JUAS 2016 – RF Exam

$\mu = \mu_0\, \mu_r$

$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)

$\varepsilon = \varepsilon_0\, \varepsilon_r$

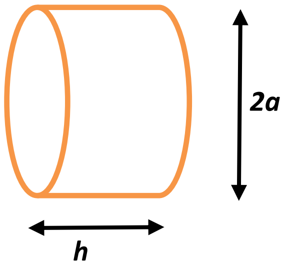
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)

$c_0 = 3 \cdot 10^8$ m/s

Name: ________________________________ Points: ______ of 20 (23 with bonus points)

Utilities: JUAS RF Course 2016 lecture script, personal notes, pocket calculator, ruler, compass, and your brain! (No cell- or smartphone, no iPad, laptop, or wireless devices, no text books or any other tools)

Please compute and write your results clear and readable, if appropriate on a separate sheet of paper. Any unreadable parts are considered as wrong.



1. “Pillbox” Cavity (8 points)

A scaled model of a simple cylindrical “pillbox” cavity is characterized in the RF laboratory (the beam-pipe ports are neglected). The cavity is made out of stainless steel ($\sigma_{ss} = 2 \cdot 10^6$ S/m, $\mu=\mu_0$), and its unloaded E_{010}-mode eigen-frequency measures 600 MHz, with a 3-dB bandwidth of 70 kHz.

a) What is the *Q*-value of the cavity? (1½ points)

$$Q = \frac{f_{E010}}{\Delta f} = \frac{600 \cdot 10^6 s}{70 \cdot 10^3 s} = 8571$$

b) What are the physical dimensions, radius (or diameter) and height, of the cylindrical resonator? (2 points)

$$a = \frac{\lambda_0}{2.61} = \frac{c_0}{2.61\, f_{E010}} = \frac{3 \cdot 10^8 ms}{2.61 \cdot 600 \cdot 10^6 s} = 191.24\, mm$$

$$Q = \frac{a}{\delta}\left[1 + \frac{a}{h}\right]^{-1} = \frac{a}{\delta\left(1 + \frac{a}{h}\right)}$$

$$1 + \frac{a}{h} = \frac{a}{\delta\, Q}$$

$$h = \frac{a}{\frac{a}{\delta\, Q} - 1}$$

$$\delta = \sqrt{\frac{2}{\omega\, \sigma_{ss}\, \mu}}$$

$$\delta = \sqrt{\frac{2 \quad s \qquad\qquad Vm \qquad\qquad Am}{2\pi \cdot 600 \cdot 10^6 \cdot 2 \cdot 10^6 A \cdot 4\pi \cdot 10^{-7} Vs}} = 14.5\, \mu m$$

$$h = \frac{191.24 \cdot 10^{-3}\, m}{\frac{191.24 \cdot 10^{-3}\, m}{14.5 \cdot 10^{-6}\, m \cdot 8571} - 1} = 357\, mm$$

$$\frac{h}{2a} = 0.933 \qquad \frac{\lambda_0}{2a} = 1.31$$

c) Sketch the *RLC* equivalent parallel circuit, and determine the values of the E_{010}-mode. (Therefore the "geometry factor", also known as "characteristic impedance" *R/Q*, needs to be calculated, based on the exact formula! Why the approximate expression cannot be applied?) (2 points)

$$\frac{R}{Q} = 128\,\Omega \frac{\sin^2(1.2024\frac{h}{a})}{\frac{h}{a}} = \frac{128\,\Omega \cdot \sin^2(1.2024 \cdot \frac{357\,mm}{191.24\,mm})}{\frac{357\,mm}{191.24\,mm}} = 41.85\,\Omega$$

$$R = \frac{R}{Q}Q = 41.85 \cdot 8571 = 358.68\,k\Omega$$

$$L = \frac{R/Q}{\omega_{res}} = \frac{358.68\,k\Omega\,s}{2\pi \cdot 600 \cdot 10^6} = 11.1\,nH$$

$$C = \frac{1}{\omega_{res}\,R/Q} = \frac{1\,s}{2\pi \cdot 600 \cdot 10^6 \cdot 358.68\,k\Omega} = 6.34\,pF$$

d) The E_{010}-mode is the fundamental eigen-mode of the pillbox cavity,

and its magnetic field has only longitudinal / **(transverse)** field components. (½ point)

(Mark the correct answer:) **(answer A)** / answer B

Assuming a ratio height-to-diameter ≅ 0.9, another resonance mode is observed in close proximity of the E_{010} frequency!

e) Which type of higher order mode (HOM) is measured close to 600 MHz? (½ point)
(Hint: The "Mode chart of a Pillbox cavity – Version 1" is a helpful tool!)

