Cyclotrons

Chapter 4 : theory versus reality

- Cyclotron versus synchrotron or linac
- Kf limitation (focusing limitation)
- Isochronism and Phase measurement
- Isochronism 2nd approach
- Resonances and tunes in a cyclotron

- Research applications
- Medical applications

Cyclotron vs other RF accelerators

	Radius	Frf	Particles
Cyclotrons	not constant	constant :CW (isochronous)	Ions limit $\gamma < 2$
Synchro-cyclotrons	not constant	not constant pulsed, Frf (t)	Ions
Synchrotrons	constant	not constant pulsed Frf (t)	Ions, electrons no limits for γ, limit €
Linacs	constant ∞	constant	Ions, electrons limit €

LINAC VERSUS *CYCLOTRON*

to get ~20 MeV protons

Proton Source

LEBT

RFQ

MEBT

DTL

Drift Tube Linac

~20 m total

~ 10 m

I cw ~ up to 1-5 mA
LINAC Cost ~ up to 10 Meuros
+ building (~ 300m2)

- Not Compact (beam lines+linac)
- specific design € € € € € € €

Internal source
H- with stripping extraction

Rextraction ~0.4 m
Total size < 2m

I cw ~ 0.05 up to 0.3 mA in H-
CYCLO Cost ~ 1.5- 3 Meuros

Several industrial manufacturers
Compact
Standard design (300 cyclo in the world)
Operation easy

Max Energy for Superconducting Cyclotrons
not limited by (B x Rextraction)

We can demonstrate that isochronism imply $n(R) = (1-\gamma^2) < 0$

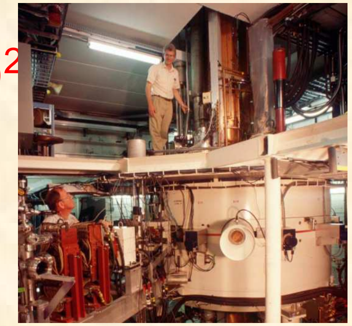
Stability : isochronous field condition *compensated* by Flutter (B(R,θ))

$$\nu_z^2 = \left[1-\gamma^2\right] + \frac{N^2}{N^2-1} F \left(1 + 2\tan^2 \varepsilon\right) > 0$$

At high energy $\left[1-\gamma^2\right] << 0$ compensation not possible

the max energy is not given by $Kb \sim 48\,(B.R_{extraction})^2$

but K_f the so-called "*focusing* factor":



- Focusing limitation (stronger than B limitation)

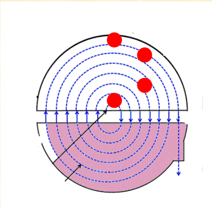
$$\left[\frac{E}{A}\right]_{max} = Kf \cdot \left\{\frac{Q}{A}\right\}^2 < Kb \cdot \left\{\frac{Q}{A}\right\}^2$$

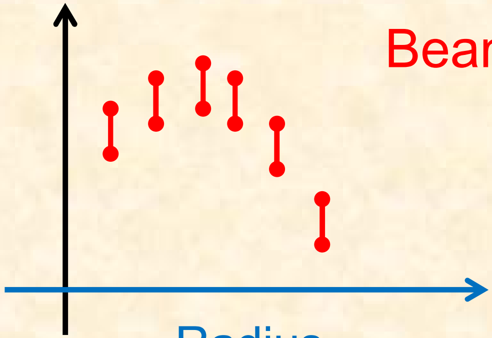
Kb~48 (B.Rextract)^2

Kf~ f(FLUTTER)

Phase measurement: n(R) check and field correction : Δ B

Relative phase
Beam vs RF



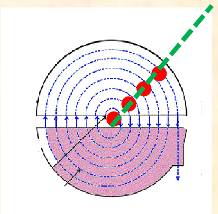


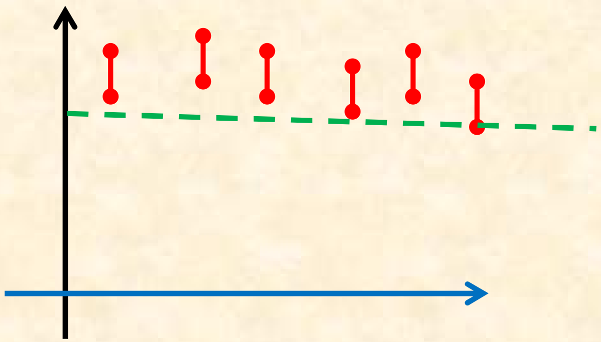
Beam not in phase with RF : B field not OK

$$B_z(R) \neq \quad B_{z0} / \sqrt{1-(R\omega_{rev})^2 / c^2}$$

Bz(R) not correct

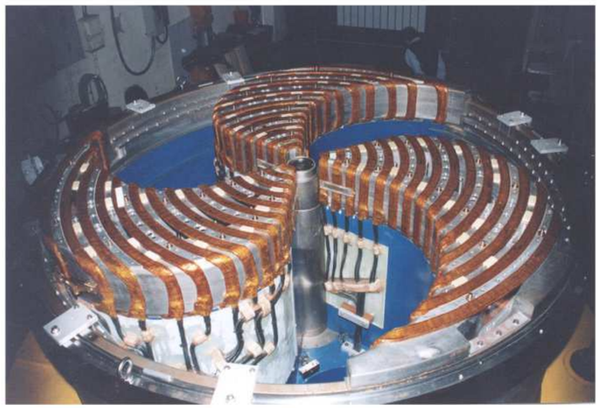
Relative phase
Beam vs RF





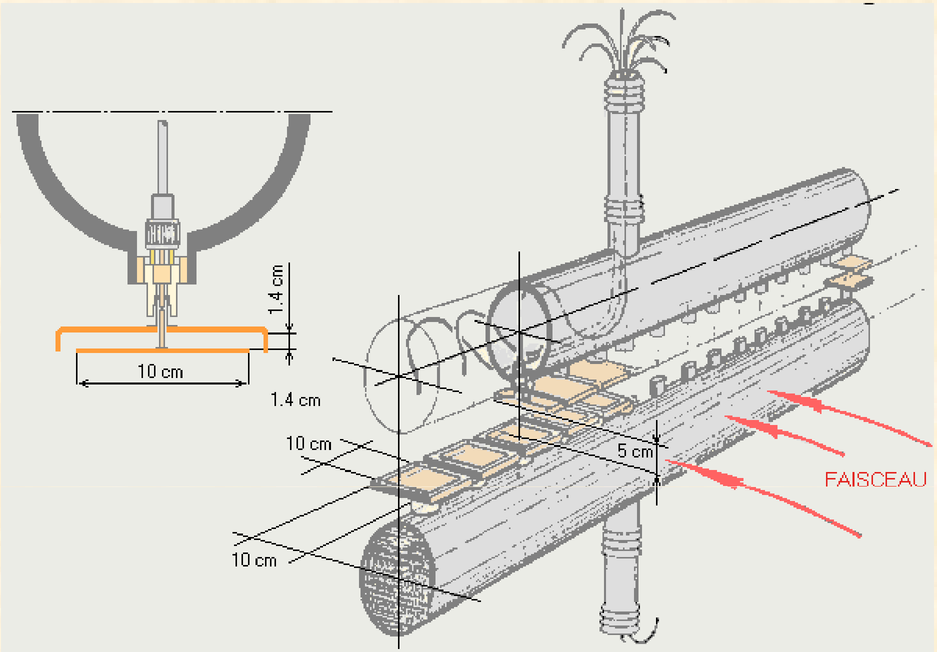
Correction of B : B($I + \Delta I$)
Bz(R) + Δ Bz

