

# JUAS 2018 – Tutorial 2

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$$\mu = \mu_0 \mu_r$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/(Am)}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\varepsilon_0 = 8.854 \cdot 10^{-12} \text{ As/(Vm)}$$

$$c_0 = 2.998 \cdot 10^8 \text{ m/s}$$

## 1.) S-Parameters

Match the following S-Matrices to the corresponding components

$$S_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad S_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad S_3 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \quad S_4 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Component	Isolator	Circulator	Transmission line, length = $\lambda/2$	3-dB attenuator
S-matrix	$S_2$	$S_4$	$S_3$	$S_1$

## 2.) Impedances in the complex plane

### Questions:

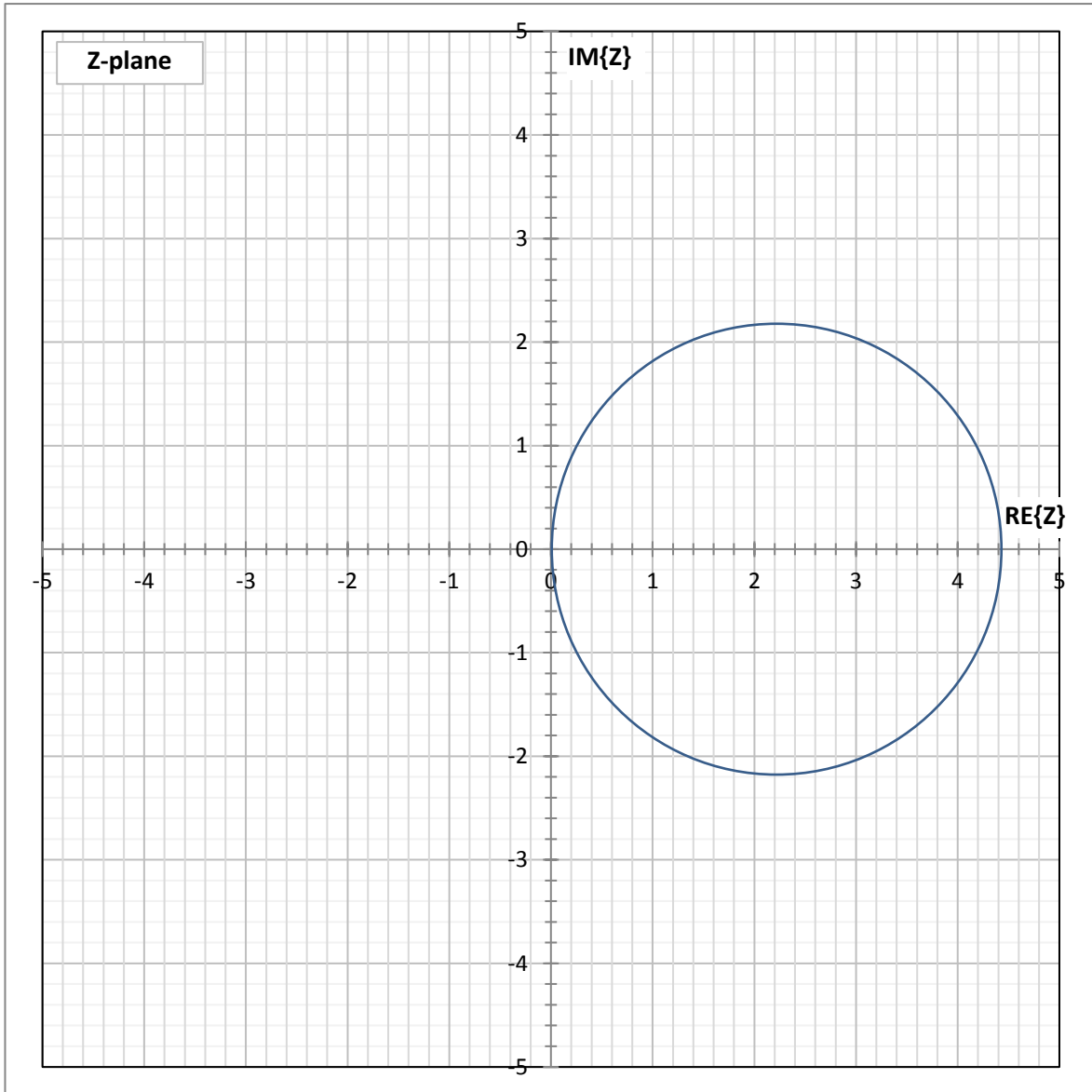
- Plot the following impedances in the Z-plane, use the plot axes on the next page:

