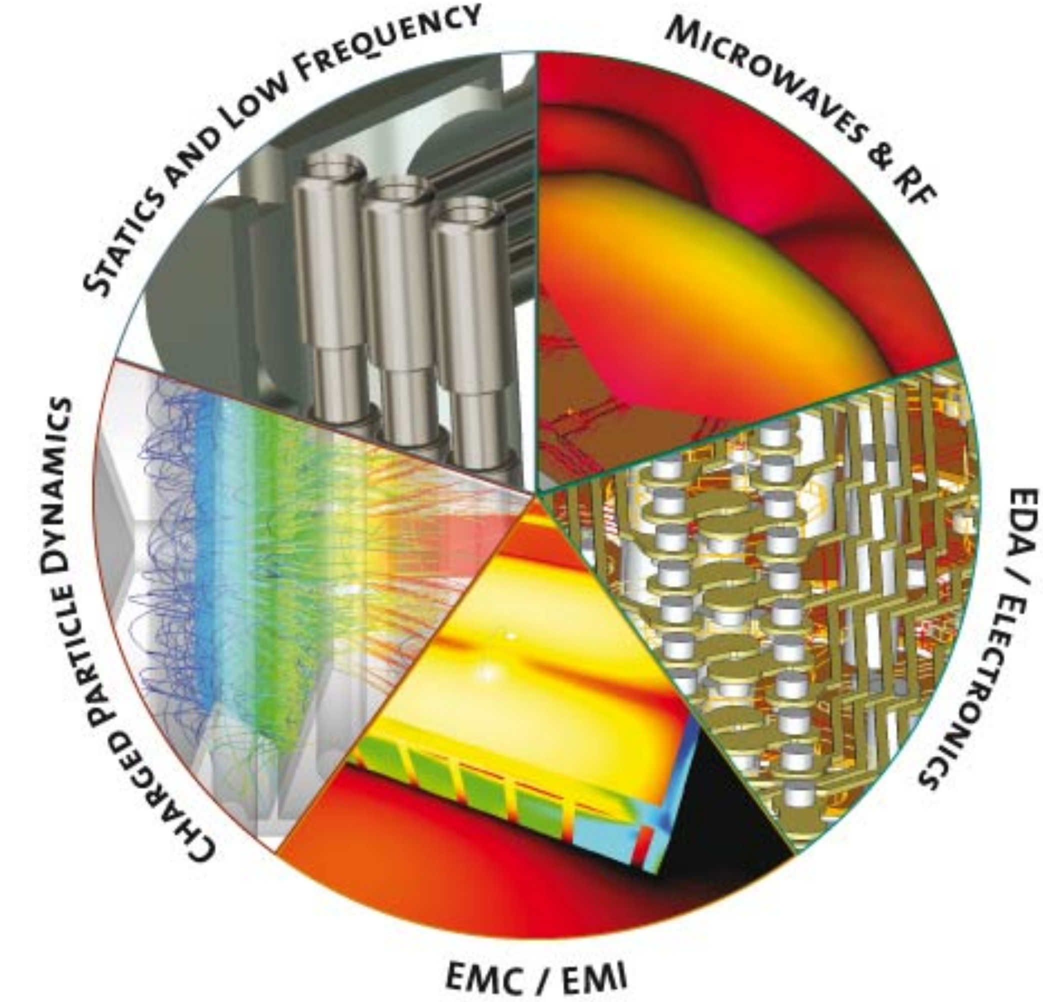
Simulation of a Pill-Box Cavity with CST



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WHY CST?

CST Studio Suite is a software used for the study of electromagnetic fields. It comprises tools for the design and optimization of accelerator devices operating in a wide range of frequencies, from static to optical. Analysis may also include thermal and mechanical effects, as well as circuit simulations.

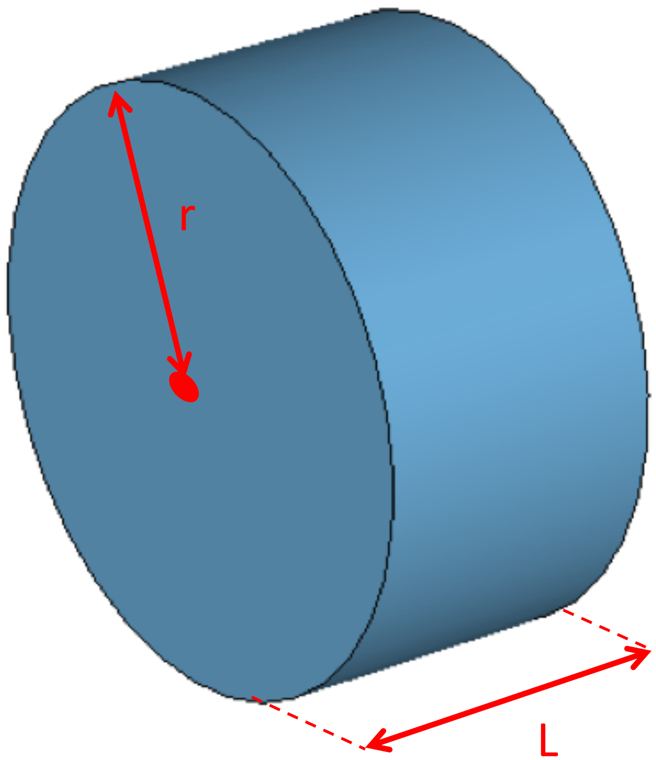
WHAT ARE WE GOING TO DO?

In this tutorial we are going to show, step by step, how to use CST in order to design a pill-box cavity and analyse its electromagnetic field. Particularly we will focus on the resonant modes of the cavity.

Simulation of a Pill-Box Cavity with CST

SOME REMARKS BEFORE STARTING: WHAT'S A PILL-BOX CAVITY?

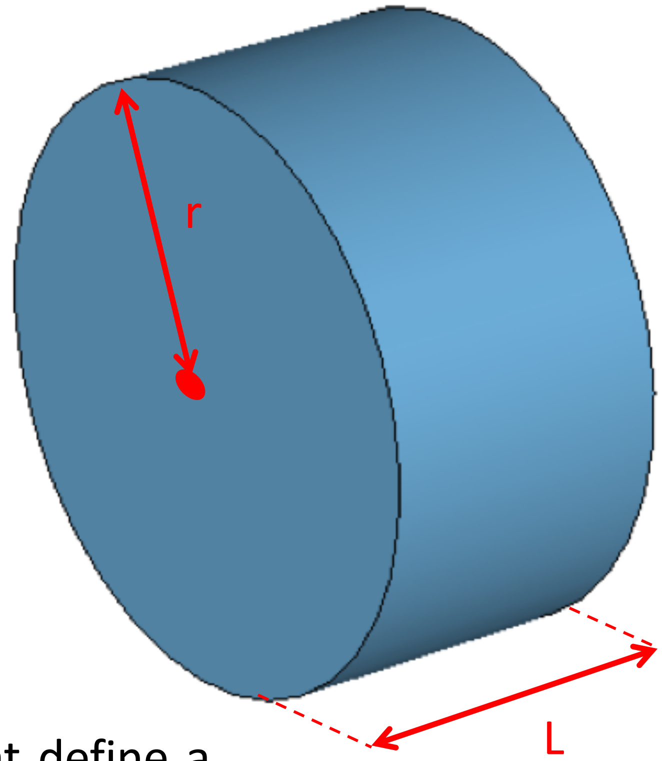
A **resonant cavity** or *radio frequency (RF) cavity* is a special type of resonator, consisting of a closed metal structure that confines electromagnetic fields inside it, storing their energy. They are used to accelerate the particle beam. The structure is either hollow or filled with dielectric material. The electromagnetic waves bounce back and forth between the walls of the cavity. At the cavity's resonant frequencies they reinforce themselves forming standing waves fields.
A **pill-box cavity** is a a particular kind of resonant cavity with cylindrical shape.



Simulation of a Pill-Box Cavity with CST

SOME REMARKS BEFORE STARTING: WHAT'S A PILL-BOX CAVITY?

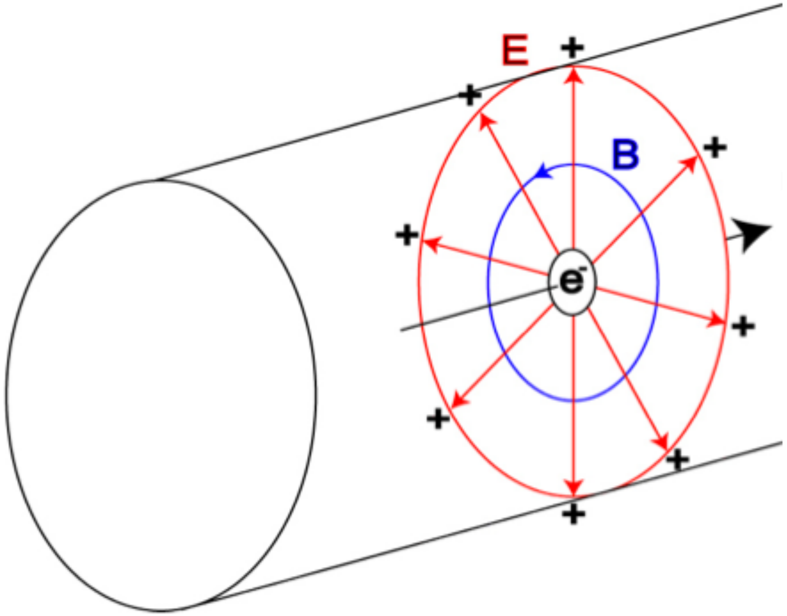
In order to accelerate the particles that pass throughout the cavity, particular kind of resonant modes are used: the so-called transverse magnetic (TM). These modes have the electric field component directed along the axis of the cylinder, and arise when the cavity is excited with particular frequencies:

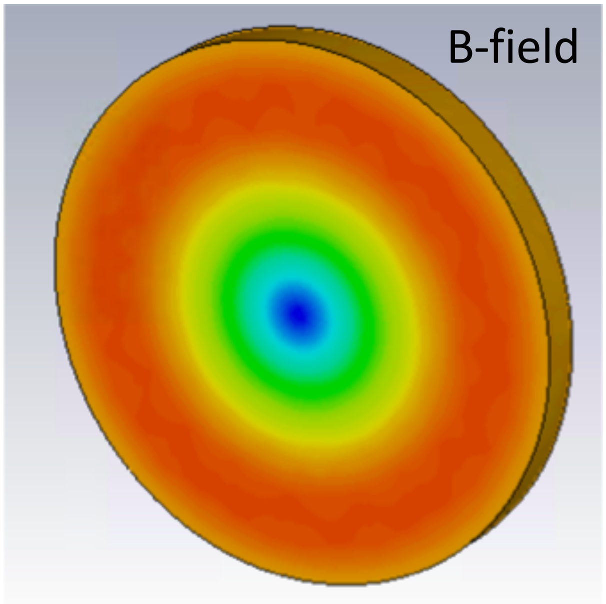


$$f_{nml} = \frac{c}{2\pi}\sqrt{\left(\frac{p_{nm}}{r}\right)^2 + \left(\frac{l\pi}{L}\right)^2}$$