Correction trim coils, AGOR

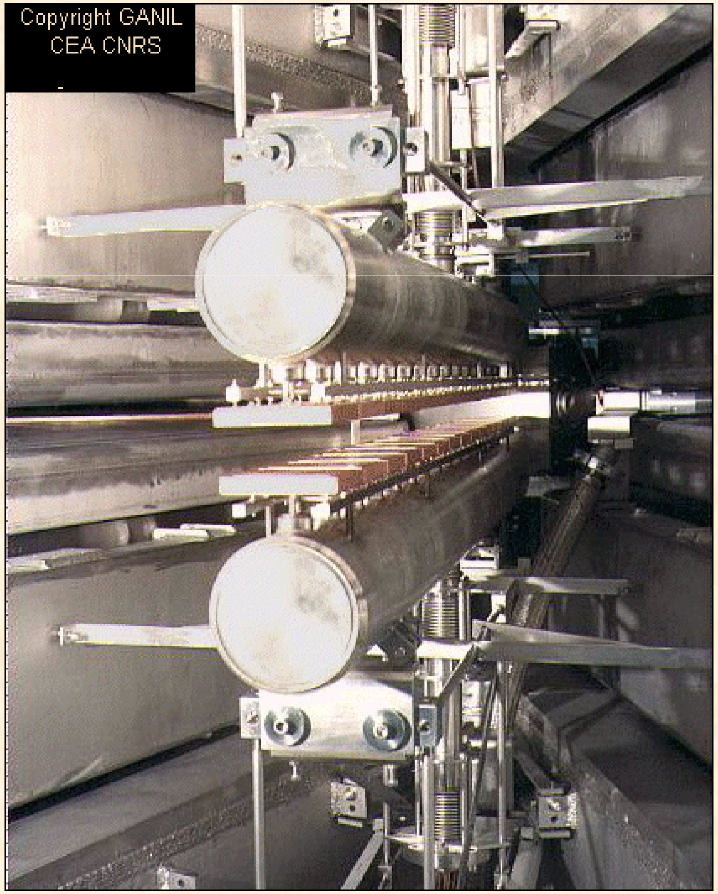


Bz(R) correct : Beam in phase with RF

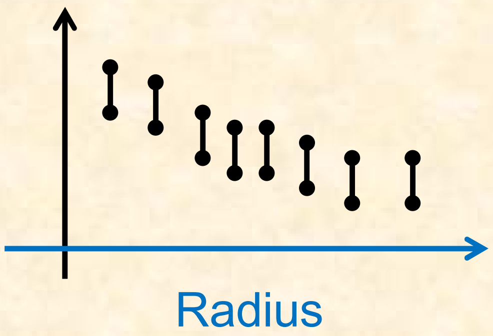
Phase measurement: Isochronism



Ganil, Caen (Fr) : CSS1

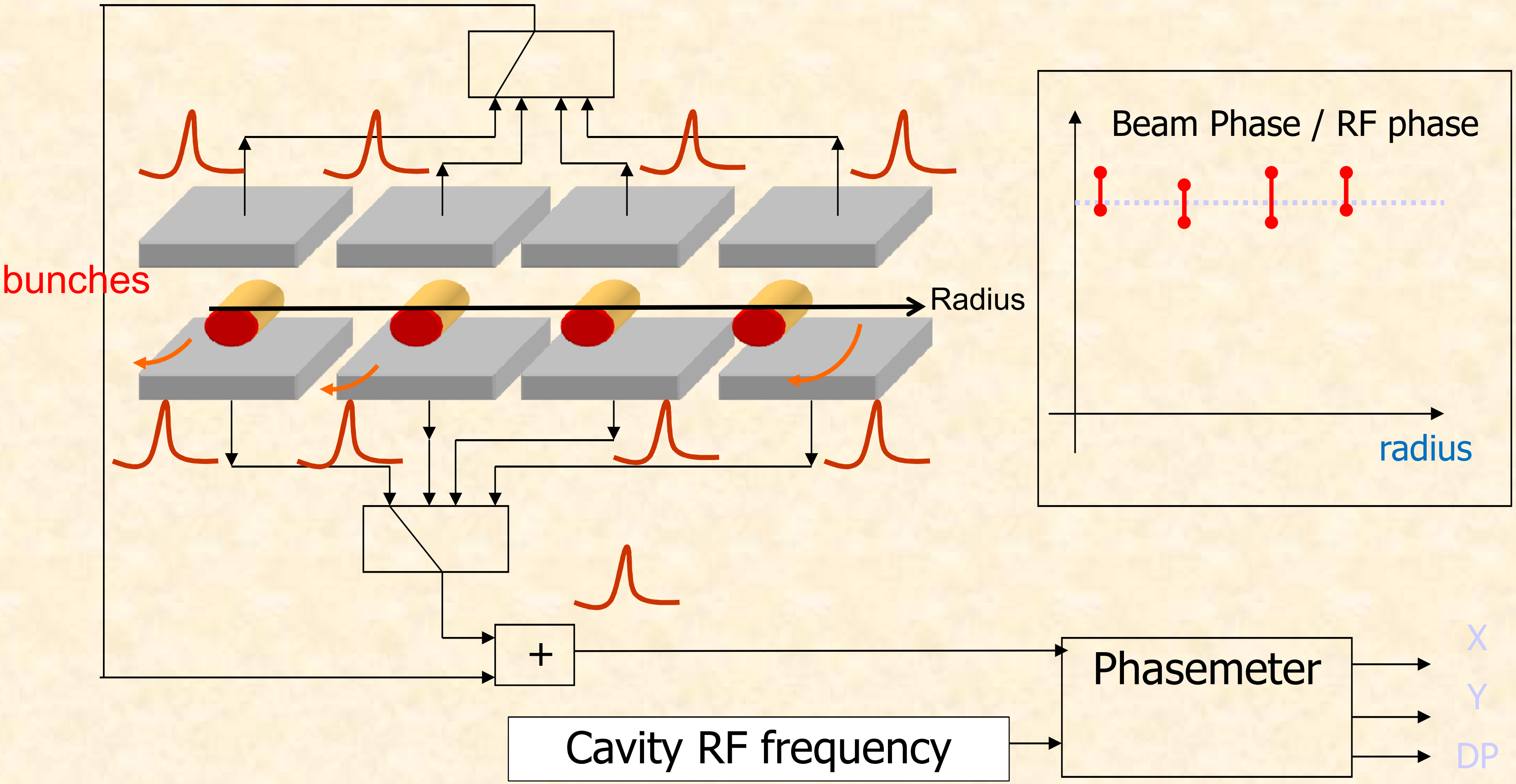


Relative phase
Beam vs RF



Isochronism & Phase measurement

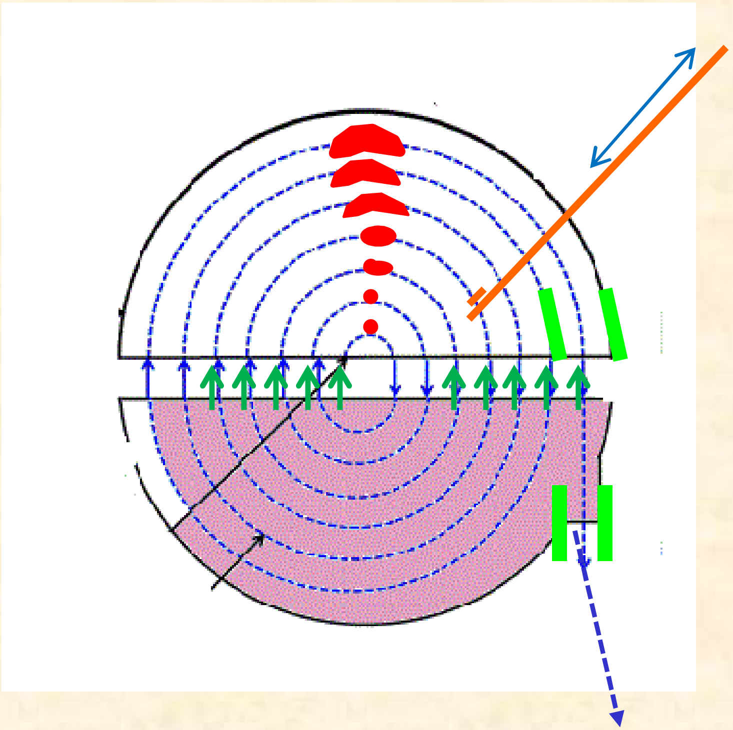
Measuring B(R) or n(R) is difficult , While the Φ (R) is more sensitive

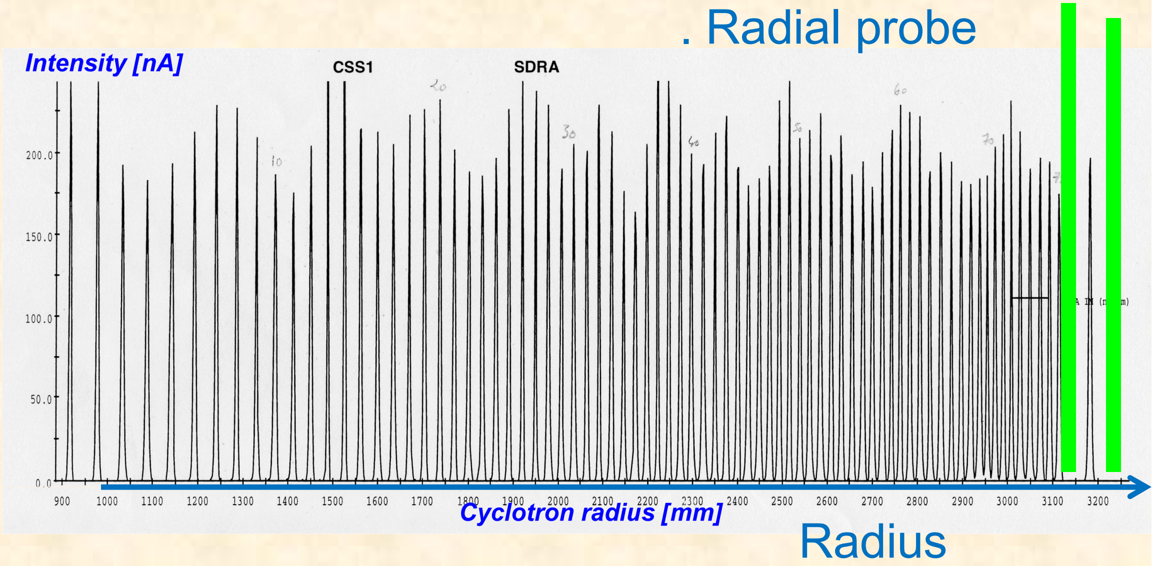


Radial probes

usefull tool for acceleration, precession study

Monitoring turns with a
Radial Probe





Radial probe : I = F(Radius)

Turn separation : δr =R(turn N) - R(turn N-1)

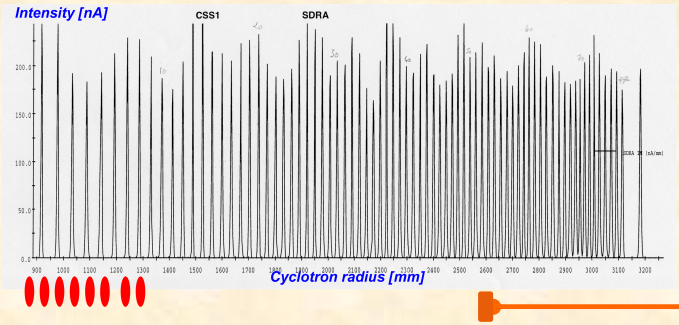
δr = Acceleration + Oscillation

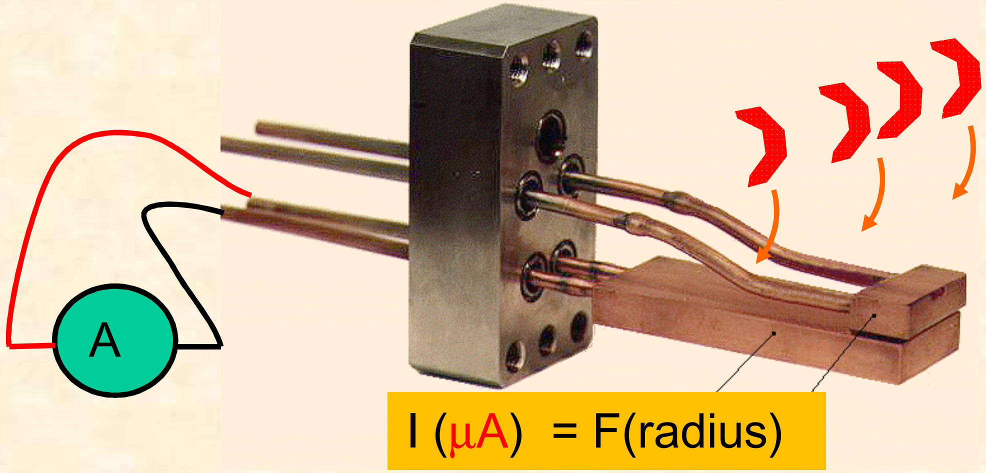
δr ~ αVRF cos (ϕ) + Oscillation

Current measurement I_{beam}=F(R): Radial probe

A Faraday cup on mobile Arm

I (µA)





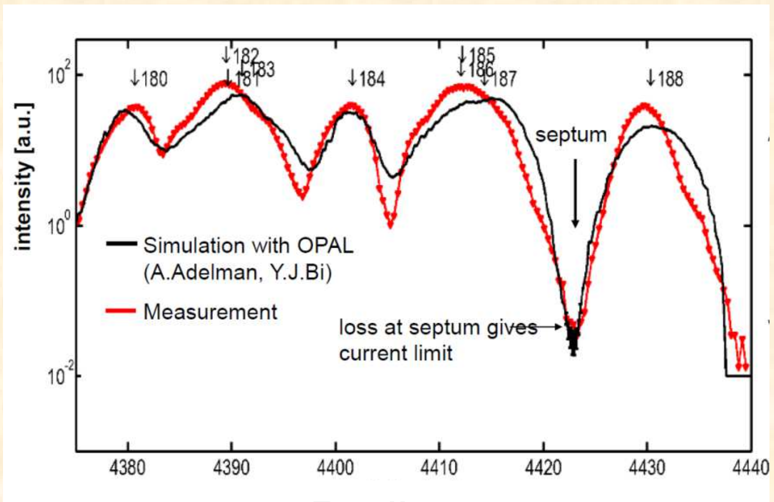
Radius

Ganil Css1 (Fr)
Kb=380 MeV , heavy ions

Check of the acceleration & extraction

PSI
Kb=590 MeV

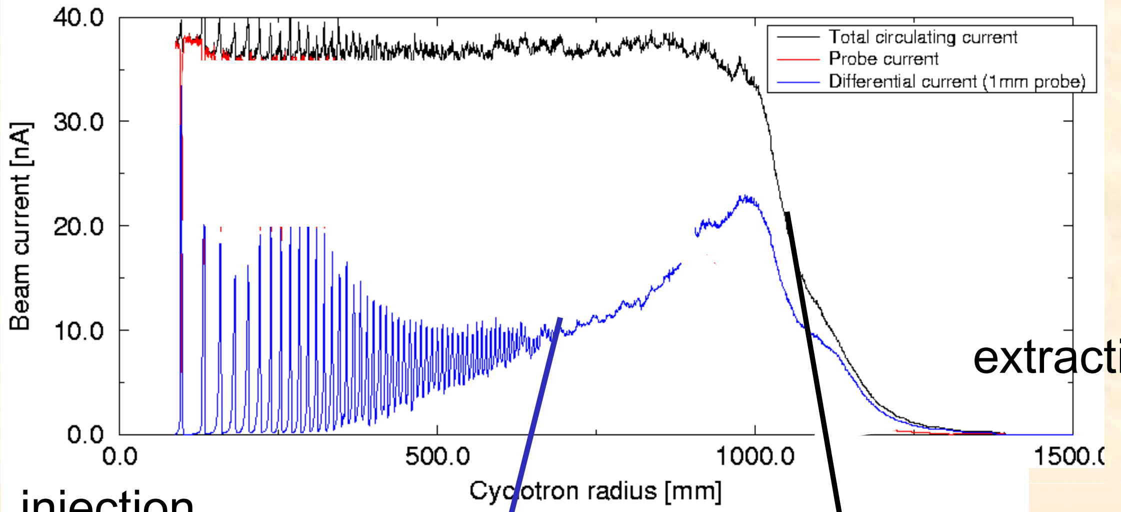
Check of the extraction



Radius

Current measurement with a Radial probe

A full check of the dynamics



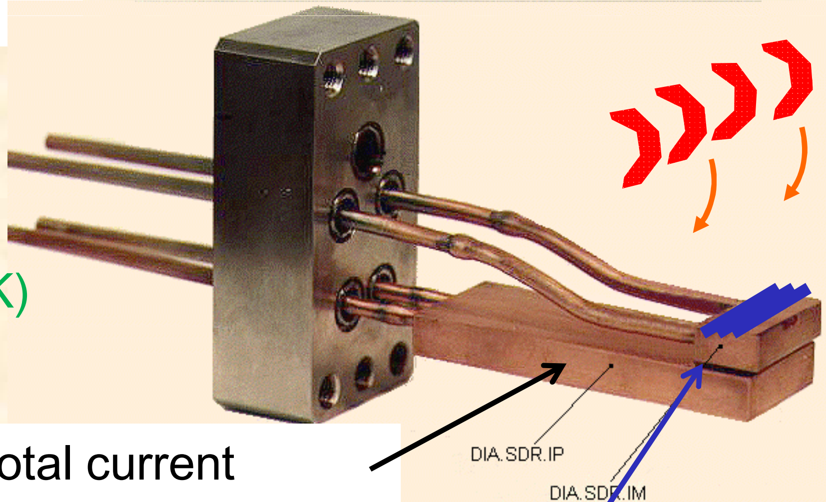
Radial probes with
2 diagnostics :
-1 small finger
-1 large plate

extraction

injection

Beam losses
Before extraction
(B field was not OK)

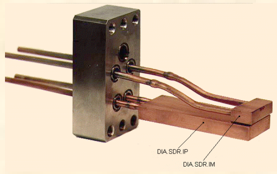
Current on small finger
of the radial probe



Total current
with a large plate

Finger current
Good resolution in r (turn separation)

