

JUAS 2017 exam

The following constant will be used in the exercises

elementary electric charge	$e = 1.6 \cdot 10^{-19} \text{ C}$
velocity of light	$c = 2.998 \cdot 10^8 \text{ m/s}$
reduced Planck constant	$\eta = h/2\pi = 1.05 \cdot 10^{-34} \text{ Kg m}^2/\text{s}$
vacuum dielectric constant	$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F/m}$
electron rest energy	0.511 MeV
proton rest energy	938 MeV
classical electron radius	$r_e = 1/(4\pi\epsilon_0) e^2/(mc^2) = 2.81 \cdot 10^{-15} \text{ m}$

$$C_q = \frac{55}{32\sqrt{3}} \frac{\eta}{mc} = 3.84 \cdot 10^{-13} \text{ m}$$

Ex. 1: [20 points]

Consider an electron storage ring operating at 3 GeV, with a lattice made of 24 DBA cells.

- give an estimate for the minimum emittance achievable in such ring (take $J_x = 1$). [3 points]
- If the magnetic field of the dipole is 1.4T calculate the critical energy of the bending magnets [4 points]

Assume we want to upgrade this lattice to a 48 DBA cells.

- give an estimate for the minimum emittance achievable in such ring. [3 points]
- Assuming that the length of each dipole has not changed in the upgrade, calculate the new critical energy for the dipole radiation [4 points]
- Describe what is the change in the characteristics of the bending magnet radiation (i.e. power, spectrum and angular distribution). [6 points]

Ex. 2: [20 points]

Consider an electron storage ring operating at 6 GeV. The injection process requires that the oscillations of the injected beam at 6 GeV are damped within 10 ms, otherwise the injected beam hits the back of the septum magnet and no injection is possible.

- What is the energy loss per turn necessary to guarantee the damping time required for injection (take $J_x = 1$ and a revolution time $T_0 = 2\mu\text{s}$). [4 points]
- If the energy of the ring reduced to 3 GeV, what is the energy loss per turn necessary to guarantee the injection with the existing injection system parameters? [4 points]
- If we want to use damping wiggler to guarantee the injection at 3GeV and $L_w = 50 \text{ m}$ are available, what is the magnetic field B_w required from the damping? [6 points]
- What are the main effects of the introduction of damping wigglers on the equilibrium beam parameters? [6 points]

Ex. 3: [20 points]

In a 3 GeV electron synchrotron, a beamline is made by an undulator of length $L = 2$ m, operating at a wavelength $\lambda = 1$ nm. The emittance of the ring is 1 nm in both planes and the optics function at the middle of the straight section are $\beta_x = \beta_y = 5$ m and $\alpha_x = \alpha_y = 0$ and the dispersion function is zero.

The beamline has a round slit aperture at 20 m downstream with radius $r = 1$ mm.

- a) is the undulator fan at $\lambda = 1$ nm passing through the slit (assume a Gaussian beam for the undulator radiation) [6 points]
- b) What happens if the optics function in the middle of the straight section are reduced to $\beta_x = \beta_y = 1$ m? Give an estimate of the change in flux through the slit. [7 points]
- c) Describe the criteria to achieve the optimal matching of the electron phase space and the photon phase space at the centre of the undulator. [7 points]

Ex. 4: [20 points]

Describe the strategies for the design of the linear optics functions β , α and dispersion D in as low emittance lattice [10 points]. What are the most common solutions? [5 points]. Sketch the optics function required to achieve the theoretical minimum emittance in a TME and DBA lattice [5 points].

