloop I // end θ loop

Endloop J // Nparticle loop



Euler algorithm (second order accurate in $d\theta$)

Numerical integration of the equations of motion

Euler algorithm (**second order** accurate in $\Delta\theta$) is not the best !!

RK4 (runge kutta order 4) is better (**4th order** accurate in $\Delta\theta$)
[See a Numerical analysis Lecture](#)

SPECIAL ATTENTION to FIELD INTERPOLATION
between the points of the field map

How to simulate a cyclotron in 4 steps

- 1) Define the basic parameters of the cyclotron (B,R,F)

START The simulation in the “middle” of cyclotron
With a defined magnetic structure

- 2) Find the closed orbit (1 particle) without acceleration at $R=R_{ref}$
- 3) Find a matched beam in the cyclotron (multiparticles) backward tracking toward injection
- 4) Forward tracking (multiparticles) toward extraction
- Extraction (multiparticles) : (deflector, precession, resonance)

Iterative process

basic parameters $R, \langle B \rangle, \text{Sectors}, \text{Flutter}$ (1/4)

Ex: cyclotron design 20 MeV/A for carbon ion 4+

What is the Max energy (MeV/A) : 20 MeV/A for carbon 4+

What are the ions $(Q/A) = 4+/12 \Rightarrow B_{p\text{extrac}} = 2 \text{ T.m}$

ION (M,Q)
FINAL ENERGY

$$B_{p\text{extrac}} = 2 \text{ T.m} \quad \gamma = 1.02$$

Reasonable Fied
 $\langle B \rangle = 1.5 \text{ T}$

$$R_{\text{extraction}} = B_{p\text{extrac}} / \langle B \rangle = 1.4 \text{ m}$$

ION (M,Q)
Source Voltage
(30kV - 100kVolts ?)

$$B_{p\text{injection}} = 0.04 \text{ T.m}$$

$$R_{\text{injection}} = B_{p\text{injection}} / \langle B \rangle = 0.04 \text{ m}$$

Vertical stability

$B(r, \theta)$ Nsector , Hill//Valley gap

$$v_z^2 = n + \frac{N^2}{N^2 - 1} F_l (1 + 2 \tan^2 \epsilon) > 0$$

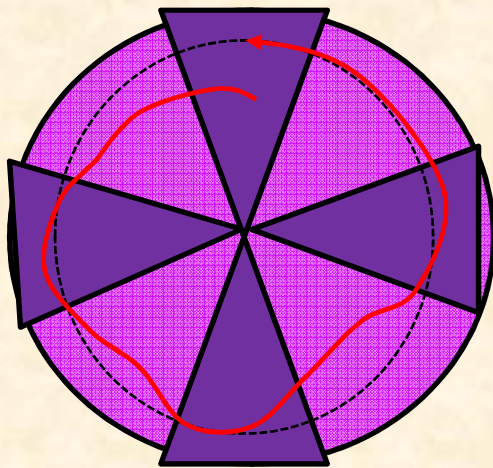
let's take 4sectors & gap hill = 12 cm // gap valley = 30 cm // ... $\epsilon = 0$

Find the closed orbit at $R=R_{ref}$ (2/4)

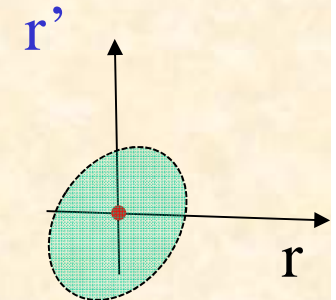
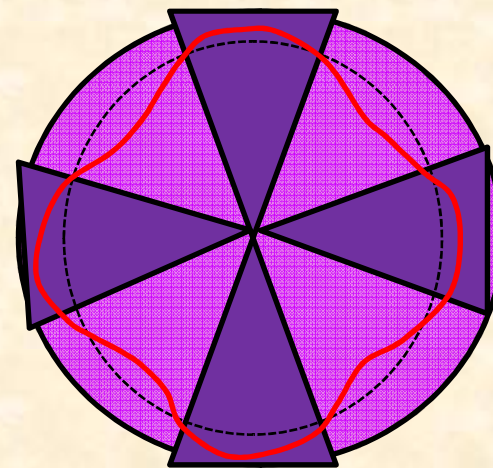
Dynamic of 1 particle in the middle of the cyclotron

- Choose reference particle (1 particle) ($M, Q, B\rho_0$)
- Choose a field Level $B(r, \theta) = k$. FIELD MAP

Find the reference radius R_{ref} ?

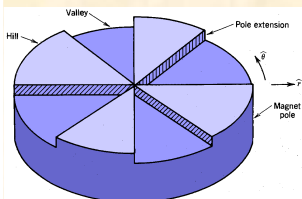


Trajectory is not a closed orbit
Not a good starting point



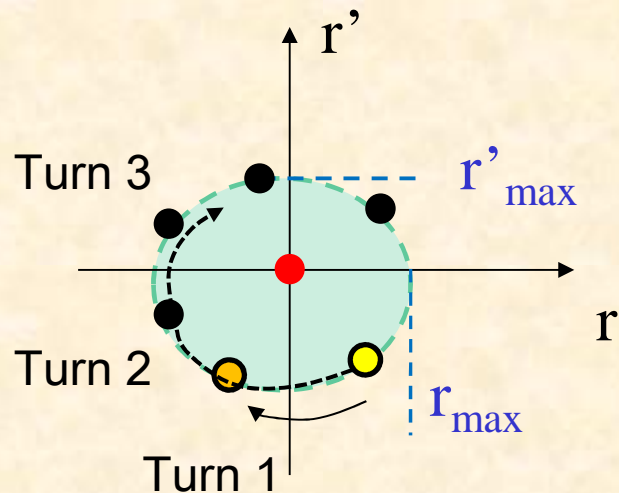
Trajectory is a closed orbit :OK

$$\langle R_{ref} \rangle = \langle B(r, \theta) \rangle / B\rho$$



Find a matched beam in the cyclotron (3/4)

