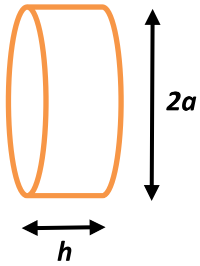
$\mu = \mu_0 \, \mu_r$
$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)
$\varepsilon = \varepsilon_0 \, \varepsilon_r$
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)
$c_0 = 2.998 \cdot 10^8$ m/s
$\sigma_{copper} = 58 \cdot 10^6$ S/m

JUAS 2015 – RF Exam

ID #: ________________________________ Points: ______ of 20

Utilities: JUAS RF Course 2015 lecture script, personal notes, pocket calculator, ruler, compass, and your brain! (No cell- or smartphone, no iPad or wireless devices, text books or any other tools)



1. "Pillbox" Cavity (6 points)

Design a simple "pillbox" (cylindrical) cavity, made out of copper ($\sigma_{copper} = 58 \cdot 10^6$ S/m).
The eigen-frequency of the lowest mode with longitudinal electric field components is 460 MHz.
(The beam-pipe ports are neglected.)

a) The **magnetic** / electric field if the TM_{010} mode das only longitudinal / **transverse** field components.

(Mark the correct answer: **true** / false) (½ point)

b) What is the radius a of the cavity? (1 point)

$$a = 0.383\,\lambda \qquad \lambda = \frac{c_0}{f}$$

$$a = \frac{0.383 \cdot 2.998 \cdot 10^8 m\, s}{460 \cdot 10^6\, s} = 249.6\, mm$$

c) What height h of the cavity has to be chosen,
to achieve an (unloaded) Q-factor of $Q = 23500$? (2 points)

$$Q = \frac{a}{\delta}\left[1 + \frac{a}{h}\right]^{-1} = \frac{a}{\delta\left(1 + \frac{a}{h}\right)}$$

$$1 + \frac{a}{h} = \frac{a}{\delta\, Q}$$

$$h = \frac{a}{\frac{a}{\delta\, Q} - 1}$$

$$\delta = \sqrt{\frac{2}{\omega\, \sigma\, \mu}}$$

$$\delta = \sqrt{\frac{2 \quad s \qquad\qquad Vm \qquad\qquad Am}{2\pi \cdot 460 \cdot 10^6 \cdot 5.8 \cdot 10^{-7} A \cdot 4\pi \cdot 10^{-7} Vs}} = 3.08\, \mu m$$

$$h = \frac{249.6 \cdot 10^{-3}\, m}{\frac{149.6 \cdot 10^{-3}\, m}{3.08 \cdot 10^{-6}\, m \cdot 23500} - 1} = 101.9\, mm$$

d) What is the 3-dB bandwidth of the resonance? (½ point)

$$Q = \frac{f_{res}}{\Delta f} \;\Rightarrow\; \Delta f = BW = \frac{f_{res}}{Q} = \frac{460 \cdot 10^6\, Hz}{23500} = 19.6\, kHz$$

e) Sketch the *RLC* equivalent circuit of this resonant mode, and determine the values. (1 point)

$$\frac{R}{Q} = 128\,\Omega \frac{\sin^2\left(1.2024\frac{h}{a}\right)}{\frac{h}{a}} = 128\,\Omega \frac{\sin^2\left(1.2024\frac{0.1019\,m}{0.2496\,m}\right)}{\frac{0.1019\,m}{0.2496\,m}} = 69.9\,\Omega$$

$$R = \frac{R}{Q}\cdot Q = 69.9 \cdot 23500 = 1.64\,M\Omega$$

$$\omega_{res} L = \frac{1}{\omega_{res} C} = \frac{R}{Q}$$

$$L = \frac{R}{Q\,2\pi f_{res}} = \frac{1.64\cdot 10^6\,V \qquad s}{23500\cdot 2\pi \cdot A \cdot 460 \cdot 10^6} = 24.1\,nH$$

$$C = \frac{1}{4\pi^2 f_{res}^2 L} = \frac{1\ \ s \qquad\qquad A}{4\pi^2\cdot(460\cdot 10^6)^2\cdot 24.1\cdot 10^{-9}\,Vs} = 4.96\,pF$$

f) The cavity is fed from a RF power amplifier with a source impedance of R_g = 50 Ω. What is required transformer ratio of the coupling loop to match the cavity shunt impedance to the generator source impedance? (½ point)

$$k = \sqrt{\frac{R}{R_{input}}} = \sqrt{\frac{1.64\cdot 10^6\,\Omega}{50\,\Omega}} = 181.1$$

g) Calculate the necessary RF power for a gap voltage of 1 MV. (½ point)

$$P = \frac{V^2}{2\,R} = \frac{10^{12}\ \ V^2 \quad A}{2\cdot 1.64\cdot 10^6\,V} = 304.9\,kW$$