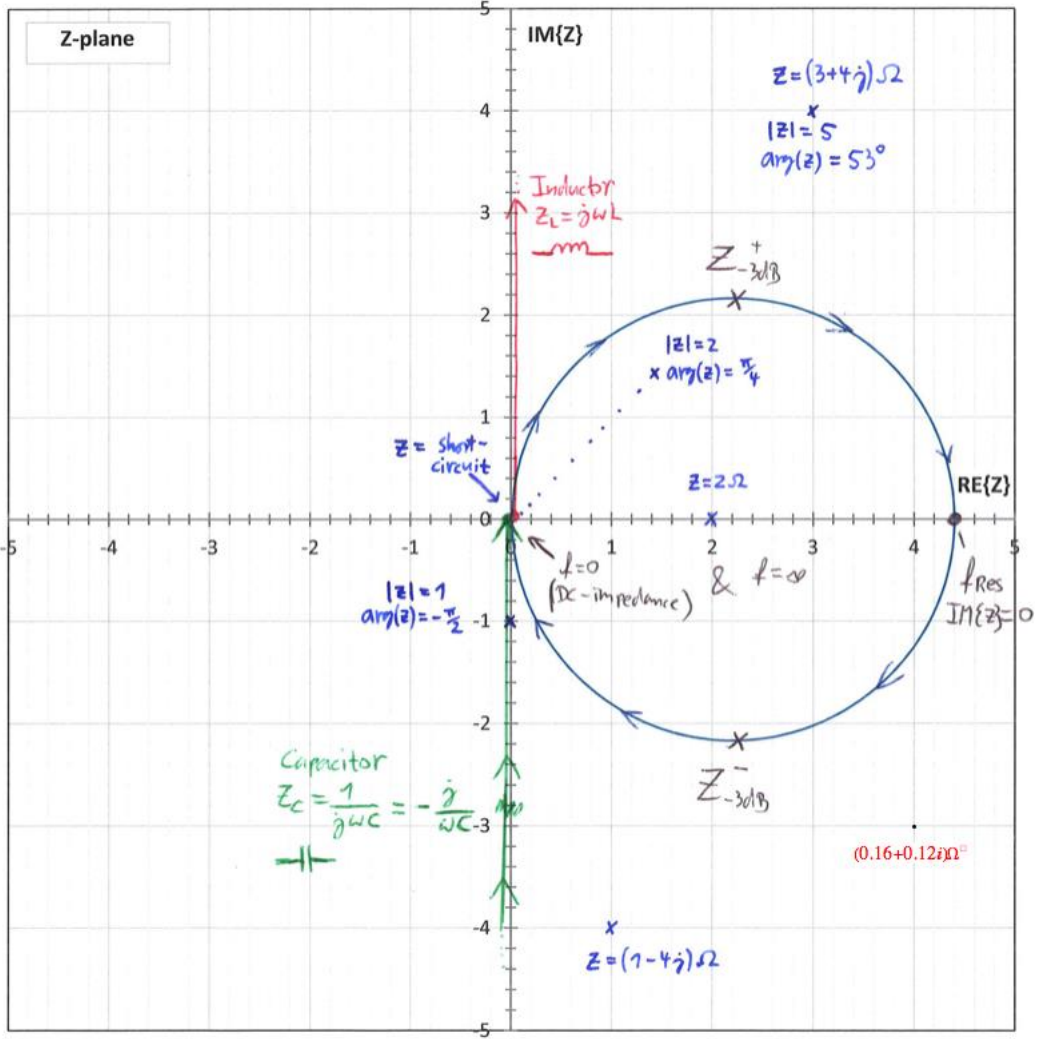
$Z = (3 + 4j) \Omega$	$ Z  = 2, \arg(Z) = \pi/4$	$Z = \text{short circuit}$
$Z = 2 \Omega$	$ Z  = 1, \arg(Z) = -\pi/2$	$Y = Z^{-1} = (0.16 + 0.12j) \Omega^{-1}$
$Z = (1 - 4j) \Omega$	$ Z  = 5, \arg(Z) = 53^\circ$	

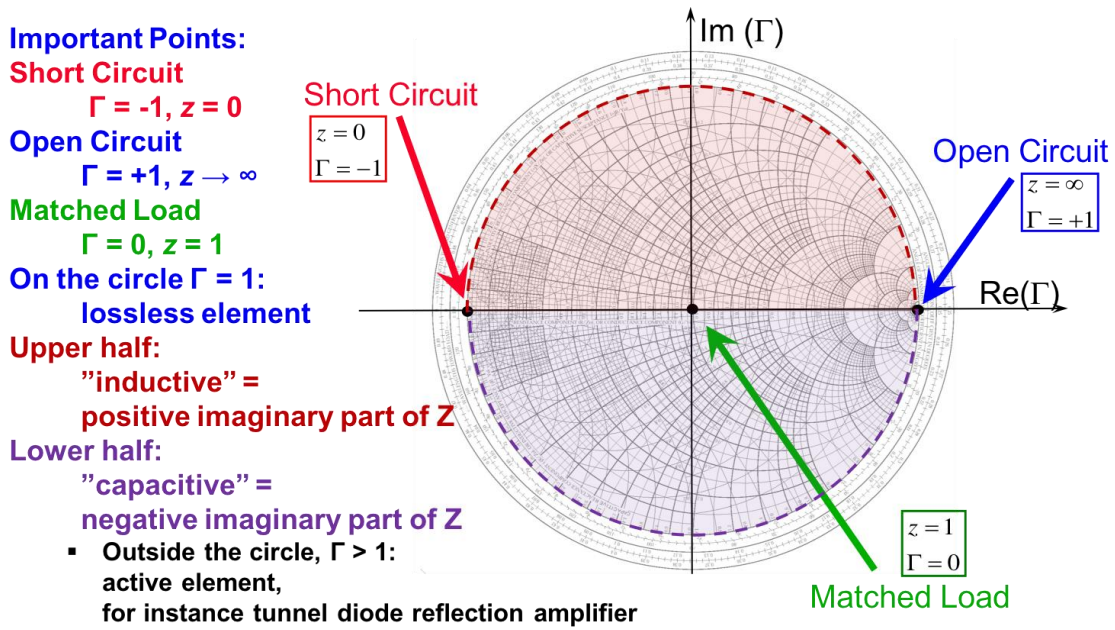
- Qualitatively, how would an inductor look like, plotted from DC to some arbitrary frequency, in the Z-plane? Hint:  $Z_L = j\omega L$
- How would a capacitor look like? Hint:  $Z_C = 1/(j\omega C)$
- The input impedance of a RLC circuit has been plotted in the Z-plane (blue circle).

