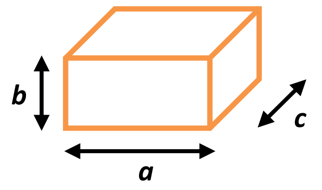
$\mu = \mu_0 \mu_r$
$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)
$\varepsilon = \varepsilon_0 \varepsilon_r$
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)
$c_0 = 3 \cdot 10^8$ m/s

JUAS 2017 – RF Exam

Name: ______________________________ Points: ______ of 20 (25 with bonus points)

Utilities: JUAS RF Course 2017 lecture script, personal notes,
pocket calculator, ruler, compass, and your brain!
(No cell- or smartphone, no iPad, laptop, or wireless devices, no text books or any other tools!!!)

Please compute and write your results clear and readable, if appropriate on a separate sheet of paper. Any unreadable parts are considered as wrong.



1. "Brick-style" Cavity (5 points)

Design a "brick"-shaped, rectangular cavity with relative dimensions $c = 0.7 \cdot a$ and $a/b = 2$, operating at the fundamental mode of 500 MHz.

a) Of what type is the fundamental mode? (½ point)
H_{101} or TE_{101}

b) What are the physical dimensions a, b and c of the cavity? (1½ points)
Hint: Free-space wavelength and frequency are linked through the speed-of-light: $\lambda_0 = \frac{c_0}{f}$

$\lambda_0 = \frac{c_0}{f} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}$ with *m=1, n=0, p=1* and *c=0.7a* gives:

a = 522.78mm; *b* = 261.34mm; *c* = 365.94mm

c) What is the (unloaded) *Q*-value of the cavity if it is made out of copper? (1 point)
($\sigma_{Cu} = 58.5 \cdot 10^6$ S/m)

$$\delta = \sqrt{\frac{2}{\omega\, \sigma_{Cu}\, \mu}} = 2.94\ \mu m$$

$$Q_{H101} = \frac{\lambda_0}{\delta} \frac{b}{2} \frac{(a^2 + c^2)^{3/2}}{c^3\,(a + 2b) + a^3\,(c + 2b)} = 38828$$

d) Determine the Q-value if the cavity is made out of stainless steel. (½ point)
($\sigma_{SS} = 1.35 \cdot 10^6$ S/m)

$$\delta = \sqrt{\frac{2}{\omega\, \sigma_{SS}\, \mu}} = 19.4\ \mu m$$

$$Q_{H101} = \frac{\lambda_0}{\delta} \frac{b}{2} \frac{(a^2 + c^2)^{3/2}}{c^3\,(a + 2b) + a^3\,(c + 2b)} = 5898$$

e) What are the mode types and frequencies of the next two higher order modes? (1½ points)

$\frac{\lambda}{a} = 1.147 \quad \frac{c}{a} = 0.7$ -> E_{110}, followed by a degenerated mode H_{011}, H_{201}

for *m=1, n=1, p=0*:

$$f_{110} = \frac{c_0}{\lambda_0} = 641.15\ MHz \quad \text{with } \lambda_0 = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}} = 0.4678\ m$$

for *m=0, n=1, p=1* or *m=2, n=0, p=1*:

$$f_{011} = f_{201} = \frac{c_0}{\lambda_0} = 704.43\ MHz \quad \text{with } \lambda_0 = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}} = 0.4254\ m$$

2. Smith chart (5 points)

a) Indicate points $P_1 \ldots P_6$ in the Smith chart, assuming a reference impedance Z_0 = 50 Ω. From the Smith chart, determine the missing Z or Γ, and complete the table. (1 point)
(Use the provided Smith chart)

Point no.	**P_1**	**P_2**	**P_3**	**P_4**	**P_5**	**P_6**
***Z* / Ω**	∞	50	**0**	30.6 – j 74.2	**50 + j 50**	**50 - j 100**
Γ	1	**0**	-1	**0.7∠-62°**	0.45∠63.4°	0.707∠-45°

b) Draw the locus of $|\Gamma| = 0.5$ in the Smith chart. (½ point)

c) Points P_5 and P_6 represent a complex load impedance Z_{load}.
Indicate the normalized z_{load} in the Smith chart, and look up

