JUAS 2015 exam

The following constant will be used in the exercises

velocity of light	$c = 2.998 \cdot 10^{8}$ m/s
reduced Planck constant	$\eta = h/2\pi = 1.05 \cdot 10^{-34}$ Kg m^2/ s
vacuum dielectric constant	$\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m
electron rest energy	0.511 MeV
proton rest energy	938 MeV
classical electron radius	$r_e = 1/(4\pi\varepsilon_0)\, e^2/(mc^2) = 2.81 \cdot 10^{-15}$ m

Solve 3 of the following 4 problems.

Ex. 1:
Consider a 3 GeV synchrotron light source. A bending magnet beamline wants to perform experiment using 8 keV photons.

a) what is the bending radius for which the critical energy is 8 keV? At what photon energy will be the peak of the radiated power? What will be the required magnetic field in the dipole?

b) What will be the total power radiated by a beam with 100 mA? What will be the power radiated per mrad along the arc of the trajectory in the dipole?

The radiation fan from the bending magnet sweeps horizontally through the aperture of the beamline, located 10 m downstream of the dipole. Assume that the aperture of the beamline is infinitely wide in the horizontal dimension and is 10 mm wide in the vertical dimension.

c) Will 8 keV photons go through such vertical aperture? (Hint: use the critical angle as the rms vertical angular divergence of the photon beam)

d) If the beamline is interested in THz radiation (corresponding to λ = 100 um, ε = 0.012 eV), should the vertical aperture be modified? If yes, how?

Ex. 2:
Consider an electron storage ring with energy 2 GeV and circumference 600m. The lattice is built with dipoles having bending radius ρ = 7m and the damping partition number is $J_x = 1$.

a) Compute the energy loss per electron per turn
b) Compute the damping time of the transverse oscillations

We want to reduce the damping time by a factor 10 by adding a damping wiggler.

c) Propose a set of realistic wiggler parameters that achieve this requirement.

Ex. 3:
Consider an undulator with period λ_u = 20 mm, N =100 periods, minimum gap 4 mm, and tip magnetic field B_r = 1.3 T, installed in a storage ring with 6 GeV energy.

Using the relation

$$K = 0.168 \cdot B_r[T] \cdot \lambda_u[mm] \exp\left(-\frac{\pi g}{\lambda_u}\right)$$

and considering the radiation emitted on-axis, compute

a) the wavelength range that is radiated in the fundamental. What is the bandwidth? Describe qualitatively how the power depends on the undulator parameter K
b) the wavelength range in the harmonics n = 2 and n = 3. What is the bandwidth? Describe qualitatively how the power varies with K.

Assume that the fundamental wavelength emitted on axis is λ_0. Consider the radiation off axis

c) Is the wavelength λ_0 emitted also off axis? If yes, at what angle? (Hint: consider the harmonics off axis)

Ex. 4: low emittance lattices
We want to build a diffraction limited light source for hard Xray (1A, 12.4 keV). What should be the emittance of the lattice?

Describe the most common design choices for a low emittance lattice and explain qualitatively why an MBA is favoured over a DBA. (<1 page)

Exercise 1 solution:
a)
The critical energy is

$$\varepsilon_c = \frac{3}{2}\frac{\eta c}{\rho}\gamma^3 = 8\,\text{keV} \tag{1}$$

For γ = 3000/0.511 = 5870 we can work out the bending radius

$$\rho = \frac{3}{2}\frac{\eta c}{\varepsilon_c}\gamma^3 = \frac{3}{2}\frac{\eta c}{8000/e}\gamma^3 = 7.44\,\text{m}$$

From the definition of rigidity at 3 GeV

$$B\rho = 10.06 \tag{2}$$

Hence the corresponding magnetic field is

$$B = \frac{10.06}{\rho} = \frac{10.06}{7.44} = 1.35\text{T} \tag{3}$$

The peak of the emission occurs at $0.3\varepsilon_c$ = 2.4 keV.

b)
The total power radiated by a beam of 100 mA will be

$$P = \frac{e\gamma^4}{3\varepsilon_0\rho} I_b = \frac{1.6e-19*5870^4}{3*8.8e-12*7.44}0.1 = 97\,\text{kW}$$

Per mrad

$$\frac{dP}{d\phi} = \frac{P}{2\pi} = 15\frac{\text{W}}{\text{mrad}}$$

c)
The critical angle at the critical frequency is

$$\theta_c = \frac{1}{\gamma} = 0.17\,\text{mrad} \tag{3}$$

At 10 m downstream the vertical width of the radiation fan is 1.7 mm so it fits in the 10 mm aperture.

d)
In the THz radiation (100 um – 0.12 eV) the critical angle (opening cone) becomes

$$\theta_c = \frac{1}{\gamma}\frac{\omega_c}{\omega} = 0.17e-3\frac{8000}{0.12} = 11\,\text{mrad} \tag{4}$$

At 10 m downstream the vertical width of the radiation fan is 110 mm so it does not fit in the 10 mm aperture. The vertical aperture must be increased.