Tune ν_r measurement with radial probes



Turn separation δr gives ν_r :

$$\nu_r = \sqrt{1-n} + ..$$

$$r(t) = \langle R(t) \rangle + X_0 \cos(\nu_r \omega_0 t)$$

$\omega_0 t$ =PHASE= θ

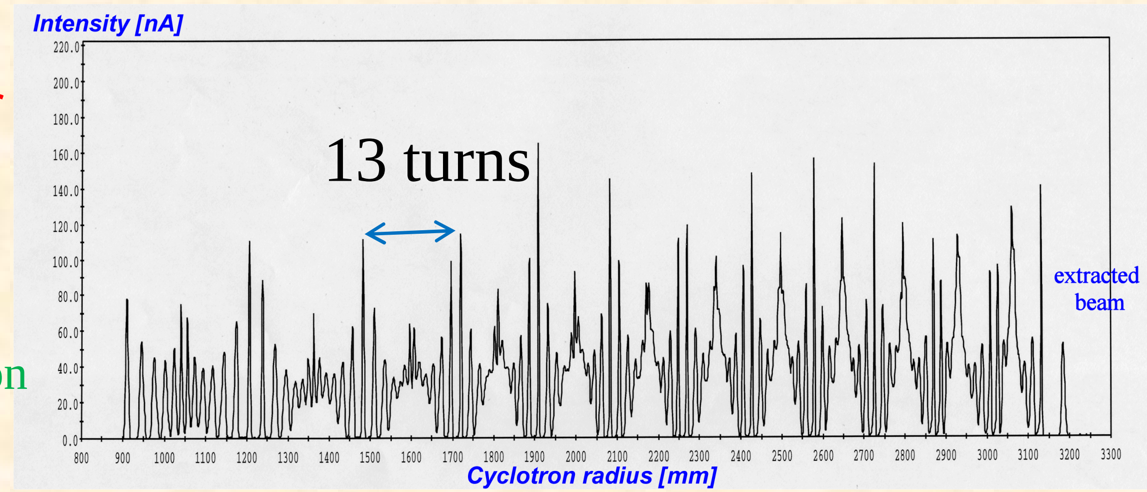
$\omega_0 \Delta t$ =360° =1 turn

Precession : $\delta r = \delta r_{acc} + X_0 \cos(\nu_r \omega_0 t)$

Centering error
At injection

Large X_0

Large Oscillation



1 period for

13 turns; so ⇨

ν_r =1/13 = 0.08

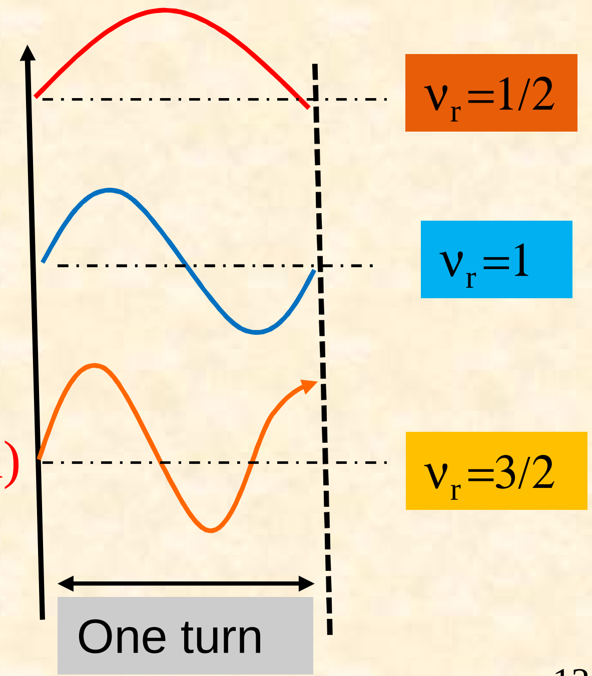
Back to dynamics and resonances at high energy

During the acceleration, ν_r and ν_z change because $\nu_{r,z} \propto B(r)$

The plot of ν_r vs ν_z is called the working point diagram.

Like any oscillatory phenomenon, the amplitude of a betatronic motion can grow uncontrolled whenever an external source excites it with its own frequency.

$$\ddot{x} + \nu_r^2 \omega_0^2 x = 0$$



Several kind of radial resonances

$$x(t) = x_0 \cos(\nu_r \omega_0 t) = x_0 \cos(\nu_r \, 2\pi \, Nturn)$$

can be excited with field defects, injection angle (unwanted)

with field bump, injection angle (desired)

What happens with P field perturbations on 1 turn

$$\ddot{z} + [\nu_z \omega_0]^2 z = \Delta \cos(P \omega_0 t)$$

Search a particular solution

$$z(t) \propto \cos(P \omega_0 t) \text{ ???}$$

$$\frac{z(t)}{\Delta} = \frac{\cos(P \omega_0 t)}{[\nu_z \omega_0]^2 - [P \omega_0]^2}$$

z/Δ diverge at νz=P (integer)

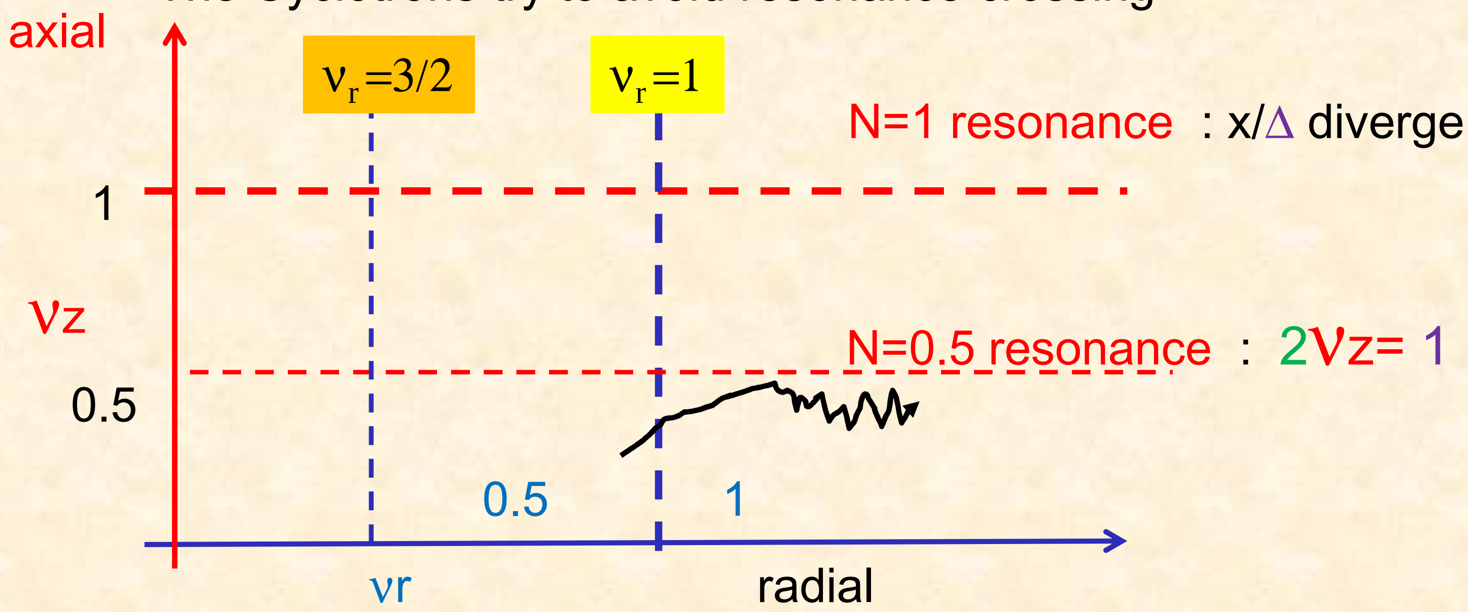
z(t) is very sensitive to any perturbation Δ

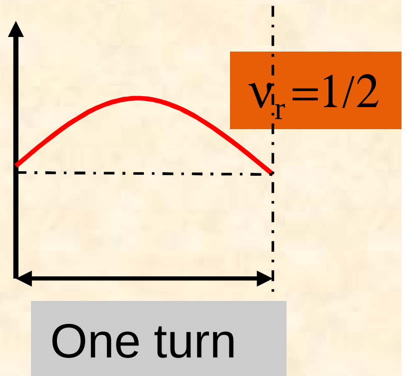
(not good : instabilities = beam losses)

With P field perturbations on L turns

z/Δ diverge at L.νz=P (integer)

The Cyclotrons try to avoid resonance crossing





Resonances

Systematic resonances : This resonance occurs as the betatronic frequency is a multiple of the "geometrical frequency" of the cyclotron. In this case, any kick given to the particle because of its particular position will be experienced again and again.

$$(K.\nu_r = P) \quad // \quad (L.\nu_z = P)$$

Coupling resonances

Under proper circumstances and frequency ratios, the 2 oscillators can be coupled and the energy stored in one motion, transferred to the other. These are coupling resonances ($K.\nu_r + L.\nu_z = P$).

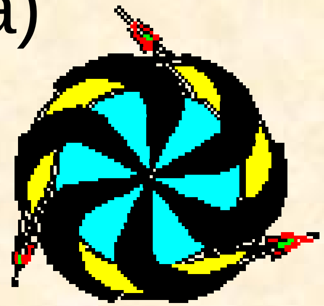
$$K.\nu_r + L.\nu_z = P$$

K, L and P integer

The particle's working point curve should avoid or cross as fast as possible those lines.

Tunes and resonances at Triumf (Canada)

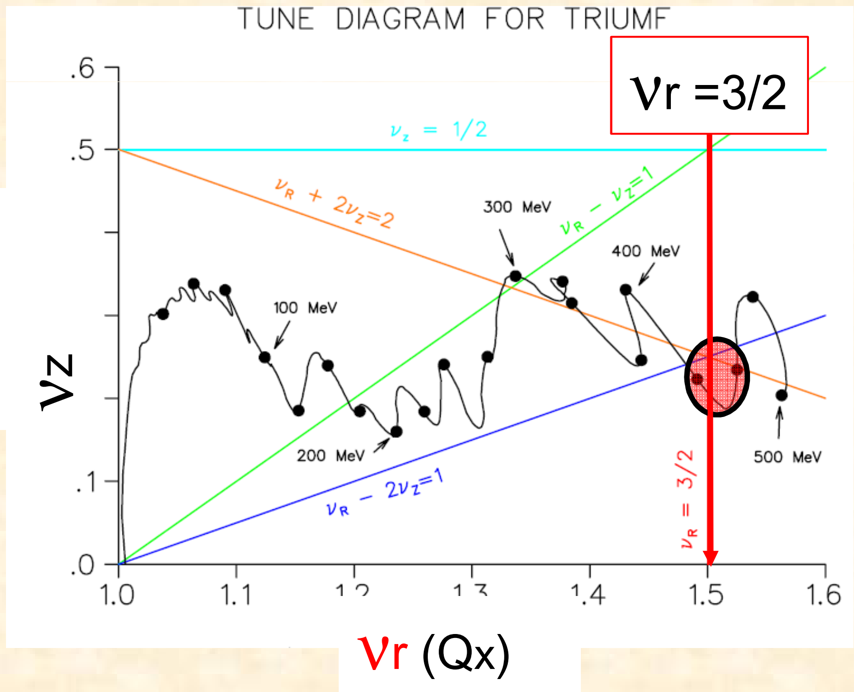
(H^- cyclo, Kb=520 MeV, 6 sectors)



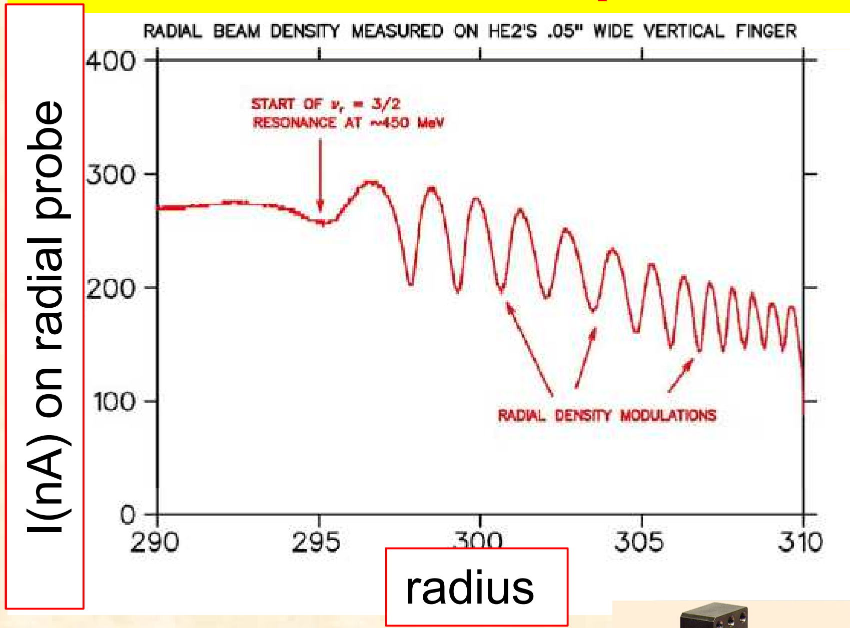
$$K.\nu_r + L.\nu_z = P$$

|K| + |L| is called the resonance order (1, 2, 3 …)

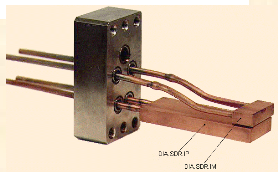
Effect on the crossing of the resonance **$2.\nu_r = 3$** **(order 2)**