- E_{011}
- **(H_{111})**
- H_{112}
- E_{010}
- E_{110}

f) What dimension needs to be changed, to keep the fundamental mode at 600 MHz, while shifting this higher order mode to higher frequencies, and staying well separated to other HOMs? (½ point)

- Increase the diameter
- increase the height
- decrease the diameter
- **(decrease the height)**

Bonus points: (the full score can be reached without examining these questions) **(1 points)**

The final version of the resonator is made of of copper, (σ_{Cu} = 58 · 10^6 S/m), with both dimensions reduced by a factor 10. (Hint: Apply the “scaling laws”)

g) What is the frequency of the fundamental mode? (¼ point)

$$f_{scaled} = 10 \cdot f_{E010} = 6\ GHz$$

h) What values for *Q*-factor and shunt impedance can be expected? (¾ point)

$$Q\frac{\delta}{\lambda_0} = const. \qquad \frac{R}{Q} = const.$$

$$\lambda_{scaled} = \frac{\lambda_0}{10} = 50\ mm$$

$$\delta_{Cu} = \sqrt{\frac{2}{\omega\sigma_{Cu}\mu}} = \sqrt{\frac{2\ s \qquad Vm \qquad Am}{2\pi \cdot 6 \cdot 10^9 \cdot 58 \cdot 10^6 A \cdot 4\pi \cdot 10^{-7} Vs}} = 0.853\mu\text{m}$$

$$Q_{scaled} = Q\frac{\delta_{ss}}{\lambda_0}\frac{\lambda_{scaled}}{\delta_{Cu}} = 8571 \cdot \frac{14.5 \cdot 10^{-6} m}{0.5\ m}\frac{0.05\ m}{0.853 \cdot 10^{-6} m} = 14596$$

$$R_{scaled} = \frac{R}{Q}Q_{scaled} = 41.85\ \Omega \cdot 14596 = 611\ k\Omega$$

2. S-Parameters (2 points)

Match the ideal S-parameters in matrix form to the corresponding components.

$$\boldsymbol{S_A} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \boldsymbol{S_B} = \begin{bmatrix} 0 & j0.1 & 0.995 & 0 \\ j0.1 & 0 & 0 & 0.995 \\ 0.995 & 0 & 0 & j0.1 \\ 0 & 0.995 & j0.1 & 0 \end{bmatrix} \quad \boldsymbol{S_C} = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix} \quad \boldsymbol{S_D} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

a) Assign the S-matrices ($\boldsymbol{S_A}$ ... $\boldsymbol{S_D}$) to the components: (1 point)

component	20 dB directional coupler	transmission line, electrical length = λ/4	circulator	isolator
S-matrix	**S_B**	**S_C**	**S_A**	**S_D**

b) Fill the missing dB and λ information (...). (1 point)

3. Resonator analysis in the complex plane (3 points)

A 500 MHz resonator is critically coupled ($Q_0 = Q_{ext}$) by means of a coupling loop to a RF generator of Z_g = 50 Ω source impedance. The coupling parameter of the loop coupler is k = 16 (Hint: See the "Equivalent Circuit" on page 50 of the course material).

a) What is the value of the shunt impedance R_p of the resonator? (1 point)

$R_p = k^2 R_g = 16^2 \cdot 50\ \Omega = 12.8\ k\Omega$

b) With help of compass and ruler, sketch the locus $Z_{resonator}(f)$ of the unloaded resonator in the complex Z-plane. (On the separate sheet provided at the end.)

- Indicated upper and lower 3-dB points, as well as the points for resonant frequency and frequency limits ($f = 0$, $f \to \infty$). (1 point)
- Use the ruler to estimate the value of the impedance Z at f_{3dB}. (½ point)

  $Z = (6.4 \pm j6.4)k\Omega$

c) Through a second, very weakly coupled loop the 3-dB bandwidth of the resonator at critical coupling was measured as 1 MHz. Determine Q_0 of the (unloaded) resonator? (½ point)

$Q_L = \frac{f_{res}}{\Delta f} = \frac{500 \cdot 10^6 s}{1 \cdot 10^6 s} = 500$

$Q_0 = 2Q_L = 2 \cdot 500 = 1000$

4. Multiple choice (4 points)

Tick the correct answer(s) like this: ☒.