Mark the points in the diagram describing:

- Impedance at the resonant frequency
- DC impedance
- 3-dB bandwidth
- Impedance at  $f \rightarrow \infty$







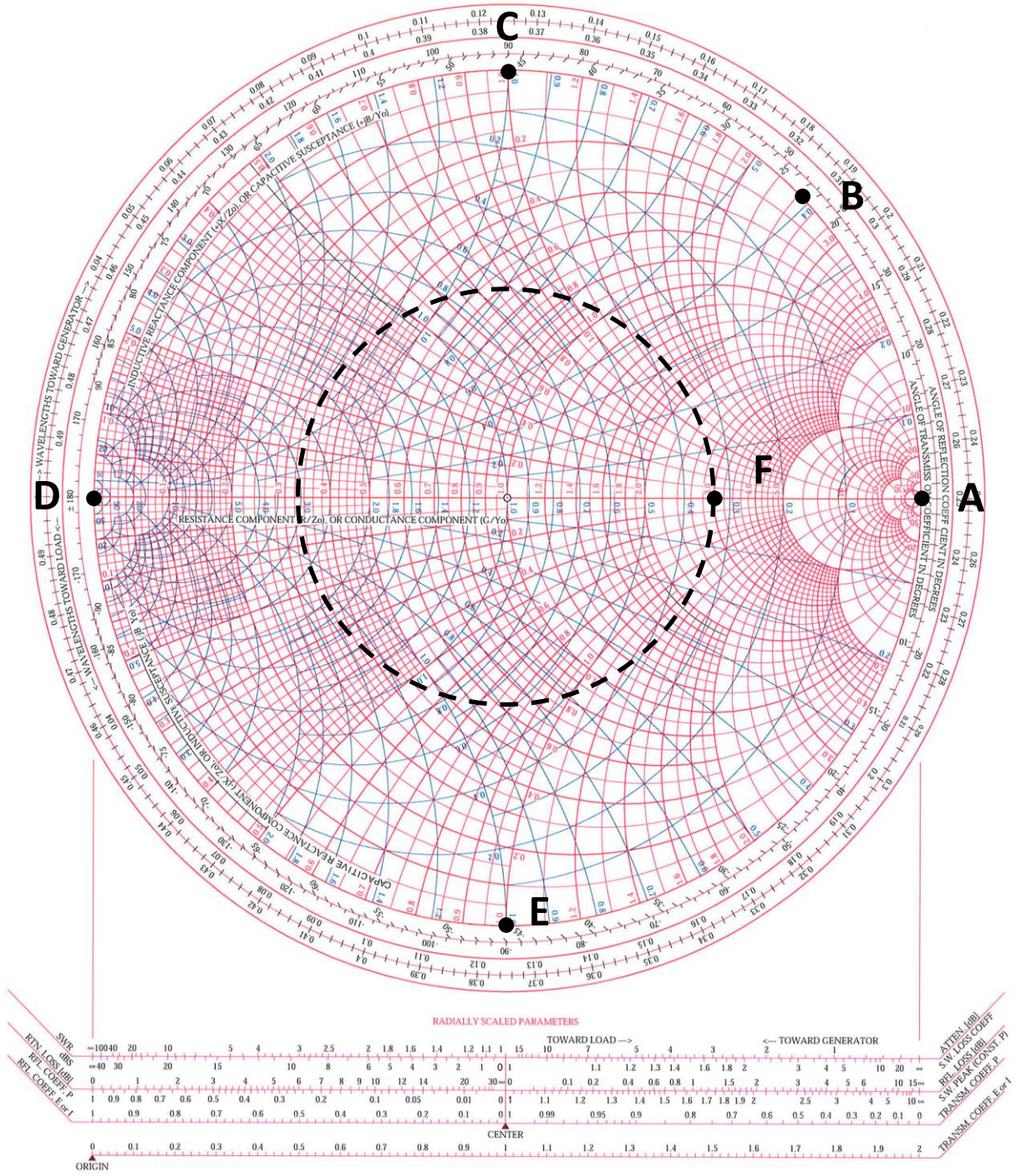
### 3.1.) Smith Chart (1)