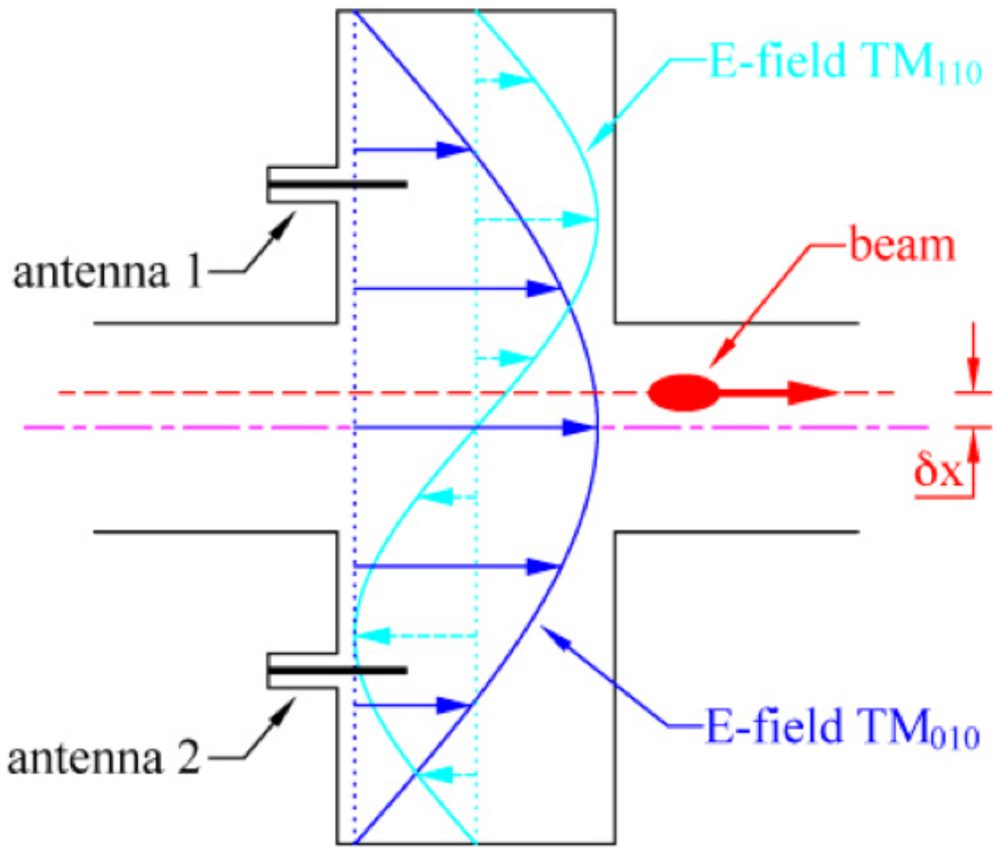
- *c* is the speed of light
- *n,m,l* are the mode numbers. They are integers that define a given TM mode. The most used is n=0 (azimuthal symmetry), m=1 (no nodes of E_z with r), l=0 (constant E_z in the z direction)
- p_{nm} is the m-th zero of the Bessel's function of n-th order
- *r,L* are the geometrical dimensions of the cavity

Simulation of a Pill-Box Cavity with CST





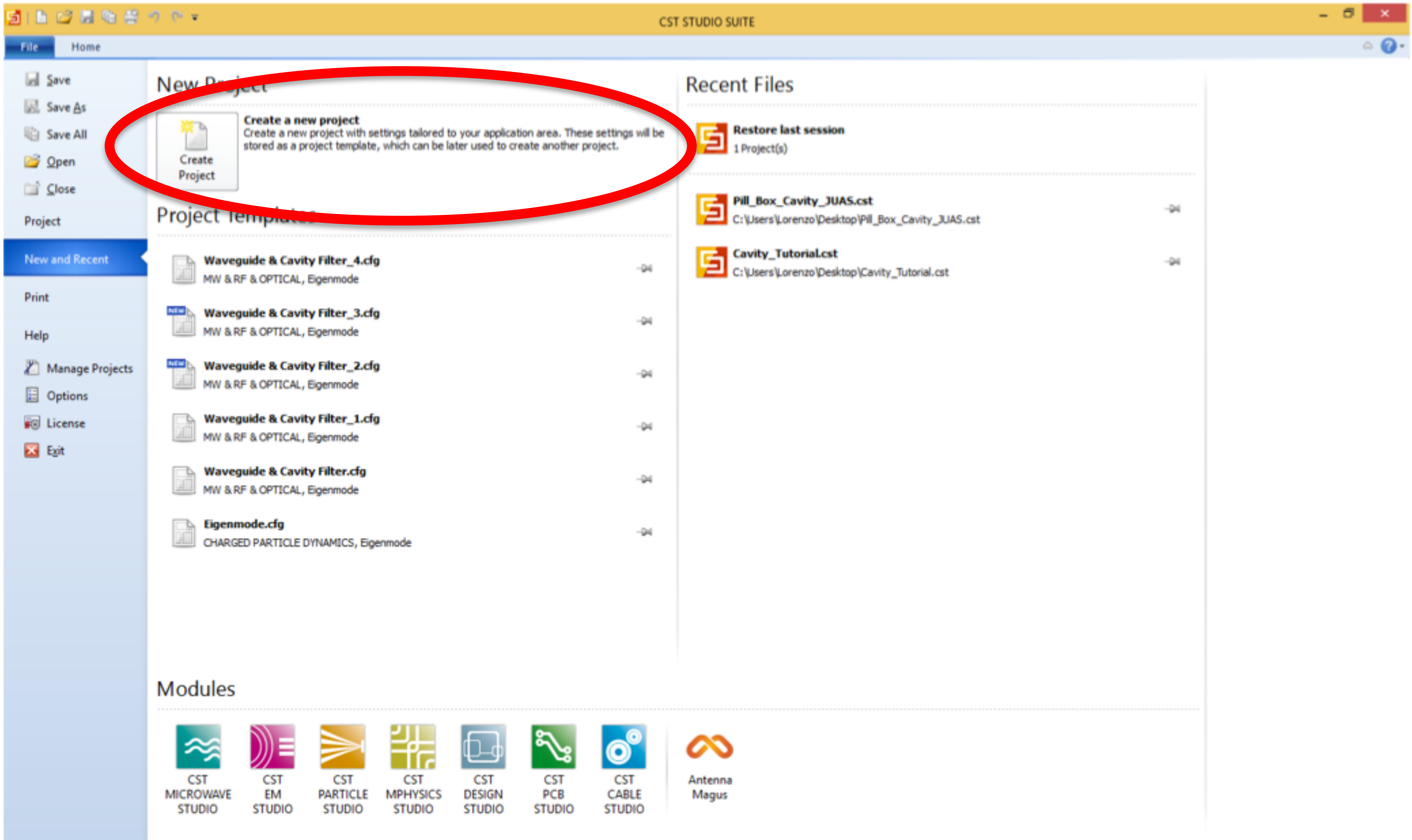
TM modes



Simulation of a Pill-Box Cavity with CST

READY TO GO!

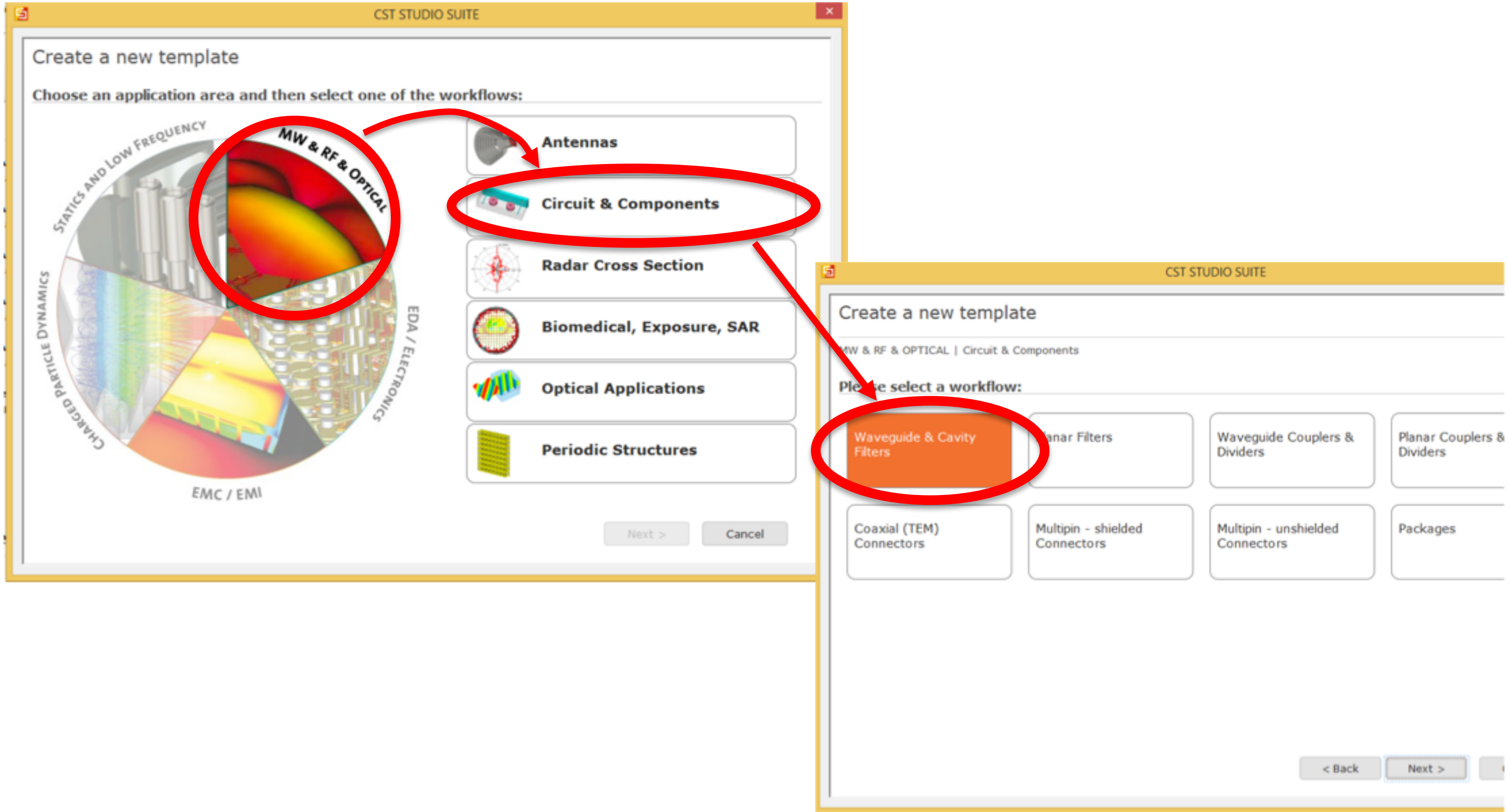
Open CST studio suite and select “Create a new Project”



Simulation of a Pill-Box Cavity with CST

READY TO GO!