Around the reference trajectory, send a particle for many turns

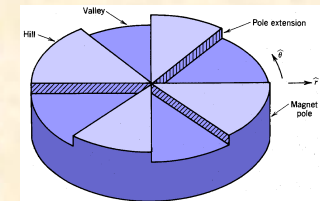
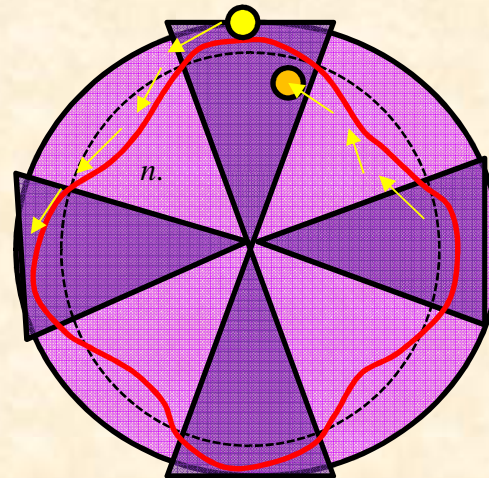


A particle trajectory follows an ellipse

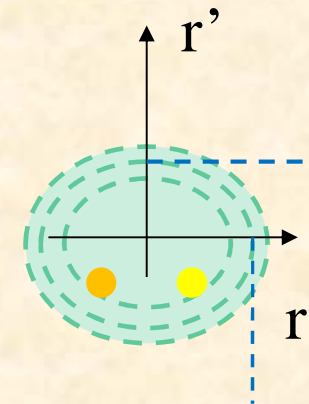
$$r(t) = r_0 + r_{\max} \cos(v_r \omega_0 t)$$

$$r'(t) = r'_{\max} \sin(v_r \omega_0 t)$$

Hill-Valley is a periodic lattice



Beam matching =
Choose a beam ellipse with
 $\Delta r' / \Delta r = r'_{\max} / r_{\max}$



This ellipse occupy the minimal size in the cyclotron

Mismatched beam recall

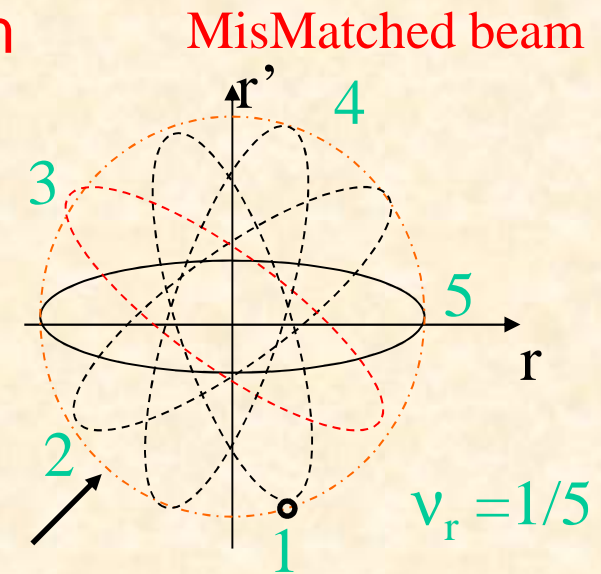
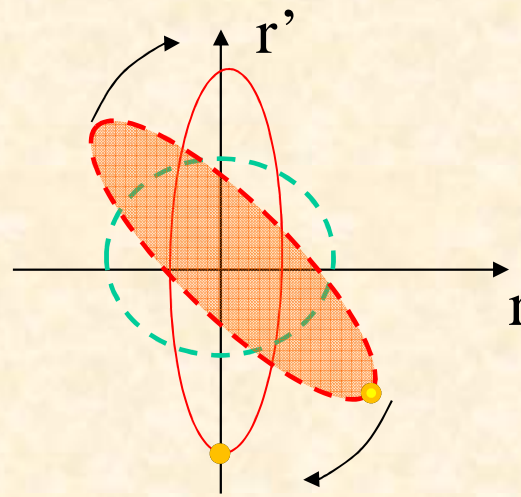
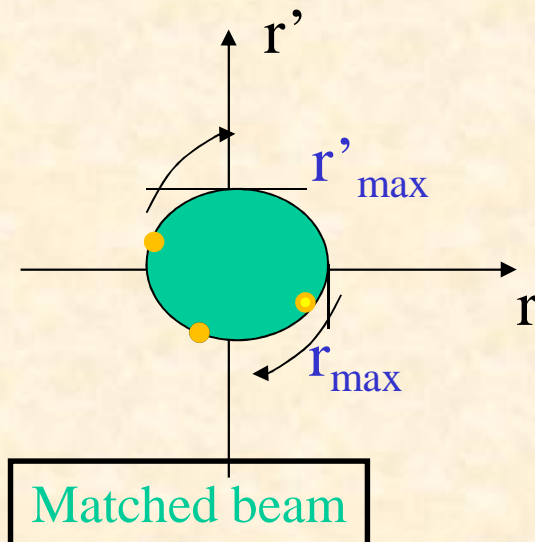
Because of each individual trajectory

$$\begin{cases} r(t) = r_0 + r_{\max} \cos(v_r \omega_0 t) \\ r'(t) = r'_{\max} \sin(v_r \omega_0 t) \end{cases}$$

it exist an optimal ellipse

for a given beam Emittance : $\varepsilon = \pi \Delta r_{\max} \cdot \Delta r'_{\max}$