- the reflection coefficient, (½ point)
- the (voltage) standing wave ratio, (½ point)
  $VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 2.62$ or 5.83
- the return loss (in dB), (½ point)
  $RL = -10log_{10}(|\Gamma|^2) = 6.99\ dB$ or $3.01\ dB$
- the reflection loss (in dB) (½ point)
  $RFL = -10log_{10}(1 - |\Gamma|^2) = 0.97\ dB$ or $3.01\ dB$

again, assuming a reference impedance of Z_0 = 50 Ω.
(Hint: Use a ruler to determine $|\Gamma|$ of z_{load}, and compare it with value found at the "radially scaled parameters" Smith chart ruler at the bottom.)

d) With a simple loss-less matching circuit, a single reactive element, the complex loads represented by points P_5 and P_6 (two independent cases!) can be matched to the reference impedance of Z_0 = 50 Ω.

- Indicate the matching as graph in the Smith chart. (½ point)
- Sketch the matching circuit. (½ point)
- Evaluate the element value, assuming an operation frequency of f = 500 MHz. (½ point)

$$C_{series} = \frac{1}{2\pi f\, X} = 6.37\, pF \;\text{ with }\; X = 50\,\Omega$$
$$L_{series} = \frac{X}{2\pi f} = 31.8\, nH \;\text{ with }\; X = 100\,\Omega$$

3. S-Parameters (2 points)

Match the ideal S-parameters in matrix form to the corresponding components.

$$\boldsymbol{S_A} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} \quad \boldsymbol{S_B} = \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{j}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{j}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{j}{\sqrt{2}} & 0 \end{bmatrix} \quad \boldsymbol{S_C} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \quad \boldsymbol{S_D} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

a) Assign the S-matrices ($\boldsymbol{S_A}$ ... $\boldsymbol{S_D}$) to the components: (1 point)

component	3 dB directional coupler	transmission line, electrical length = λ/2	resistive power divider	isolator
S-matrix	$\boldsymbol{S_B}$	$\boldsymbol{S_C}$	$\boldsymbol{S_A}$	$\boldsymbol{S_D}$

b) Fill the missing dB (coupler) and λ (transmission-line) information (...). (1 point)

4. Multiple choice (5 points)

Tick ***one*** correct answer like this: ☒.

1. Using an electromagnetic simulation software, the mesh cells in regions with electric field concentration should be made: (½ point)
   - ☒ smaller
   - ○ larger
   - ○ doesn't matter
2. A sinusoidal RF signal is measured with an oscilloscope, having and internal 50 Ω termination. The cursors displays a peak-to-peak voltage of 500 mV. What is the signal power in dBm? (½ point)
   - ○ -1 dBm
   - ☒ -2 dBm
   - ○ +4 dBm

3. For a "E"(or "TM") mode, the following is true: (½ point)
   - ☒ Its magnetic field has only transverse components
   - ○ Its magnetic field has transverse and longitudinal components
   - ○ Its electric field has only transverse components

4. Changing the height *h* of a cylindrical cavity operating on the TM_{110} dipole-mode will **NOT** change: (½ point)
   - ☒ its resonant frequency
   - ○ its quality factor
   - ○ its R/Q

5. The GSM standard specifies a minimum sensitivity requirement of -100 dBm for the signal reception, while the output power of the cell phone transmitter is typically in the order of 1 W. This corresponds to how many orders of magnitude in power? (½ point)
   - ○ 5
   - ○ 10
   - ☒ 13

6. A 4-port directional coupler has 10 dB coupling and 20 dB directivity. What is the relative level between the desired coupled output and the input signal? (½ point)
   - ☒ -10 dB
   - ○ +20 dB
   - ○ -30 dB

7. In a RF accelerating cavity, the transit time factor expresses: (½ point)
   - ○ The time it takes for the energy to transfer from the electric field to the magnetic field
   - ☒ The time variation of the accelerating field during the bunch passage
   - ○ The time it takes the bunch to travel through the cavity

8. Examples of TEM transmission lines are: (½ point)

- ○ Waveguides operating below cut-off frequency
- ☒ Coaxial cables
- ○ Resonant cavities with input and output coupler

9. Critical coupling (impedance match at resonance) between resonator and generator occurs at (½ point)

- ○ $Q_L = Q_{ext}$
- ☒ $Q_L = Q_0/2$
- ○ $Q_L = 2 \cdot Q_0$

10. A network analyser is used to (½ point)

- ○ Analyse signals in the frequency domain
- ☒ To characterize the S-parameters of an RF element (DUT = device under test)
- ○ Measure and calibrate signals from the internet communication structure

5. Skin-Effect (3 points)

A beam pipe with circular cross-section is given. It has a diameter of 10 cm and is made out copper (σ_{Cu} = 58.5 · 10^6 S/m), with a wall thickness of 2 mm.

a) Determine the skin depth at f = 1 GHz. Determine the real part of the impedance per meter at this frequency. (1½ points)
(Hint: This is the surface resistance as defined for lossy coaxial cables.)