Radial Oscillations : $2.\nu_r = 3$



$\nu r^2 = 1 - n + \ldots = 1 - (1 - \gamma^2) + \ldots \sim \gamma^2 + \ldots$



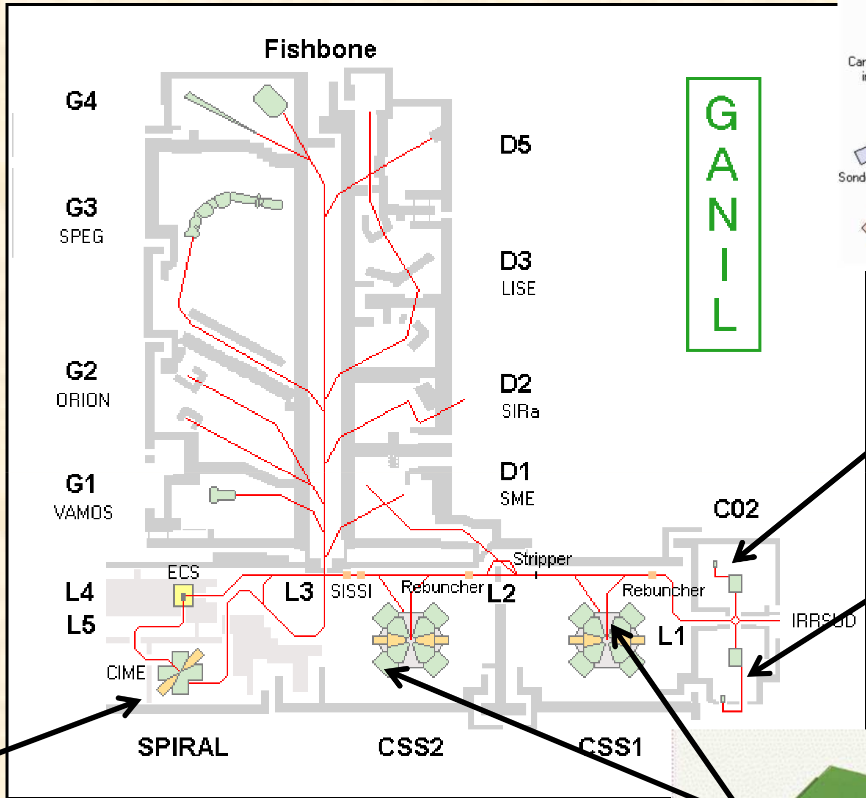
Cyclotrons in the world

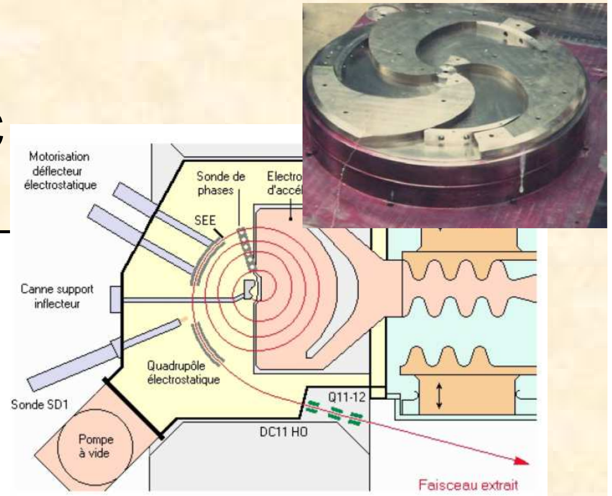
Some research laboratories with Cyclotron(s)

Some of the Research Facility in the world using cyclotrons

GANIL Facility (Caen, Fr)
5 cyclotrons + a new Linac

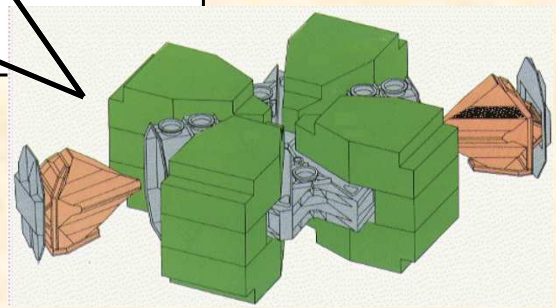
-Nuclear physics
-Atomic physics
-Solid state
-Radiobiology
- Irradiation for industry





2 compact cyclotrons
Kb=28 MeV

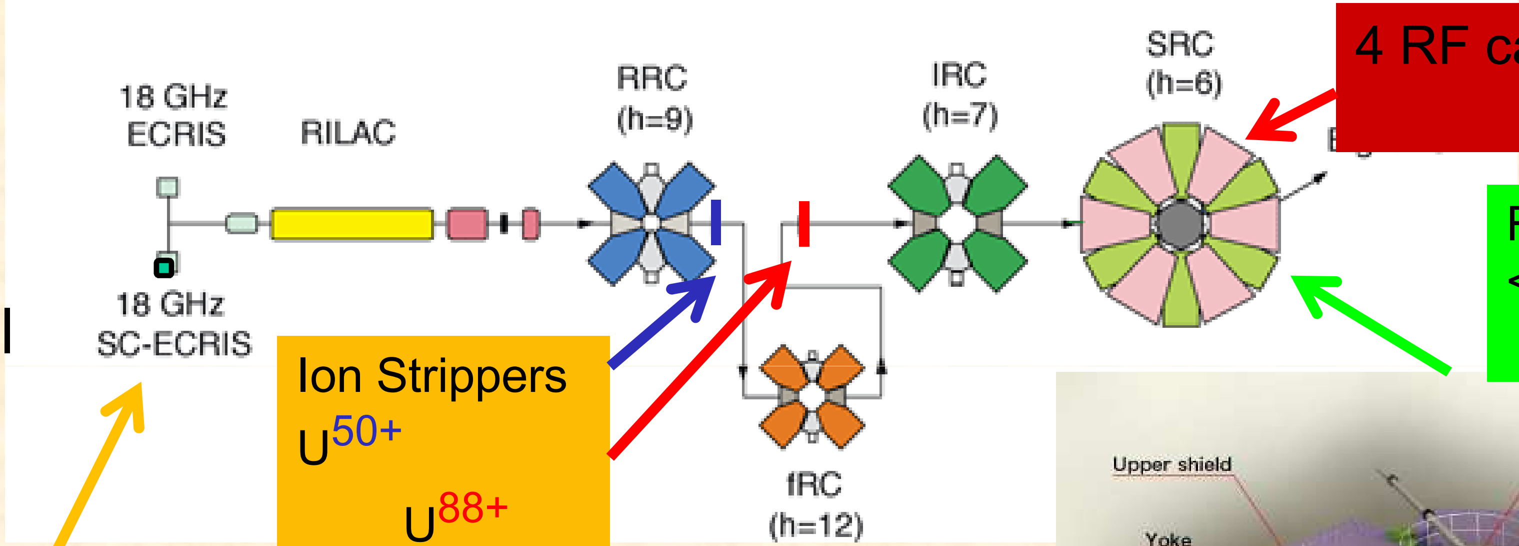
1 compact cyclo Kb =265
For radioactive ions



2 Separated Sectors cyclotrons
Kb =380 MeV

RIBF (Tokyo, Japan) :
Uranium beam $^{238}U^{88+}$ @345 MeV/A cw

Mode (1): RILAC + RRC + (stripper2) + fRC + (stripper3) + IRC + SRC



4 RF cavities

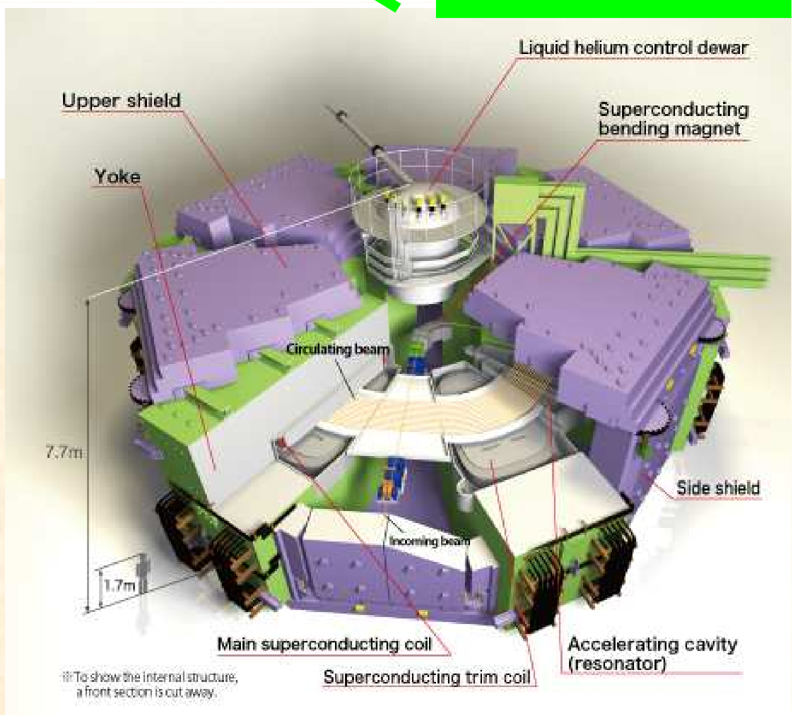
Radius=6m
<B>=3.8T

Ion Strippers
U^{50+}
U^{88+}

« external »
ECR source
U^{30+}

5 accelerators

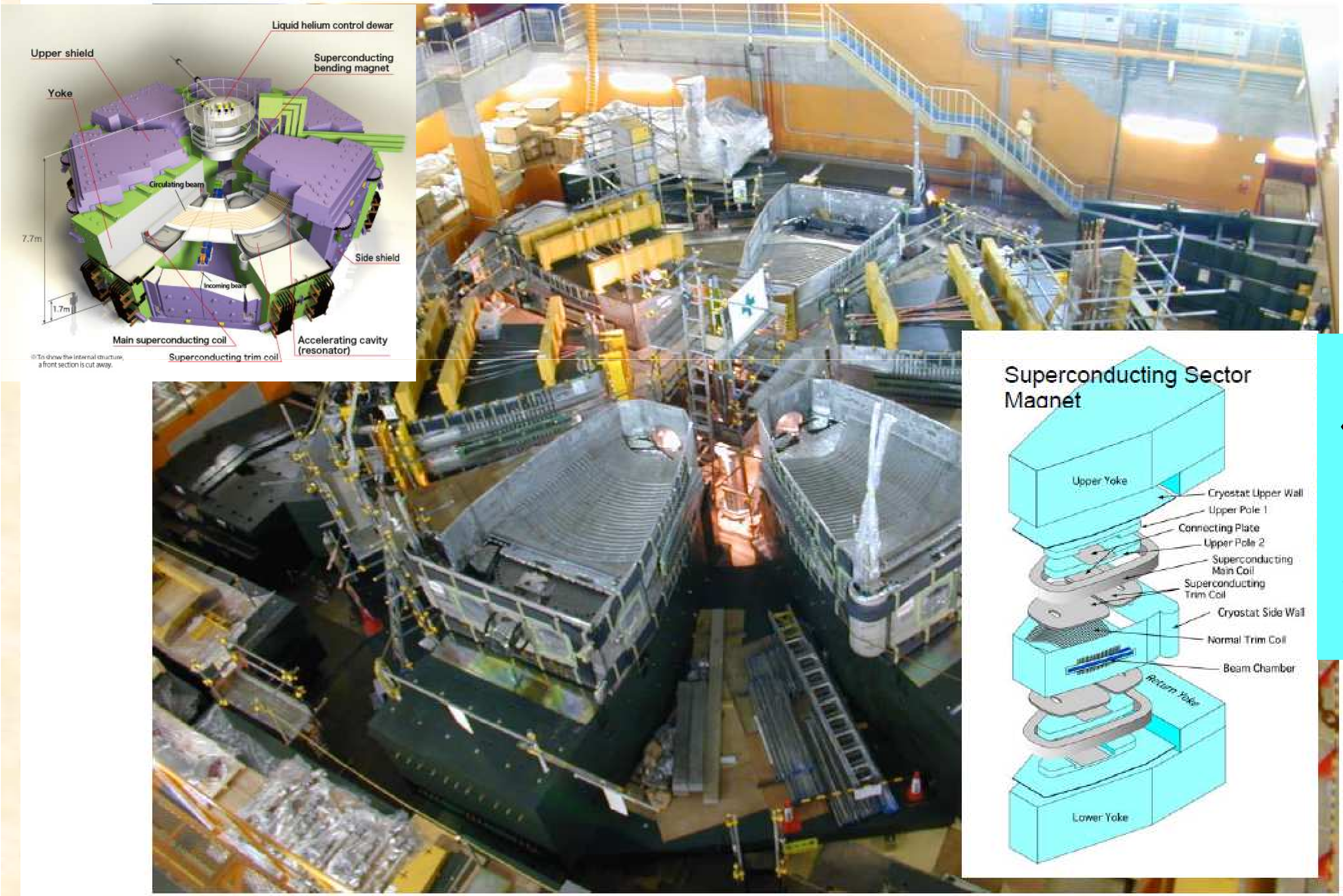
1 LINAC
+ 4 Cyclotrons



RIBF (Japan) : SRC (K=2600 MeV)

the largest cyclotron in the world

Superconducting Ring Cyclo

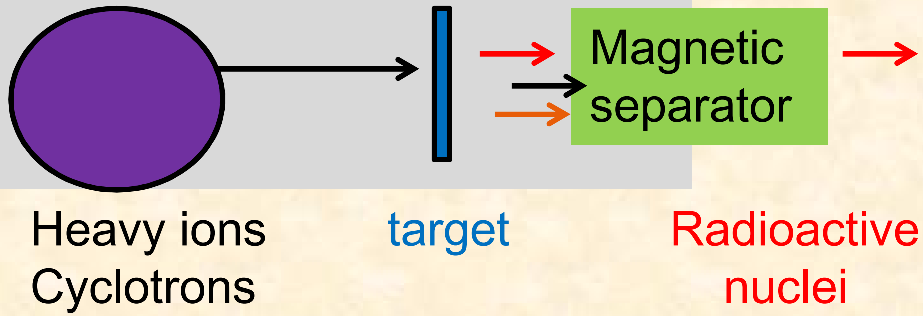


6 sectors :
« Iron dominated magnets »
With superconducting coils

Completed November 2005 - the 140-ton cold mass cooled to 4.5K.