Betatron oscillation with mismatched beam



Larger acceptance required for the cycle : not good

Matched beam recall

$$\begin{cases} \mathbf{r}(t) = \mathbf{r}_0 + \mathbf{r}_{\max} \cos(\nu_r \omega_0 t) \\ \mathbf{r}'(t) = d\mathbf{r}/ds = d\mathbf{r} / R \omega_0 dt = -(\mathbf{r}_{\max} \nu_r / R) \sin(\nu_r \omega_0 t) \end{cases}$$

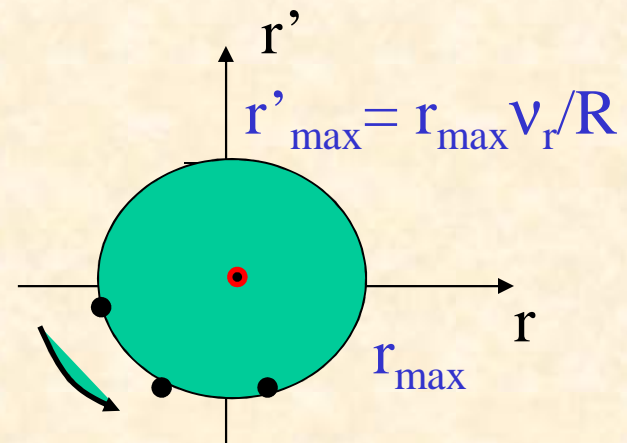
the matched ellipse $|\mathbf{r}'_{\max}| = |\mathbf{r}_{\max} \nu_r / R|$

⇒ Initial beam conditions depend of the tune (ν_r) of the cyclotron at the matching point.

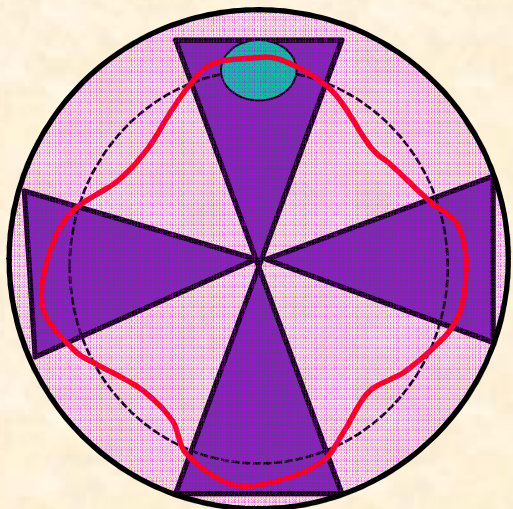
⇒ Betatron oscillation disappears

⇒ Matched beam

⇒ Minimum of acceptance

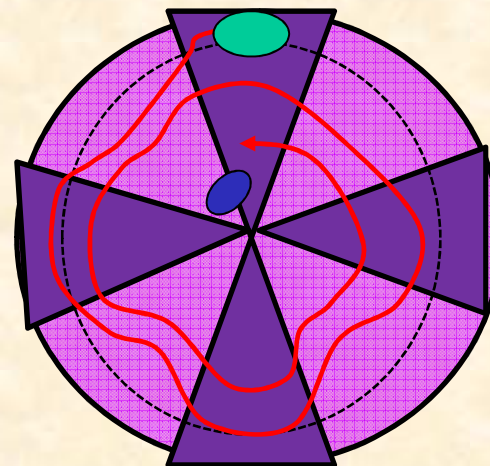


backward tracking toward injection (3/4)

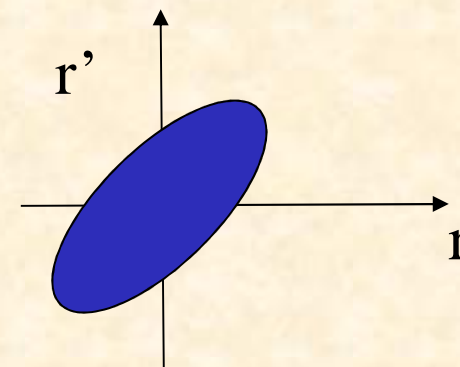
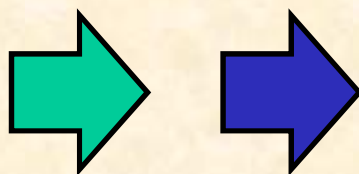
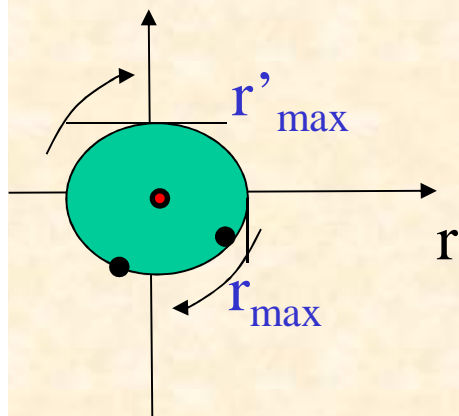


Central Trajectory is a closed orbit :OK

Beam matched : OK



turn on RF : backward toward injection
Adjust V_{rf} , central field.....