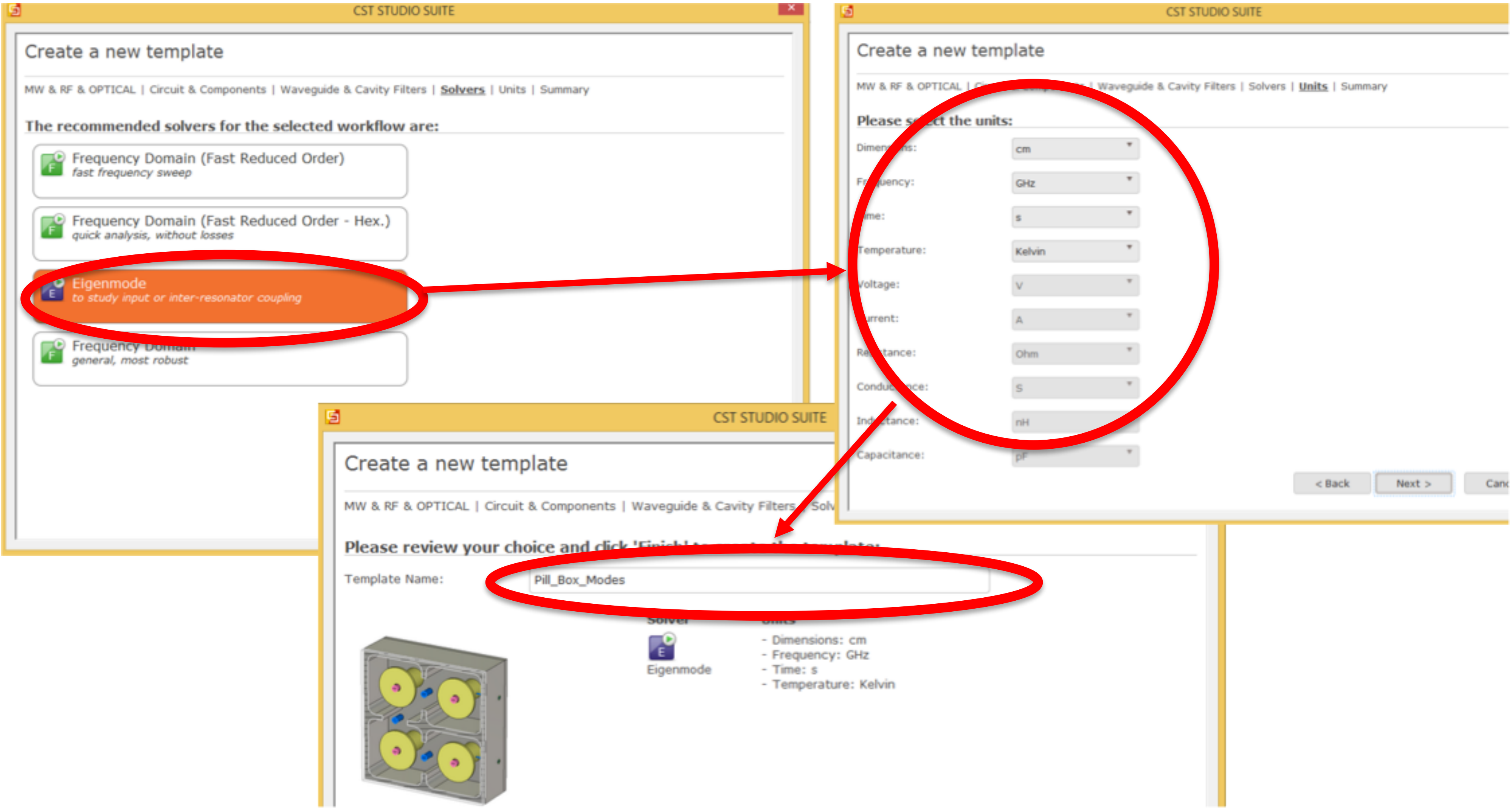
Thus let's select MW & RF & Optical -> Circuit & Components -> Waveguides & Cavity Filters



Simulation of a Pill-Box Cavity with CST

READY TO GO!

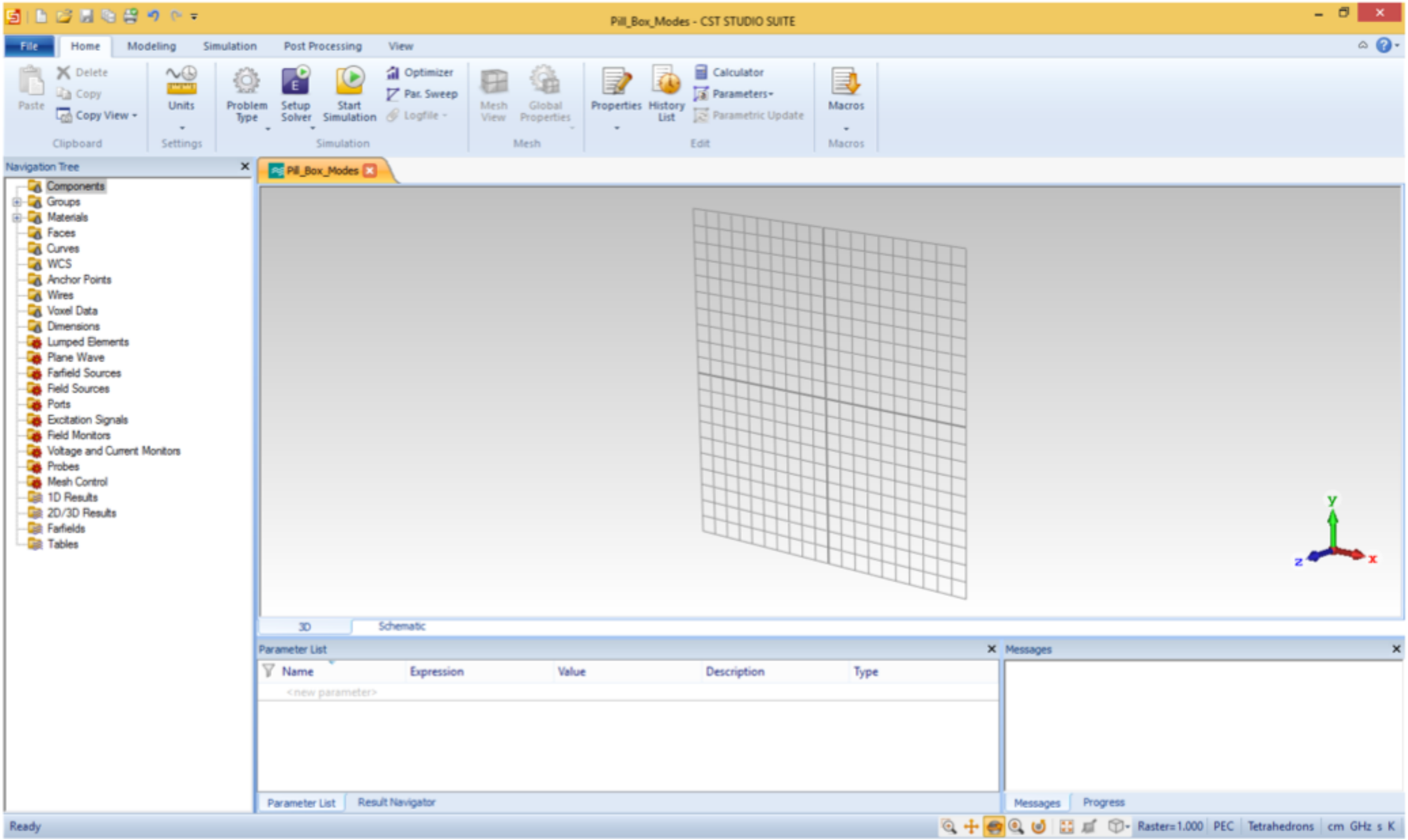
Then we select "Eigenmode". After that we chose the working dimensional units (cm, GHz, s, Kelvin) and name the file as: Pill_Box_Modes



Simulation of a Pill-Box Cavity with CST

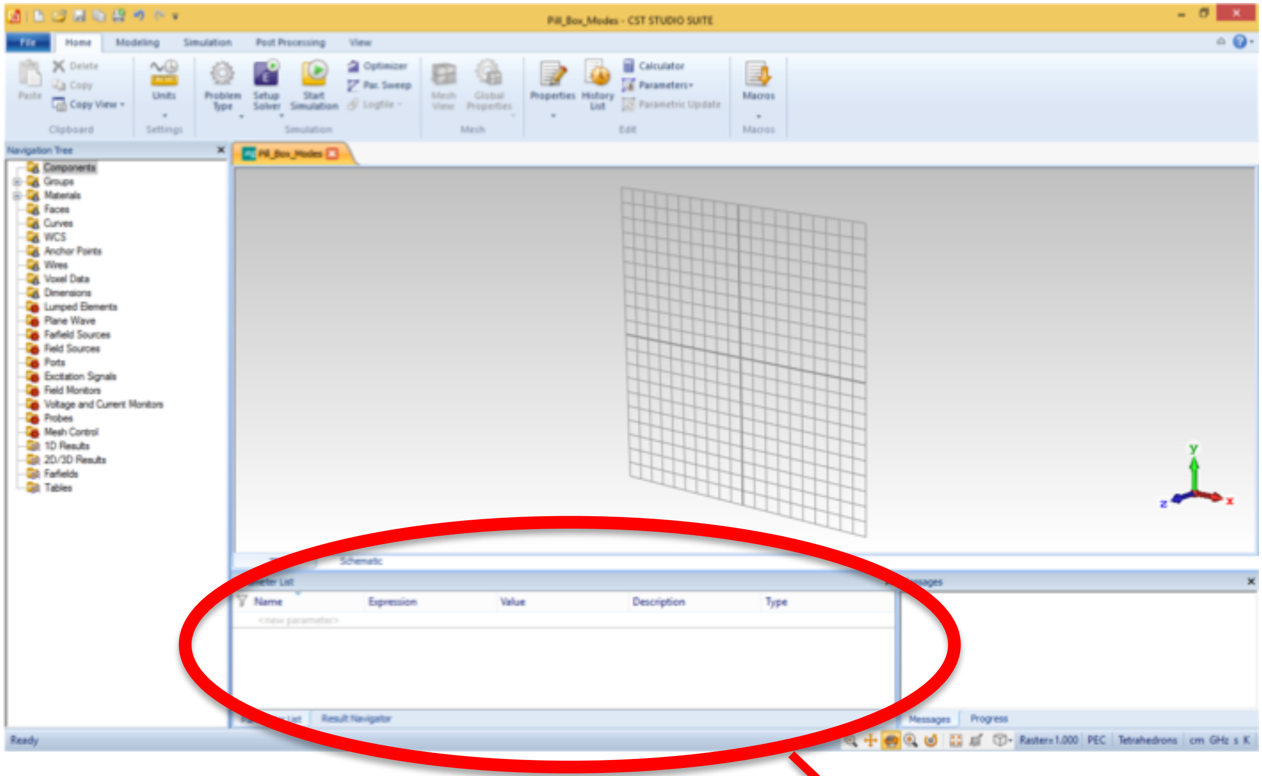
READY TO GO!

You should now see a screen like this one.

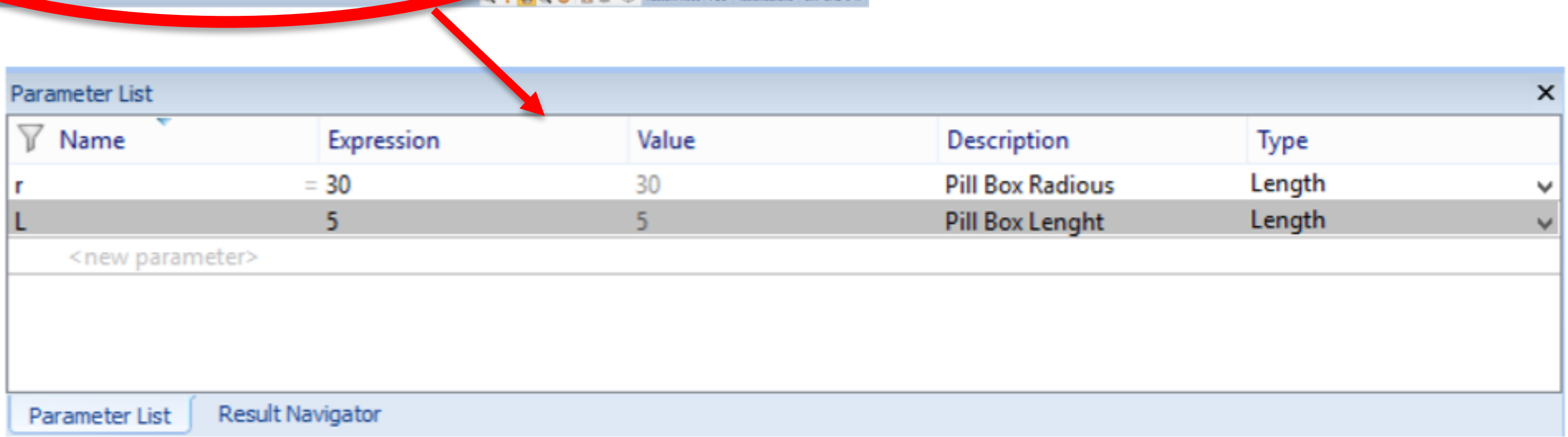


Simulation of a Pill-Box Cavity with CST

MODELLING



Let's now introduce the geometrical dimension of the Pill Box as parameters. This will allow to modify the entire model in future. It is possible to enter the parameter name, its expression (it could be a value, according to the dimensional units previously selected, or a combination of other parameters), its description and its type.