2. Resonator analysis in the complex plane (2 points)

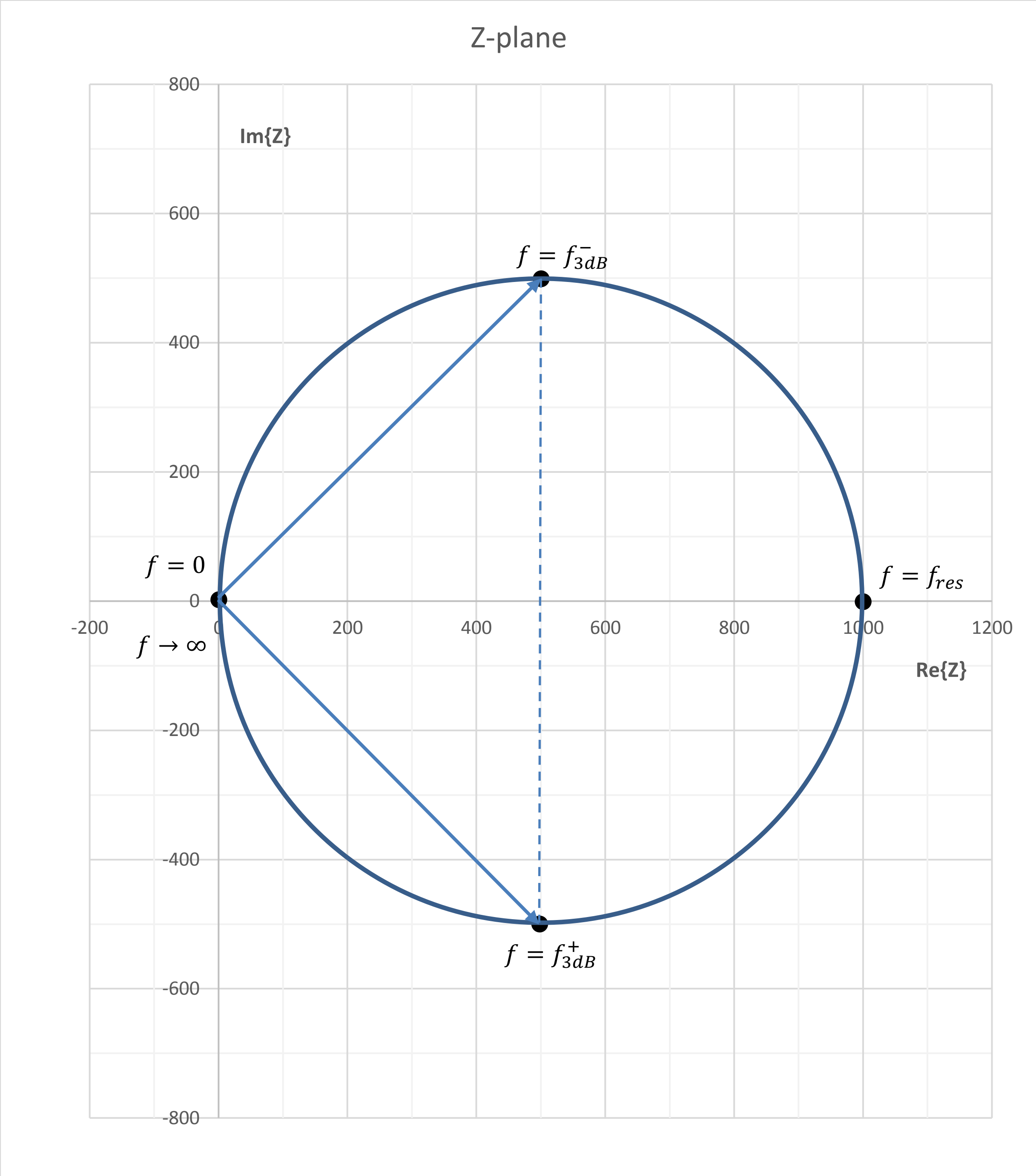
At the upper 3-dB point (definition see RF lecture script) of a 1 GHz resonator, the complex impedance measures $|Z(\omega=6.289\ 10^9 s^{-1})|$ = 707 Ω.

a) With help of compass and ruler, sketch the locus of *Z(f)* in the complex Z-plane

- Indicated upper and lower 3-dB points, as well as the points for resonant frequency and frequency limits (f = 0, $f \rightarrow \infty$). (1 point)
- Estimate the value of the shunt impedance *R*. (½ point)

$$R = 1\,k\Omega$$

$$f_{res}$$



b) Determine the Q-value of the resonator. (½ point)

$$f_{3dB}^+ = \frac{\omega_{3dB}^+}{2\pi} = 1.000925\ GHz$$

$$BW = \Delta f = 2(f_{3dB}^+ - f_{res}) = 2 \cdot (1.000925 \cdot 10^9 - 10^9)\ Hz = 1.85\ MHz$$

$$Q = \frac{f_{res}}{\Delta f} = 540.8$$

3. Smith chart (6 points)

a) Indicate points **P₁…P₅** in the Smith chart, assuming a reference impedance Z_0 = 50 Ω.
From the Smith chart, determine the missing *Z* or *Γ*, and complete the table. (1½ points)