TREND in Nuclear physics : Physics at RIBF (Tokyo)

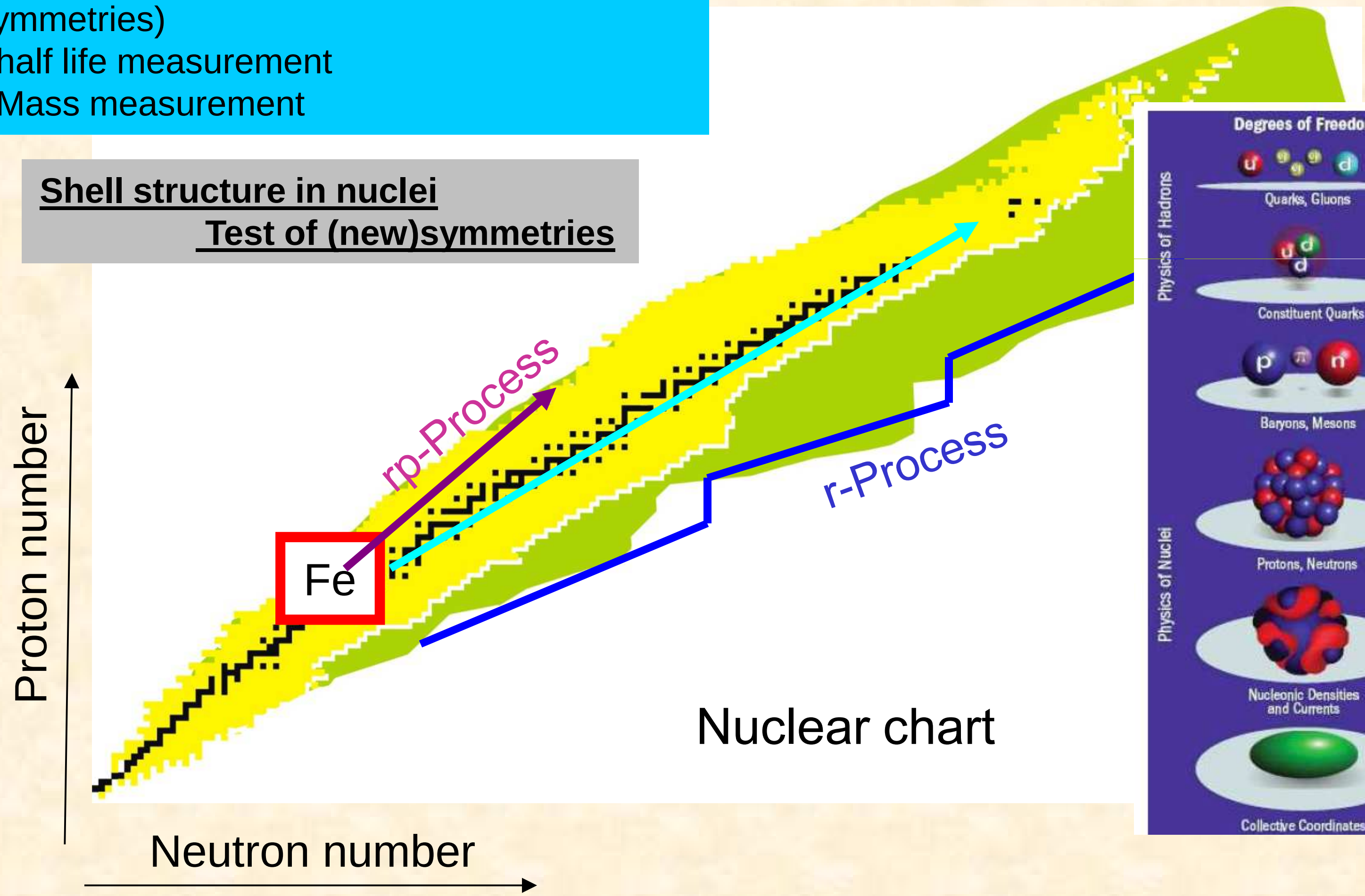
Create Unknown "exotic" nuclei
by nuclear fragmentation
or by fission

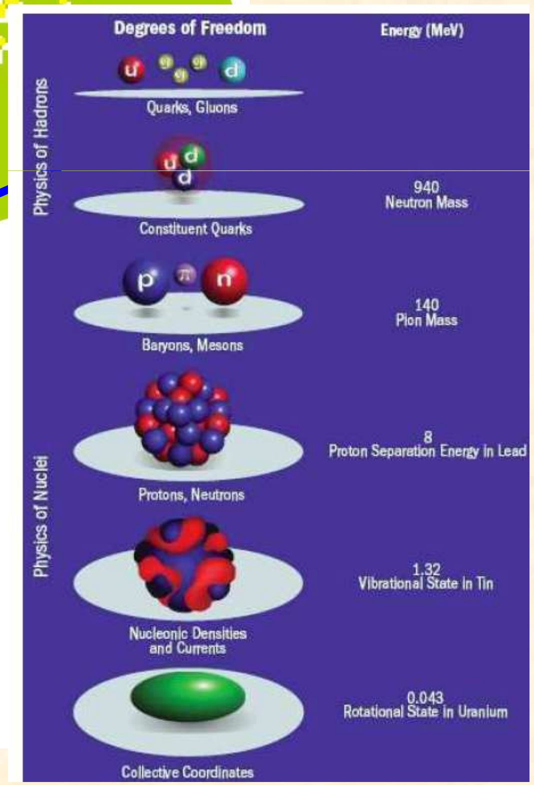


Exotic Nuclei (Very large proton-neutron asymmetries)

- half life measurement
- Mass measurement

Shell structure in nuclei
Test of (new)symmetries

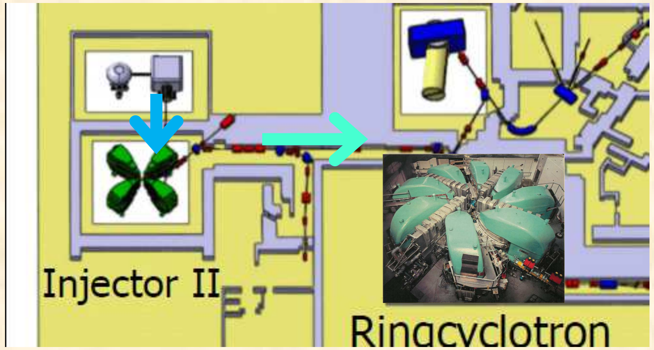


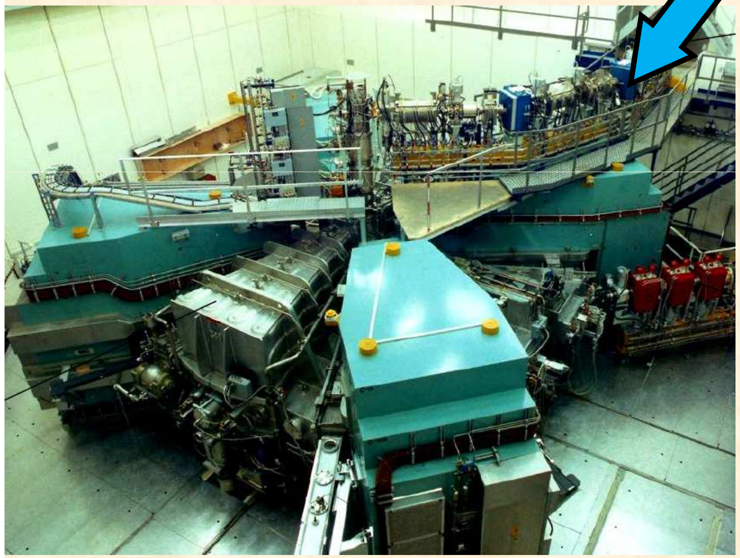


PSI 590Mev proton (Ch)

870 keV

Cokcroft-Walton



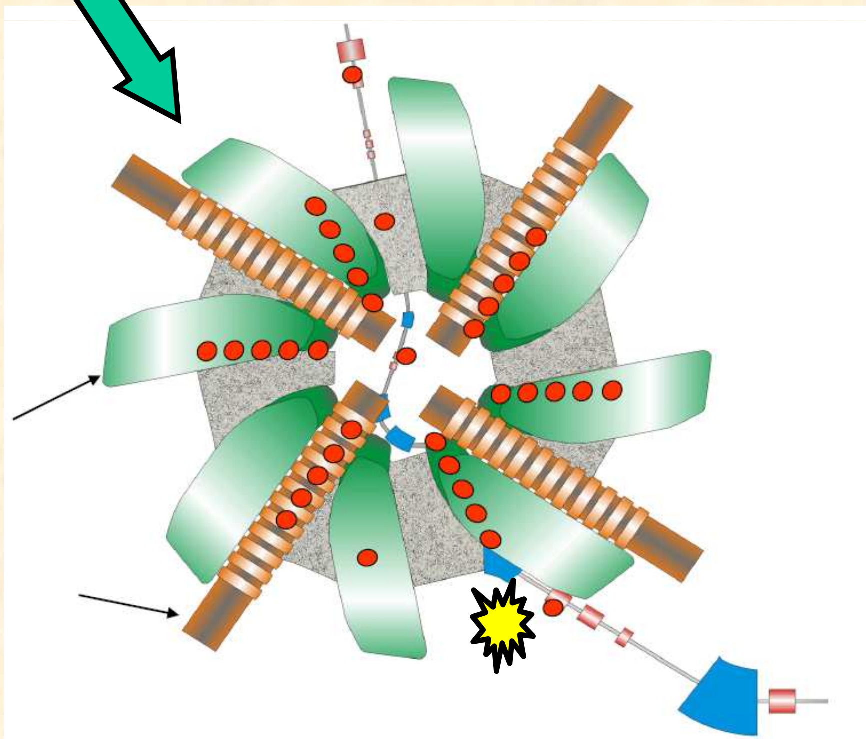


kb=72 MeV (injector 2)

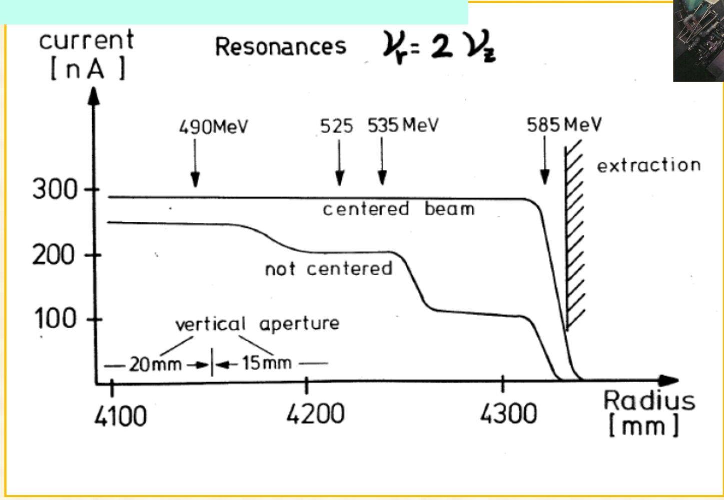
Coupling of 2 cyclotrons :

$$\left[\frac{F_{HF}.R_{extrac}}{h}\right]_{cycloA} = \left[\frac{F_{HF}.R_{injection}}{h}\right]_{cycloB}$$

590 MeV Ring cyclotron



PSI :K= 590 MeV ring cyclotron P_{beam}= 1Mwatt

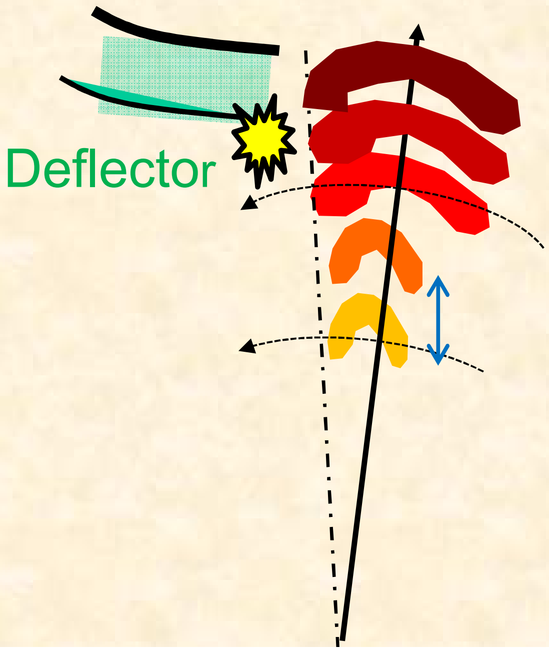


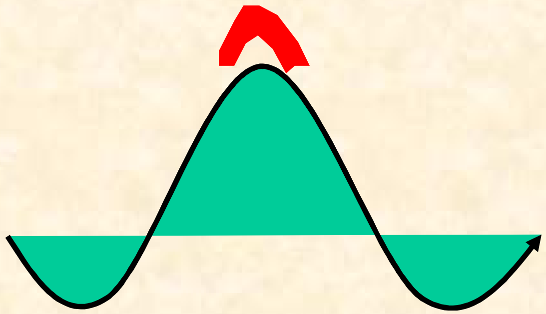
1) Injection centering :

Watch the resonance $2.\nu_z + \nu_r = 2$

2) Extraction : Watch the beam losses !