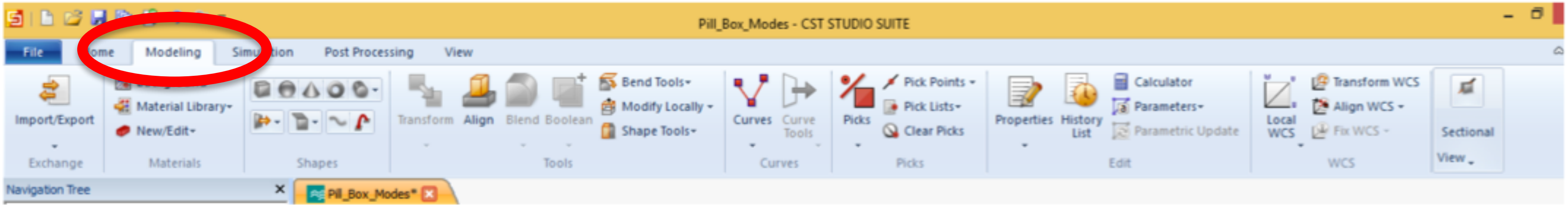
Parameter List

Name	Expression	Value	Description	Type
r	= 30	30	Pill Box Radious	Length
L	5	5	Pill Box Lenght	Length
<new parameter>				

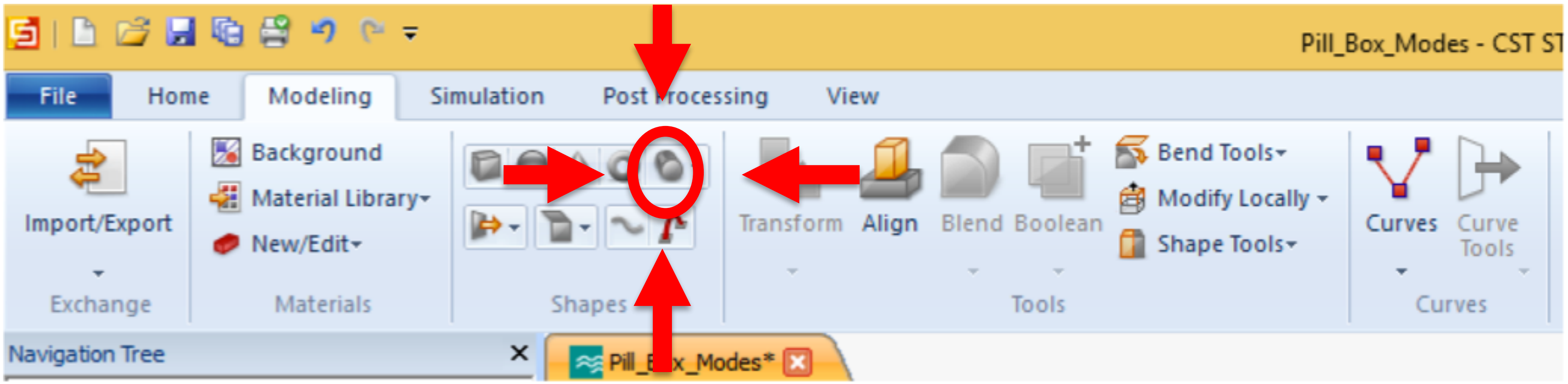
Simulation of a Pill-Box Cavity with CST

MODELLING

Now let's click on Modelling item on the top menu



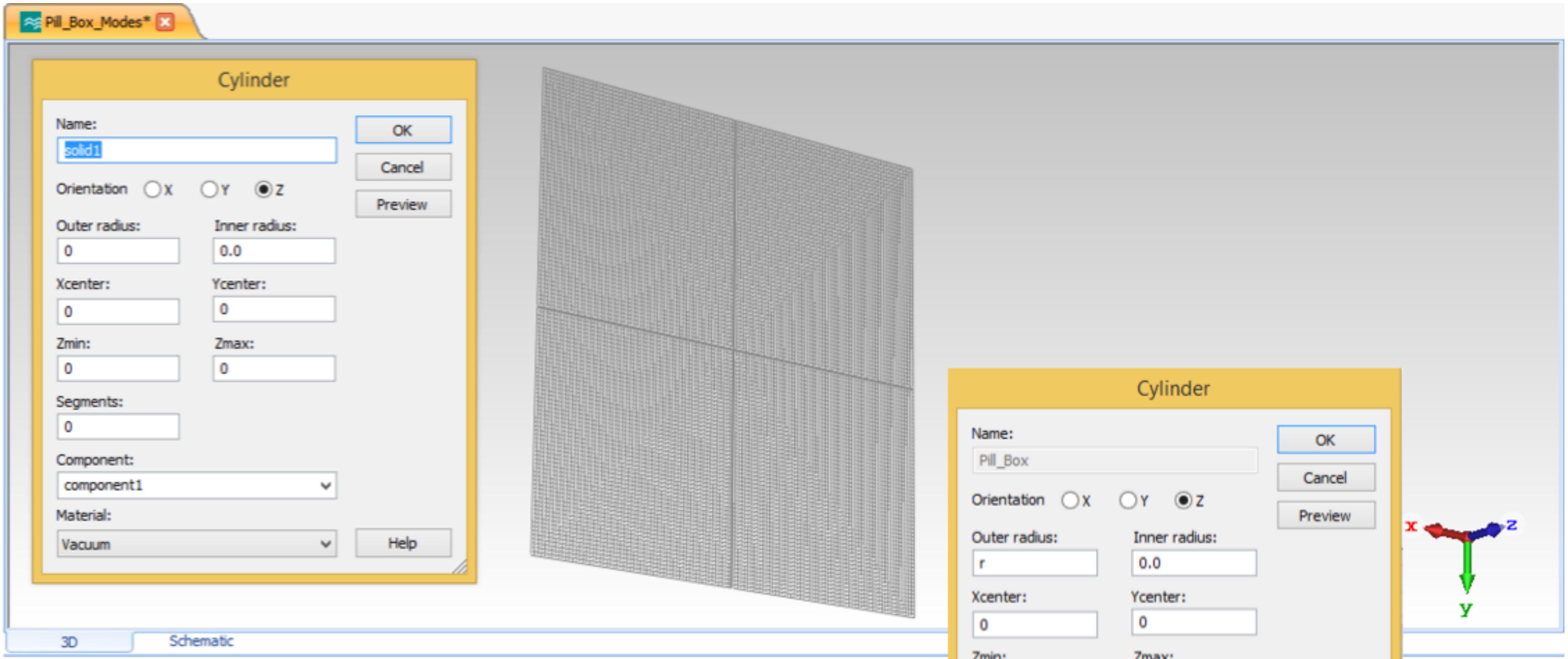
Then let's select the Cylinder



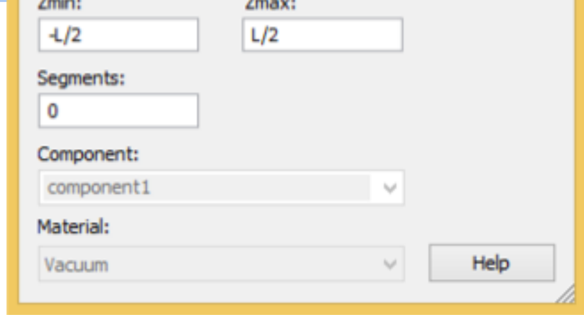
Simulation of a Pill-Box Cavity with CST

MODELLING

Press esc. You should see the following window.



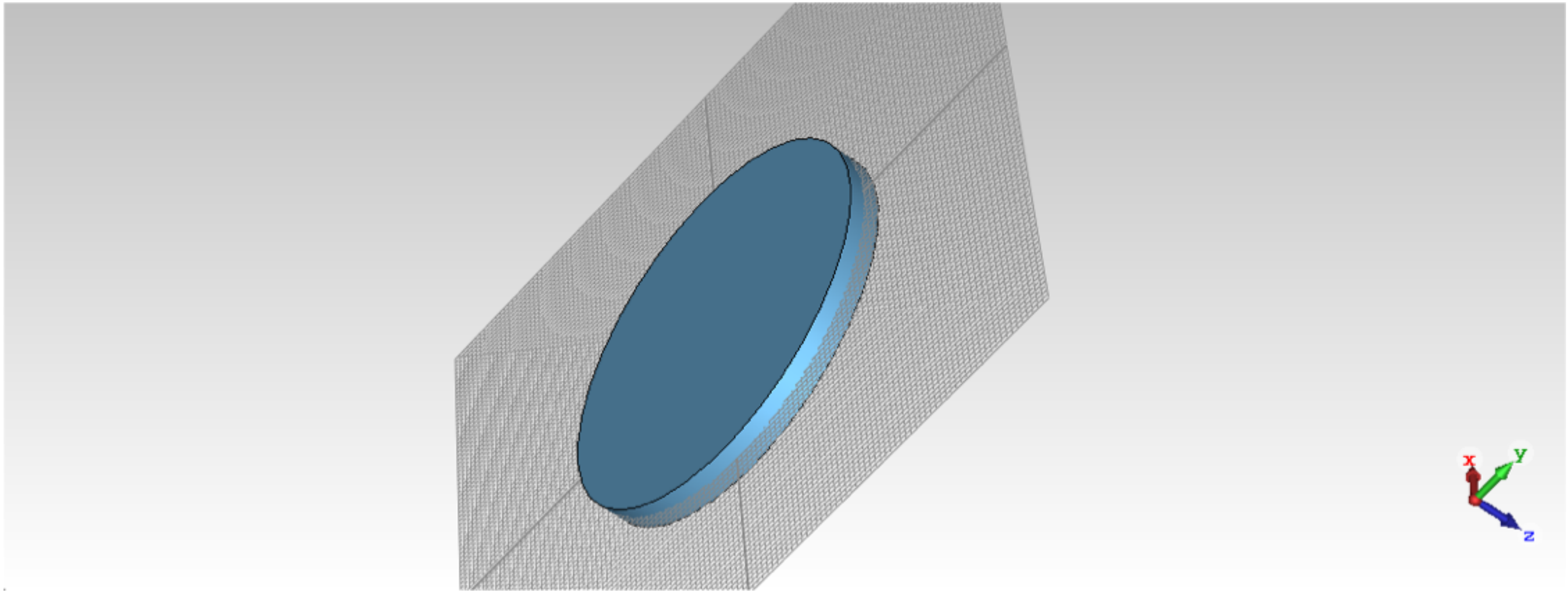
Fill the Windows as shown and press OK



Simulation of a Pill-Box Cavity with CST

MODELLING

This is our Pill Box!