1. Mark the reflection factors  $\Gamma$  of points A to F in the Smith Chart and find approximate values for the corresponding (normalized) impedances  $z$ :

Point	Reflection factor $\Gamma$	Normalized impedance $z$
A	$1 \angle 0^\circ$	$\infty$
B	$1 \angle 45^\circ$	$0 + j2.4$
C	$1 \angle 90^\circ$	$0 + j1.0$
D	$1 \angle 180^\circ$	$0$
E	$1 \angle -90^\circ$	$0 - j1$
F	$0.5$	$3.0$

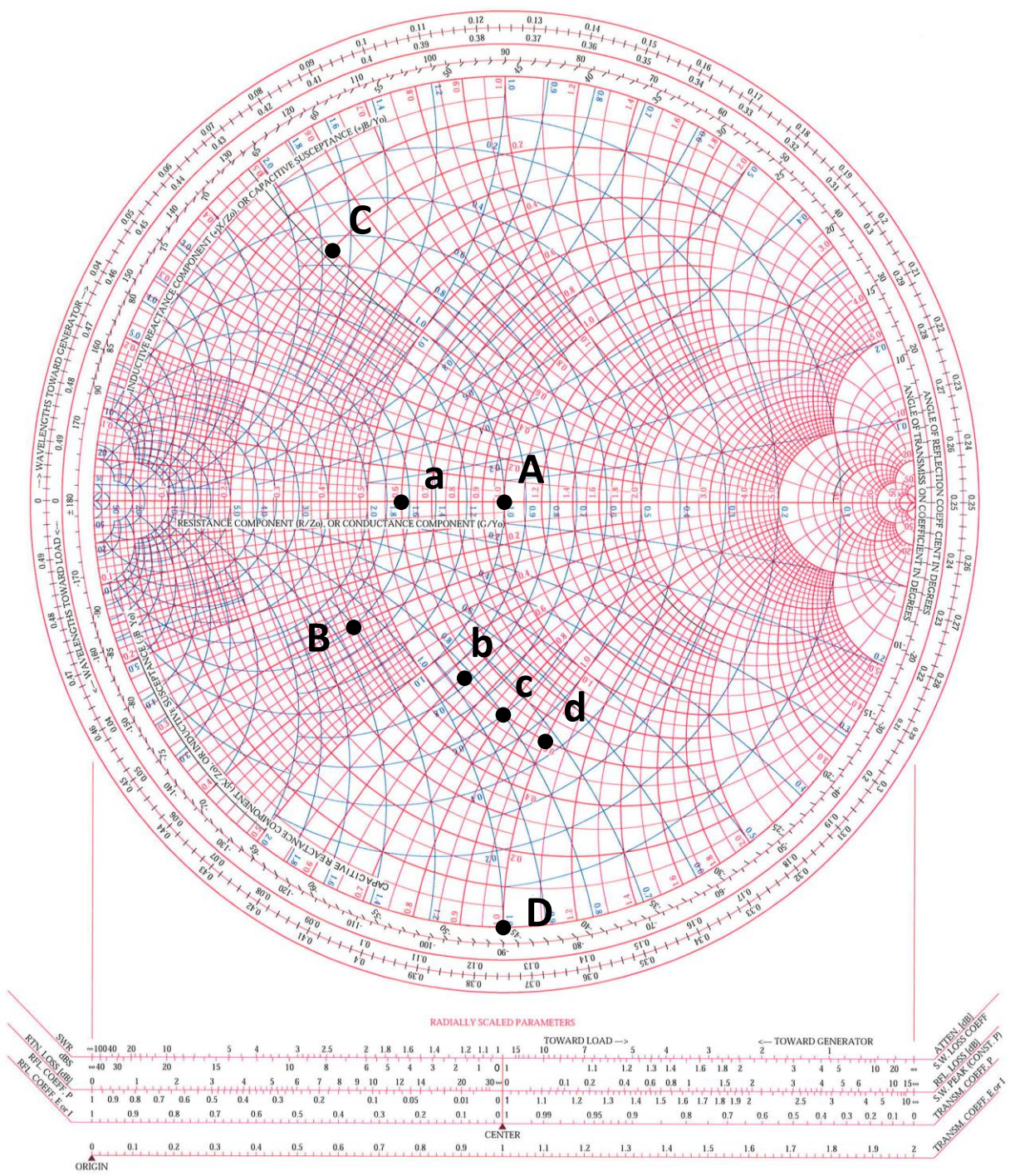


# NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES





# NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



### 3.2) Smith Chart (2)

1. Plot the following **normalized** impedances  $z$  into the Smith Chart:

Point	a	b	c	d
<b>z</b>	$0.6 + j0$	$0.6 - j0.6$	$0.6 - j0.8$	$0.6 - j1.0$

2. Plot the following impedances  $Z$  into the Smith Chart, assuming a reference impedance of  $Z_0 = 50 \Omega$

Point	A	B	C	D
<b>Z</b>	$50 + j0$	$20 - j15$	$10 + j25$	$0 - j50$

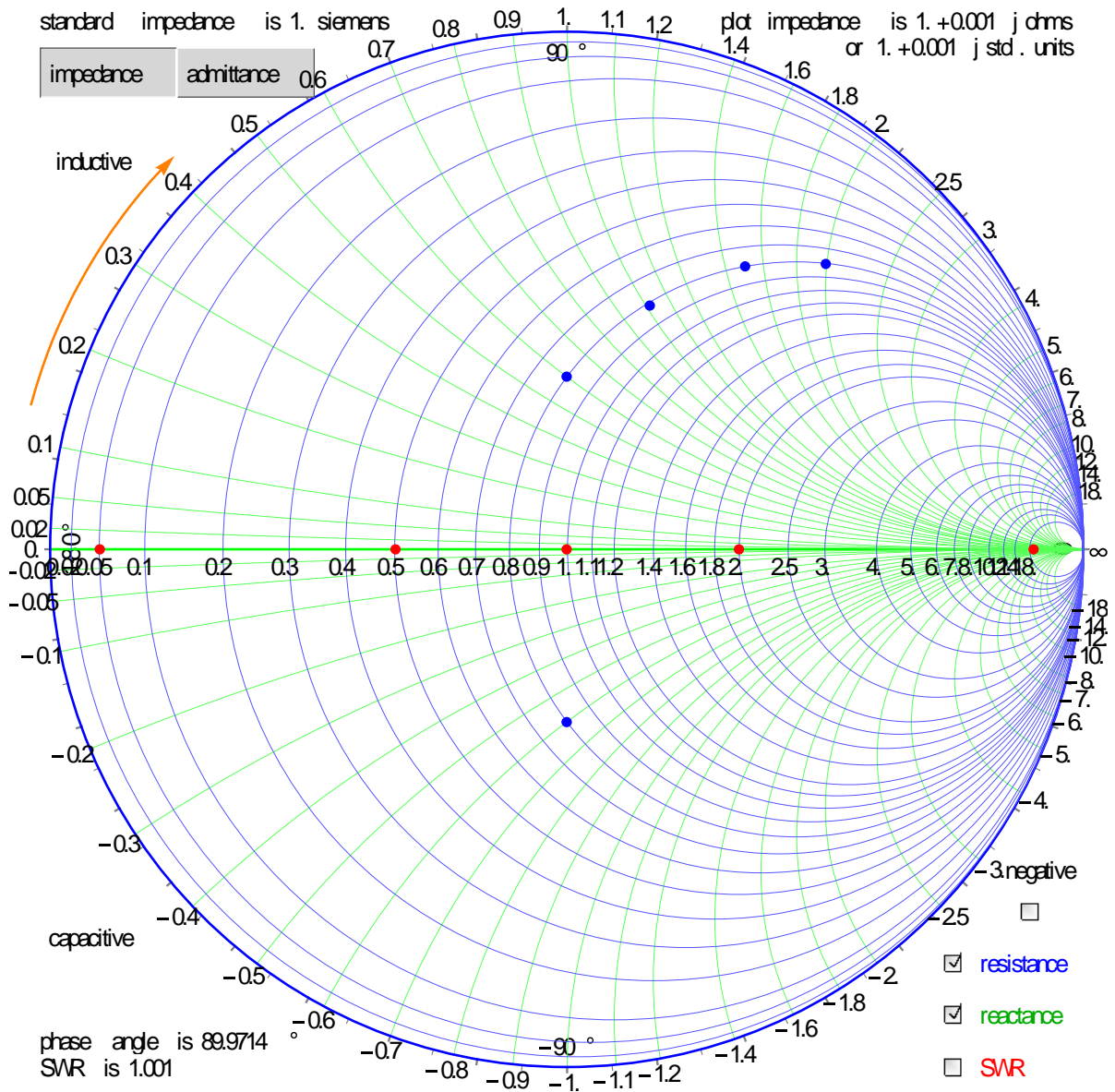
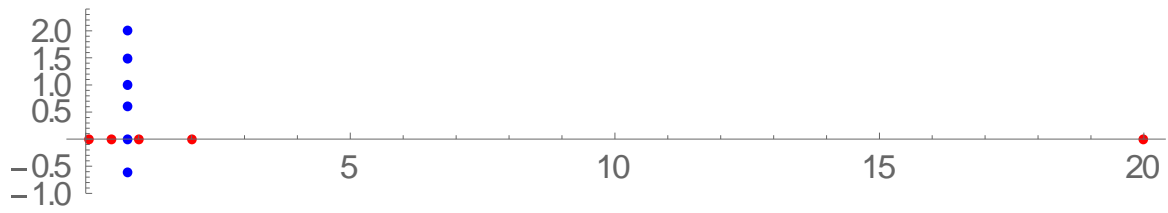
### 3.3.) Impedances in the complex plane and in the Smith chart

Plot the following impedances as points (marks)

- in a “normal” Cartesian coordinate system (complex plane)
- as reflection factors in the *Smith* chart (the reflection factor coordinates are given for convenience)