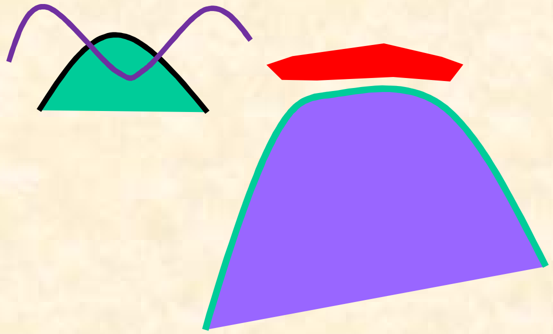
A Flat top RF cavity has been added to reduce losses: V_{RF}= Cos(ωt) – α.cos (3ωt)





Standard cavity :

V_{RF}= cos(ω_{RF} t)



Flat top cavity

cos(ω_{RF} t) - cos (3ωt)

Some Commercial Cyclotrons : manufacturers

IBA (Belgium)			Cost estimate 2008
Cyclone 5/10	proton/D		1 Meuros
Cyclone 9/18	H-/D-		1.5 Meuros
Cyclone 30	H-/D-		….
ProteusOne 250 MeV	p	synchro-cyclo superconducting	
C70	p/D or H-/D-		

Sumitomo HI (Japan)

HM-12	p
HM-18	p

EBCO (Canada)

TR 9/ 18	H-/D-
TR 15/ 30	H-/D-

GE-Scanditronix (USA-Sweden)

MINI TRACE 9/18

Accel // VARIAN ……

250 MeV proton cyclo superconducting

>300 commercial cyclotrons in the World

- (10-20MeV) protons
- (230MeV) protons :develloping market

Commercial Cyclotron

Radioisotope production (5-20 MeV)

Radiotracer $^{18}F^*$: « Beta+ » emitter

Fluorine 18

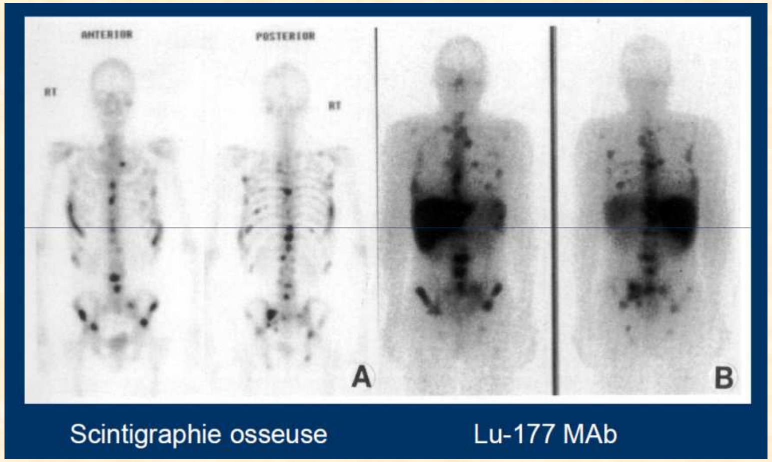
$T_{1/2}$ =109.7 min

$^{18}F^*$ ⇨ ^{18}O + positon + neutrino

↳ γ + γ

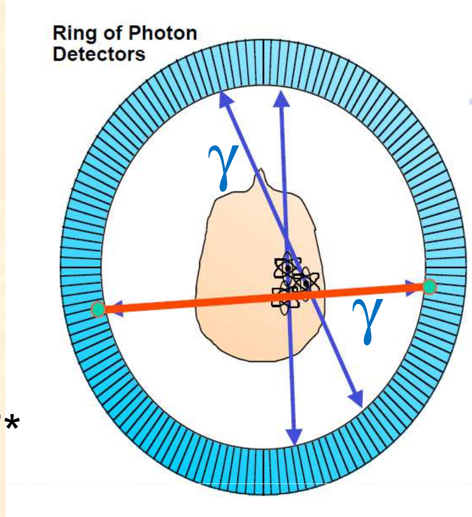
Production With cyclotrons

Proton 10-20MeV + ^{18}O ⇨ $^{18}F^*$ + n

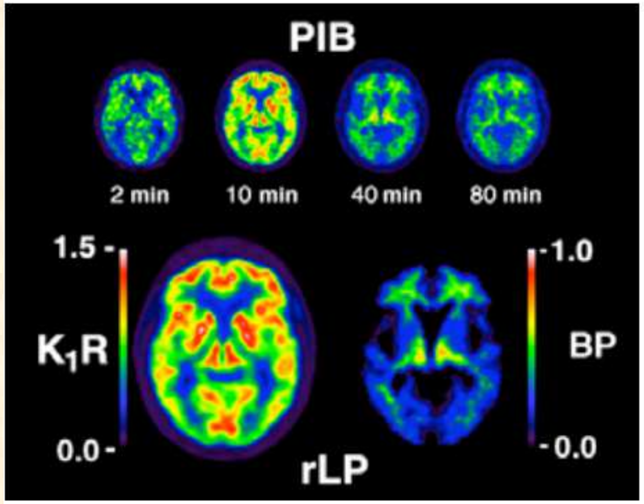


Injection of $^{18}F^*$

TEP camera



Reconstruction

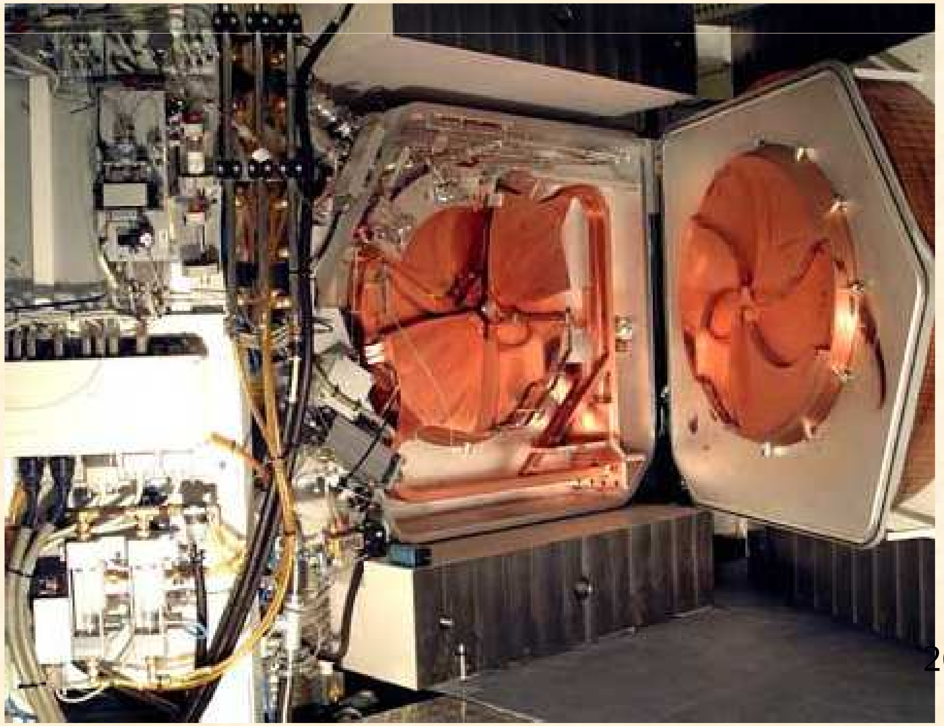


Cyclone 10/5 Mev

= 10 MeV proton

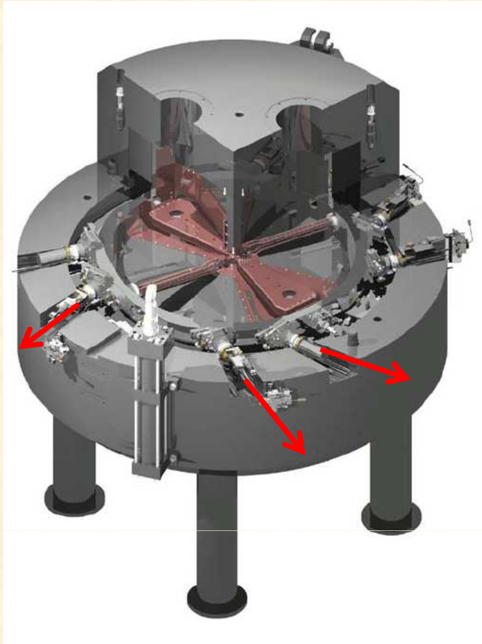
=5 MeV D

cyclone 3D



CYCLONE 18/9 (IBA) : H^- 18 MeV

Designed for medical applications (radiotracers production)

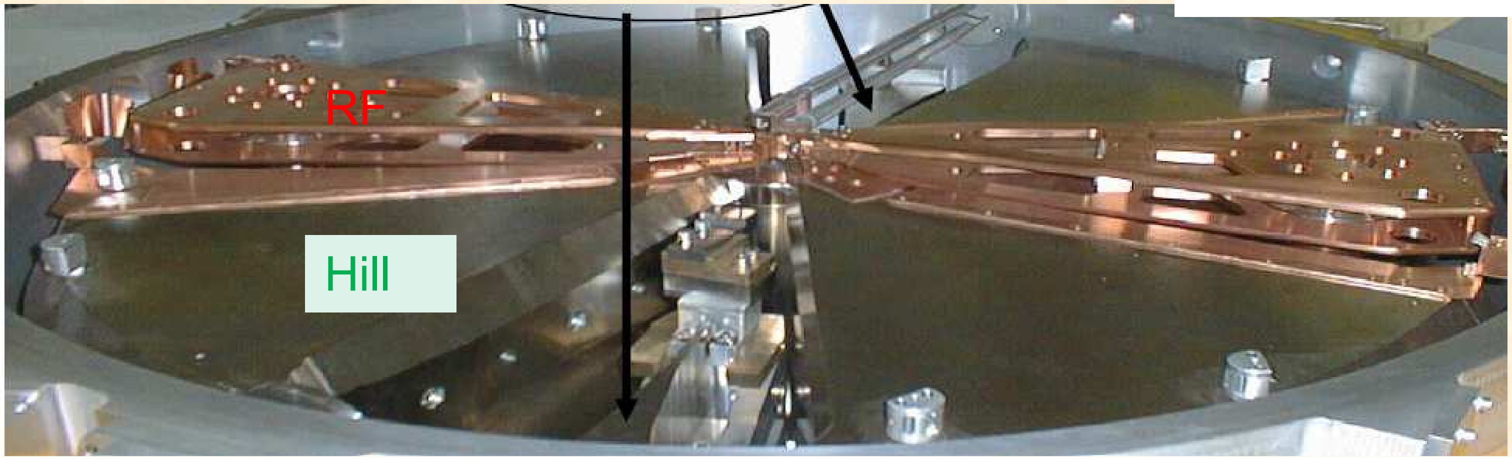


Internal PIG source, $\mathbf{H}^-$ stripping extraction

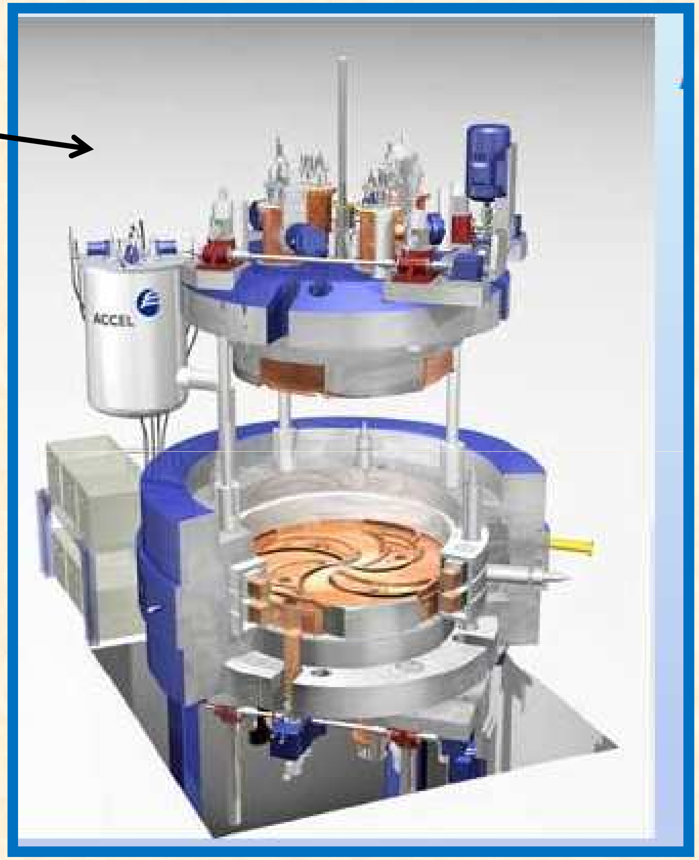
Kb=18 MeV
Fixed energy ;
4 straight sector 50°
<B> =1.35 Tesla
Hill //valley gap 3cm// 67cm
fixed Frf =42Mhz

2 Dee = 30° , 32 kV
Harmonic h=2(p) ,4 (D)
Internal source
Rextraction=0.46 m

Bρmax=0.46x 1.35=0.62 T.m



Commercial Cyclotron :
proton therapy (230-250 MeV)