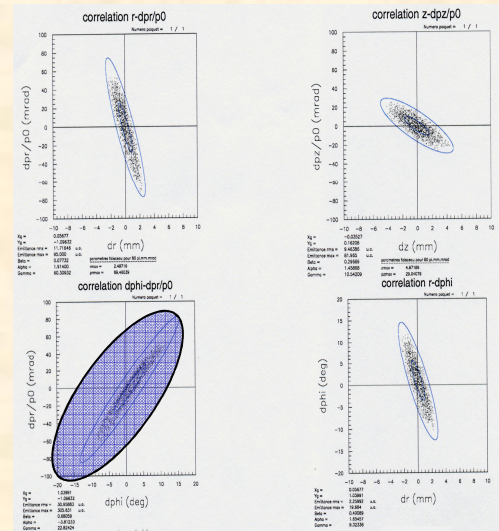
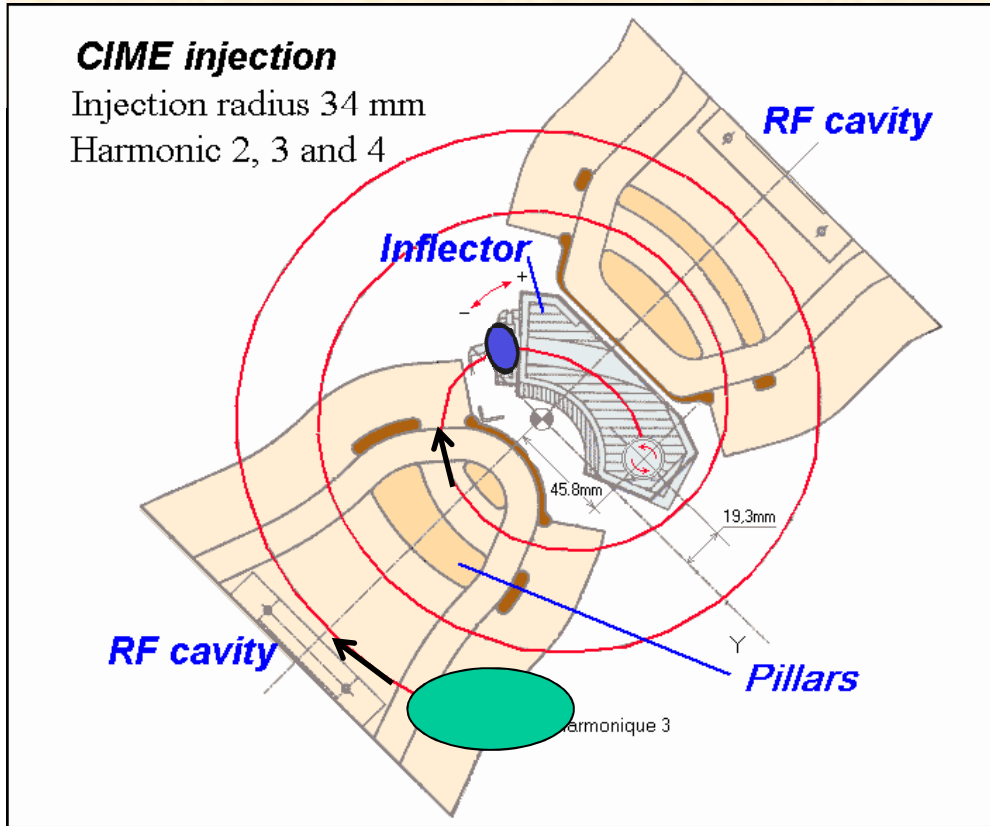
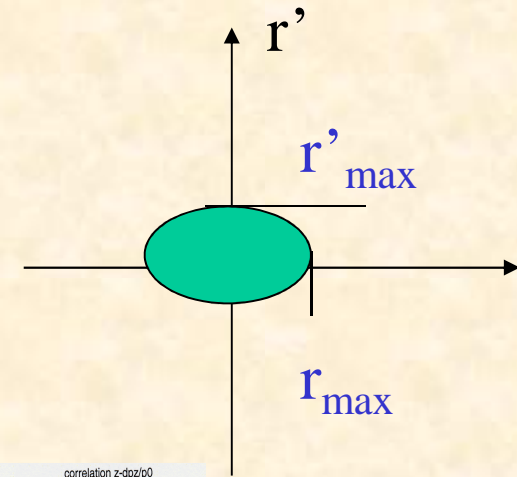


Corresponding optimal beam at injection radius

Drive the beam backward

Start with matched beam in the cyclotron (multiparticles) at large radius
 Then Adjust V_{rf} , central field to reach injection Radius