Ex 1: solution

a)

A 24 cell DBA lattice has 48 dipoles therefore the bending angle is

$$\theta = \frac{2\pi}{48} = 131 \text{ mrad}$$

The theoretical minimum emittance of a DBA lattice is (including the breaking of the achromatic condition – many have not used this one)

$$\varepsilon = \frac{1}{12\sqrt{15}} \frac{C_q \gamma^2 \theta^3}{J_x}$$

where

$$C_q = \frac{55}{32\sqrt{3}} \frac{\eta}{mc} = 3.84 \cdot 10^{-13} \text{ m}$$

For a 3 GeV beam ($\gamma = 5871$) and $J_x = 1$ we have

$$\varepsilon = \frac{1}{12\sqrt{15}} \frac{C_q \gamma^2 \theta^3}{J_x} \sim \frac{3.84 \cdot 10^{-13}}{12\sqrt{15}} (5870)^2 (0.131)^3 = 640 \text{ pm}$$

b)

The critical energy is

$$\varepsilon_c = \frac{3\eta c \gamma^3}{2\rho}$$

In order to compute the bending radius we use

$$\frac{1}{B\rho} = \frac{e}{p} \quad \rightarrow \quad \rho = \frac{p}{eB} = \frac{\beta E}{ecB} \sim \frac{3 \text{ GeV}}{ec \cdot 1.4 \text{ T}} = \sim 7.14 \text{ m}$$

The critical energy reads

$$\varepsilon_c = \frac{3\eta c \gamma^3}{2\rho} = \frac{3\eta c (5870)^3}{2 \cdot 7.14} = 8.37 \text{ keV}$$

c)

In a 48 DBA cells lattice there are 96 dipoles hence

$$\theta = \frac{2\pi}{96} = 65 \text{ mrad}$$

The theoretical minimum emittance achievable is

$$\varepsilon = \frac{1}{12\sqrt{15}} \frac{C_q \gamma^2 \theta^3}{J_x} \sim \frac{3.84 \cdot 10^{-13}}{12\sqrt{15}} (5870)^2 (0.65)^3 = 80 \text{ pm}$$

d)

The length of the dipole does not change therefore the bending radius is doubled and the magnetic field is halved. The new critical energy is also halved

$$\varepsilon_c = \frac{3\eta c \gamma^3}{2\rho} = \frac{3\eta c (5870)^3}{2 \cdot 3.57} = 4.18 \text{ keV}$$

e)

The total instantaneous power is reduced by a factor 4 (since ρ is doubled)

$$P = \frac{e^2}{6\pi\epsilon_0} \frac{\gamma^4}{\rho^2}$$

The high energy end of the spectrum is reduced as the critical energy decreases from 8 keV to 4 keV.

The critical angle

$$\theta_c = \frac{1}{\gamma} \left(\frac{\omega_c}{\omega} \right)^{1/3}$$

increases as the critical frequency decreased

Ex. 2 solution

a)

The damping time in the horizontal plane is

$$\frac{1}{\tau_x} = \frac{J_x U_0}{2T_0 E_0}$$

For a 6GeV beam ($T_0 = 2 \mu\text{s}$, $J_x = 1$) and $\tau_x = 10 \text{ ms}$ we must have

$$U_0 = \frac{2T_0 E_0}{\tau_x J_x} = \frac{2 \cdot 2\mu\text{s} \cdot 6\text{GeV}}{0.01} = 2.4 \text{ MeV/turn}$$

b)

The energy loss per turn is

$$U_0 (\text{keV}) = 88.46 \frac{E(\text{GeV})^4}{\rho(\text{m})}$$

If the ring energy is reduced from 6 GeV to 3 GeV, the energy loss per turn is reduced by a factor $2^4 = 16$ therefore

$$U_0 = 0.15 \text{ MeV/turn}$$

and the damping time becomes

$$\frac{1}{\tau_x} = \frac{J_x U_0}{2T_0 E_0} = \frac{0.15}{2 \cdot 2\mu\text{s} \cdot 3000} = 12.5 \quad \rightarrow \quad \tau_x = 80 \text{ ms}$$

In order to guarantee injection, the energy loss per turn has to be increased by a factor of 8, going from 0.15 MeV/turn to 1.2 MeV/turn.

c)

The damping wiggler must provide the additional

$$\Delta U_0 = 1.2 - 0.15 = 1.05 \text{ MeV/turn}$$

The energy loss per turn do to a wiggler of field $B_w = B_0/\sqrt{2}$ and length L_w is

$$E_w = \frac{2}{3} \frac{r_e e^2}{m_e^3 c^4} E^2 B_w^2 L_w$$

i.e.

