- JUAS 2018 -

GUIDED STUDY AND TUTORIAL ON RF LINEAR ACCELERATORS

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Problem 1: *Pillbox cavity*

Suppose that you have to design a TM_{010} mode pillbox cavity (see Fig.3) with a square-wave electric field distribution and you are free to choose the length. If the length is too short, the voltage gain across the cavity is small; if it is too long, the transit-time factor is small.

- **a.** Plot the transit-time factor versus $\frac{g}{\beta\lambda}$: the ratio of length to $\beta\lambda$ (for $0 < \frac{g}{\beta\lambda} < 2$).
- **b.** Find the ratio of the length to $\beta\lambda$ that maximizes the value of the energy gain for the cavity.
- **c.** Calculate the transit-time factor at this length.
- **d.** What should be the length of the cavity to maximize the energy gain per unit length? Is it a practical solution?

Problem 2: *Drift-tube linac (DTL)*

Suppose that we want to design a continuous wave (CW) room-temperature drift-tube linac to accelerate a 100 mA proton beam from 3 to 20 MeV. Assume for the RF power we can purchase 350 MHz klystrons of 1 MW capacity each. Suppose we run the SUPERFISH electromagnetic field-solver code and obtain for all β values the following results: transit-time factor $T=0.8$, effective shunt impedance $ZT^2=50\ M\Omega/m$ and the ratio of the peak surface electric field to the average axial electric field $E_s/E_0=6$. We restrict the peak surface electric field at a bravery factor $b=E_s/E_K=1.2$ (For E_K see the Kilpatrick limit criterion plot in Fig.4). For adequate longitudinal acceptance we choose the synchronous phase $\phi_s = -30^\circ$.

- **a.** Calculate the average axial electric field E_0.
- **b.** Calculate the length of the linac assuming it consists of a single tank.
- **c.** Calculate the structure power P_S (power dissipated in the cavity), beam power P_B and the total RF power required.
- **d.** Calculate the structure efficiency ε_s (ratio of beam power to total RF power).
- **e.** How many klystrons do we need for our structure?

Problem 3: *Cell length and Energy gain*

For the DTL of Problem 2;

- **a.** Calculate the wavelength at 350 MHz.

b. Calculate first the relativistic γ and then the relativistic β of the protons at the entrance (3MeV) and at the exit (20MeV) of the DTL tank.
c. Estimate the length of the first and the last cell of the DTL tank using the values calculated in **a.** and **b.**
d. Estimate and compare the energy gain at the first and the last cell of the DTL tank.

Problem 4: *Transverse phase advance and RF focusing/defocusing*

If the cavity is not fed with RF power, there won't be any electric field available to accelerate the beam. However, this cavity can still be used for the transportation of the beam without acceleration. For example, DTL structure of Linac4 is composed of 3 tanks and will accelerate the beam up to 50MeV. During the commissioning of the DTL, the beam parameters will be measured at a measurement bench installed after the 3rd tank of the DTL. At one stage of the commissioning, the 3rd tank will be switched off (no RF power will be sent into the tank) and 30MeV beam will be measured at the measurement bench.

For the DTL of Problem 2, assume that all the quadrupoles in the tank have the same length $l_q = 45\text{mm}$ and quadrupole gradient G=106.3T/m.

a. Calculate the zero current transverse phase advance per focusing period at the injection energy when the RF is switched off.
b. Calculate the zero current transverse phase advance per focusing period at the injection energy when the cavity is fed with RF power. What effect do the RF fields have in transverse planes (focusing or defocusing)?
c. Now assume that we are experimenting with RF and shift the synchronous phase to $\phi_s = 30^\circ$. Calculate the zero current transverse phase advance per focusing period at the injection energy for the new synchronous phase $\phi_s = 30^\circ$ What effect do the RF fields have in transverse planes (focusing or defocusing)?
d. Is there any synchronous phase which will result in both transverse and longitudinal focusing?

Problem 5: *Longitudinal acceptance*

Suppose that for the designed DTL the longitudinal acceptance is an upright ellipse in the longitudinal phase-space and can be expressed as $A = \Delta\phi\Delta W$. Suppose it is smaller than the emittance of the beam. Which parameter and how should be changed to increase the longitudinal acceptance of the accelerator?

Problem 6: *Longitudinal phase advance*

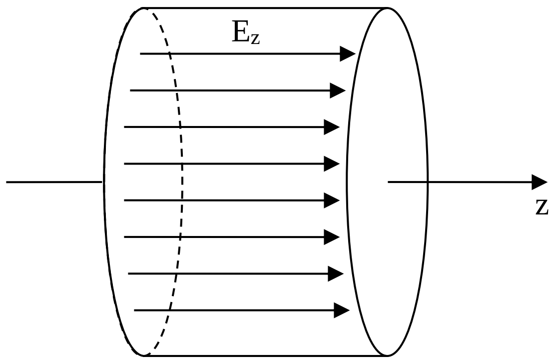
For the DTL of Problem 2 calculate the zero current longitudinal phase advance per focusing period σ_{0l} in degrees at the injection energy for a FODO focusing lattice (period $P = 2\beta\lambda$) and assuming that β does not change through one period.