Find the optimal beam at injection radius

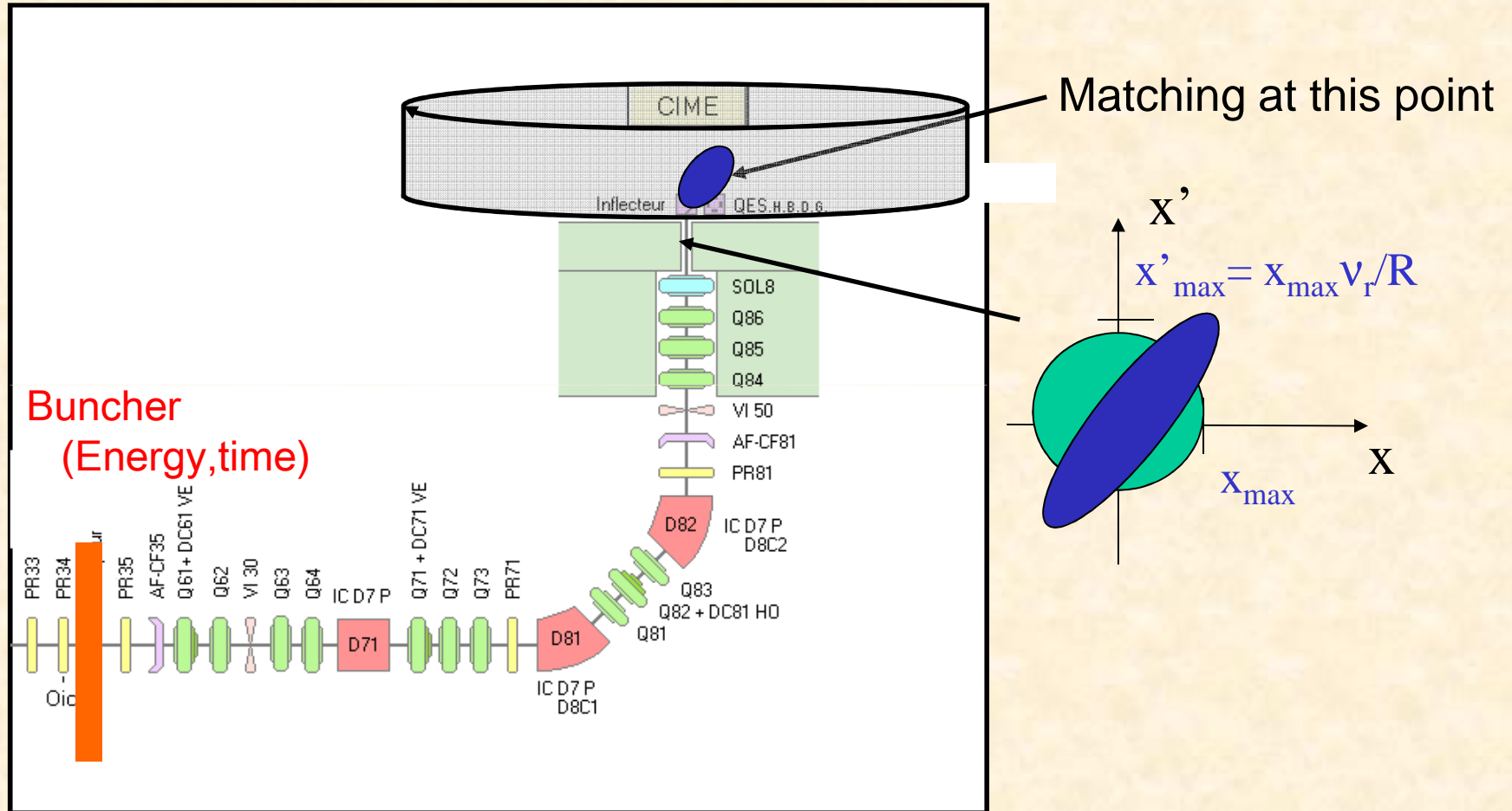


Beam
 Obtained
 With
 backward
 tracking

(r, r', z, z', E, ϕ)

Simulate the injection beam line

to get the perfect beam at injection



Classical transport line problems :

Adjust quad to get desired beam at injection (r,r') (z,z') (t,E)

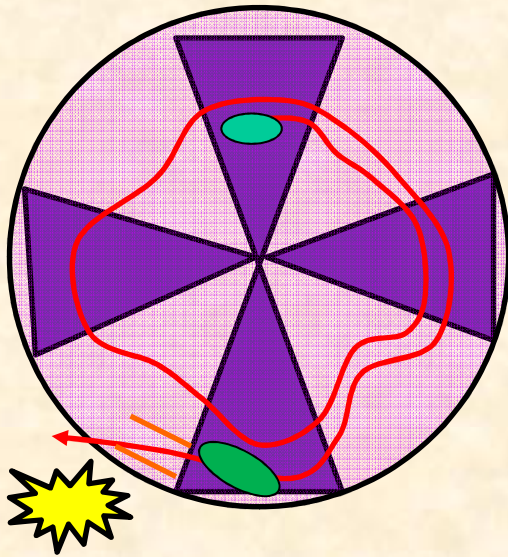
Forward tracking up to extraction (4/4)

turn on RF : Forward toward extraction

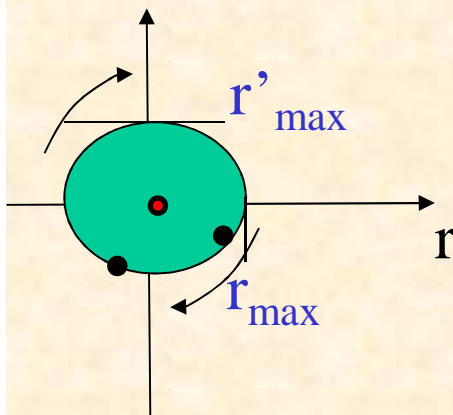
tune the isochronism $\langle B(r) \rangle = \langle B \rangle \gamma(r)$

Extraction

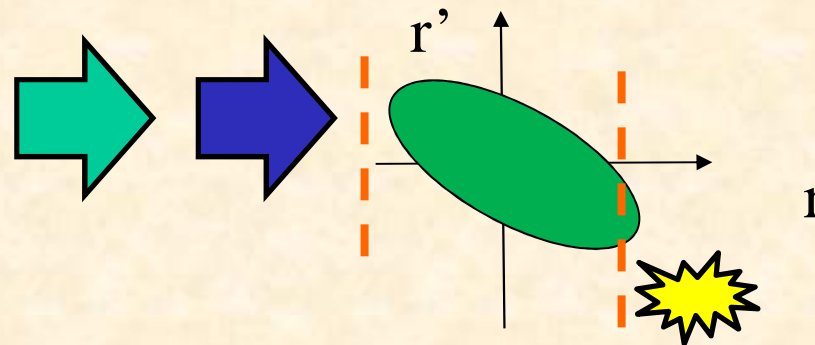
- design the extraction (deflector+..)
- turn separation (RF +precession? + magnetic bump?)
- beam losses ?



Beam matched



beam at extraction radius : watch beam losses



THE CYCLOTRON IS COMPUTED, Let's construct it !