$X_{rect}$	$X_{polar}$	$\Gamma_{rect}$	$\Gamma_{polar}$
0.05	$0.05 \angle 0^\circ$	-0.904	$0.904 \angle 180^\circ$
0.5	$0.5 \angle 0^\circ$	-0.333	$0.333 \angle 180^\circ$
1	$1.0 \angle 0^\circ$	0	0
2	$2.0 \angle 0^\circ$	0.333	$0.333 \angle 0^\circ$
20	$20 \angle 0^\circ$	0.904	$0.904 \angle 0^\circ$
0.8	$0.8 \angle 0^\circ$	-0.111	$0.111 \angle 180^\circ$
$0.8 + j0.6$	$1.00 \angle 36.9^\circ$	0 + j0.333	$0.333 \angle 90^\circ$
$0.8 + j1.0$	$1.28 \angle 51.3^\circ$	$0.159 + j0.472$	$0.459 \angle 72.3^\circ$
$0.8 + j1.5$	$1.70 \angle 61.9^\circ$	$0.344 + j0.546$	$0.645 \angle 57.8^\circ$
$0.8 + j2.0$	$2.15 \angle 68.2^\circ$	$0.502 + j0.552$	$0.747 \angle 47.7^\circ$
$0.8 - j0.6$	$1.00 \angle -36.9^\circ$	0 - j0.333	$0.333 \angle -90^\circ$

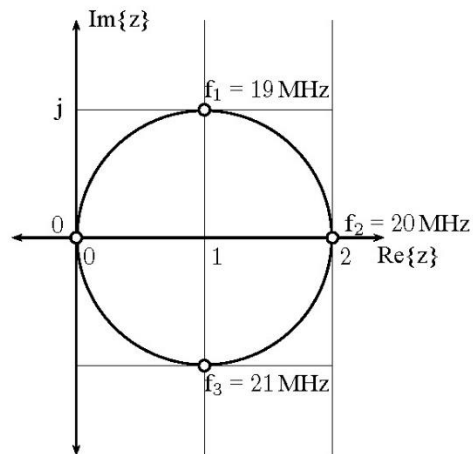
Convince yourself with a few examples that  $\Gamma(1/X) = -\Gamma(X)$





### 3.4) Smith Chart (3)

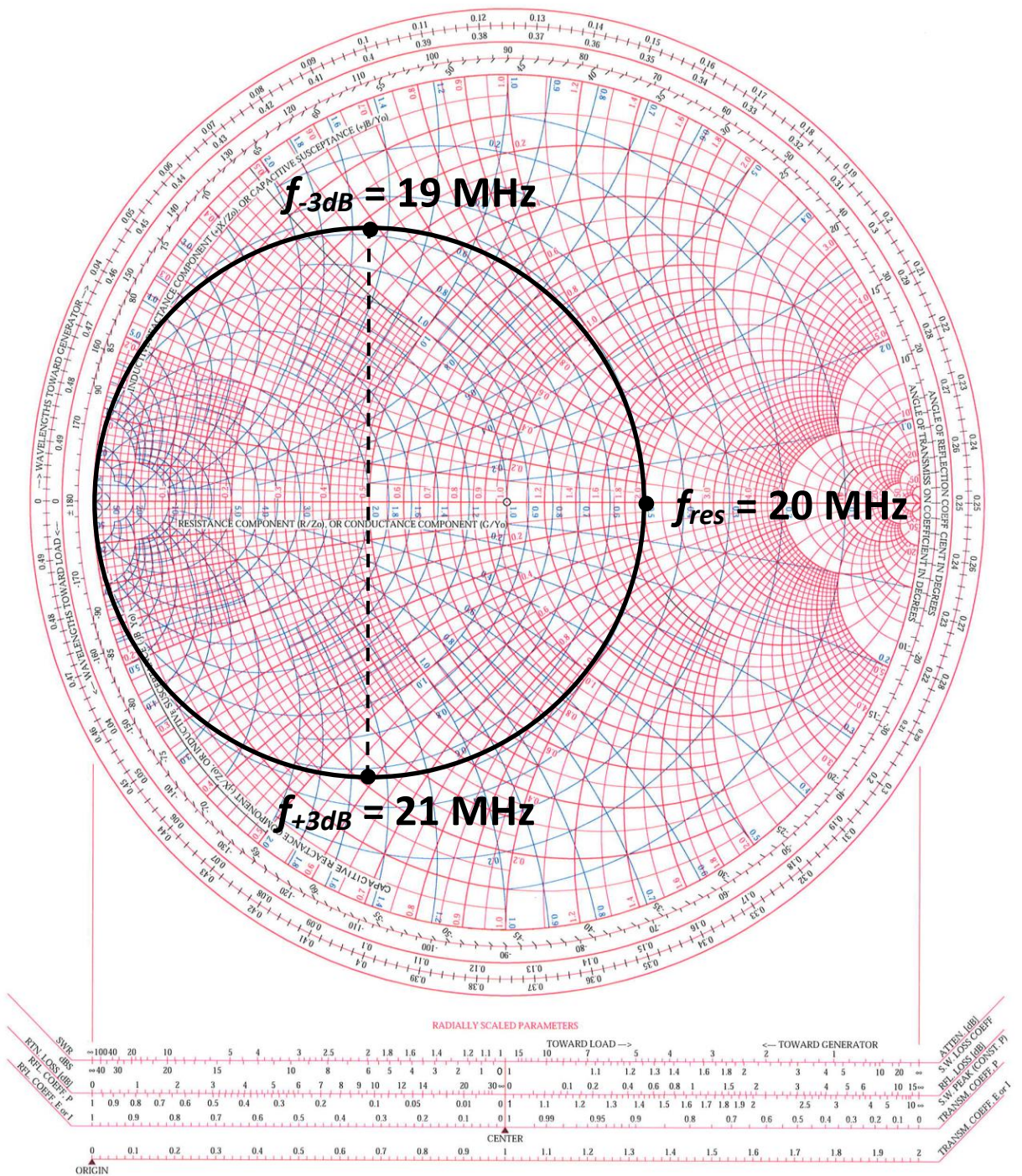
The locus of impedance of a parallel RLC resonant circuit is given in the complex z-plane (z-plane = normalized Z-plane, normalization to  $50 \Omega$ ;  $z = Z / 50 \Omega$ ).