Problem 7: *Energy acceptance*

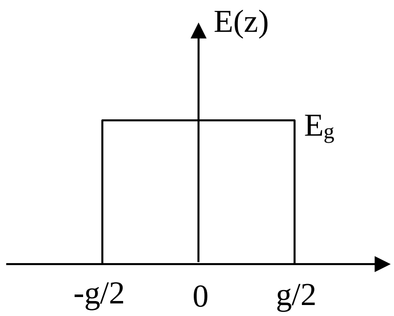
For the DTL of Problem 2 derive and calculate the maximum energy acceptance ΔW_{max} (see Fig.5) for the synchronous phase ϕ_s = -30° at the injection energy from the equation of the separatrix in the longitudinal phase-space:

$$\frac{\omega}{2m_0c^3\beta_s^3\gamma_s^3}\Delta W^2 + qE_0T[\sin(\phi_s + \Delta\phi) + \sin\phi_s - (2\phi_s + \Delta\phi)\cos\phi_s] = 0 \, .$$

Figures



TM_{010} mode in a pillbox cavity



Square-wave electric field distribution

Fig. 1*Pillbox cavity*

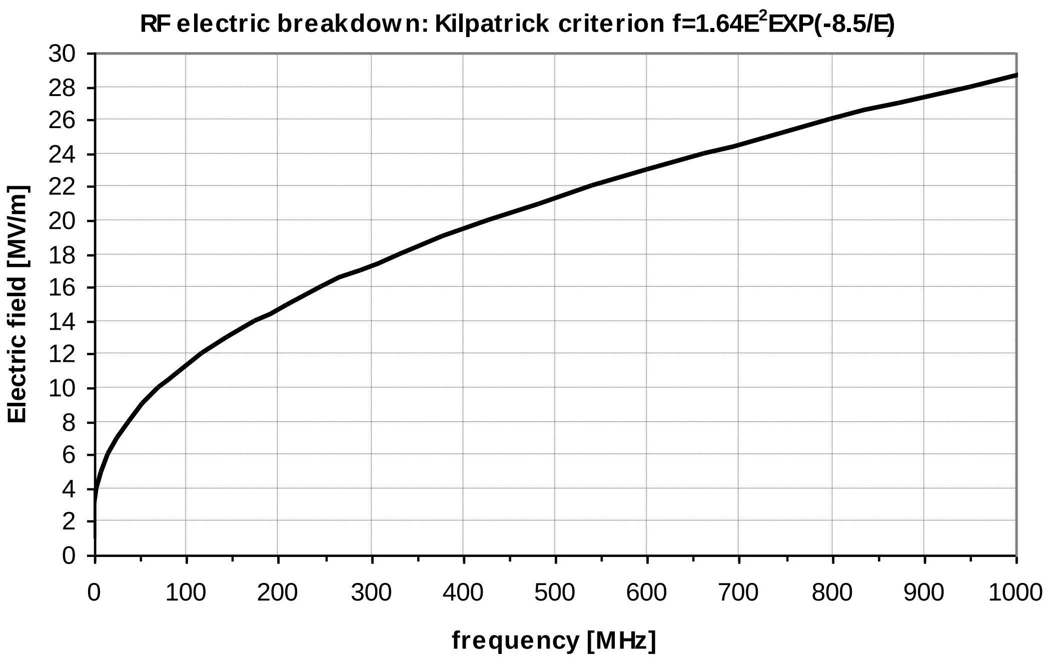


Fig. 2 *Kilpatrick limit for RF electric breakdown*

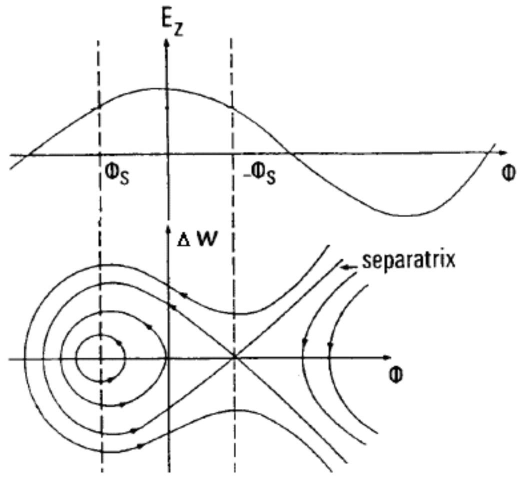


Fig. 3 *Separatrix in the longitudinal phase-space*

Constants

Proton rest mass $E_r = m_0 c^2$ =938.27 *MeV*

c =299792458 m/s

Useful expressions

$$f_{RF} = \frac{\omega}{2\pi} \text{ (RF frequency)}$$

$$\gamma = 1 + \frac{W}{E_r},\ \beta = \sqrt{1 - \frac{1}{\gamma^2}} \text{ (relativistic factors)}$$

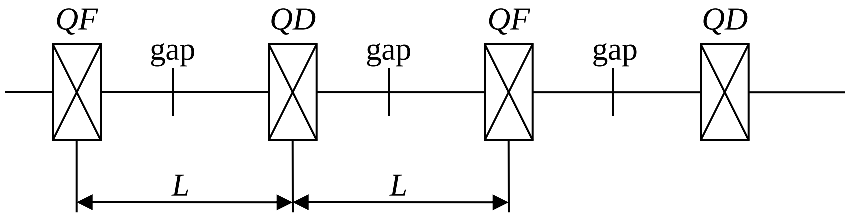
Energy gain in a cavity: $\Delta W = qE_0 TL \cos\phi_s$

Effective shunt impedance: $ZT^2 = \dfrac{(E_0 T)^2}{P_S / L}$

Beam power (CW): $P_B = I\Delta W / q$

Transverse phase advance per period (FODO lattice):

$$\sigma_{0t} \approx \sqrt{\left(\frac{qGl_qL}{m_0c\gamma\beta}\right)^2 - \frac{\pi qE_0T\sin(-\phi)(2L)^2}{m_0c^2\lambda(\beta\gamma)^3}}$$



FODO period P=2L

Longitudinal phase advance per unit length: $k_{0l} = \sqrt{\frac{2\pi qE_0T\sin(-\phi_s)}{m_0c^2\beta_s^3\gamma_s^3\lambda}}$