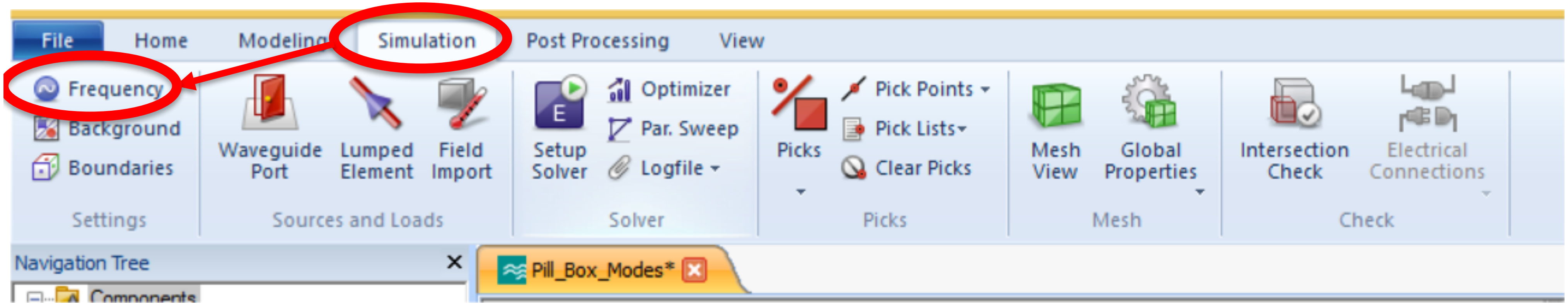


Geometry is done!

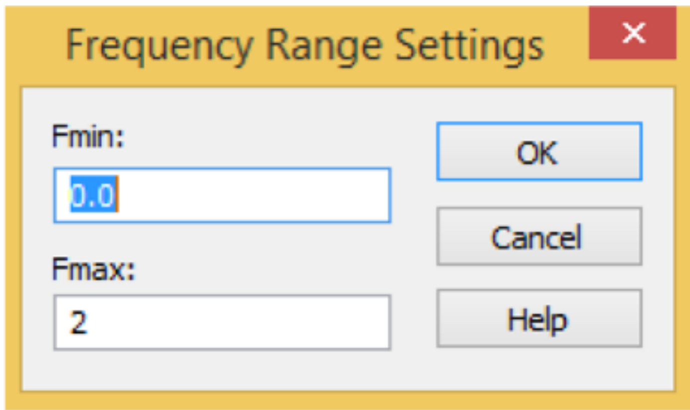
Simulation of a Pill-Box Cavity with CST

ANALYSIS

Let’s move to the electromagnetic field simulations.
Click on “Simulations” on the top menu



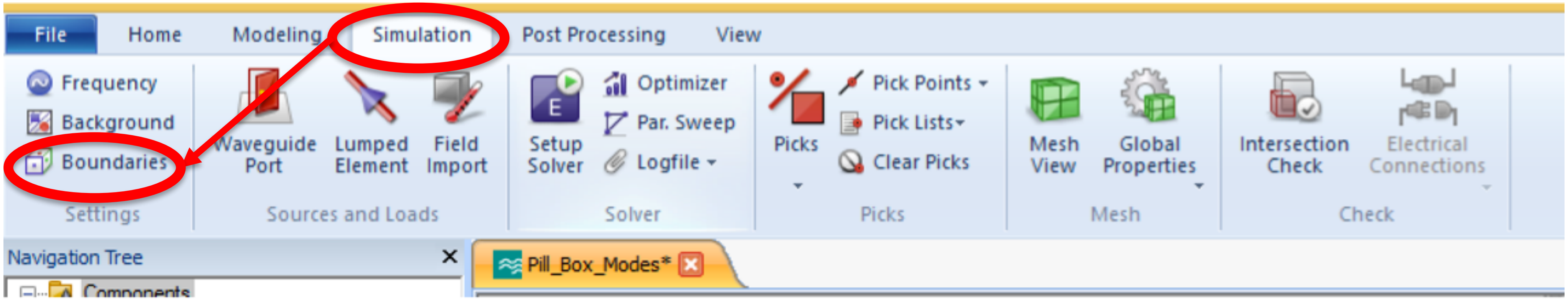
Then select “Frequency”. It will appear a window to set the frequency range where you would like to study the field. We consider frequencies it between 0 and 2 GHz



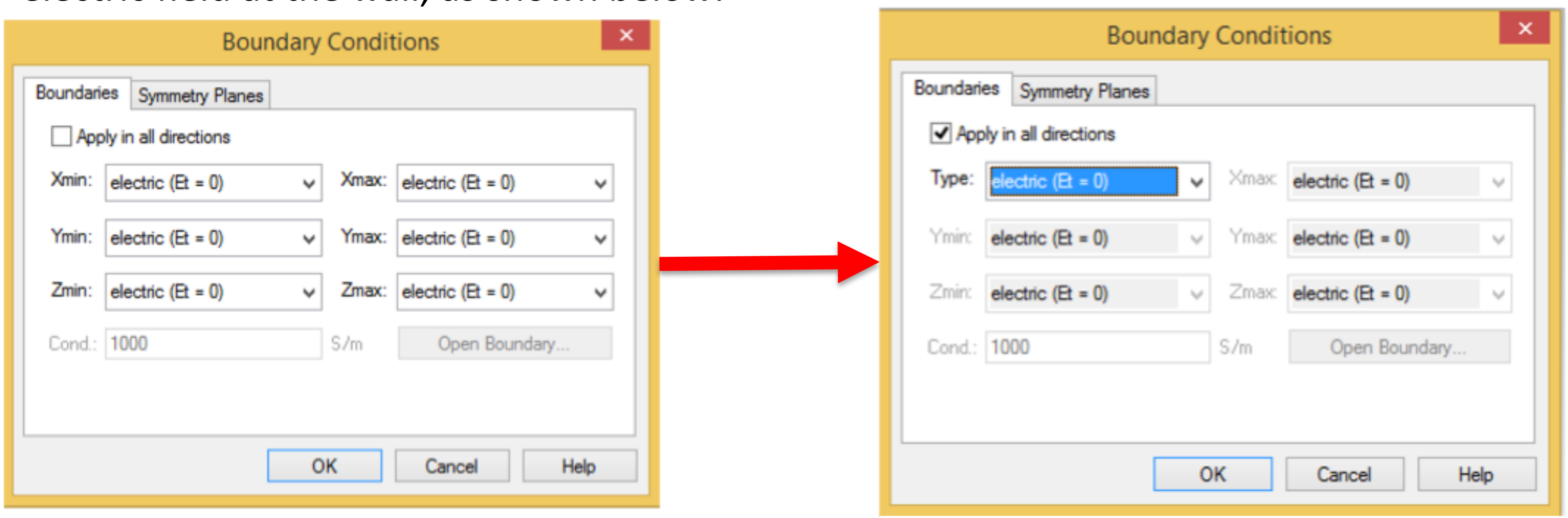
Simulation of a Pill-Box Cavity with CST

ANALYSIS

Still from the "Simulation" bar click on Settings -> Boundaries in order to set the boundary conditions



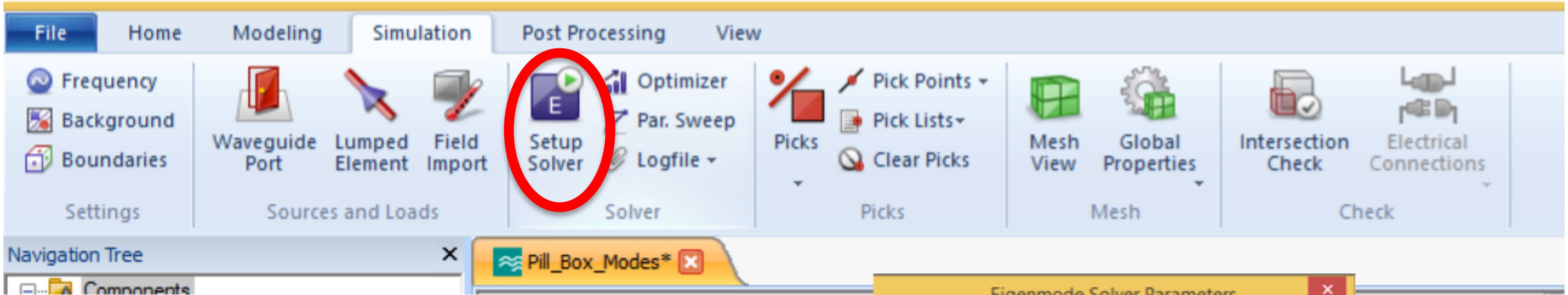
Particularly, in order to find the TM modes let's impose the condition of zero tangential electric field at the wall, as shown below:



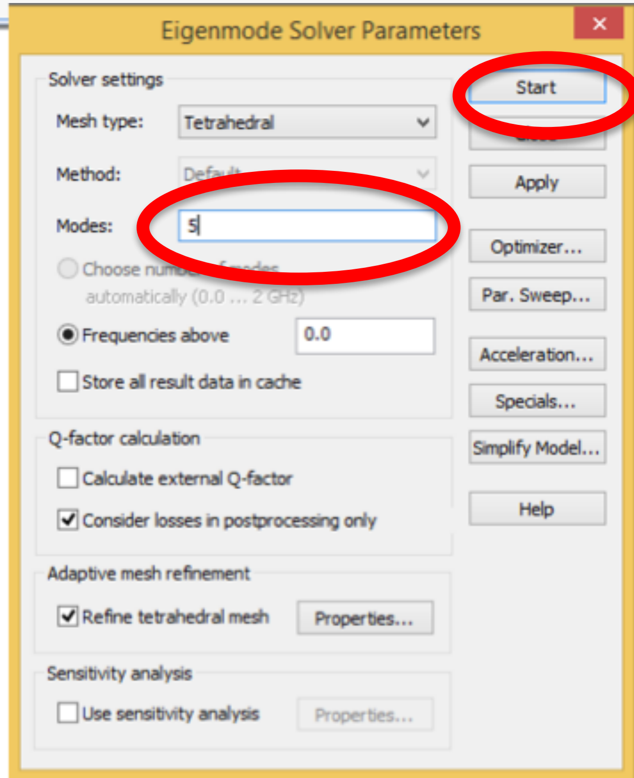
Simulation of a Pill-Box Cavity with CST

ANALYSIS

Now we are ready, from the “Simulation” bar click on ”Setup Solver”