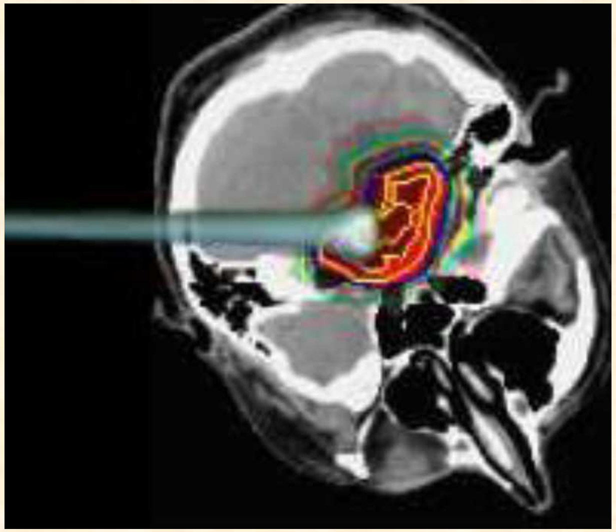


Superconducting cyclotron

or

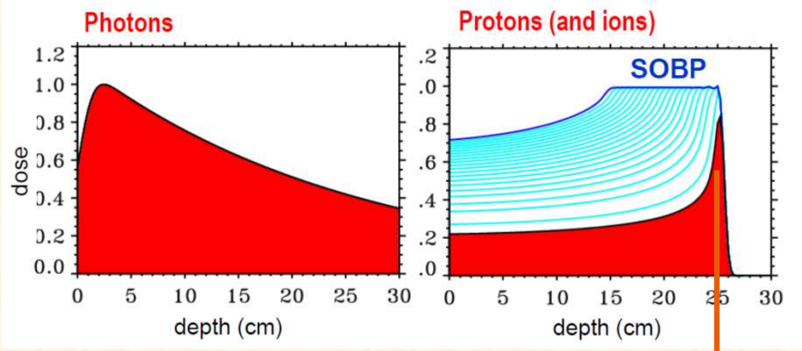
Superconducting Synchro-cyclotron

Brain tumor treatment with protons



Commercial Cyclotron :
For proton therapy (230 MeV)

Photon :
(Radiotherapy)
A Dose in the
whole body

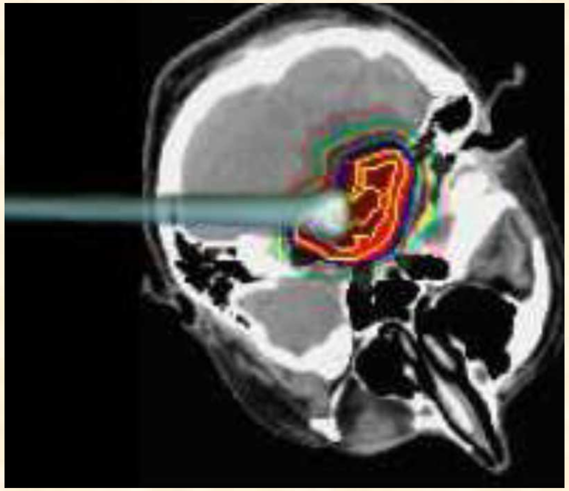


Protons :
Better than
Radiotherapy (photons)

Dose inside the tumor

Bragg Peak

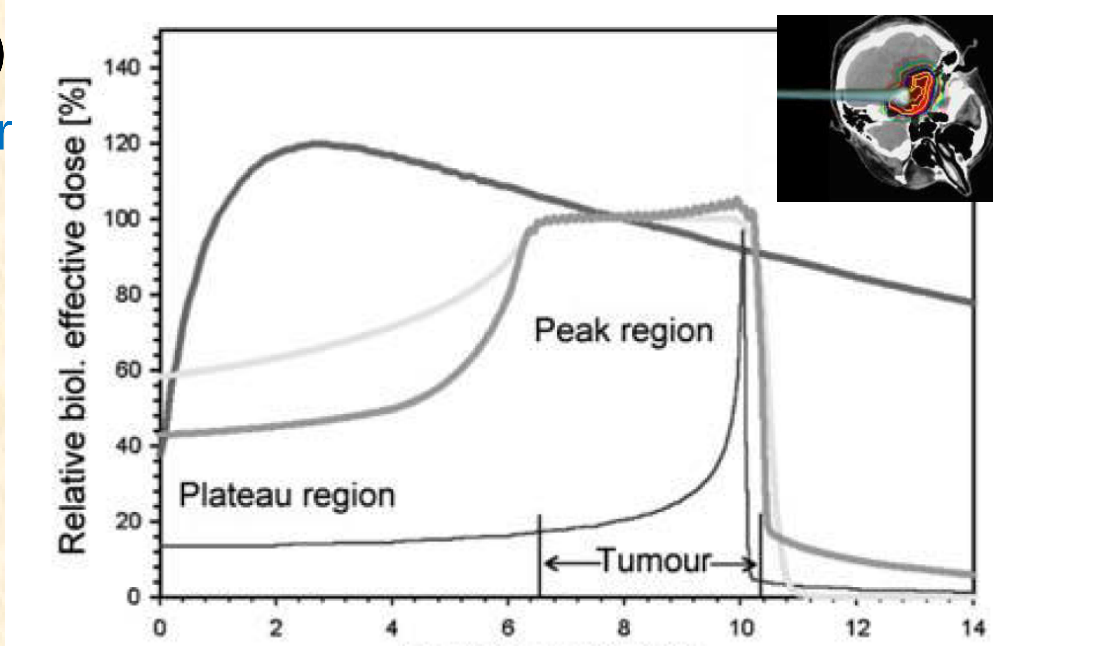
Brain tumor treatment with protons



Eyes tumor treatment

Proton therapy (230 MeV)

-Energy variation with degrader

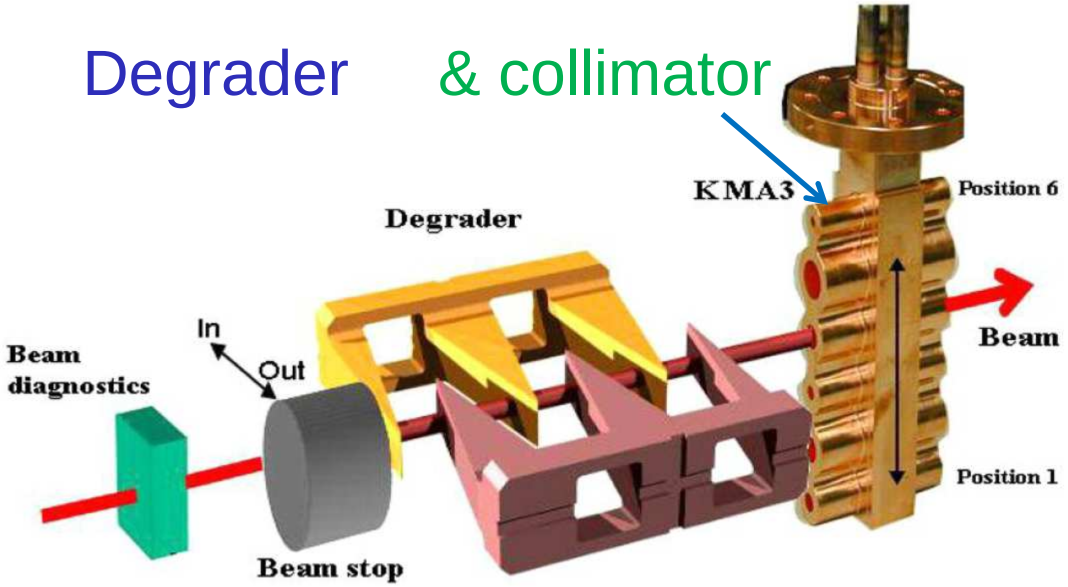


Depth (cm)

Scanning the tumors :

With energy variations induce by a degrader

Degrader & collimator



Energy variation

=

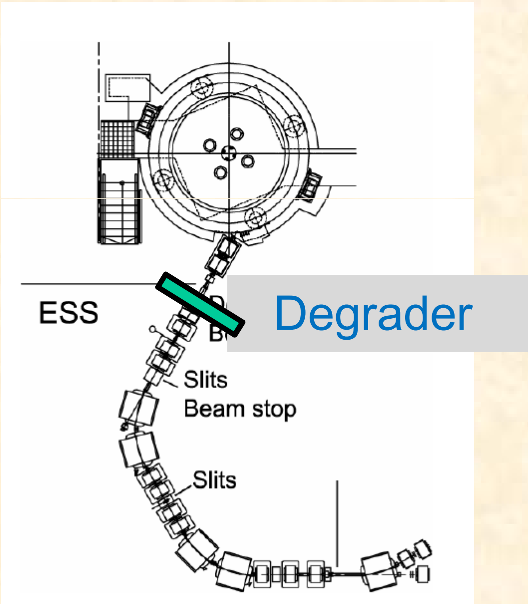
Range variation (tumour scanning)

Proton therapy (230 MeV)

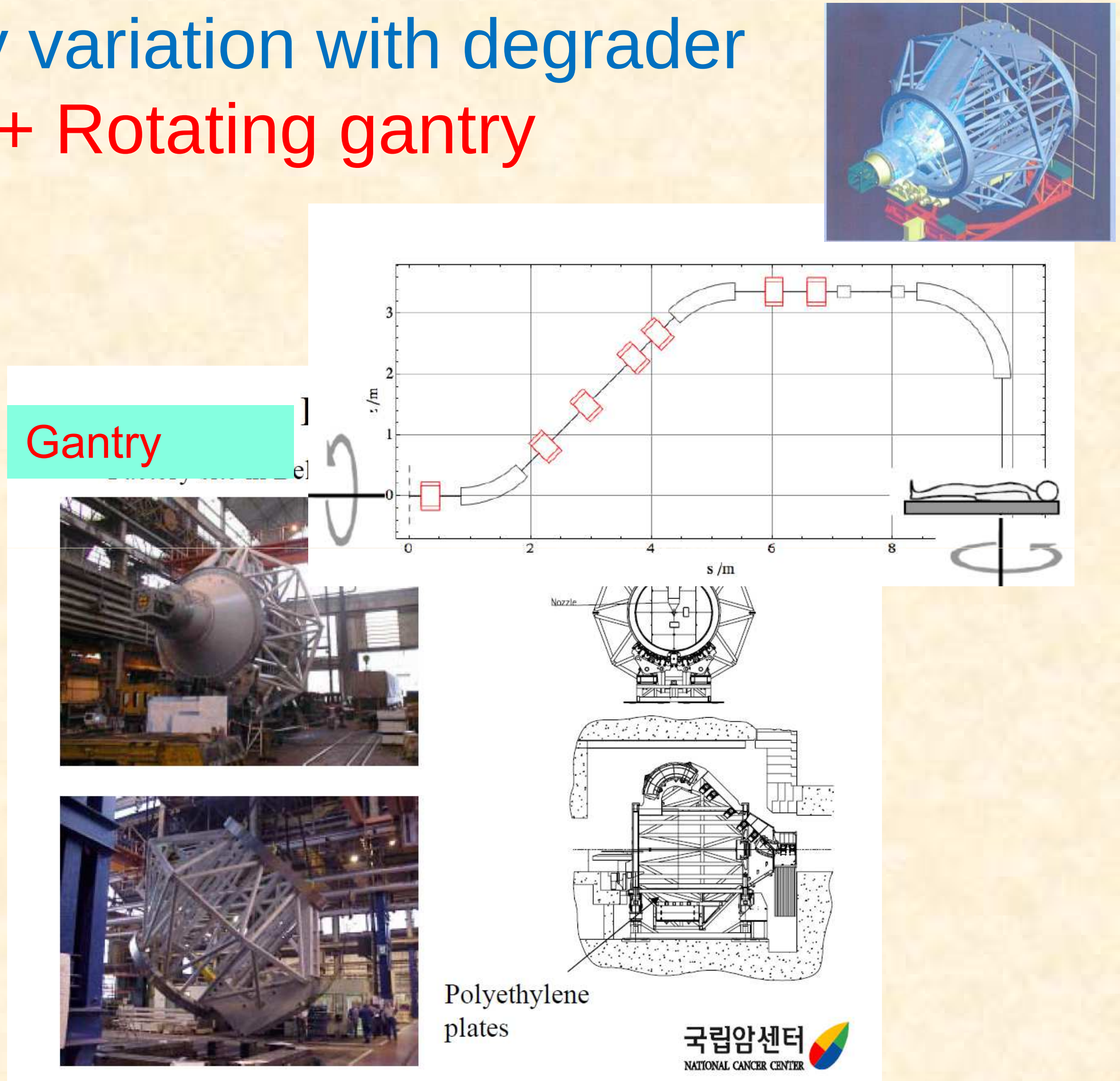
Energy variation with degrader

+ Rotating gantry

Optimal dose delivery
Scan the tumors :



Depth variation in patiuent body:
Tumor scanning



The gantry reduces dose in healthy tissues

END

References & Acknowledgements :

F. Chautard, Juas 2015
M. Craddock lecture on Cyclo //FFAG
W.Joho lecture on PSI facility
S. Brandenburg lecture on beam optics