Point no.	**P_1**	**P_2**	**P_3**	**P_4**	**P_5**
***Z* / Ω**	**50**	**0**	∞	69.1+j65.1	*R* = 75 Ω, *C* = 4.25 pF, (@ *f* = 500 MHz)
Γ	0	-1 1∠180°	**1∠0°**	**0.5∠45°**	0.542∠-40.6° (series) 0.416∠-85.3° (parallel)

b) Indicate |*Γ*| = 0.75 in the Smith chart. (Hint: It is not a point) (½ point)

c) At 400 MHz a complex load impedance measures Z_{load} = (25+j67) Ω.

i. Indicate the normalized z_{load} in the Smith chart, and look up

- the reflection coefficient, (¼point)
  $\Gamma = 0.258 + j0.662 = 0.711\angle 68.7°$
- the (voltage) standing wave ratio, (¼ point)
  $SWR = 5.92$
- the return loss (in dB), (¼ point)
  $ReturnLoss = 2.96\ dB$
- the reflection loss (in dB) (¼ point)
  $ReflectionLoss = 3.06\ dB$

for a reference impedance of Z_0 = 50 Ω.
(Hint: Use a ruler to determine |*Γ*| of z_{load}, and compare it with value found at the "radially scaled parameters" Smith chart ruler at the bottom.)

ii. With help of the Smith chart, sketch a lossless matching network, and determine the component values to adapt to a 50 Ω source impedance of the RF generator.

- Define the locus path lossless elements to route from z_{load} to the normalized reference impedance. (1 point)
  (Hint: Remember the Dellsperger Smith Chart computer exercises, only 2 lossless elements are required. Different solutions are possible.)
- Determine the values of the lossless circuit elements. (2 points)

4. S-Parameters (2 points)

Match the ideal S-parameters in matrix form to the corresponding components.

$$\boldsymbol{S_A} = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad \boldsymbol{S_B} = \frac{1}{10}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \boldsymbol{S_C} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix} \qquad \boldsymbol{S_D} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

a) Assign the S-matrices ($\boldsymbol{S_A}$ ... $\boldsymbol{S_D}$) to the components: (1 point)

component	3 dB directional coupler	transmission line, length = λ/2	6 dB resistive power divider	20 dB attenuator
S-matrix	$\boldsymbol{S_C}$	$\boldsymbol{S_D}$	$\boldsymbol{S_A}$	$\boldsymbol{S_B}$

b) Fill the missing dB and λ information (...). (1 point)

5. Multiple choice (4 points)

Tick the correct answer(s) like this: ☒.
(Except for questions 7. and 8., only one answer is correct)

1. A coaxial line is filled homogeneous with a dielectric material, e.g. PTFE (“Teflon”). The signal velocity of a coaxial line of same physical length, but filled with air is (½ point)
   - ○ identical
   - ☒ higher
   - ○ lower
2. TEM stands for (½ point)
   - ○ Transient Electro-Magnetics
   - ☒ Transverse Electro-Magnetic mode
   - ○ Turbo Electric Motor
3. For a cylindrical (“pillbox”) cavity, the eigen-frequencies are independent of the cavity height *h* dimension: (½ point)
   - ○ False, for any eigen-mode the resonance frequency depends on height *h* and radius *a*
   - ○ True only for the fundamental mode
   - ☒ True only for TM_{010} and TM_{110} modes
4. When comparing with a charged particle passing the cavity gap with infinite velocity, the integrated field in the cavity gap, seen by a particle of finite velocity is (½ point)
   - ○ increased due to
   - ☒ reduced due to
   - ○ independent of

the transit time factor.

5. Critical coupling (match at resonance) between resonator and generator occurs at (½ point)
   - ○ $Q_L = Q_{ext}$
   - ☒ $Q_L = Q_0/2$
   - ○ $QL = 2\ Q_0$
6. A 10 W RF generator is connected via a 20-dB attenuator to a 50 Ω load impedance. At the load we measure: (½ point)
   - ○ 1 W
   - ○ -20 dBW
   - ☒ +20 dBm
7. What is true for 2-conductor transmission-lines? (½ point)
   - ☒ Ideal for broadband (down to DC), low level signal transmission.
   - ○ The signal transmission is based on “modes”.
   - ○ Low losses at high frequencies, therefore ideal for high power RF transmission.
8. What is true for waveguides? (½ point)
   - ○ Ideal for broadband (down to DC), low level signal transmission.
   - ☒ The signal transmission is based on “modes”.
   - ☒ Low losses at high frequencies, therefore ideal for high power RF transmission.