JUAS 2014 exam

The following constant will be used in the exercises

velocity of light	$c = 2.998 \cdot 10^8$ m/s
reduced Planck constant	$\eta = h/2\pi = 1.05 \cdot 10^{-34}$ Kg m^2/ s
vacuum dielectric constant	$\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m
electron rest energy	0.511 MeV
proton rest energy	938 MeV
classical electron radius	$r_e = 1/(4\pi\varepsilon_0)\, e^2/(mc^2) = 2.81 \cdot 10^{-15}$ m

Ex. 1:
The LHC is a proton synchrotron with

proton beam energy (at injection):	450 GeV
proton beam energy (at collision):	7 TeV
LHC dipole bending radius:	2803.9 m

a) compute the critical energy of the synchrotron emission for the LHC proton beam at collision energy;

b) the energy is ramped from the injection energy to the collision energy. Is there a value of the energy during the ramp such that the synchrotron emission is peaked on the blue light (2.8 eV)?

Ex. 2:
The diamond synchrotron stores a 3 GeV electron beam of 300 mA. Knowing that

diamond dipole bending radius:	7.1 m
diamond average radius:	89.4 m

a) compute the energy loss per turn for a single electron;

b) compute the total power lost by a stored beam of 300 mA; what should be the energy of a proton beam with 300 mA in order to radiate the same synchrotron power?

c) assume the RF trips off. Knowing that the maximum horizontal dispersion is 25 cm and the horizontal aperture is ±2 cm. Compute the number of turns the beams survives in the ring before hitting the wall.

Ex. 3:
Consider again Diamond with the data in exercise 2. Assume we add a wiggler with peak field 3.5 T and length 2 m.

a) Compute the new value for the total energy loss per turn per electron

b) Assuming the RF voltage is set to 3 MV, What is the corresponding shift of the synchronous phase?

c) Assuming $J_x \sim 1$, compute the transverse damping time with and without the wiggler.

Ex. 4: low emittance lattices

Describe the physical causes that determine the emittance of an electron beam. What are the basic strategies to build a low emittance lattice (<1 page)?

Exercise 1 solution:
a) From the definition of critical energy

$$\varepsilon_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3 \qquad (1)$$

we compute

$$\gamma = \frac{E}{E_0} = 7462.7$$

and

$$\varepsilon_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3 = 7\cdot 10^{-18}\ \text{J} = 43.7\ \text{eV}$$

b) we know that the peak of the synchrotron radiation emission occurs approximately at 0.3*critical photon energy. For the peak of the emission to be in the blue spectral region (2.8 eV) we require that

$$0.3\ \varepsilon_c = 2.8\ \text{eV} = 4.5\cdot 10^{-19}\ \text{J}$$

inverting (1) we get

$$\gamma = \left(\frac{2}{3}\frac{\rho}{\hbar c}\varepsilon_c\right)^{1/3} = 4459$$

i.e. the emission on the blue is reached at an energy

E = 4.2 TeV

within the LHC energy ramp.

Exercise 2 solution:
a) we know that the energy loss per electron are

$$U_0(keV) = \frac{e^2\gamma^4}{3\varepsilon_0\rho} = \frac{88.46E[GeV]^4}{\rho[m]} = 1.01\ \text{MeV/turn}$$

b) we know that the power loss by a beam with average current I_b is

$$P(kW) = \frac{88.46E[GeV]^4 I[A]}{\rho[m]} = 300\ \text{kW}$$

The proton energy of a 300 mA beam that would radiate 300 kW can be obtained inverting

$$P = \frac{e\gamma^4}{3\varepsilon_0\rho} I_b$$

in

$$\gamma = \left(\frac{3\varepsilon_0 \rho P}{eI_b} \right)^{1/4} = 5859$$

i.e. E = 5.5 TeV. Notice that the LHC proton beam at 7 TeV radiates a power of only 3.7 kW with a stored current to 530 mA: this is due to the large LHC dipole bend radius.

c) When the RF trips the energy loss per turn is not replaced by the RF, therefore the electron beam start loosing energy continuously at a rate given by (point a))

U_0 = 1.01 MeV/turn

If the dispersion is D_x the beam changes its horizontal position according to

$$x = D_x \delta$$

where δ is the relative energy variation. Given a maximum dispersion of D_x = 0.25 cm, to cover an horizontal offset x = 2 cm the energy deviation must be

$\delta = x/D_x = 0.08$

which correspond to an absolute energy loss of

ΔE = 0.08 E = 240 MeV

Since we lose about 1 Mev per turn it will take about 240 turns for the beam to hit the wall. The wall will be hit in the inside of the vacuum pipe towards the centre of the ring.

Exercise 3 solution:
a) we know that the energy loss per electron generated by a wiggler is

$$E_w = \frac{2}{3} \frac{r_e e^2}{m^3 c^4} E^2 B^2 L_w$$

*** Mind who is B here $B = B_w = B_0/\sqrt{2}$ this must be the average field

For the wiggler data in question we have

E_w = 280 keV/turn

Which adds to the dipole contribution U_0 computed in Ex 2a) to make a total energy loss per turn

$$E_t = U_0 + E_w = 1.28 \text{ MeV/turn}$$

b) the synchronous phase shift is defined as

$$U_0 = eV\sin(\varphi_s)$$

adding the wiggler we get

$$U_0 + E_w = eV\sin(\varphi_{sw})$$

therefore

$$\sin(\varphi_{sw}) = \sin(\varphi_s) + E_w/e$$

We have

$$\varphi_s = 19.7 \text{ deg}$$
$$\varphi_{sw} = 51.3 \text{ deg}$$
$$\Delta\varphi = 31 \text{ deg}$$

c) diamond transverse damping time is

$$\tau_x = \frac{2ET_0}{J_x U_0} = 11 \text{ ms}$$

If the energy loss per turn increases from 1.01 MeV/turn to 1.28 MeV/turn the damping time will be

$$\tau_x = 8.6 \text{ ms}$$

Exercise 4)
A short answer by the students.