Let’s modify the number of modes from one to five and click on Start

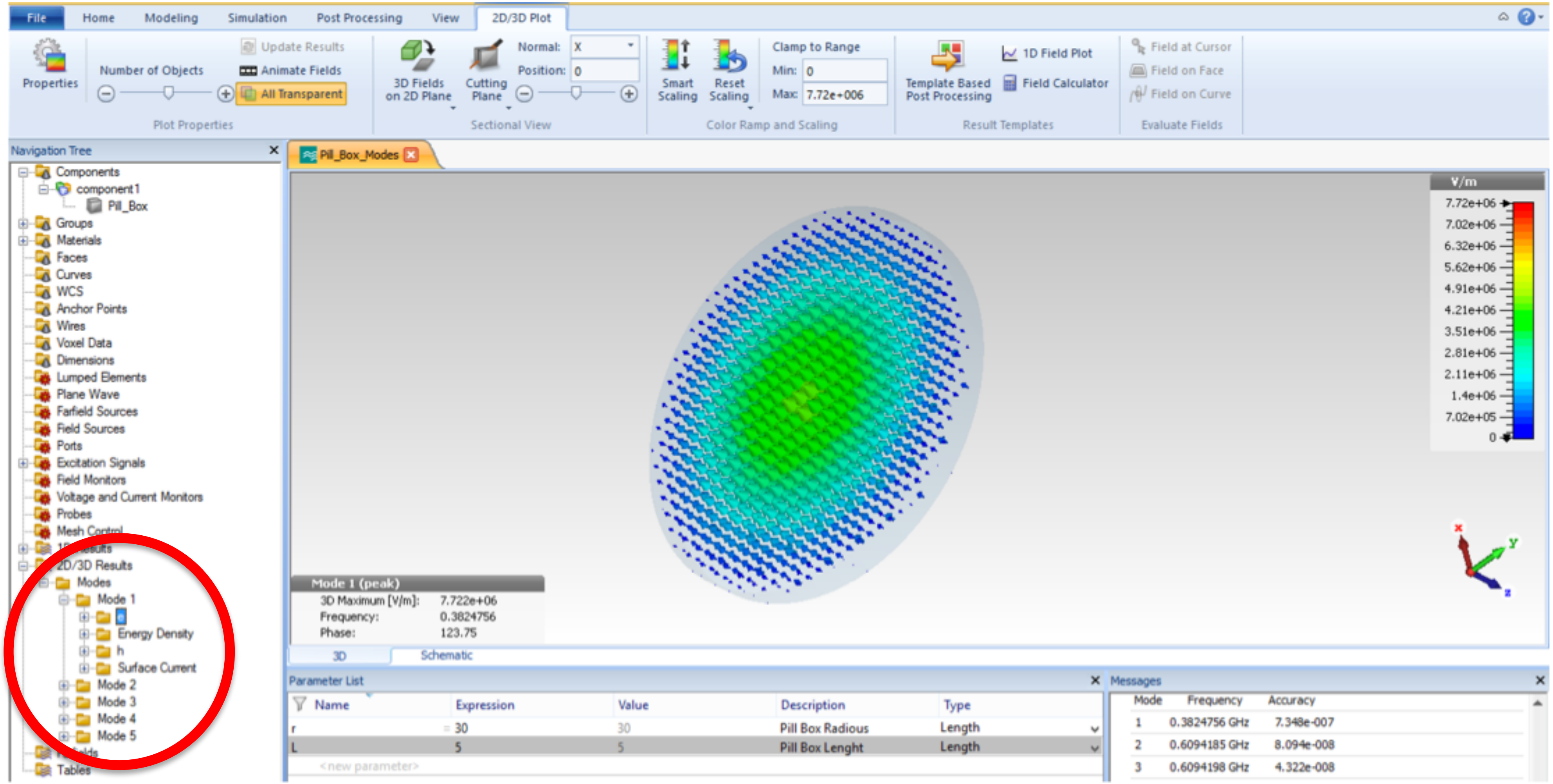


Simulation of a Pill-Box Cavity with CST

ANALYSIS

The results are on the navigation tree.

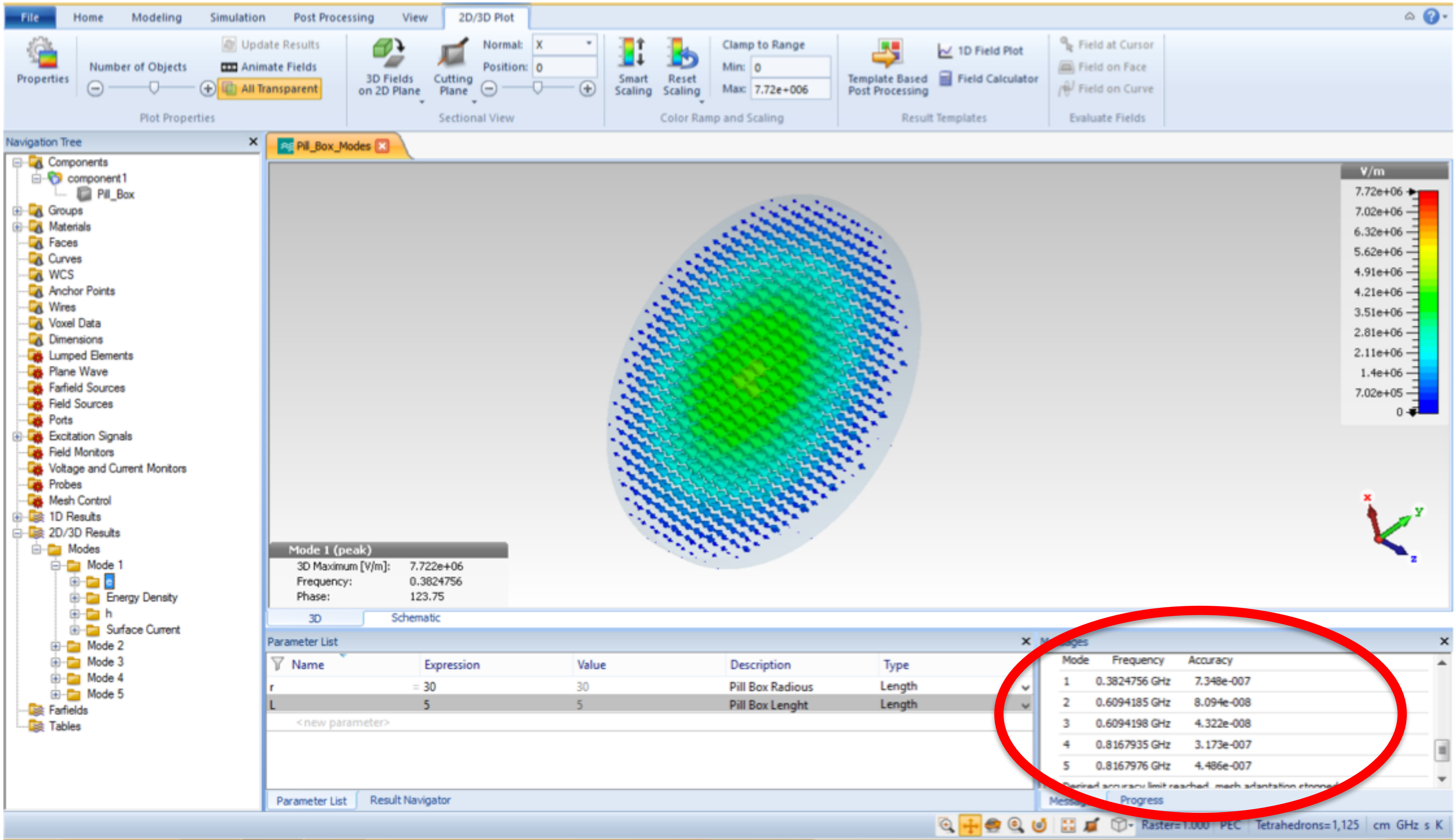
Clicking on the “2D/3D Results” folder we can see the E-field (e), the B-field(h) and some other features for every computed mode. Below the E-field of the mode 1 is reported.



Simulation of a Pill-Box Cavity with CST

ANALYSIS

Let's note that CST gives also the mode frequencies



Simulation of a Pill-Box Cavity with CST

ANALYSIS

In this simple case we can compare some analytical results with the numerical ones. Let's start with the frequencies. Recalling their expression:

$$f_{nml} = \frac{c}{2\pi}\sqrt{\left(\frac{p_{nm}}{r}\right)^2 + \left(\frac{l\pi}{L}\right)^2}$$

Using the values

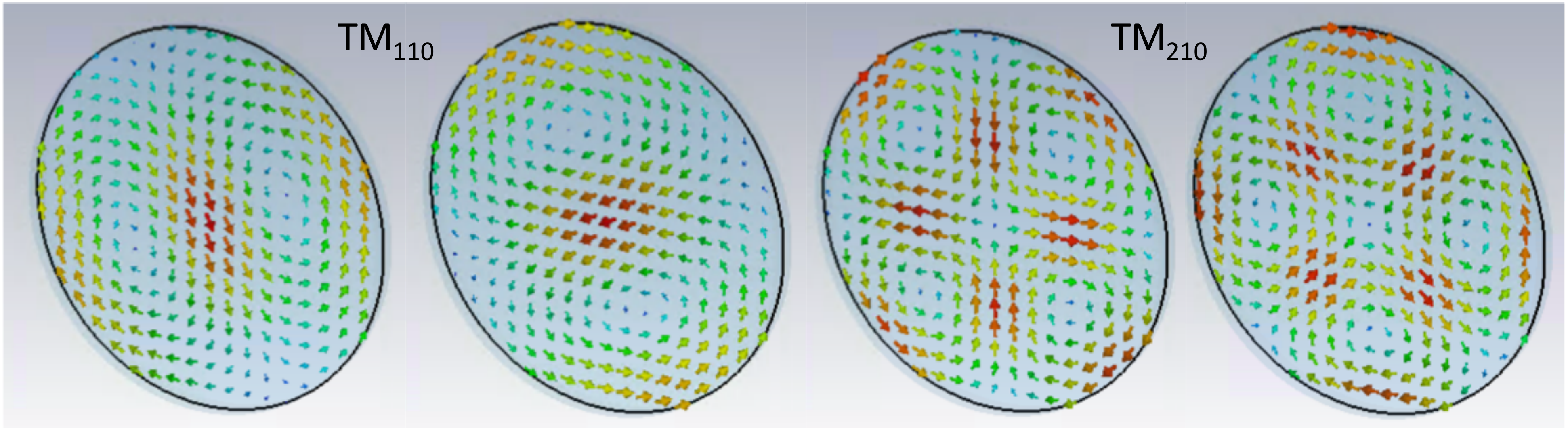
p_{nm} (m \ n)	0	1	2	3	4	5
1	2.4048	3.8317	5.1356	6.3802	7.5883	8.7715
2	5.5201	7.0156	8.4172	9.7610	11.0647	12.3386
3	8.6537	10.1735	11.6198	13.0152	14.3725	15.7002
4	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801
5	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178

Simulation of a Pill-Box Cavity with CST

ANALYSIS

Mode	CST Frequency (GHz)	CST Frequency Accuracy (GHz)	Analytical Frequency (GHz)
TM_{010} (n=0,m=1,l=0)	0.3824756	7.348e-007	0.3824751
TM_{110} (n=1,m=1,l=0)	0.6094185	8.094e-008	0.6094131
TM_{210} (n=2,m=1,l=0)	0.8167935	3.173e-007	0.8167942

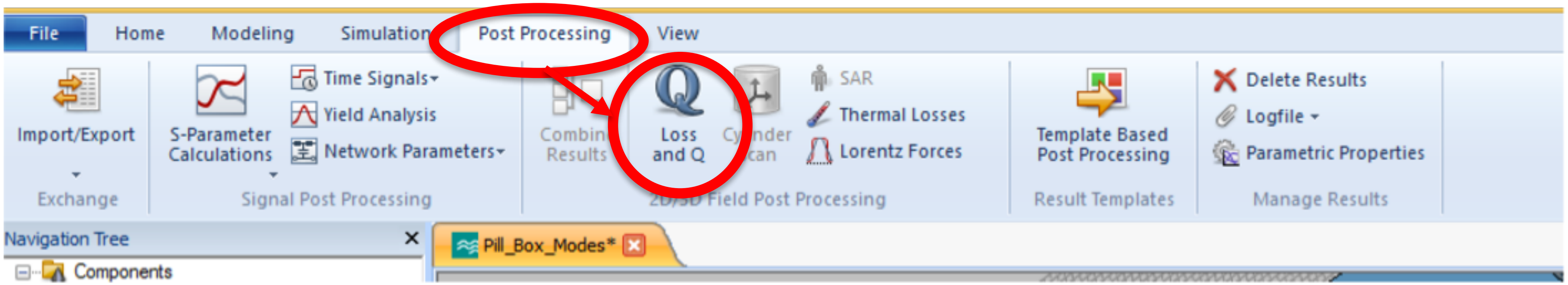
NB: the modes TM_{110} and TM_{210} are not azimuthally symmetric and have two states of polarization:

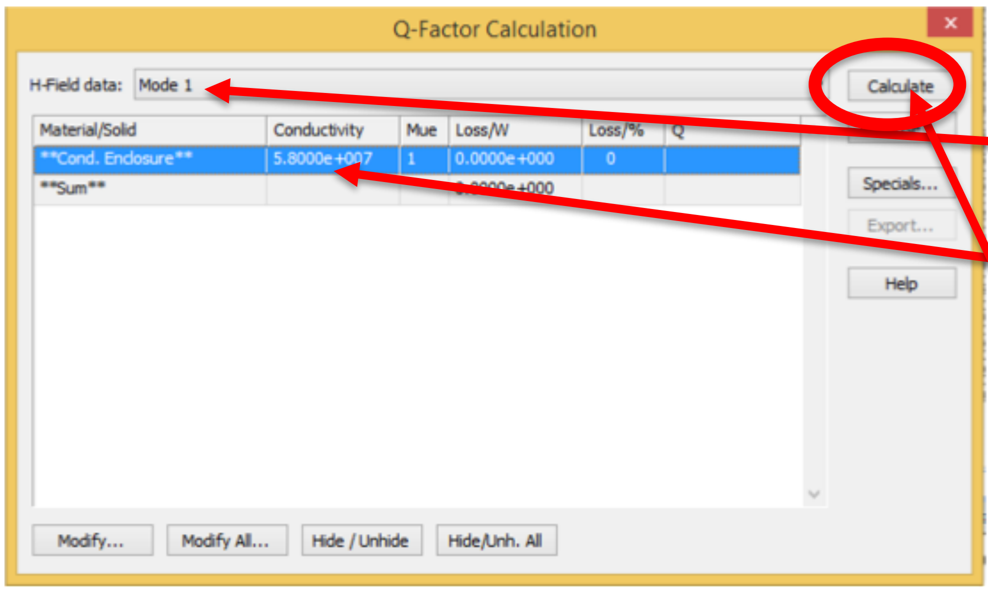


Simulation of a Pill-Box Cavity with CST

ANALYSIS

CST is also capable of computing the Q-factor of a mode if the material conductivity is known. In order to do this let's click on Post Processing (top menu) -> Loss and Q





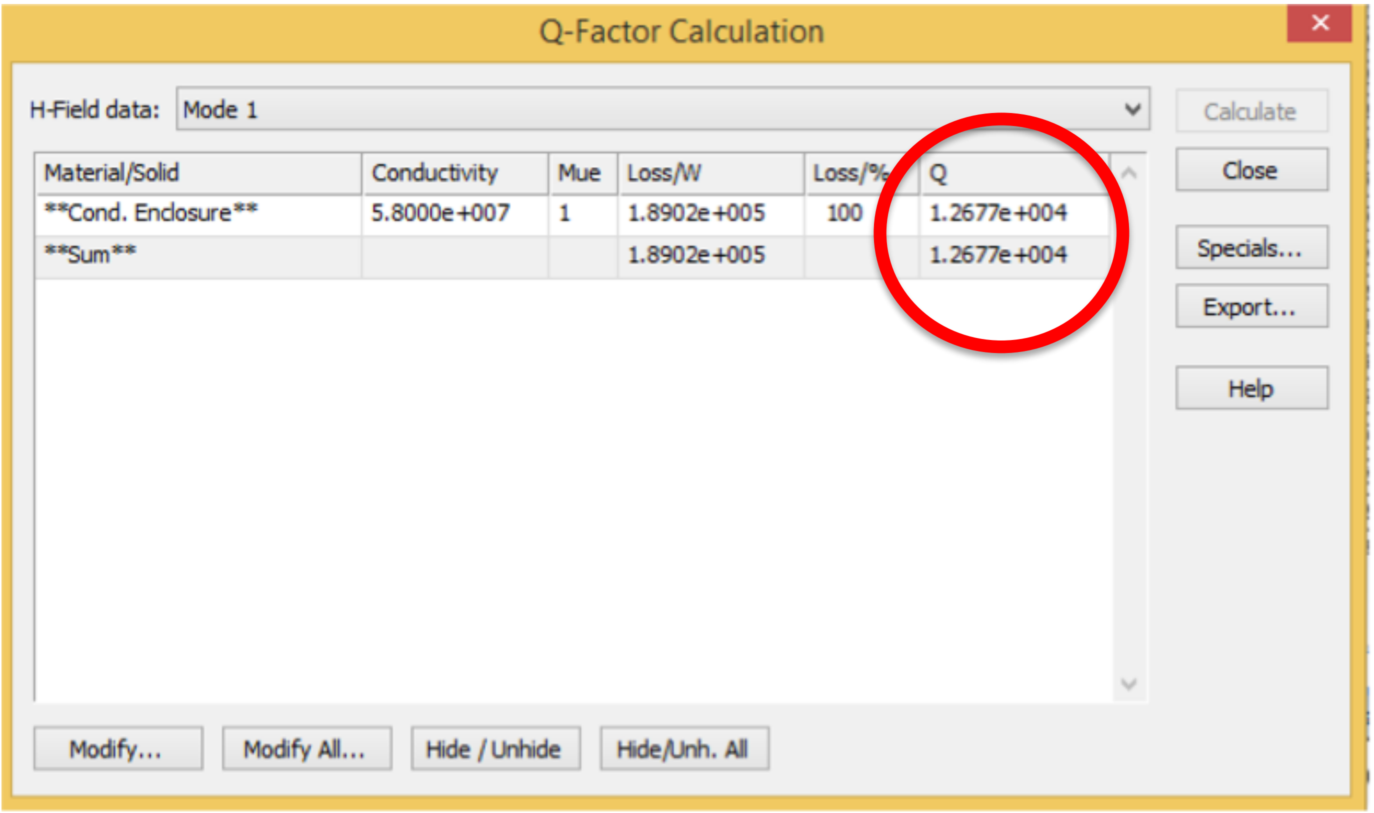
Then, from the appearing window:

- select the mode you want to analyse
- Set the conductivity of the cavity material (according to the dimensional units previously selected), copper is by default.
- Finally click on "Calculate"

Simulation of a Pill-Box Cavity with CST

ANALYSIS

Then, on the Q column, you have the quality factor of the mode.



Simulation of a Pill-Box Cavity with CST

ANALYSIS

As for the frequencies, we can compare the obtained value of the Q-factor with the computed ones. In order to do this, let's briefly recall some theoretical results for a pill-box. In particular, in cylindrical coordinates, the magnetic and electric fields can be written as (From CERN-94-01-V1, page 253 and following):

$$E_z = k_2^2 \cos(k_1 z) J_n(k_2\rho)\cos(n\theta)$$

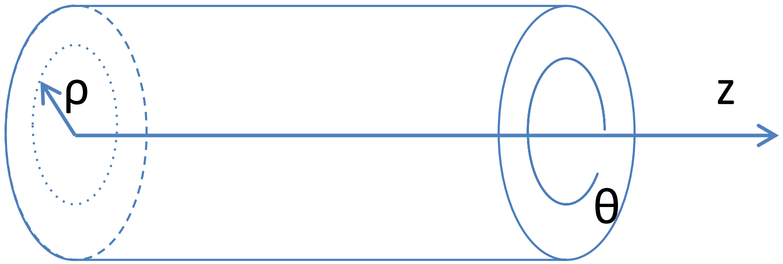
$$E_\rho = -k_1 k_2 \sin(k_1 z) J_n'(k_2\rho)\cos(n\theta)$$

$$E_\theta = \frac{nk_1}{\rho}\sin(k_1 z) J_n(k_2\rho)\sin(n\theta)$$

$$H_z = 0$$

$$H_\rho = -i\frac{nk}{Z_0\rho} J_n(k_2\rho)\sin(n\theta)$$

$$H_\theta = -i\frac{kk_2}{Z_0} J_n'(k_2\rho)\cos(n\theta)$$



In order to have the TM modes the following boundary conditions have to be respected:

$$E_\rho = E_\theta = 0 \quad for \quad z=0 \quad and \quad z=L$$

$$E_z = E_\theta = 0 \quad for \quad \rho = r$$

they lead to:

$$k_1 = \frac{l\pi}{L}, \quad k_2 = \frac{p_{nm}}{r}, \quad k = \frac{2\pi}{c} f_{nml}$$

Here Z_0 is the impedance of free space. The reported field components have a multiplying time dependent term $\cos(2\pi f_{nml} t)$ omitted because of no interest for our purposes.

Simulation of a Pill-Box Cavity with CST

ANALYSIS

The Q factor is defined as: $Q = 2\pi \dfrac{\text{Stored energy}}{\text{Energy lost during one period}}$

The stored energy in the cavity volume is given by: $W_s = \dfrac{\mu}{2}\int_V |H|^2 dV = \dfrac{\varepsilon}{2}\int_V |E|^2 dV$

The energy lost in one period is due to the power dissipated on the walls (Joule effect)

$$P = \frac{1}{2}\int_V \frac{|j|^2}{\sigma_c} dV = \frac{1}{2}\int_S \frac{|j|^2}{\sigma_c} \delta dS$$

with δ the skin depth evaluated at the resonant frequency of the mode: $\delta = \dfrac{1}{\sqrt{\pi\mu\sigma_c f_{nml}}}$

After some manipulation the power loss can be written as: $P = \dfrac{1}{2}\int_S R_w |H|^2 dS$

with the surface resistance: $R_w = \dfrac{1}{\sigma_c \delta}$

The energy lost in one period is the power divided by the resonant frequency

$$W_d = \frac{P}{f_{nml}} = \frac{\pi\mu\delta}{2}\int_S |H|^2 dS$$

Simulation of a Pill-Box Cavity with CST

ANALYSIS

Finally:

$$Q = \frac{2}{\delta} \frac{\int_V |H|^2 dV}{\int_S |H|^2 dS}$$

Considering the TM_{010} mode we get:

$$Q = \frac{L}{\delta} \frac{r}{(r+L)}$$

And considering the values:

L = 0.05 [m]

r = 0.3 [m]

σ_c = 5.8e7 [S/m]

μ = 4πe-7 [H/m]

δ = 3.3791e-06 [m]

CST Quality Factor TM_{010}	Analytical Quality Factor TM_{010}
1.2677e+004	1.2683e+004

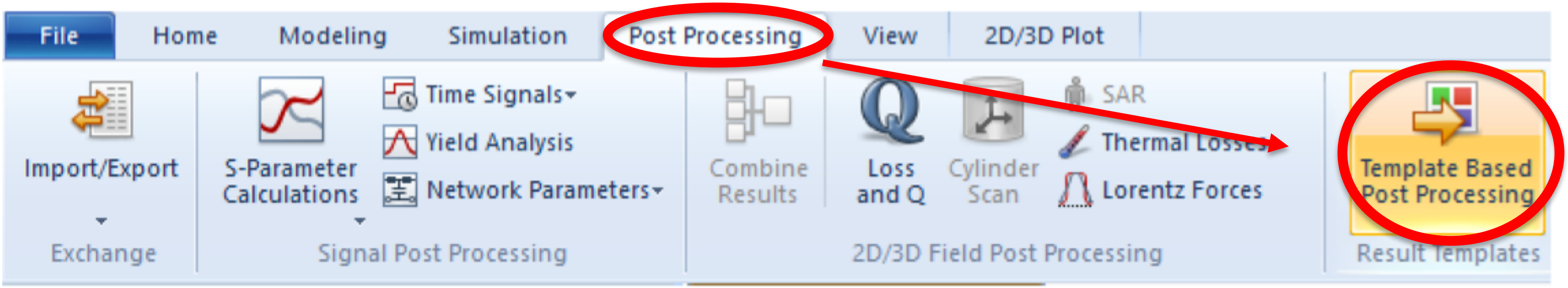
Simulation of a Pill-Box Cavity with CST