1. A sinusoidal RF signal is measured with an oscilloscope, having and internal 50 Ω termination. The cursors display a peak-to-peak voltage of 500 mV. What is the signal power in dBm? (½ point)

   - ○ -1 dBm
   - ☒ -2 dBm
   - ○ +4 dBm

2. For a "H"(or "TE") mode, the following is true: (½ point)

   - ○ Its magnetic field has only transverse components
   - ○ Its magnetic field has transverse and longitudinal components
   - ☒ Its electric field has only transverse components

3. Changing the height h of a cylindrical cavity oscillating on the E_{010}-mode will: (½ point)

   - ○ change its resonant frequency
   - ☒ change its quality factor
   - ☒ change its R/Q

4. For which material an AC current flows closest to its surface (assume μ_r = 1)? (½ point)
   - ○ Aluminium ($\sigma = 35 \cdot 10^6$ S/m)
   - ☒ Silver ($\sigma = 63 \cdot 10^6$ S/m)
   - ○ Copper ($\sigma = 58 \cdot 10^6$ S/m)

5. What is the limiting factor when using air-filled coaxial lines for transmitting signals of high frequencies? (½ point)
   - ○ There is no frequency limit
   - ☒ The propagation of higher order (waveguide) modes
   - ○ Signal leakage (escaping electrons)

6. In a RF accelerating cavity, the transit time factor expresses: (½ point)
   - ○ The time it takes for the energy to transfer from the electric field to the magnetic field
   - ☒ The time variation of the accelerating field during the bunch passage
   - ○ The time it takes the bunch to travel through the cavity

7. Which coaxial cable has the highest propagation velocity (for TEM)? (½ point)
   - ○ Filled with Teflon (PTFE) dielectric
   - ○ Filled with Polyethylene (PE) dielectric
   - ☒ Filled with air dielectric

8. A gridded tube (triode) operates on the principle of (½ point)
   - ☒ Electron density modulation
   - ○ Electron temperature modulation
   - ○ Electron velocity modulation

5. Smith chart (6 points)

a) Indicate points **P_1…P_5** in the Smith chart, assuming a reference impedance Z_0 = 50 Ω.
From the Smith chart, determine the missing Z or Γ, and complete the table. (1½ points)
(Use the provided Smith chart)

Point no.	**P_1**	**P_2**	**P_3**	**P_4**	**P_5**
Z / Ω	**∞**	**50**	**0**	**31 - j 74**	**100 + j 100**
Γ	**1∠0° (+1)**	**0**	**1∠180° (-1)**	**0.7∠-62°**	**0.62∠30°**

b) Indicate $|\Gamma|$ = 0.5 in the Smith chart. (Hint: It is not a point) (½ point)

c) Point **P_5** represents a complex load impedance Z_{load}.