#### Questions:

1. Transform this locus of impedance into the Smith Chart
2. Mark the resonance frequency, both, in the z-plane and in the Smith Chart.
3. Mark the 3-dB points (for the unloaded Q), both, in the z-plane and in the Smith Chart.

### NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



## 5.) Various questions

1. What is the difference between a *Stripline* and a *Microstripline*?

Striplines have a ground plane above and below the central conductor. Microstriplines have a groundplane only below the conductor, the other side is left open.

2. Name 3 disadvantages of *Microstriplines* compared to *Striplines*.

The open structure of a Microstripline resembles an antenna structure. RF power is radiated into the environment and electromagnetic interference is easily picked up. Crosstalk and mutual coupling becomes an issue if several Microstriplines are placed close to each other. The asymmetric, open structure causes a frequency-dependent characteristic impedance and a considerable dispersion.

3. A RF signal needs to be guided from a power amplifier on the surface to the cavity of a particle accelerator in the tunnel below ground.

The distance is  $l = 100$  m. The signal has parameters:  $f = 50$  MHz,  $P = 100$  kW

- a. Would you use waveguide or coaxial transmission line? Why?
- b. What would you use if the signal would have a frequency of 500 MHz? Why?

While a waveguide is in general more power efficient (less attenuation), the dimensions of this structure would be impractically large for 50 MHz. The free space wavelength at 50 MHz is  $\lambda = \frac{c}{f} = 6$  m. So the waveguide needs to be at least 3 m wide. Coaxial transmission line does not have a lower cut-off frequency and would be the better choice.

4. Why are some accelerator cavities (for frequencies in the MHz range) loaded with ferrite? Explain how the resonant frequency of those cavities can be tuned without moving parts. Looking at the RLC equivalent circuit of the cavity, the ferrite increases the inductance part L and thus lowers the resonant frequency compared to air filled cavities. This allows to reduce the resonant frequency or alternatively the size of the structure. Placing the ferrite in a static magnetic bias field, its  $\mu$  can be varied which leads to change in L and in the resonant frequency. So, by changing the bias current, the cavity can be electronically tuned.
5. After deploying a new accelerating cavity, a RF-engineer starts pounding on it with a hammer. What is he doing? He is in fact fine tuning the resonant frequency of the cavity. This punch-tuning or dimple tuning procedure was used for example in the CERN Linear Injector for LEP. The structure was specially designed so it could be slightly deformed at certain points with a hammer.

## 6.) Scaling laws

A cavity shall be scaled from existing designs for a frequency  $f_x = 318.32$  MHz and  $C_x = 10$  pF. There are three test designs, with the following parameters:

Cavity	$f_{res}$ / MHz	C / pF	Q	Diameter / mm
A	100	7.957	10000	600
B	500	3.18	5000	200
C	3000	1.061	2000	25

### Questions:

1. Which cavity is suitable as reference design?
  2. Calculate the diameter of the new design.
  3. Calculate the expected Q factor of the new design, provided it will be build out of the same material as the reference design.
1. For the right candidate design, the parameter  $r/Q$  has to agree with the one calculated for the actual cavity. Using  $r/Q = \frac{1}{\omega C}$  we find an  $r/Q$  of 50 and 200, 100, 50 for the actual cavity and the three candidate designs, respectively. Therefore, cavity C is chosen for the scaling.
  2. All cavity dimension are proportional to  $\lambda = 1/f$ .  

$$D_x = D \frac{f}{f_x} = 0.236 \text{ m}$$
  3. Using the third scaling law,  $Q \frac{\delta}{\lambda} = \text{const}$ , we find  $Q \propto \frac{\lambda}{\delta}$  with the skin depth  $\delta$  decreasing linearly with frequency.  

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \propto \frac{1}{\sqrt{f}} \propto \sqrt{\lambda}.$$
Therefore,  $Q \propto \sqrt{\lambda} \propto \frac{1}{\sqrt{f}}$ .  $\Rightarrow Q_x = Q \sqrt{\frac{f}{f_x}} = 6140$



## 7.) Thermal expansion and scaling laws

An accelerator cavity heats up under high RF power load.