ANALYSIS

Another factor of interest for a cavity is R, the shunt resistance, (for further analysis about it see the Fritz Caspers lessons in the second course). Its expression is given by (Linac convention):

$$R = 256\ Q\ \frac{r \sin\left(1.2024 \frac{L}{r}\right)^2}{L}$$

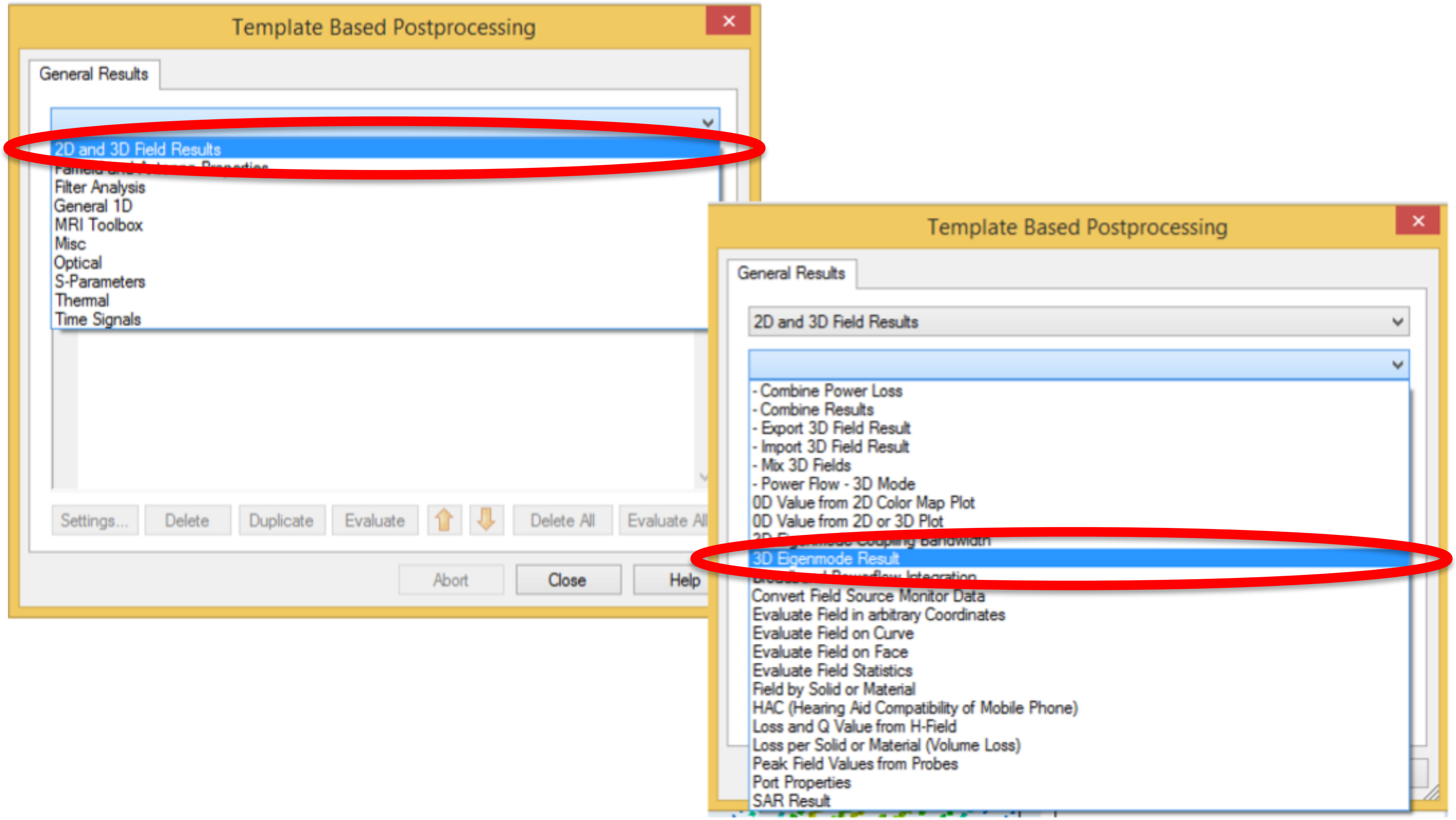
In order to compute it with CST let's go to Post Processing -> Template Based Post Processing



Simulation of a Pill-Box Cavity with CST

ANALYSIS

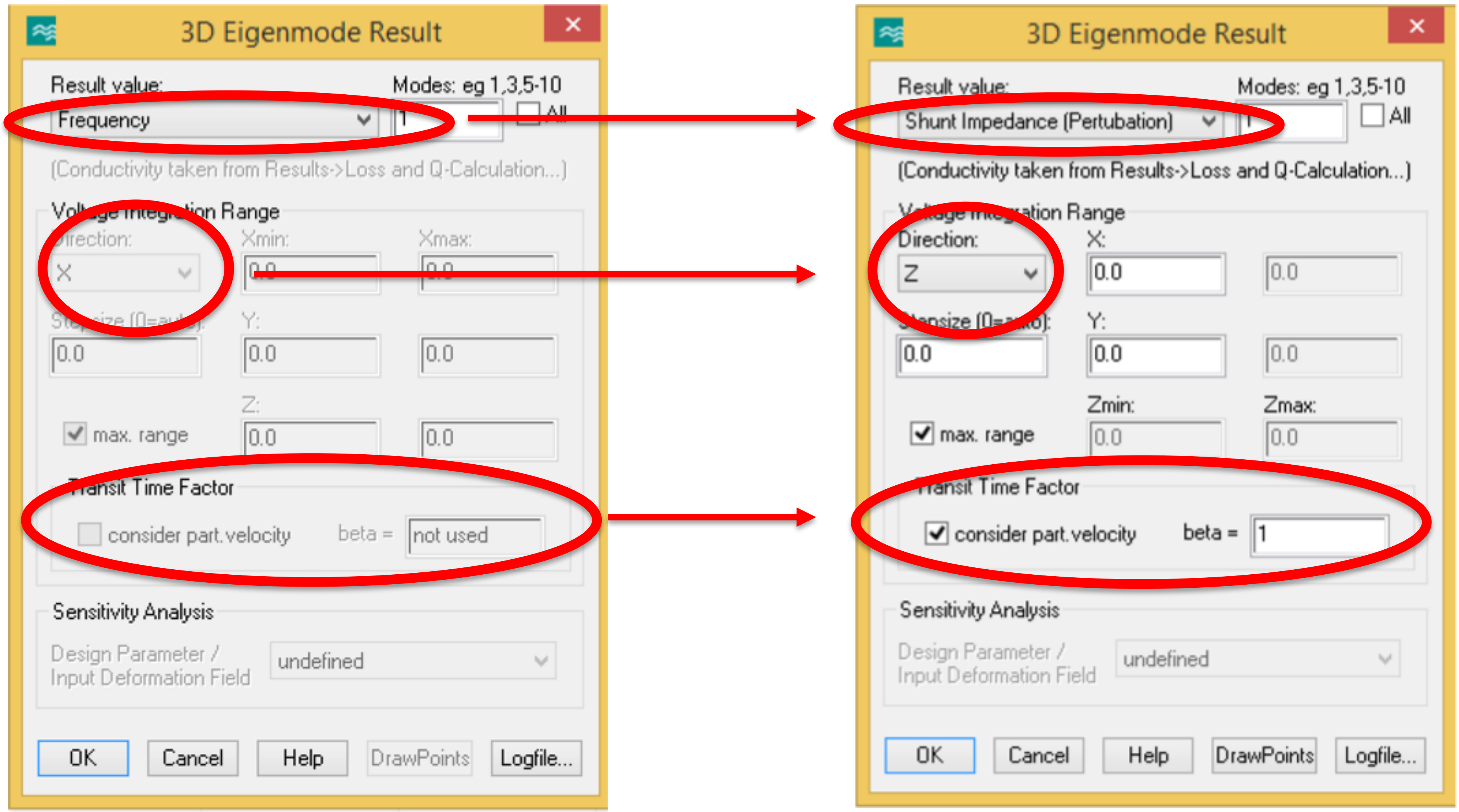
Select “2D and 3D Field Results” from the first drop-down menu and then “3D Eigenmode Result” from the second one.



Simulation of a Pill-Box Cavity with CST

ANALYSIS

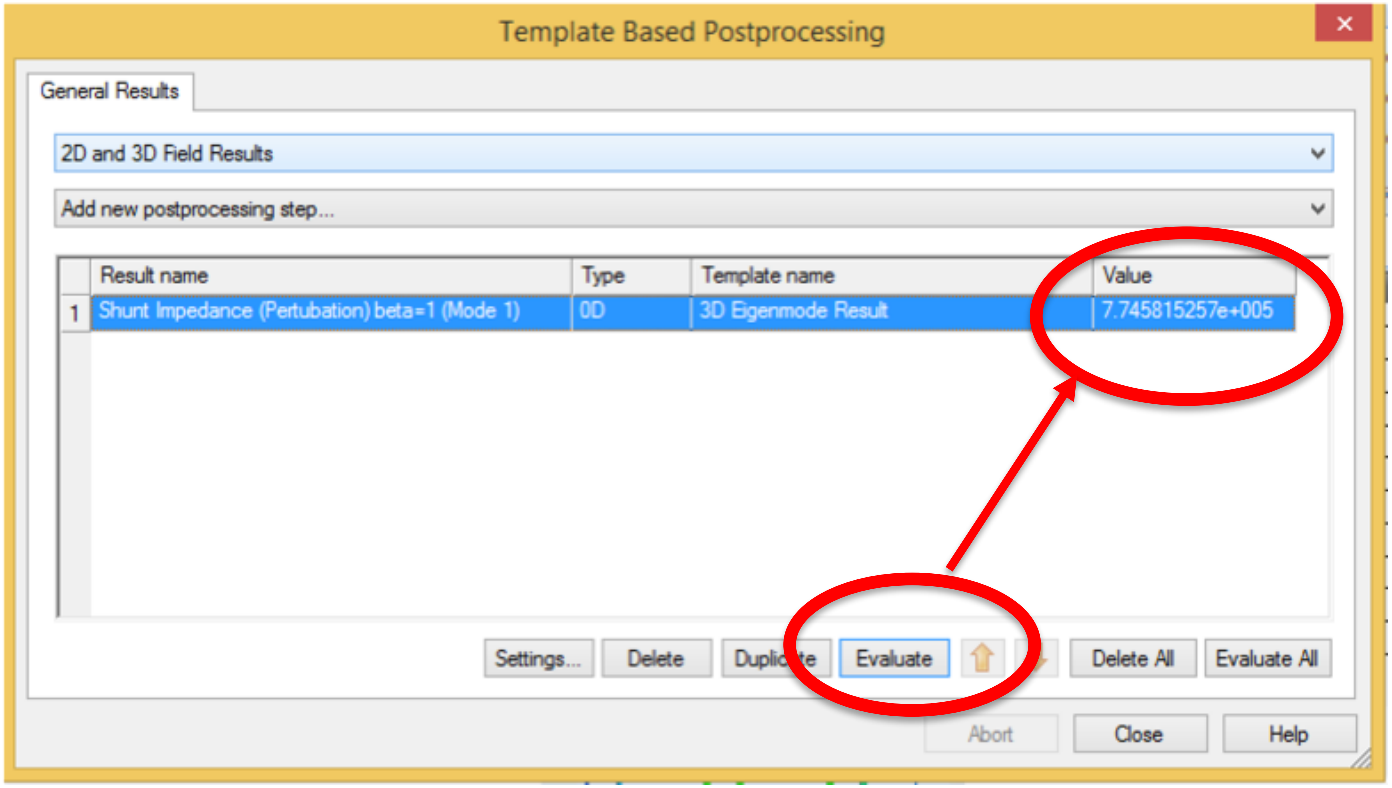
Then set the value of the appearing window as shown below and press ok.



Simulation of a Pill-Box Cavity with CST

ANALYSIS

You should see the following window, click Evaluate and then you will get the value of the shunt impedance on the “Value” column.



CST Shunt Impedance TM_{010}	Analytical Shunt Impedance TM_{010}
7.745815257e+05 [Ω]	7.719408360e+05 [Ω]