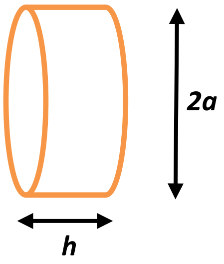
$$\delta = \sqrt{\frac{2}{\omega\, \sigma_{Cu}\, \mu}} = 2.08\ \mu m \quad R_s = \frac{\rho_{Cu}}{\delta} = 8.21\ m\Omega/m$$

b) Now assume the beam pipe is made from a stainless steel, which has a 43 times higher bulk resistivity compared to copper. (½ point)
What are the skin depth and the surface resistance (= resistance per unit square) now?

$$\delta = \sqrt{\frac{2}{\omega\, \sigma_{SS}\, \mu}} = 13.6\ \mu m \quad R_s = \frac{\rho_{SS}}{\delta} = 53.9\ m\Omega/m$$

c) At which frequency (case: stainless steel) the wall thickness equals the skin depth? (1 point)

$$f = \frac{1}{\pi\, \rho_{SS}\, \mu\, \delta^2} = 46.55\ kHz \quad \text{with } \delta = 2\ mm$$



6. "Pillbox" Cavity (5 bonus points)

Analyze a simple cylindrical "pillbox" cavity (the beam-pipe ports are neglected). The cavity is made out of copper (σ_{Cu} = 58.5 · 10^6 S/m), has a dimension ratio *h/a* = 0.2 (*a* = radius, *h* = height), and operates at 200 MHz for the fundamental mode (TM_{010} = E_{010}).

a) Calculate the *R/Q* of the cavity! (½ point)

$$\frac{R}{Q} \approx 185\frac{h}{a} = 37$$

b) What is the is radius *a* of this resonator? (½ point)

$$a = 0.383\,\lambda_0 = 0.383\frac{c_0}{f} = 573.71\,mm$$

c) Determine the lumped elements R, L, and C of the equivalent parallel R-C-L circuit. (1½ points)

$$h = 0.2\,a = 114.74\,mm$$

$$\delta = \sqrt{\frac{2}{\omega\,\sigma_{Cu}\,\mu}} = 4.65\,\mu m$$

$$Q = \frac{a}{\delta}\left(1+\frac{a}{h}\right)^{-1} = 20550$$

$$R = \frac{R}{Q}Q = 760.4\,k\Omega$$

$$L = \frac{R/Q}{2\pi f} = 29.4\,nH$$

$$C = \frac{1}{2\pi f\,R/Q} = 21.5\,pF$$

d) Now assume that this cavity is made from a stainless steel, which has a 43 times higher resistivity than copper. For the same geometrical dimensions, re-compute the resonance frequency, R/Q, and the unloaded Q-value. Which values of the parallel equivalent circuit has changed? (1½ points)

$$\delta = \sqrt{\frac{2}{\omega\,\sigma_{Cu}\,\mu}} = 30.5\,\mu m$$

$$Q = \frac{a}{\delta}\left(1+\frac{a}{h}\right)^{-1} = 3134$$

$$R = \frac{R}{Q}Q = 115.9\,k\Omega$$

$$L = \frac{R/Q}{2\pi f} = 29.4\,nH$$

$$C = \frac{1}{2\pi f\,R/Q} = 21.5\,pF$$

e) The copper cavity is now scaled to a resonance frequency of 100 MHz. Indicate the scaling ratio of all linear dimensions. What is the new Q-value? (1 point)

$\frac{f_1}{f_2} = 0.5$

$a_1 = \frac{a_2}{f_1/f_2} = 1.147\ m$

$h_1 = const.$ Does not influence the frequency of the fundamental mode ($TM_{010} = E_{010}$)!

$Q_1 = Q_2\sqrt{\frac{f_2}{f_1}} = 29062$