P. Heikinnen, CAS Jyväskylä 1992, CERN 94-01 (1994) “Cyclotrons“ and “Injection and Extraction“

T. Stammbach, CAS La Hulpe, 1994, CERN 96-02 (1996) “Introduction to Cyclotrons“

... & Many others

Cyclotron Summary : with formulas

Isochronous cyclotron = constant revolution frequency

$$\omega_{rev} = \frac{qB_z(R)}{\gamma(R)\, m} = const$$

field index n<0

$$Bz(R) = < B_0 > R^{-n}$$

$$< R > = \frac{B\rho}{< B_z >} = \frac{\gamma\, mv}{q < B_z >}$$

$$\omega_{rev} h = \omega_{RF}$$

$E/A = Kb \,.\, (Q/A)^2$

Vertical stability in isochronous cyclotron $Bz = F(R,\theta)$

requires Azimuthal Field Modulation (N sectors)

$$\ddot{z} + [\nu_z \omega_{revolution}]^2 z = 0$$

$$\nu_z^2 = n + \ldots < 0$$

$z(t) \sim z_0 \exp(-i \;\nu z\, \omega\, t)$: vertical tune νz; real for stability

$$\nu_z^2 = n + \frac{N^2}{N^2 - 1} F_l \,(1 + 2 \tan^2 \varepsilon) > 0$$

Few other slides for questions……

Isochronous fied B(R) = good field index n(R)

$$n = -\frac{R}{B_{0z}}\frac{\partial B_z}{\partial R} \iff \frac{dB}{B} = -n\frac{dR}{R}$$

$$B\rho = \langle B \rangle . \langle R \rangle = \frac{p}{q}$$

$$\frac{dp}{p} = \frac{dB}{B} + \frac{dR}{R} = (1-n)\frac{dR}{R}$$

Longitudinal dynamics lecture

$$\frac{dp}{p} = \gamma^2\frac{d\beta}{\beta} = \gamma^2\frac{d(\omega_{rev}.R)}{\omega_{rev}.R} = \gamma^2\frac{dR}{R}$$

$$1 - n = \gamma^2$$

$$n(R) = (1-\gamma^2)$$

« At high energy » isochronism requires $n << 0$

Bz Azimutal modulations are not sufficient
It is a (Focusing) limit for high energy isochronous cyclotron

Max Energy for Superconducting Cyclotrons

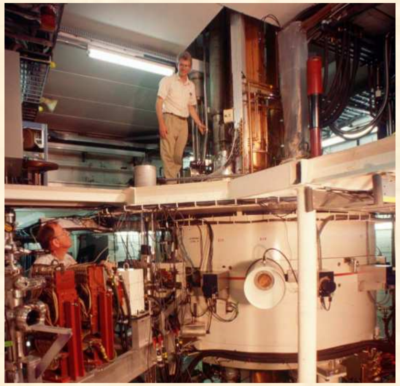
not limited by (B x Rextraction)

Because of the focusing limitation due to the Flutter dependence on the B field, the max energy is not given by *Kb* ~48 (*B.R*extraction)^2

but K_f the so-called "*focusing* factor":

$$\left[\frac{E}{A}\right]_{max} \neq Kb . \left\{\frac{Q}{A}\right\}^2$$

vertical oscillation+ isochronous field condition



$$\nu_z^2 = \left[1 - \gamma^2\right] + \frac{N^2}{N^2 - 1} F (1 + 2 \tan^2 \varepsilon) > 0$$

- Focusing limitation (stronger than B limitation)

$$\left[\frac{E}{A}\right]_{max} = Kf . \left\{\frac{Q}{A}\right\}^2 < Kb . \left\{\frac{Q}{A}\right\}^2$$

Kb~48 (B.Rextract)^2

Kf~ f(FLUTTER)

Tutorial

The axial oscillations of ion beams in a isochronous cyclotron is described by

Where the vertical tune should respect ν_z^2 >0,

$$\ddot{z} + [\nu_z \omega_0]^2 z = 0$$

a. Why ? (1)

b. Give a particular solution of the differential equation when ν_z^2 < 0 (1)

c. What is the axial physically νz

Answer

a.Otherwise the beam is unstable (beam size increase exponentially)

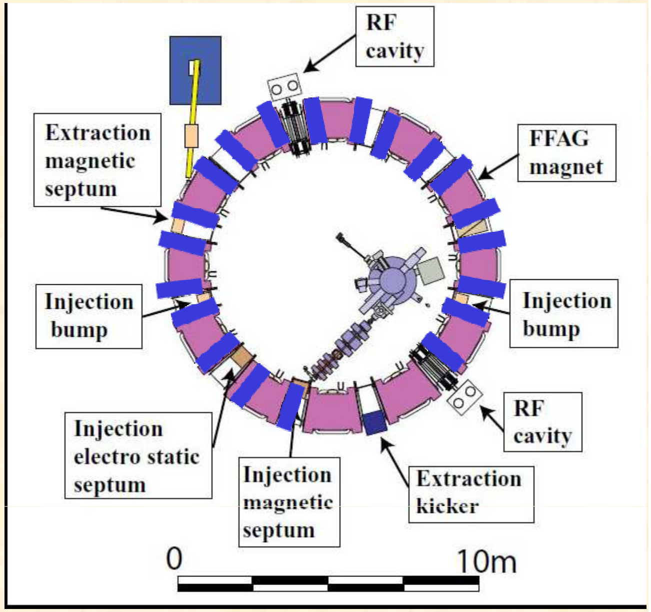
$$z(t) \sim Z_0 \exp(+/- i \ \nu_z \ wt)$$

b. $z(t) = Z_0 \exp(+/- |\nu_z| \ wt)$

c. ν_z correspond to number of oscillations per turn for beam not injected at the reference orbit.

An exotic cyclotron = the FFAG

FFAG ="Fixed-Field Alternating-Gradient"



A kind of synchro-cyclotron

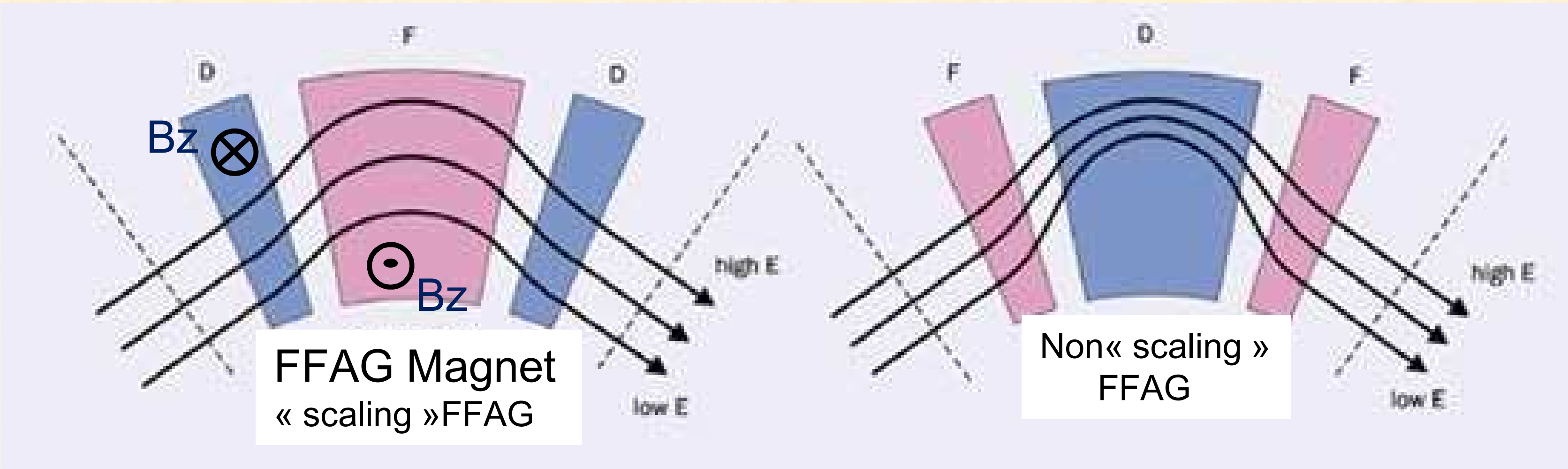
- pulsed beam

$F_{rf}= f(t)$

- not Isochronous
- focusing DFD or FDF with alternating magnet

High repetition Rate (100Hz-1kHz)

Large momentum acceptance



R&D an exotic cyclotron =FFAG

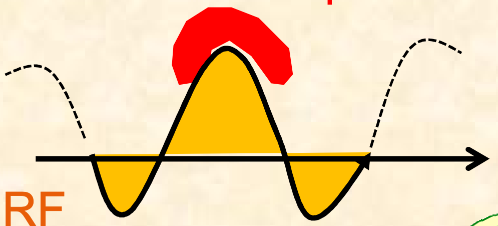
FFAG =Fixed-field alternating-gradient: Higher repetition rate than a synchrotron

2 coupled FFAG =150 MeV proton in Kyoto

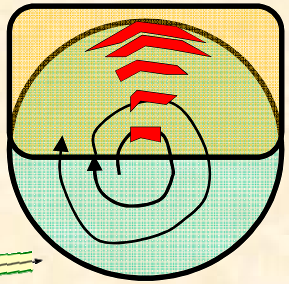
R&D for : - Accelerator Driven System (nuclear reactor)
- muon acceleration

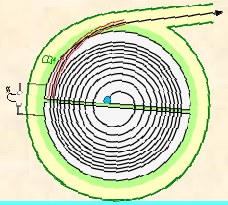
Cyclotron Summary : with pictures

Bunch at the top of the RF



RF





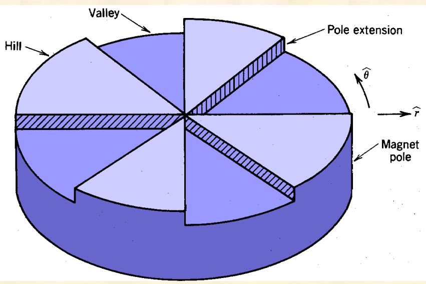
harmonics =h bunches by turn

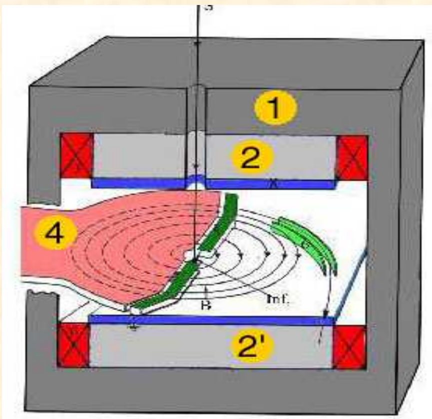
$$\omega_{rf} = \mathbf{h}\ \omega_{rev}$$

$$\omega_{rev} = \frac{qB_z}{\gamma m}$$

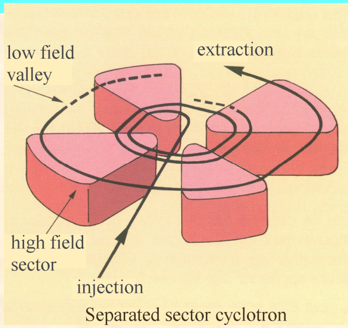
Kb= 30 MeV-200MeV
Compact cyclotrons (with Hills //Valleys)
Straight or Spiralled sectors

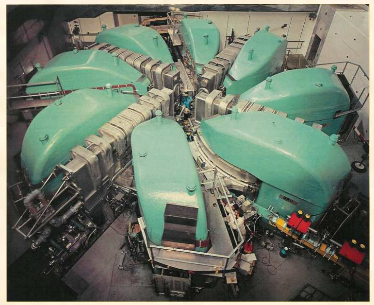
Kb= 300-600 MeV
« Ring cyclotrons » :
Straight or Spiralled
Separated sectors





30-200 MeV protons





300-600 MeV protons

The Cyclotron Family

isochronous cyclotron
(Azimuthally Varying Field)

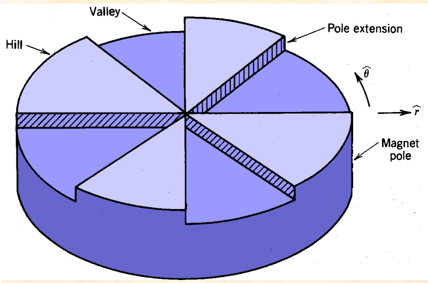
B*z (R, θ)* = NOT uniform
F*rev* = Constant
F*RF* = constant

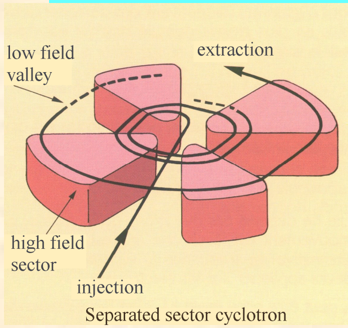
Isochronous

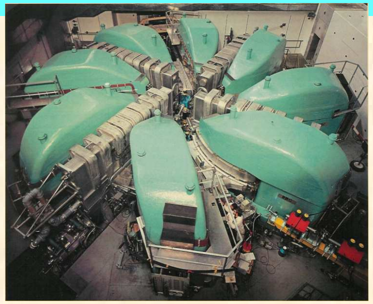
$$\omega_{rev} h = \omega_{RF}$$

Compact cyclotrons (with Hills //Valleys)
Straight or Spiralled sectors

Ring cyclotrons :
Straight or Spiralled
Separated sectors







Synchrocyclotrons

$$\omega_{rev}(R).h = \omega_{RF}(t)$$

F*rev* = NOT Constant
F*RF* = NOT Constant = beam pulsed

Not Isochronous