Exercises on Space Charge

Exercise 1

Compute the transverse space charge forces and the incoherent tune shifts for a cylindrical beam in a circular beam pipe, having the following longitudinal distributions: parabolic, sinusoidal modulation, Gaussian.

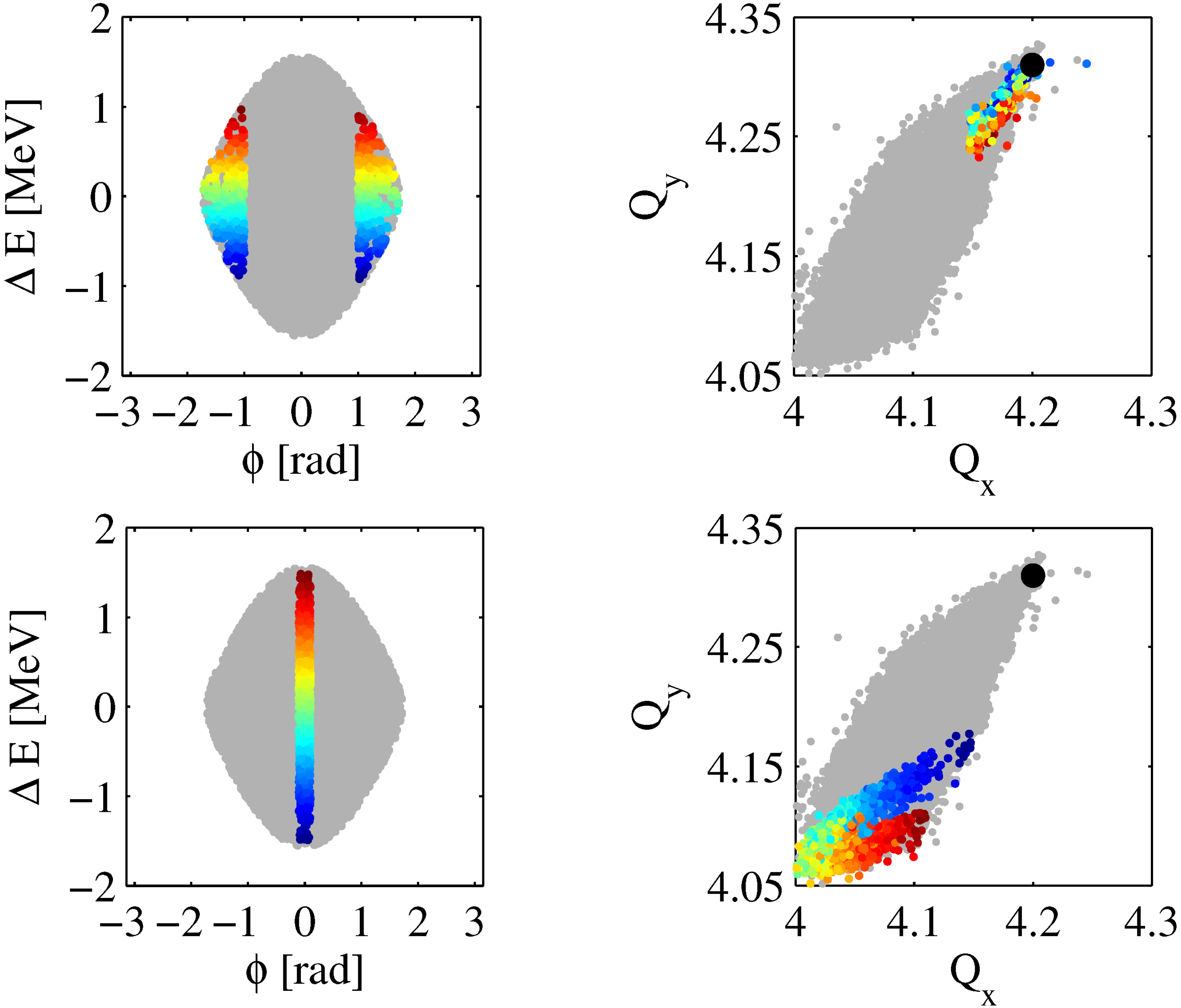
Evaluate also the tune spread (max tune shift – min tune shift) produced by the space charge forces with the same distributions.

parabolic
$$\lambda(z) = \frac{3Ne}{2l_o}\left[1 - \left(\frac{2z}{l_o}\right)^2\right]$$

sinusoidal modulation
$$\lambda(z) = \lambda_o + \Delta\lambda \cos(k_z z) \quad ; \quad k_z = 2\pi / \lambda_w$$