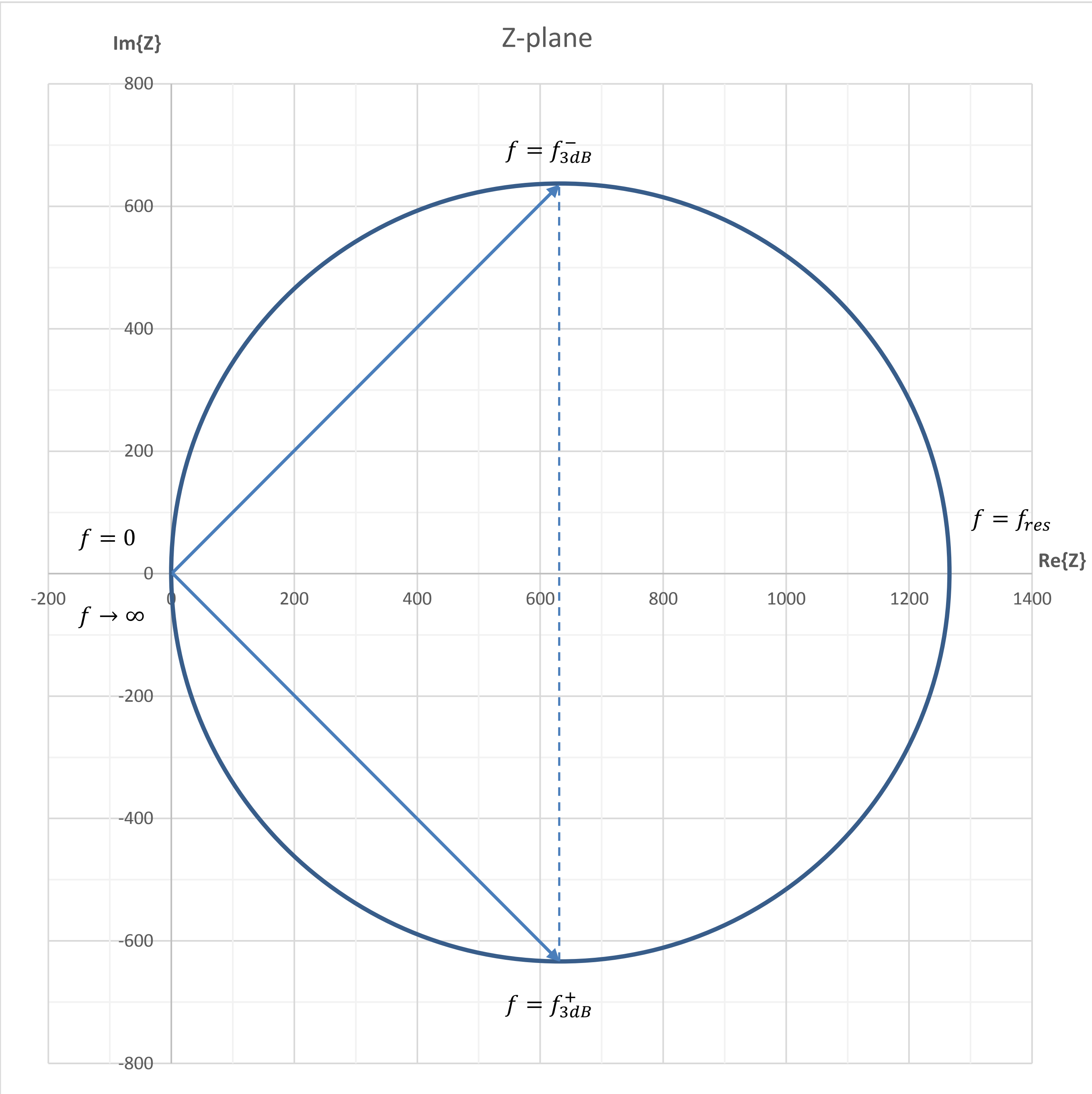
   i. Indicate the normalized z_{load} in the Smith chart, and look up
   - the reflection coefficient, $|\Gamma|$ = 0.62 (½ point)
   - the (voltage) standing wave ratio, SWR = 4.3 (½ point)
   - the return loss (in dB), $RTN.\ LOSS$ = 4.1 dB (½ point)
   - the reflection loss (in dB) $RFL.\ LOSS$ = 2.1 dB (½ point)

again assuming a reference impedance of Z_0 = 50 Ω.
(Hint: Use a ruler to determine $|\Gamma|$ of z_{load}, and compare it with value found at the "radially scaled parameters" Smith chart ruler at the bottom.)

Bonus points: (the full score can be reached without examining these questions) **(2 points)**

ii. With help of the Smith chart, design a passive compensation network to match Z_{load} of point **P_5** for f = 400 MHz to a 50 Ω source impedance of the RF generator.

- Define the locus path of two circuit elements to route from z_{load} to the normalized reference impedance. (1 point)
  (Hint: Remember the Dellsperger Smith Chart computer exercises, and the Smith chart navigation examples pages 178-180. Only 2 circuit elements are required. Different solutions are possible.)
- Determine the values of the circuit elements, and sketch the circuit of the matching network. (1 points)



Z-plane
Im{Z}
Re{Z}
800
600
400
200
0
-200
-400
-600
-800
-200
0
200
400
600
800
1000
1200
1400
$f = f_{3dB}^-$
$f = f_{res}$
$f = 0$
$f \to \infty$
$f = f_{3dB}^+$