Exercise 2 solution:
a)
we know that the energy loss per electron are

$$U_0(\text{keV}) = \frac{e^2\gamma^4}{3\varepsilon_0\rho} = \frac{88.46\,E[\text{GeV}]^4}{\rho[\text{m}]} = \frac{88.46 * 2^4}{7} = 202 \text{ keV/turn}$$

b)
The revolution time is

T = 600/3e8 = 2 us

the transverse damping time is

$$\tau_x = \frac{2ET_0}{J_x U_0} = \frac{2 * 2e9 * 2e-6}{1 * 202\,e3} = 39.6 \text{ ms}$$

c)
If the energy loss per turn increases from 202 keV/turn to 2.02 MeV/turn the damping time will be

τ_x = 3.96 ms

We need an additional loss per turn

U_0 = 2.02 – 0.2 = 1.82 MeV/turn

We know that the energy loss per electron generated by a wiggler is

$$E_w = \frac{2}{3}\frac{r_e e^2}{m^3c^4}E^2B^2L_w$$

in practical units

$$E_w(\text{eV}) = \frac{2}{3}\frac{r_e e^2}{m^3c^4}E^2B^2L_w = 1.5\frac{2.8e-15*(1.6e-19)\text{^}2}{(9.1e-31)^3(2.998\,e8)^4}(2e9*1.6e-19)^2B^2L_w$$
$$= 11274 \cdot B(T)^2 L_w(m)$$

To reach 1.82 MeV we need E_W(eV) = 1.82e6, i.e.

$$B(T)^2L_w(m) = \frac{E_w(\text{eV})}{11274} = \frac{1.82e6}{11274} = 161$$

e.g.

B = 3T $\qquad$ L_W = 18 m

Exercise 3: solution

a-b)

The wavelength radiated in the fundamental is

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}(1+\frac{K^2}{2})$$

and K depends on the gap with the suggested relation

$$K = 0.168 \cdot B_r \lambda_u \exp\left(-\frac{\pi g}{\lambda_u}\right)$$

Since λ_u = 20 mm, and remnant field B = 1.3 T

$$K\ (g = 4mm) = 0.168 \cdot 1.3 \cdot 20 \cdot \exp\left(-\frac{\pi * 4}{20}\right) = 2.33$$

K min = 0.5

At 6 GeV, γ = 6/0.511 = 11740

The radiation scans

$$\lambda_n(max) = \frac{\lambda_u}{2n\gamma^2}(1+\frac{K_{max}^2}{2}) = \frac{20e-3}{2n(11740)^2}(1+\frac{2.33^2}{2}) = \frac{2.65A}{n}$$

$$\lambda_n(min) = \frac{\lambda_u}{2n\gamma^2}(1+\frac{1}{8}) = \frac{9*20e-3}{16n(11740)^2} = \frac{0.8A}{n}$$

The bandwidth is

$\Delta\omega/\omega$ = 1/nN = 1/n/100 = 1% / n

c)

Assume that the fundamental wavelength emitted on axis is λ_0. Is the wavelength λ_0 emitted also off axis? if yes, at what angle?

$$\lambda_0 = \frac{\lambda_u}{2\gamma^2}(1+\frac{K^2}{2})$$

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}(1+\frac{K^2}{2}+(\gamma\theta)^2)$$

We have

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}(1+\frac{K^2}{2}+(\gamma\theta)^2) = \frac{\lambda_u}{2\gamma^2}(1+\frac{K^2}{2}) = \lambda_0$$

For

$$(\gamma\theta)^2 = (1+\frac{K^2}{2})(n-1)$$

$$\theta = \frac{1}{\gamma}\sqrt{(1+\frac{K^2}{2})(n-1)}$$

Exercise 4)

A short answer by the students from the slides of SR_III