$$\begin{aligned} E_w \text{ (eV)} &= \frac{2}{3} \frac{r_e c^2}{(m_e c^2)^3} E^2 B_w^2 L_w = \frac{2}{3} \frac{2.81 \cdot 10^{-15} 9 \cdot 10^{16}}{(0.511 \cdot 10^6)^3} E \text{ (eV)}^2 B_w^2 L_w \text{ (m)} = \\ &= 1263 \cdot 10^{-18} E \text{ (eV)}^2 B_w^2 L_w \text{ (m)} = 1263 \cdot 10^{-18} (3 \cdot 10^9)^2 B_w^2 L_w \text{ (m)} = 11367 \cdot B_w^2 L_w \text{ (m)} \end{aligned}$$

In order to achieve 1.05 MeV we have/turn

$$E_w \text{ (eV)} = 11367 \cdot B_w^2 L_w \text{ (m)} = 1.05 \cdot 10^6$$

hence

$$B_w^2 L_w \text{ (m)} = \frac{1.05 \cdot 10^6}{11367} = 92.4$$

with a length of 50 m available $B_w = 1.34$ T.

d)

Beyond the energy loss per turn and the damping time, the damping wiggler also change the emittance and the energy spread (see slides)

Ex. 3 solution

a)

The source size and divergence of the undulator are given by the quadratic sum of the electron and photon beam size

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_{ph}^2} \quad \Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{ph'}^2}$$

Given the emittance $\epsilon_x = \epsilon_y = 1$ nm and the optics function ($\beta_x = \beta_y = 5$ m) quoted

$$\begin{aligned} \sigma_x &= \sqrt{\epsilon_x \beta_x} &&= 71 \text{ um} \\ \sigma_{x'} &= \sqrt{\frac{\epsilon_x}{\beta_x}} &&= 14 \text{ urad} \end{aligned}$$

The values in the vertical plane are the same.

An undulator of $L = 2$ m at $\lambda = 1$ nm will have

$$\begin{aligned} \text{Photon beam size} \quad \sigma_{ph} &= \frac{\sqrt{2L\lambda}}{2\pi} && 10.1 \text{ um} \\ \text{Photon beam divergence} \quad \sigma_{ph}' &= \sqrt{\frac{\lambda}{2L}} && 15.8 \text{ urad} \end{aligned}$$

The total beam size and divergence read

$$\begin{aligned} \Sigma_x &= \sqrt{\sigma_x^2 + \sigma_{ph}^2} = 71.7 \text{ um} \\ \Sigma_{x'} &= \sqrt{\sigma_{x'}^2 + \sigma_{ph'}^2} = 21.1 \text{ urad} \end{aligned}$$

At 20 m downstream the spot size will be

$$dx = 21.1 \text{ urad} * 20\text{m} = 0.422 \text{ mm}$$

Summed with 72.6 μm yields 0.493 mm rms. The aperture has a radius of 1 mm therefore about $\pm 2 \sigma$ of the Gaussian beam pass through.

b)

If the optics function are reduced to $\beta_x = \beta_y = 1\text{m}$

$$\sigma_x = \sqrt{\epsilon_x \beta_x} = 31.6 \text{ } \mu\text{m}$$

$$\sigma_{x'} = \sqrt{\frac{\epsilon_x}{\beta_x}} = 31.6 \text{ urad}$$

The values in the vertical plane are the same.

The total beam size and divergence read

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_{ph}^2} = 33.2 \text{ } \mu\text{m}$$

$$\Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{ph'}^2} = 35.3 \text{ urad}$$

At 20 m downstream the spot size will be

$$dx = 35.3\text{urad} * 20\text{m} = 0.706 \text{ mm}$$

summed with the beam size gives 0.738 mm. In an aperture of 1 mm radius this is about $\pm 0.5 \sigma$. So the radiation is clipped and the flux reduced ($\pm 1 \sigma$ corresponds to 66% of the flux, $\pm 2 \sigma$ corresponds to 95% of the flux).

c)

the ellipses in phase space have to be aligned and

$$\beta_x \sim \frac{L}{\pi} = 0.63 \text{ m}$$

Ex. 4:

See slides (minimise β_x and put a waist in the dipole, control D_x , MBA lattices).