Cyclotron Design strategies

Radio-Isotopes production

cost & reliability

Medical applications : Cancer treatment

cost & reliability

Nuclear physics & Research facility

performance , intensity

Strategy for Radio-Isotopes production medical applications

10 MeV Protons / 5 MeV Deutons : @ low cost

$$B_{pmax} = 0.458 \text{ T.m} = \langle B \rangle R_{extract}$$

$$R_{extract} = 0.34 \text{ m}$$

$$\langle B \rangle = 1.35 \text{ Tesla} \quad [\text{hill} = 1.8 \text{ T} // \text{valley} = 0.5 \text{ T}]$$

AVF with 4 straight sectors (sufficient z-focusing)

$$I \sim 0.1 - 0.05 \text{ mA}$$

Rf Dees : 2 (so 4 gaps)

2 possibilities for extraction

Extraction By stripping :
external target (18F, radiotracer)

No Extraction :
internal target (in yoke)



A « low energy » industrial Cyclotron Cyclone 10/5 : H & D

K_b=10 MeV

Fixed energy ;
4 straight sectors 50°
fixed **F_{rf} =42Mhz**

** =1.35 Tesla**

Harmonic h=2(p) ,4 (D)

Internal source

Recontraction=0.33m

B_pmax=0.33x 1.35=0.45 T.m

$$\left[\frac{E}{A} \right]_{\max} (\text{MeV} / \text{nucleon}) = K_b \left\{ \frac{Q}{A} \right\}^2$$

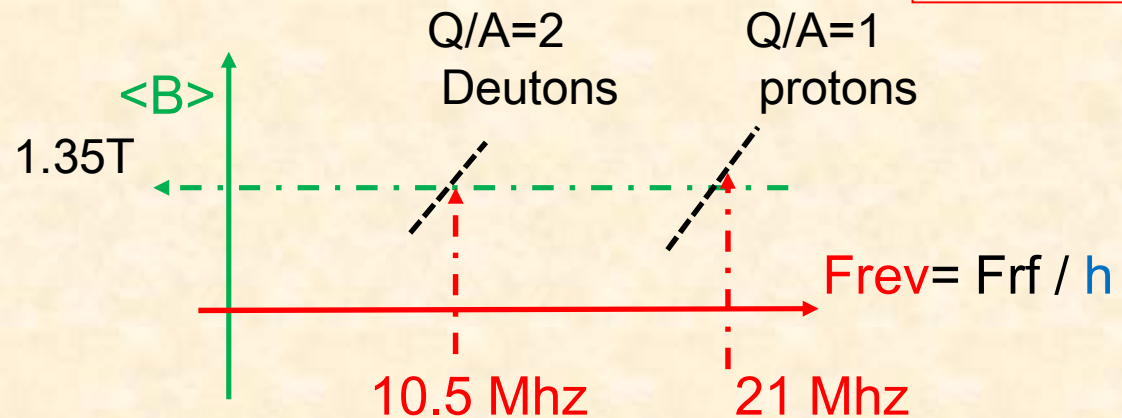
E_{protons}=10 MeV protons= ¹H¹⁺ A=1 Q=1
(E/A=K_b*1² =10MeV/A)

RF Harmonic =2 **F_{rev}=42 Mhz/ h= 21 Mhz**

E_{Deutons}=5 MeV Deutons= ²H¹⁺ A=2 Q=1
(E/A=K_b*0.5² = 2.5 MeV/A)

RF Harmonic =4

$$F_{rev} \propto \frac{Q \cdot B_{cyclo}}{A}$$



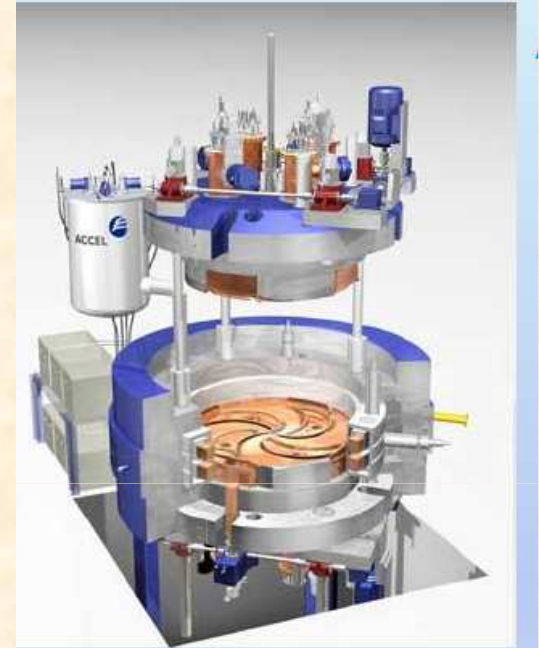
Strategy for cancer treatment proton therapy (>80 facilities in the world)

-250 MeV Protons

Accel VARIAN Isochronous cyclotron

Superconducting $\langle B \rangle = 2.2$ Tesla

Rextrac ~1.2m



-230 MeV Protons

IBA Synchro cyclotron

Superconducting $\langle B \rangle = 5$ Tesla

Rextract ~0.6 m

Very compact

Hill/valley not needed



Strategy for a Cyclotron in a research facility

High energy

$$E/A_{max} = K_b \cdot (Q/A)^2$$

$$\text{High } K_b \sim (R \cdot B)^2$$

Large magnet
(Radius)

High B_z
(superconducting)

Vertical stability
Increase « Flutter »
Separated sectors

High ion charge
 Q

« external »
ECR source

Ion stripping at high energy
« 2 Stages accelerators »

Strategy for a Cyclotron in a research facility

High energy

$$E/A = Kb \cdot (Q/A)^2$$

$$\text{High } Kb \sim (R \cdot B)^2$$

Large magnet
(Radius)

High Bz
(superconducting)

Vertical stability
Increase « Flutter »
Separated sectors

High ion charge
Q

High Intensity

Good ion source

« external »
ECR source

Ion stripping at high energy
« 2 Stages accelerators »

No losses at
extraction

High RF
voltage

Many
gaps

Coupling of 2 Cyclotrons : velocity matching

Two cyclotrons can be used to reach higher energy :