Gaussian
$$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

Effect of longitudinal distribution



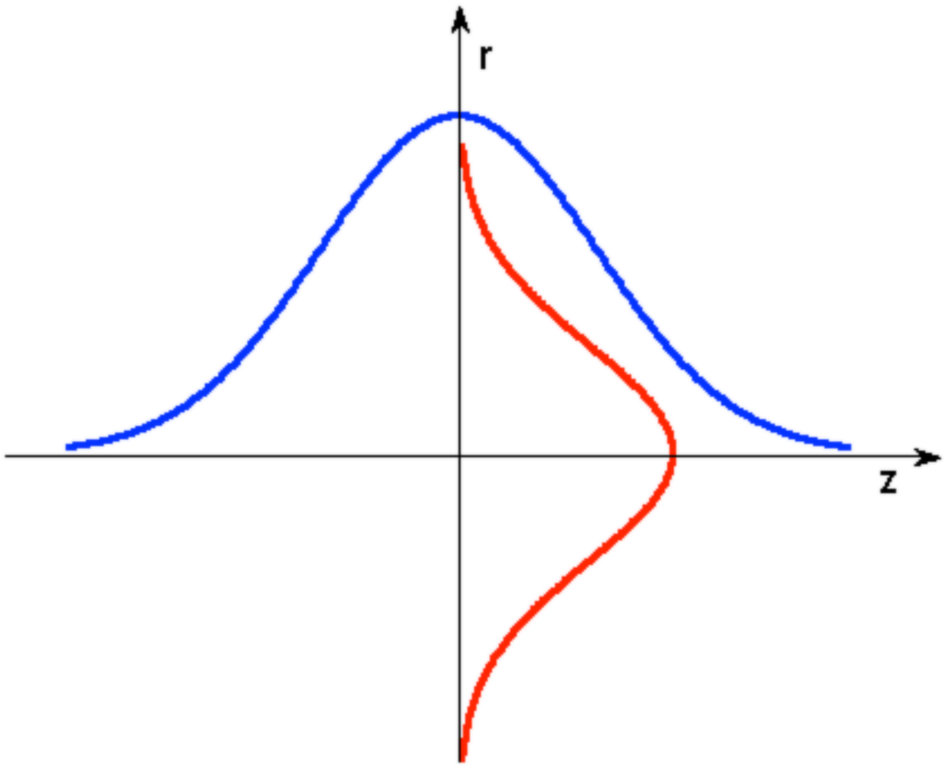
Longitudinal phase-space ($\Delta\phi$, ΔE) scatter plot of the bunch tune footprint. The black dot is the bare tune. Particles at the edges of the bunch have tunes close to the bare tune in the necktie.

Indeed, in this longitudinal region, the beam line density is smaller with respect to the centre of the bunch, therefore also the space charge detuning is small.

(Courtesy of V. Forte, 'Performance of the CERN PSB at 160 MeV with H- charge exchange injection', PhD thesis, Université Blaise Pascal, Clermont-Ferrand, France, 2016)

Exercise 2

Compute the transverse space charge force and the incoherent tune shift for a cylindrical beam in a circular beam pipe, having a bi-Gaussian longitudinal and transverse distribution.



$$bi-Gaussian$$

$$\lambda(z)=\frac{Ne}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

$$\rho(r,z)=\frac{\lambda(z)}{2\pi\sigma_r^2}\exp\left(\frac{-r^2}{2\sigma_r^2}\right)$$

Exercise 3

Evaluate the dependence of the longitudinal and transverse space charge force with z at fixed r (e.g. << σ_r) for the bi-Gaussian distribution

Exercise 4

Compute the longitudinal space charge force of a transverse uniform cylindrical beam in a circular perfectly conducting beam pipe

Exercise 5

Compute the longitudinal space charge forces for a cylindrical beam in a circular beam pipe, having the following longitudinal distributions: parabolic, sinusoidal modulation, Gaussian

parabolic

$$\lambda(z) = \frac{3Ne}{2l_o}\left[1 - \left(\frac{2z}{l_o}\right)^2\right]$$

sinusoidal modulation

$$\lambda(z) = \lambda_o + \Delta\lambda \cos\left(k_z z\right) \quad ; \quad k_z = 2\pi / l_w$$

Gaussian

$$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

Exercise 6

Compute the incoherent betatron tune shift of a uniform proton beam inside two perfectly conducting parallel plates