The cavity used is constructed from a material having a:

thermal expansion coefficient:  $\Delta l/l = 20\text{e-}6/^{\circ}\text{C}$  (per degree Centigrade)

thermal resistivity coefficient:  $\Delta\rho/\rho = 4\text{e-}3/^{\circ}\text{C}$  (per degree Centigrade)

At room temperature the cavities resonance frequency is  $f_1 = 100$  MHz, and has a 3-dB bandwidth of  $BW_1 = 100$  kHz.

Under RF power the cavity temperature increases by  $100^{\circ}\text{C}$  (subscripts 2 apply).

### Questions:

Determine

- the ratio  $\lambda_2/\lambda_1$
- the ratio  $L_2/L_1$
- the ratio  $C_2/C_1$
- the ratio  $Q_2/Q_1$  (hint: the skin depth  $\delta$  is proportional to  $\sqrt{\rho/f}$ )
- the resonance frequency  $f_2$  under load
- and the 3-dB bandwidth  $BW_2$  of the resonance under load

**thermal expansion coefficient:**

$$\Delta l / l = 20 \times 10^{-6};$$

**thermal resistivity coefficient:**

$$\Delta \rho / \rho = 4 \times 10^{-3};$$

**At room temperature the cavity resonates at a frequency**

$$f_1 = 100 \times 10^6;$$

**with a bandwidth of**

$$\Delta f_1 = 100 \times 10^3;$$

**Under RF power, the cavity temperature is increased by**

$$\Delta t = 100;$$

**(subscripts 2 apply)**

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## Questions

**Determine**

- the ratio  $\lambda_2 / \lambda_1$

The wavelength scales proportional with the cavity dimension  $d$ , as  $\lambda$  is inverse proportional to  $f$

$$\frac{\lambda_2}{\lambda_1} = \frac{d_2}{d_1} = \frac{d_1(1 + \Delta l / l \Delta T)}{d_1} = 1 + \frac{\Delta l}{l} \Delta T$$

$$\lambda_2 / \lambda_1 = N [1 + \Delta l / l \Delta t]$$

$$1.002$$

see also page 85

- the ratio  $L_2 / L_1$

The characteristic impedance  $R/Q$  stays constant, therefore:

$$\frac{L_2}{L_1} = \frac{R/Q}{\omega_2} = \frac{\omega_1}{R/Q} = \frac{\omega_1}{\omega_2} = \frac{\lambda_2}{\lambda_1}$$

$$L_2 / L_1 = \lambda_2 / \lambda_1$$

$$1.002$$

see also pages 67 and 85

- the ratio  $C_2 / C_1$

The characteristic impedance  $R/Q$  stays constant, therefore:

$$\frac{C_2}{C_1} = \frac{1}{R/Q \omega_2} = \frac{R/Q \omega_1}{1} = \frac{\omega_1}{\omega_2} = \frac{\lambda_2}{\lambda_1}$$

$$C_2 / C_1 = \lambda_2 / \lambda_1$$

$$1.002$$

see also pages 67 and 85

- the ratio  $Q_2/Q_1$

The quality factor  $Q \propto \frac{\lambda_0}{\delta}$ , and the skin depth follows  $\delta \propto \sqrt{\frac{\rho}{f}}$ , there-

fore:

$$\frac{Q_2}{Q_1} = \frac{\text{const} \lambda_2}{\delta_2} \frac{\delta_1}{\text{const} \lambda_1} = \frac{\lambda_2}{\lambda_1} \frac{\delta_1}{\delta_2} = \frac{\lambda_2}{\lambda_1} \sqrt{\frac{\rho_1 f_2}{\rho_2 f_1}} = \frac{\lambda_2}{\lambda_1} \sqrt{\frac{\rho_1 \lambda_1}{\rho_2 \lambda_2}} = \sqrt{\frac{\rho_1 \lambda_2}{\rho_2 \lambda_1}}$$

$$\rho_1 \text{ over } \rho_2 = N[(1 + \Delta \rho \text{ over } \rho \Delta t)]$$

1.4

$$q_2 \text{ over } q_1 = \sqrt{\rho_1 \text{ over } \rho_2 \lambda_2 \text{ over } \lambda_1}$$

1.1844

see also pages 59 and 85

- the resonance frequency  $f_2$

$$\frac{f_2}{f_1} = \frac{\lambda_1}{\lambda_2}$$

$$f_2 = \frac{f_1}{\lambda_2 \text{ over } \lambda_1}$$

$9.98004 \times 10^7$

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$99.8004 \times 10^6$

- the bandwidth  $\Delta f_2$

$$\Delta f_1 = \frac{f_1}{Q_1}, \Delta f_2 = \frac{f_2}{Q_2} \implies \frac{\Delta f_2}{\Delta f_1} = \frac{f_2}{Q_2} \frac{Q_1}{f_1}$$

$$\Delta f_2 = \frac{f_2}{f_1} \frac{1}{q_2 \text{ over } q_1} \Delta f_1$$

84262.5

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$84.2625 \times 10^3$