- Harmonic & Radius of the 2 cyclotrons have to be matched

$$\frac{v}{2\pi} = \left[\frac{F_{HF} \cdot R_{ejec}}{h} \right]_{cycloA} = \left[\frac{F_{HF} \cdot R_{inj}}{h} \right]_{cycloB}$$

The **velocity** of extraction CycloA

= velocity of injection CycloB

- Ion stripping can be used, to increase Q before injection into the second cyclo

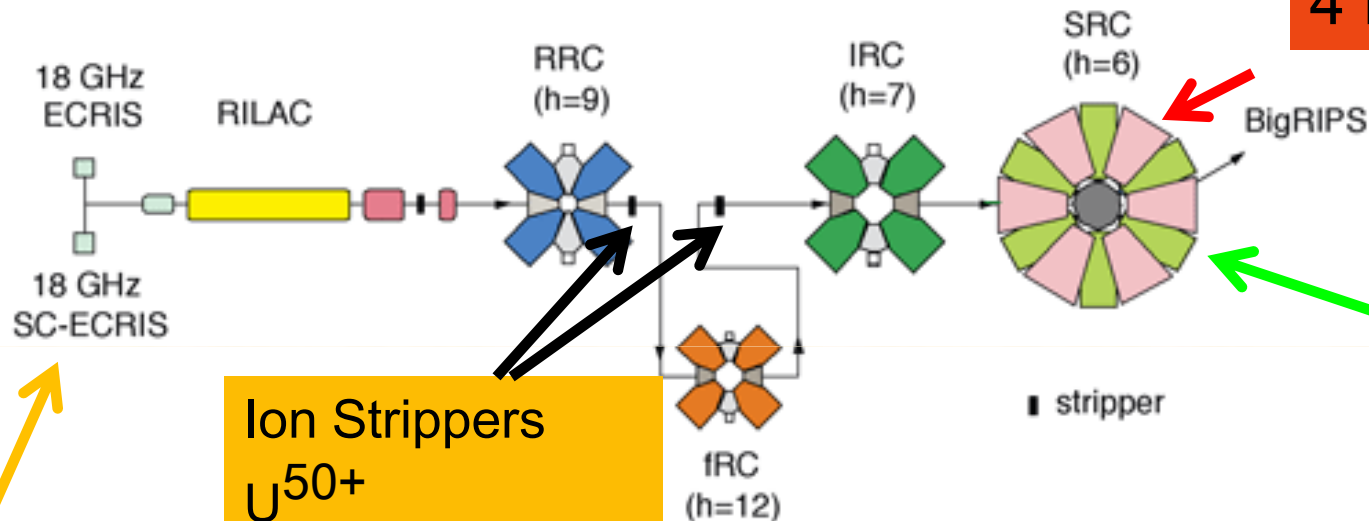
large Q \Rightarrow large E_{max}

$$\left[\frac{E}{A} \right]_{\max} = K_b \left\{ \frac{Q}{A} \right\}^2$$

RIBF (Japan) : SRC (K=2600 MeV) –the biggest cyclo

Uranium beam $^{238}\text{U}^{88+}$ @345 MeV/A cw

Mode (1): RILAC + RRC + (stripper2) + fRC + (stripper3) + IRC + SRC



4 RF cavities

Radius=6m
=3.8T

Ion Strippers
 U^{50+}
 U^{88+}

ECR source
 U^{30+}

5 Stages accelerators
1 LINAC
+ 4 Cyclotrons

Ion Stripping at high energy

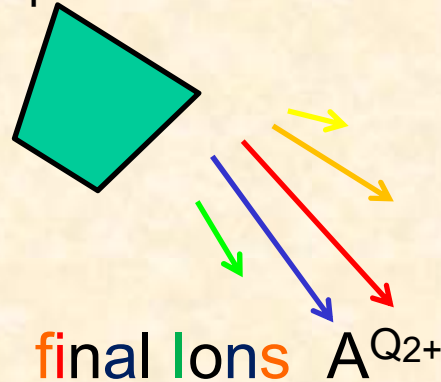
Heavy ions are not fully stripped by ion sources :

Incoming Ions



Stripping some of residual electrons

Magnetic spectrometer

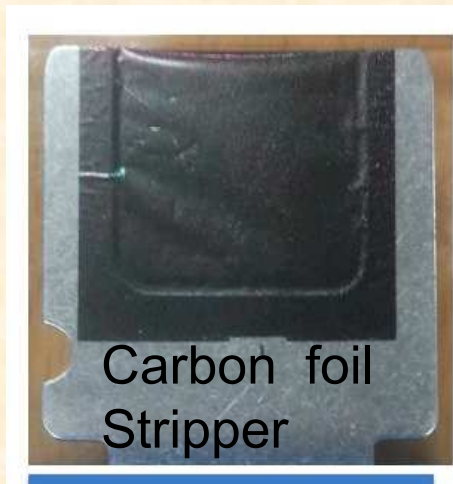
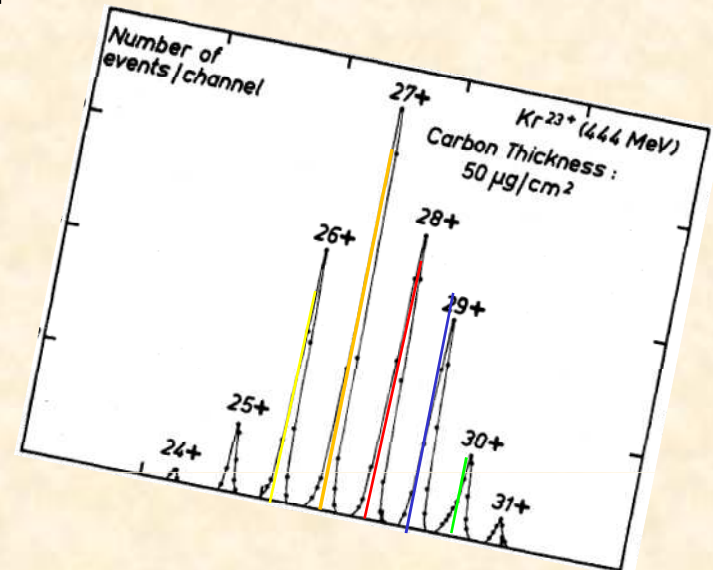


