

Exercises on Wake Fields and Instabilities

Exercise 1:

Show that the impedance of an RLC parallel circuit is that of a resonant mode and relate R , L and C to Q , R_s and ω_r

Exercise 2:

Calculate the amplitude of the resonator wake field given $R_s = 1 \text{ k}\Omega$, $\omega_r = 5 \text{ GHz}$, $Q = 10^4$

Calculate the ratio $|Z(\omega_r)| / |Z(2\omega_r)|$ for $Q = 1, 10^3, 10^5$

Exercise 3: Beam Break Up

Consider a beam in a linac at 1 GeV without acceleration. Obtain the growth of the oscillation amplitude after 3 km if:

$$N = 5e10, w_{\perp}(-1 \text{ mm}) = 63 \text{ V}/(\text{pC m}), L_w = 3.5 \text{ cm}, k_y = 0.06 \text{ 1/m}$$

Exercise 4: Beam Break Up (2)

Consider the same beam of the previous exercise being now accelerated from 1 GeV with a gradient $g = 16.7$ MeV/m. Obtain the growth of the oscillation amplitude

$$E_f = E_0 + gL_L \approx gL_L = 50 \text{ GeV}$$

$$\left(\frac{\Delta \hat{y}_2}{\hat{y}_2} \right)_{\max} = \frac{c N e w_{\perp}(z) L_L}{4 \omega_y (E_f / e) L_w} \ln \frac{E_f}{E_0} = ?$$

Exercise 5: Evaluate the energy lost per unit length by a charge due to the longitudinal wake field of the space charge and compare it with the longitudinal space charge force in $r=0$

Exercise 6: Evaluate the energy spread ($U_{\max}-U_{\min}$) of a Gaussian bunch of RMS length σ due to the longitudinal wake field of the space charge in a structure of length L

Exercise 7: Evaluate the energy lost by a charge inside a uniform beam of length l_0 due to the longitudinal wake field of a pill box cavity of length g at high frequency $\omega \gg c/b$ (diffraction model), with b the pipe radius

$$w(z) = \frac{Z_0 c \sqrt{2g}}{2\pi^2 b} \frac{1}{z^{1/2}}$$