final Ions A^{Q_2+}

$$Q_2 > Q_1$$

$$B\rho_2 < B\rho_1$$

$$B\rho = \frac{P}{q} = \frac{\gamma m v}{q}$$



Carbon foil Stripper

Ion Stripping help to increase the maximal energy of a given cyclotron....

$$[E/A]_{\max} = Kb \left[\frac{Q}{A} \right]^2$$

- End Chapter 3

important facts for cyclotron :

- 1) Simulations are done with realistic magnetic field
(not transport matrices)
- 2) Magnetic structure should provide the vertical stability (field index n compensated by sector)
- 3) The Beam matching at injection for better transverse acceptance

Additive slides

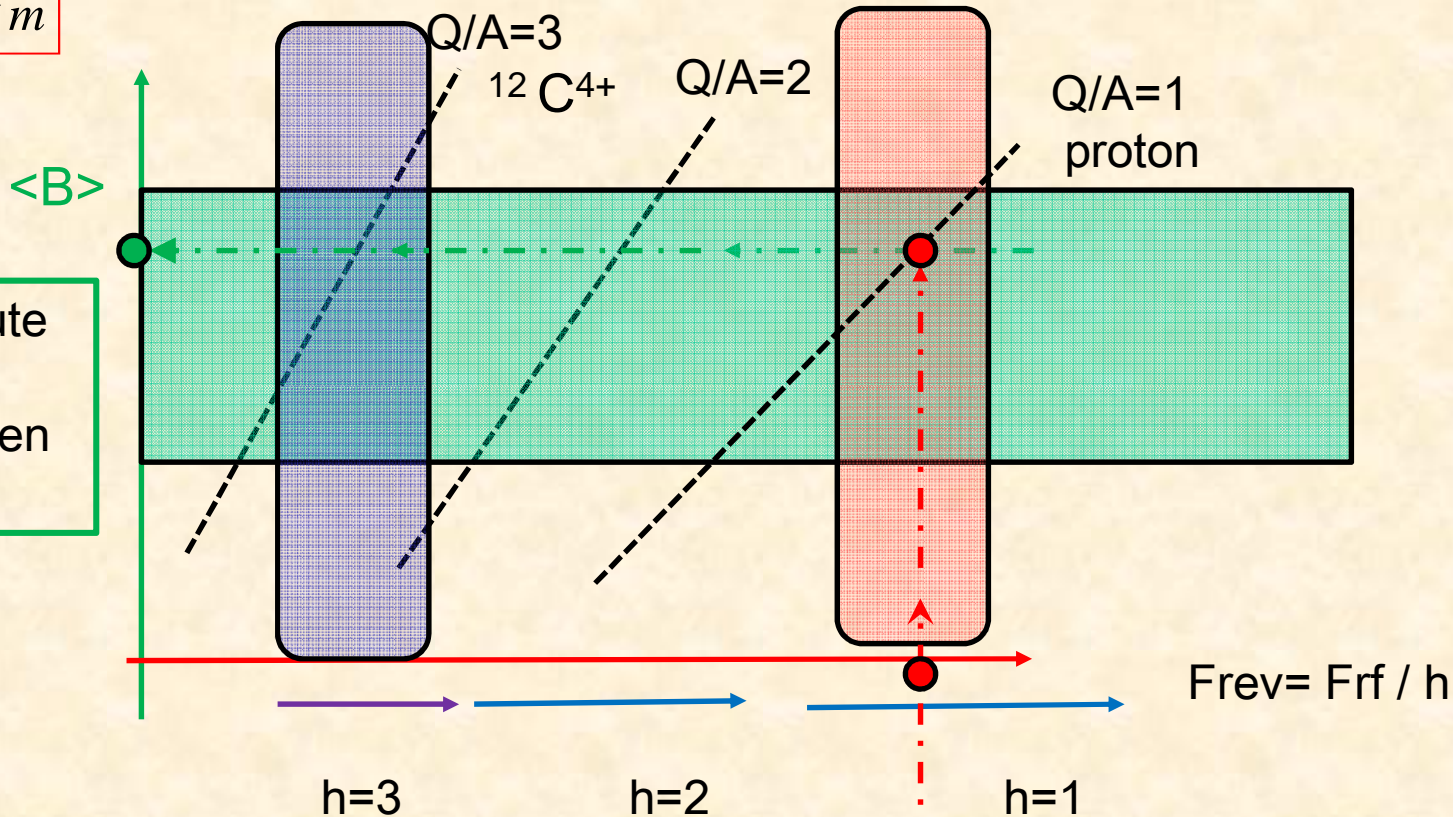
Diagram for The variable energy cyclotrons

$$F_{rev} \propto \frac{Q \cdot B_{cyclo}}{A} \propto h \cdot F_{RF}$$

$$\omega_{rev} = \frac{qB}{\gamma m}$$

A= nucleon number
Q= charge number

2) Compute $\langle B \rangle$
For a given (Q,A)



1) Select the energy (F_{rev})
Select the ions (Q,A)
Adjust $\langle B \rangle$

B_p # $\langle B \rangle$ Rextract

E/A (MeV/A) # $K (Q/A)^2$